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On the origin of the dimensional forms of the laws of physics

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The physical quantities are used in physics based on quantity calculus within the SI convention. This research aims at the construction of a rigorous mathematical framework for explaining the origin of the laws of physics. Here, we show that each SI physical quantity, that is represented by a lattice point in a seven dimensional integer lattice, has a *unique* 7-sphere. This 7-sphere is the geometrical representation of a seven dimensional Helmholtz differential equation of a *unique* scalar wave function associated to the physical quantity. The solutions of the differential equation are given by the lattice points of the integer lattice that are incident on the 7-sphere. The lattice points incident on the 7-sphere are forming rectangles containing the origin $\boldsymbol{o}$, the lattice point $\boldsymbol{z}$ representing the selected physical quantity and the lattice point representations of a pair of *compatible* physical quantities $\boldsymbol{x}, \boldsymbol{y}$ where $\boldsymbol{z} = \boldsymbol{x} + \boldsymbol{y}$. The resulting rectangles are the geometric representation of the *laws of physics* of the dimensional form $[z] = [x][y]$ for the selected physical quantity $[z]$.

1. Outline of the paper

Section 2 introduces the research topic. In section 3, we present the axioms of quantity calculus. We demonstrate in section 4 the isomorphism between the classes of physical quantities and the seven dimensional integer lattice $\mathbb{Z}^7$. In section 5, we present the decomposition of a vertex in pairwise orthogonal vertices. We show in section 6 that the 7-sphere of a physical quantity has an alternative representation being a seven dimensional Helmholtz wave equation of the physical quantity. In section 8 we discuss the physical interpretation of the wave equation of a physical quantity. Section 9 contains the conclusion of the present research.

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2. Introduction

The SI [1] is used worldwide as language in the domains of science and technology. Physical quantities occur in the scientific literature as tensors of rank m with $m \leq 0$. An algebraic structure for *quantity calculus* was proposed by R. Fleischmann [2]. This research aims at finding the origin of the occurrence of physical quantities in the description of physics and infer from their relations those that are to be called *laws of physics*.

3. Axioms of the SI physical quantities

We posit from the 8th edition of the SI [1] a set of axioms derived from *elevating* some of the SI conventions to mathematical axioms.

Axiom 1. *The base quantities are length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity.*

Axiom 2. *The base quantities are independent.*

Axiom 3. *The physical quantities are organized according to a system of dimensions.*

Axiom 4. *For each base quantity of the SI, there exists one and only one dimension.*

Axiom 5. *The product of two quantities is the product of their numerical values and units.*

Axiom 6. *The quotient of two quantities is the quotient of their numerical values and units.*

The symbols of the dimensions of the SI base quantities are given in Table 1.

Table 1: Base quantities and dimensions used in the SI

Base quantity	Symbol of the quantity	Symbol of the dimension
length	$l, x, r \dots$	L
mass	m	M
time	t	T
electric current	I, i	I
thermodynamic temperature	T	Θ
amount of substance	n	N
luminous intensity	I_ν	J

The base quantities have no unique symbols. Only the dimensions of the base quantities have a unique symbol that is written in roman font. The uniqueness of the SI symbols are forming an alphabet that is a that base of any physics expression.

Definition 1. The dimension of a physical quantity q is expressed as a dimensional product [1] :

$$\dim q = \mathrm{L}^{\alpha}\mathrm{M}^{\beta}\mathrm{T}^{\gamma}\mathrm{I}^{\delta}\Theta^{\epsilon}\mathrm{N}^{\zeta}\mathrm{J}^{\eta}\,; \tag{3.1}$$

where the exponents $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta \in \mathbb{Z}$ are called *dimensional exponents.*

The dimensional exponents of the *simple* SI8 physical quantities take integer values in the following sets:

- α takes the values from $\{-3, -2, -1, 0, 1, 2, 3, 4\}$;

- β takes the values from $\{-3,-2,-1,0,1\}$;
- γ takes the values from $\{-4,-3,-2,-1,0,1,2,3,4,6,7,10\}$;
- δ takes the values from $\{-2,-1,0,1,2,3,4\}$;
- ϵ takes the values from $\{-4,-1,0,1\}$;
- ζ takes the values from $\{-1,0,1\}$;
- η takes the values from $\{0,1\}$.

