

DESIGN AND ANALYSIS OF A MULTIVARIABLE FRACTIONAL ORDER CONTROLLER FOR A NON-MINIMUM PHASE SYSTEM

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ABSTRACT

Two control strategies for multivariable processes are proposed that are based on a decentralised and a steady state decoupling approach. The designed controllers are fractional order PIs. The efficiency and robustness of the proposed strategies is tested and validated using a non-minimum phase process. Previous research for the same non-minimum phase process has proven that simple decentralised or decoupling techniques do not yield satisfactory results and a multivariable IMC controller has been proposed as an alternative solution. The simulation results presented in this paper show that the proposed fractional order multivariable control strategies ensure an improved closed loop performance and disturbance rejection, as well as increased robustness to modelling uncertainties, as compared to traditional multivariable IMC controllers.

KEYWORDS: fractional order control, non-minimum phase system, water tanks, decentralized control, decoupling control, Internal Model Control

INTRODUCTION

The large majority of chemical processes are multivariable in nature, exhibiting some strong couplings and occasionally a non-minimum phase character that makes the control design problem a challenging task [1,2]. In general, for such systems, the objective of a control system is to maintain several controlled variables at independent set points. Despite the coupling problems associated with multivariable systems, a non-minimum phase system is even more difficult to control. None of the techniques that are based upon model inversion can be used since such an inversion leads to an unstable closed loop system. Multivariable controllers have been previously designed for such systems. However, simplified algorithms are generally preferred. In contrast to the centralised multivariable control, decentralised control is widely preferred in practice and industrial applications especially because of its main advantage that allows for an easy implementation and tuning, if a sufficient number of sensors and actuators exist. It is also highly reliable.

For highly interacting processes, a decoupling control is usually preferred instead of a decentralized algorithm. Decoupling is a procedure that reduces multivariable interactions [3] and sets the premises for an improved design of the decentralized control. The mathematical procedure to decouple a MIMO system consists in a transformation of the original transfer function matrix of the process into a diagonal one, achieved by using an additional controller, also called a decoupler, which is designed in order to compensate for process interactions. Then, for the resulting pseudo-plant, consisting of the original model of the multivariable process and the decoupler, SISO techniques can directly be used in designing the controllers.

The quadruple tank process, considered as a case study in this paper, is a multivariable process with a multivariable zero located in the right half plane. Apart from the non-minimum phase character, it also exhibits elegantly complex dynamics, including interactions and transmission zero [4]. This particular process has been the focus of numerous papers, since it exhibits characteristics of interest in both control and research education. Bearing in mind that in process control industry, more than 95% of the control loops are of PI/PID (Proportional plus Integral /Proportional plus Integral plus Derivative) type, the most widely used control method for this type of processes has been the classical PID, either

in a decentralised or in a decoupling approach [5]. For example, a recent paper compares the two approaches, while the tuning of the PI controllers is performed in several ways, ranging from direct synthesis, sequential relay with ZN settings, to more advanced methods such as the IMC (Internal Model Control) [5]. To reduce the interaction effects, a partial decoupling method for MIMO systems has also been proposed and implemented for the non-minimum phase quadruple tank system [6, 7].

Other more advanced methods have also been employed for the quadruple tank process, such as fuzzy control algorithms implemented in a decentralised version [4], with the results clearly showing the closed loop performance improvement compared to the traditional decentralised PI control. Fuzzy logic has also been used to tune a combined state-feedback sliding-mode controller for quadruple tank system [8]. The simulation results showed that the proposed version achieved better closed loop performance than the stand alone versions of state-feedback controller or sliding-mode controller. Sliding mode control has been designed and tested on an experimental setup, providing increased robustness and excellent set point tracking [9].

For the particular setup considered in this paper, the quadruple tank system from Quanser, a decentralised, decoupling and multivariable IMC strategies have been proposed [10], however the experimental results obtained showed the necessity of more complex control algorithms when stringent performance is envisaged and coupling, as well as RHP zeros need to be tackled efficiently. For this particular process, both decentralized and decoupling controls achieved poor performance for disturbance rejection tests, which motivated the application of the more advanced IMC control and even a possible future work regarding model predictive control.

The purpose of this paper is to design a simple control algorithm that is based on combining fractional order controllers with a decentralised as well as decoupling approach that allow for a SISO interpretation of the controller tuning, but that can also achieve improved performance compared to the multivariable IMC control (MIMO IMC). The fractional order PID (FOPID) controller was proposed as a generalization of the traditional integer order PID controller. The use of fractional order controllers is expected to enhance the performance of the closed loop system and increase the robustness of the system [11,12,13], being used in a wide area of applications [14].

Several fractional order techniques have been proposed in literature for controlling multivariable processes, such as the extension of the CRONE algorithm [15], MIMO-QFT robust synthesis methodology combined with CRONE control [16], sliding mode control based on the selection of a special fractional-order sliding variable [17]. Different methods for tuning multivariable fractional PID have been proposed, such as an approach to consider the tuning formulated as an H_∞ problem with a controller structure constraint [18], the LMI (Linear Matrix Inequality) approach [19], as well as a genetic algorithm for determining the gains and orders of the fractional order PID controllers [20]. Contrary to these multivariable fractional order control algorithms, the present paper proposes simpler approaches, also based on robust fractional order control algorithms that enable the use of SISO control techniques for multivariable processes.