Observe that the dimensional exponents are *small* integers. One could object that some derived physical quantities (rms of a quantity, noise spectral density, specific detectivity, thermal inertia, thermal effusivity, ...) are defined as the square root of some product or quotient of other physical quantities. We define these derived physical quantities as *fractional physical quantities* that do not comply with the SI8 definition for a physical quantity. Each of these fractional physical quantities are, by a *proper exponentiation*, transformed to a physical quantity having integer dimensional exponents. The linear operators occurring in quantum physics have also dimensions. The expectation value $E(q)$ of a physical quantity q is given by

$$E(q)=\int_{-\infty}^{+\infty}\Psi^* q_{op}\Psi dx \tag{3.2}$$

where q_{op} is the linear operator representing the physical quantity q. The wave function Ψ is dimensionless and thus the expectation value $E(q)$ will have the dimension derived from the product of the dimensions of the operator q_{op} and the quantity dx. When all the dimensional exponents are zero, we call the physical quantity *dimensionless* or a physical quantity of *dimension one*. These dimensionless quantities occur in the celebrated *Buckingham theorem* [3] also known as the π-theorem. Functions $F(q)$ of a physical quantity occur in 2 types:

- q is dimensionless and thus $F(q)$ can always be formed as is the case for all mathematical formulas (e.g. $\sin(\theta), \exp(\frac{kT}{h\nu}),\ldots$);
- the function $F(x)=\ln(x)$ defined in integral form as $\ln(q)=\int_1^q\frac{du}{u}$ is *dimensionless* for all physical quantities q and thus one may always write the dimensionless quantities Π_i occuring in the Buckingham theorem as $\Pi_i=\ln(q)$ where q is a physical quantity.

4. Isomorphism between classes of physical quantities and the 7-dimensional integer lattice

Let the set of *all* physical quantities be denoted by Q. As physical quantities occur in the form of scalars, vectors, multi-vectors, matrices and tensors, we can without loss of generality consider a component of a physical quantity and denote it as q. We know that *concepts* in physics are labeled in many ways. The physics concept energy receives dedicated words as: potential energy, kinetic energy, work, Lagrange function, Hamilton function, Hartree energy, ionization energy, electron affinity, electro-negativity, dissociation energy... in our formulations of physical relations. To cope with this multitude of *labels*, we define an equivalence relation between the physical quantities $a,b\in\mathsf{Q}$ with notation $a\sim b$ meaning "a is *dimensionally* equivalent with b". The set of all equivalence classes in Q, given the equivalence relation $\sim$, is the quotient set $\mathsf{Q}/\!\sim$. The equivalence class for the *concept* energy has notation $[energy]_\sim$. We define the surjective function $\dim(q)$ from Q to $\mathsf{Q}/\!\sim$ given by $\dim(q)=[q]_\sim=\mathrm{L}^\alpha\mathrm{M}^\beta\mathrm{T}^\gamma\mathrm{I}^\delta\Theta^\epsilon\mathrm{N}^\zeta\mathrm{J}^\eta$. In the sequel of this article we omit the symbol for the equivalence relation $\sim$ and denote the equivalence class as $[q]$. The class of dimensionless physical quantities is denoted $[\mathit{1}]$. We consider a multiplicative binary operator $\cdot$ between the equivalence classes of $\mathsf{Q}/\!\sim$. The algebraic properties of the composition of the equivalence classes result in a *multiplicative commutative group* $\mathsf{Q}/\!\sim,\cdot$. We now consider the set of integer septuples $\mathbb{Z}^7\doteq\{(f_1,\ldots,f_7):f_i\in\mathbb{Z}\}$. We know that $\mathbb{Z}^7,+$ is an *additive commutative group*. We define a mapping dex ():

Definition 2 (Mapping $\mathrm{dex}\,()$)**.** The mapping $\mathrm{dex}\,()$ is defined from $\mathsf{Q}/\!\sim$ into $\mathbb{Z}^7$ and formally as $\mathrm{dex}\,() : \mathsf{Q}/\sim \to \mathbb{Z}^7 : \mathrm{dex}\,([q]) \doteq \boldsymbol{f} = (f_1, \ldots, f_7)$ where $f_i \in \mathbb{Z}$.

We relabel f_i such that $f_1 = \alpha$, $f_2 = \beta$, $f_3 = \gamma$, $\ldots f_7 = \eta$ being the dimensional exponents *taken in the correct order* of a physical quantity q. Observe that we map the unit element $[1]$ of $\mathsf{Q}/\sim, \cdot$ on the unit element $\boldsymbol{o} = (0, \ldots, 0)$ of $\mathbb{Z}^7, +$ and thus we have $\mathrm{dex}\,([1]) \doteq \boldsymbol{o} = (0, \ldots, 0)$. Each element of $\mathbb{Z}^7$ is the image of *one and only one* class $[q]$ of dimensionally equivalent physical quantities. We define the inverse mapping $\mathrm{dex}^{-1}\,()$:

Definition 3 (Mapping $\mathrm{dex}^{-1}\,()$)**.** The inverse of the $\mathrm{dex}\,()$ mapping is a mapping of $\mathbb{Z}^7$ into $\mathsf{Q}/\!\sim$, and defined as $\mathrm{dex}^{-1}\,() : \forall \boldsymbol{a} \in \mathbb{Z}^7, \exists [a] \in \mathsf{Q}/\sim : \mathrm{dex}^{-1}\,(\boldsymbol{a}) \doteq [a]$.

A homomorphism $\mathrm{f} : \mathsf{Q}/\sim \to \mathbb{Z}^7$ is an *isomorphism* if there exists a homomorphism $\mathrm{g} : \mathbb{Z}^7 \to \mathsf{Q}/\sim$ such that $\mathrm{f} \circ \mathrm{g}$ and $\mathrm{g} \circ \mathrm{f}$ are the identity mappings of $\mathbb{Z}^7$ and $\mathsf{Q}/\sim$ respectively [4]. We identify $\mathrm{f} = \mathrm{dex}\,()$ and $\mathrm{g} = \mathrm{dex}^{-1}\,()$ and infer that a *group isomorphism* exists between $\mathsf{Q}/\!\sim$ and $\mathbb{Z}^7$ that we denote $\mathbb{Z}^7 \approx \mathsf{Q}/\sim$ [4]. The set $\mathbb{Z}^n$ is known as the n-dimensional integer lattice [5]. The properties of $\mathbb{Z}^n$ are found in several publications [5]. In this specific case we put $n = 7$. We are free to select seven basis lattice points of $\mathbb{Z}^7$ and choose an orthonormal base and define:

- $\boldsymbol{e}_1 = \mathrm{dex}\,([\mathit{length}]) = (1, 0, 0, 0, 0, 0, 0)$
- $\boldsymbol{e}_2 = \mathrm{dex}\,([\mathit{mass}]) = (0, 1, 0, 0, 0, 0, 0)$
- $\boldsymbol{e}_3 = \mathrm{dex}\,([\mathit{time}]) = (0, 0, 1, 0, 0, 0, 0)$
- $\boldsymbol{e}_4 = \mathrm{dex}\,([\mathit{electric\ current}]) = (0, 0, 0, 1, 0, 0, 0)$
- $\boldsymbol{e}_5 = \mathrm{dex}\,([\mathit{thermodynamic\ temperature}]) = (0, 0, 0, 0, 1, 0, 0)$
- $\boldsymbol{e}_6 = \mathrm{dex}\,([\mathit{amount\ of\ substance}]) = (0, 0, 0, 0, 0, 1, 0)$
- $\boldsymbol{e}_7 = \mathrm{dex}\,([\mathit{luminous\ intensity}]) = (0, 0, 0, 0, 0, 0, 1)$

with $\boldsymbol{e}_i \in \mathbb{Z}^7$. We will adopt the *Conway abbreviation* [5] for the lattice points and write

Example 1. $\boldsymbol{e}_3 \doteq \mathrm{dex}\,([\mathit{time}]) = (0, 0, 1, 0, 0, 0, 0)$ as $(0^2, 1, 0^4)$.

(a) Useful identities of the mappings $\mathrm{dex}\,()$ and $\mathrm{dex}^{-1}\,()$

We claim without giving proofs the following $\mathrm{dex}\,()$ identities:

$$\forall [a], [b] \in \mathsf{Q}/\sim : \mathrm{dex}\,([a][b]) = \mathrm{dex}\,([a]) + \mathrm{dex}\,([b]) \ , \tag{4.1a}$$

$$\forall [a], [b] \in \mathsf{Q}/\sim : \mathrm{dex}\left(\frac{[a]}{[b]}\right) = \mathrm{dex}\,([a]) - \mathrm{dex}\,([b]) \ , \tag{4.1b}$$

$$\forall [a], [b], [c] \in \mathsf{Q}/\sim : \mathrm{dex}\,([a][b][c]) = \mathrm{dex}\,([a]([b][c])) = \mathrm{dex}\,(([a][b])[c]) \ , \tag{4.1c}$$

$$\forall p \in \mathbb{Z}, \forall [a] \in \mathsf{Q}/\sim, \forall \boldsymbol{a} \in \mathbb{Z}^7 : \mathrm{dex}\,\big([a]^p\big) = p\,\mathrm{dex}\,([a]) = p\boldsymbol{a} \, . \tag{4.1d}$$

We claim without giving proofs the following $\mathrm{dex}^{-1}\,()$ identities:

$$\forall \boldsymbol{a}, \boldsymbol{b} \in \mathbb{Z}^7 : [a][b] = \mathrm{dex}^{-1}\,(\boldsymbol{a} + \boldsymbol{b}) \ , \tag{4.2a}$$

$$\forall \boldsymbol{a}, \boldsymbol{b} \in \mathbb{Z}^7 : \frac{[a]}{[b]} = \mathrm{dex}^{-1}\,(\boldsymbol{a} - \boldsymbol{b}) \ , \tag{4.2b}$$

$$\forall \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{Z}^7 : \mathrm{dex}^{-1}\,(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}) = \mathrm{dex}^{-1}\,(\boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c})) = \mathrm{dex}^{-1}\,((\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c}) \ , \tag{4.2c}$$

$$\forall p \in \mathbb{Z}, \forall [a] \in \mathsf{Q}/\sim : \mathrm{dex}^{-1}\,(p\boldsymbol{a}) = (\mathrm{dex}^{-1}\,(\boldsymbol{a}))^p = [a]^p \, . \tag{4.2d}$$

5. Decomposition of a vertex in pairwise orthogonal vertices

Two physical quantities $[x]$ and $[y]$ are called by J. Schwinger *compatible* [6] when the measurement of $[x]$ does not destroy the knowledge gained by the prior measurement of $[y]$. The property of compatibility of physical quantities is related to the orthogonality of $\mathrm{dex}([x])$ and $\mathrm{dex}([y])$. The decomposition of a vertex $\boldsymbol{z}$ in two pairwise orthogonal vertices $\boldsymbol{x}$ and $\boldsymbol{y}$ assumes the existence of a system of Diophantine equations:

$$\boldsymbol{x}+\boldsymbol{y}-\boldsymbol{z}=0\,, \tag{5.1a}$$

$$\boldsymbol{x}\cdot\boldsymbol{y}=0\,, \tag{5.1b}$$

where $\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}\in\mathbb{Z}^7$. We eliminate $\boldsymbol{y}$ from the equation (5.1b) and find:

$$\boldsymbol{x}\cdot\boldsymbol{x}-\boldsymbol{x}\cdot\boldsymbol{z}=0\,. \tag{5.2}$$

We apply the method of "completing the square" and write equation (5.2) as:

$$(\boldsymbol{x}-\frac{\boldsymbol{z}}{2})^2=(\frac{\boldsymbol{z}}{2})^2\,, \tag{5.3}$$

that represents a seven-dimensional hypersphere with center at $\dfrac{\boldsymbol{z}}{2}$ with radius $\|\dfrac{\boldsymbol{z}}{2}\|_2$.

6. Wave equation of the laws of physics

(a) Mathematical preliminaries

(i) Properties of 7D unit hypersphere

The hypersurface area of a n-sphere of radius $R=1$ is:

$$S_n=\frac{2\left(\sqrt{\pi}\right)^n}{\Gamma\left(\frac{n}{2}\right)} \tag{6.1}$$

where the gamma function is given by

$$\Gamma(m)=2\int_0^{+\infty} r^{2m-1}\mathrm{e}^{-r^2}\,\mathrm{d}r\,. \tag{6.2}$$

For $n=7$ we find $S_7=\dfrac{16\pi^3}{15}$. The n-dimensional volume of a n-sphere is:

$$V_n=\frac{S_nR^n}{n} \tag{6.3}$$

and thus for a unit 7-sphere we have $V_7=\dfrac{16\pi^3}{105}$.

(ii) n-dimensional Fourier transform

We define the n-dimensional Fourier transform relations for $\boldsymbol{k},\boldsymbol{x}\in\mathbb{R}^n$ as:

$$F(\boldsymbol{x})=\underbrace{\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}}_{n} f(\boldsymbol{k})\mathrm{e}^{-2\pi i\boldsymbol{k}\cdot\boldsymbol{x}}\,\mathrm{d}^n\boldsymbol{k} \tag{6.4}$$

and

$$f(\boldsymbol{k})=\underbrace{\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}}_{n} F(\boldsymbol{x})\mathrm{e}^{+2\pi i\boldsymbol{k}\cdot\boldsymbol{x}}\,\mathrm{d}^n\boldsymbol{x} \tag{6.5}$$

From the article of A.W Lohmann et al. [7] and the article of J. Foadi and G. Evans [8] we construct a method that relates the 7-sphere to a seven dimensional Helmholtz wave equation.

We recall that the seven dimensional integer lattice $\mathbb{Z}^7$ can be represented as:

$$Q(\boldsymbol{r}) = \sum_{f_1=-\infty}^{+\infty} \dots \sum_{f_7=-\infty}^{+\infty} \delta^7(\boldsymbol{r} - \sum_{i=1}^{7} f_i \boldsymbol{e}_i)\,, \tag{6.6}$$

where δ^7 is the 7D Dirac distribution. The Fourier transform of $Q(\boldsymbol{r})$ is the 7-dimensional integral:

$$q(\boldsymbol{u}) = \int_{allspace} Q(\boldsymbol{r}) \times \exp\left[2\pi i \boldsymbol{u}\cdot\boldsymbol{r}\right] d\boldsymbol{r} \tag{6.7}$$

It can be shown [8] that :

$$q(\boldsymbol{u}) = \sum_{f_1=-\infty}^{+\infty} \exp(2\pi i f_1 \boldsymbol{e}_1\cdot\boldsymbol{u}) \dots \sum_{f_7=-\infty}^{+\infty} \exp(2\pi i f_7 \boldsymbol{e}_7\cdot\boldsymbol{u}) \tag{6.8}$$

and that we have:

$$\sum_{f_j=-\infty}^{+\infty} \exp(2\pi i f_j \boldsymbol{e}_j = \sum_{m_j=-\infty}^{+\infty} \delta(\boldsymbol{e}_j\cdot\boldsymbol{u} - m_j) \tag{6.9}$$

where $j = 1, \dots, 7$. The Fourier transform $q(\boldsymbol{u})$ has values different from 0 in the u-space where $\boldsymbol{e}_j \cdot \boldsymbol{u} = m_j$ where m_j are *integers* . The u-space is known as *reciprocal space.* We define in the u-space a reciprocal basis $\boldsymbol{e}_1^*, \dots, \boldsymbol{e}_7^*$ and write:

$$q(\boldsymbol{u}) = \sum_{m_1=-\infty}^{+\infty} \delta(\boldsymbol{e}_1\cdot\boldsymbol{u} - m_1\boldsymbol{e}_1\cdot\boldsymbol{e}_1^*) \dots \sum_{m_7=-\infty}^{+\infty} \delta(\boldsymbol{e}_7\cdot\boldsymbol{u} - m_7\boldsymbol{e}_7\cdot\boldsymbol{e}_7^*) \tag{6.10}$$

that can be transformed to:

$$q(\boldsymbol{u}) = \sum_{m_1=-\infty}^{+\infty} \dots \sum_{m_7=-\infty}^{+\infty} \delta^7(\boldsymbol{e}_1\cdot(\boldsymbol{u} - m_1\boldsymbol{e}_1^*), \dots, \boldsymbol{e}_7\cdot(\boldsymbol{u} - m_7\boldsymbol{e}_7^*)) \tag{6.11}$$

The seven inner products under the 7D-Dirac delta can be considered as a linear transformation represented by a 7×7 matrix T on a lattice point $\boldsymbol{u} - \sum_{j=1}^{7} m_j \boldsymbol{e}_j^*$ with $det(\mathsf{T}) = V$ where V is the cell volume. We find [8] that :

$$q(\boldsymbol{u}) = \frac{1}{V} \sum_{m_1=-\infty}^{+\infty} \dots \sum_{m_7=-\infty}^{+\infty} \delta^7(\boldsymbol{u}^* - \boldsymbol{u}^*_{m_1,\dots,m_7}) \tag{6.12}$$

(b) Seven dimensional Helmholtz wave equation of a physical quantity

We use the above results in our method and begin with multiplying both sides of the equation (5.3) with $(2\pi i)^2$ and write:

$$\left(2\pi i(\boldsymbol{x} - \frac{\boldsymbol{z}}{2})\right)^2 - \left(2\pi i(\frac{\boldsymbol{z}}{2})\right)^2 = 0\,. \tag{6.13}$$

We construct the integral:

$$\underbrace{\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty}}_{7} \Big[(2\pi i(x_1 - \frac{z_1}{2})^2 + \dots + (2\pi i(x_7 - \frac{z_7}{2})^2 + k^2\Big]$$
$$\times\, Q\left((x_1 - \frac{z_1}{2}), \dots, (x_7 - \frac{z_7}{2})\right) \times \exp\left[2\pi i \left(u_1(x_1 - \frac{z_1}{2}) + \dots + u_7(x_7 - \frac{z_7}{2})\right)\right] \mathrm{d}x_1 \dots \mathrm{d}x_7 = 0\,, \tag{6.14}$$

where

$$[q(u_1,\ldots,u_7)] = \\ \underbrace{\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}}_{7} Q\left((x_1-\frac{z_1}{2}),\ldots,(x_7-\frac{z_7}{2})\right) \\ \times \exp\left[2\pi i\left(u_1(x_1-\frac{z_1}{2})+\ldots+u_7(x_7-\frac{z_7}{2})\right)\right]\,\mathrm{d}x_1\ldots\mathrm{d}x_7\,, \tag{6.15}$$

and

$$k^2 = -(2\pi i\frac{\boldsymbol{z}}{2})^2\,. \tag{6.16}$$

We recognize the equation (6.15) as the Fourier transform representation of the physical quantity $[q]$ and write the seven dimensional Helmholtz differential equation:

$$\nabla^2[q(u_1,\ldots,u_7)] + k^2[q(u_1,\ldots,u_7)] = 0\,, \tag{6.17}$$

where the physical quantity $[q]$ is represented as a wave in the 7D u-space and where k^2 is determined by the physical quantity, with representation $Q(z_1,\ldots,z_7)$ in $\mathbb{Z}^7$, for which the laws of physics are to be found.

7. Distribution of the laws of physics

To find the laws of physics we have to search for the number of *unique* rectangles as function of increasing infinity norm. A computer search reveals the following distribution of laws of physics:

Table 2: Number of laws of physics in each hypercubic shell of infinity norm $\|\boldsymbol{x}\|_\infty = s$.

Infinity norm $\|\boldsymbol{x}\|_\infty = s$	Number of laws of physics
0	0
1	1
2	7
3	26
4	79
5	182
6	333
7	693

The results are represented in the Fig. 1.

[Figure 1 about here.]

We find in the hypercube where $\|\boldsymbol{x}\|_\infty \leq 7$ a total of 1321 laws of physics.

8. Discussion

We show in section 6 that the 7-sphere of a physical quantity has an alternative representation being a seven dimensional Helmholtz wave equation of the physical quantity. This method gives a new interpretation to the use of the seven dimensional integer lattice $\mathbb{Z}^7$, where a 7-sphere represents *all* the fundamental physical relations between the different *spectral* coordinates $(f_1,\ldots,f_7)$. These spectral coordinates are nothing else than the differences between the dimensional exponents of the physical quantity $[x]$ and 1/2 of the dimensional exponents of the selected physical quantity $[z]$. The u-space is a restriction of the universe to its 7D mathematical model. We model the universe as a 7D unit hypersphere S^7 because it is known that the 7D unit hypersphere S^7 has the largest surface area of all n-dimensional unit hyperspheres S^n [9]. This choice for modeling the universe as a 7D unit sphere S^7 is based on the minimalization of the surface energy of the universe and thus maximalization of the surface area.

9. Conclusion

We show that each SI physical quantity, that is represented by a lattice point in a seven dimensional integer lattice, has a *unique* 7-sphere. This 7-sphere is the geometrical representation of a seven dimensional Helmholtz differential equation of a *unique* scalar wave function associated to the physcial quantity. The solutions of the differential equation are given by the lattice points of the integer lattice that are incident on the 7-sphere. The lattice points incident on the 7-sphere are forming rectangles containing the origin $\boldsymbol{o}$, the lattice point $\boldsymbol{z}$ representing the selected physical quantity and the lattice point representations of a pair of *compatible* physical quantities $\boldsymbol{x}, \boldsymbol{y}$ where $\boldsymbol{z} = \boldsymbol{x} + \boldsymbol{y}$. The resulting rectangles are the geometric representation of the *laws of physics* of the dimensional form $[z] = [x][y]$ for the selected physical quantity $[z]$.

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References

1. Bureau International des Poids et Mesures, BIPM 2006 Le Système International d'unités The International System of Units SI, 8th edn. Paris, STEDI Media.(http://www.bipm.org/utils/common/pdf/si_brochure_8.pdf)
2. Fleischmann R. 1953 Physikalisches Begriffssystem und Dimensionen. *Phys. Bl.* **9** , 301–313. (doi: 10.1002/phbl.19530090702)
3. Buckingham E. 1914 On physically similar systems; illustrations of the use of dimensional equations, *Phys. Rev.* **4**, 345–376.
4. Lang S. 2005 *Algebra, Revised Third edition.* New York, NY: Springer Science+Business Media.
5. Conway J.H., Sloane N.J.A. 1999 Certain Important Lattices and Their Properties. In *Sphere Packings, Lattices and Groups, Third Edition*, Chapter 4, 106–108. Berlin Heidelberg New York, 1999. Springer-Verlag.
6. Schwinger J. 1959 The algebra of microscopic measurement. *Proc. N.A.S.* **45** (10), 1542–1553.
7. Lohmann A.W., Mendlovic D., Zalevsky Z., Shabtay G. 1997 The use of Ewald's surfaces in triple correlation optics. *Optics Communications* **144**, 170–172.
8. Foadi J., Evans G. 2008 Elucidations on the reciprocal lattice and the Ewald sphere. *Eur. J. Phys.* **29**, 1059–1068.
9. E.W. Weisstein, Hypersphere, in MathWorld - A Wolfram Web Resource, [cited 2012 Mar 08]. Available from: http://mathworld.wolfram.com/Hypersphere.html.

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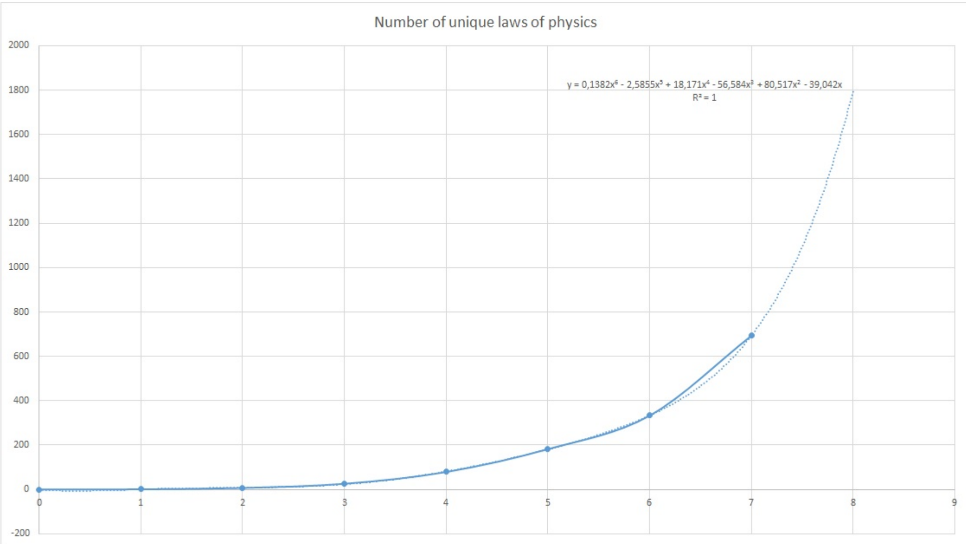

Figure 1. Number of laws of physics in each hypercubic shell of infinity norm $\|\boldsymbol{x}\|_\infty = s$.