ALTERNATIVE DESIGNS OF A FRACTIONAL ORDER CONTROLLER FOR MULTIVARIABLE PROCESSES

The two alternative designs for the fractional order controller proposed in this paper consist in a decentralised, as well as a steady state decoupling approach.

Decentralised approach

The decentralised approach in controlling MIMO systems consists in a proper selection of the input-output pairings, with the purpose of dividing the initial control problem into several SISO control

loops, while aiming to reduce the amount of interaction. The first step in the decentralised approach consists in a RGA (Relative Gain Array) analysis of the multivariable process that allows for a proper pairing of the input-output signals [21, 22]. The next step consists in the design of the individual fractional order PI controllers for each input-output pairing by neglecting the effect of the interaction loop. The transfer function of the fractional order PI controller, proposed in this paper, is given as:

$$H_{FO-PI}(s) = k_p \left(1 + \frac{k_i}{s^\mu} \right) \quad (1)$$

with $\mu \in (0 \div 2)$ the fractional order. To tune the fractional order PI controller, three performance specifications are imposed: a) a certain gain crossover frequency - ω_{gc} , b) a phase margin - φ_m - of the open loop system, denoted $H_d(s)$ and c) a robustness condition to gain variations. Considering that the open loop transfer function is written as:

$$H_d(s) = H_{FO-PI}(s)H_p(s) \quad (2)$$

where $H_p(s)$ is the process transfer function, the tuning of the controller is done based on the following set of equations [23,24,25]:

$$\left| \frac{1}{K + jL} \right|_{\omega_{gc}} \left| k_p \left[1 + k_i \omega_{gc}^{-\mu} \left(\cos \frac{\pi\mu}{2} - j \sin \frac{\pi\mu}{2} \right) \right] \right| = 1 \quad (3)$$

$$\frac{k_i \sin\left(\frac{\pi \cdot \mu}{2}\right)}{\omega_{gc}^\mu + k_i \cos\left(\frac{\pi \cdot \mu}{2}\right)} = \text{tg}\left(\pi - \varphi_m - \text{atan}\left(\frac{L}{K}\right)\right) \quad (4)$$

$$\frac{\mu k_i \omega_{gc}^{-\mu-1} \sin \frac{\pi\mu}{2}}{1 + 2k_i \omega_{gc}^{-\mu} \cos \frac{\pi\mu}{2} + k_i^2 \omega_{gc}^{-2\mu}} - \frac{\dot{L}K - L\dot{K}}{L^2 + K^2} = 0 \quad (5)$$

where K is the real part and L is its imaginary part of the process $H_p(j\omega_{gc})$. To simplify the computation of the fractional order PI controller parameters, the values for k_i and μ are determined graphically using (4) and (5) [23,24,25], while k_p is then computed using (3).

Decoupling approach

In case of a highly coupled MIMO system, the decentralised approach may result in poor closed loop performance due to the multiple input-output interactions. A decoupling solution could then be used instead. In this paper, a steady state decoupling is employed. Given the $n \times n$ MIMO system:

$$G_p(s) = \begin{pmatrix} H_{p11}(s) & H_{p12}(s) & \dots & H_{p1n}(s) \\ H_{p21}(s) & H_{p22}(s) & \dots & H_{p2n}(s) \\ \dots & \dots & \dots & \dots \\ H_{pn1}(s) & H_{pn2}(s) & \dots & H_{pnn}(s) \end{pmatrix} \quad (6)$$

the steady state decoupler is the inverse of the process transfer function gain matrix in (6), denoted as $G_m^\#$. The steady state decoupled process is then computed as:

$$G_D(s) = G_m^\# \begin{pmatrix} H_{p11}(s) & H_{p12}(s) & \dots & H_{p1n}(s) \\ H_{p21}(s) & H_{p22}(s) & \dots & H_{p2n}(s) \\ \dots & \dots & \dots & \dots \\ H_{pn1}(s) & H_{pn2}(s) & \dots & H_{pnn}(s) \end{pmatrix} \quad (7)$$

The tuning of the fractional order PI controllers is then performed for each diagonal element in the decoupled process $G_D(s)$ using the same tuning procedure based on (3)-(5). The final multivariable FO-PI controller computed as:

$$G_C(s) = G_m^\# \begin{pmatrix} H_{FO-PI_1}(s) & 0 & \dots & 0 \\ 0 & H_{FO-PI_2}(s) & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & H_{FO-PI_n}(s) \end{pmatrix} \quad (8)$$

CASE STUDY. CONTROL STRATEGIES FOR NON_MINIMUM PHASE QUADRUPLE TANK SYSTEM

The schematic representation of the quadruple water tanks system is given in Fig. 1. The system is a multivariable one, with two inputs, the voltages applied to the two pumps, denoted as $V_{p1}(t)$ and $V_{p2}(t)$, and two outputs, the water levels of the lower tanks, Tank2 and Tank4, denoted as $L_2(t)$ and $L_4(t)$, respectively. There is a strong coupling effect between the inputs and the outputs. Such a coupling may be observed in Tank2 which has two inputs: the flow from Pump1 ($V_{p1}(t)$) through Out2, marked with dashed red line, and the flow from Pump2 ($V_{p2}(t)$) through Out1, denoted with green continuous line, that is the output flow from Tank1). Hence, the controlled level in Tank2 (L_2) is influenced by the two inputs.

By a simple adjustment of the percentage of water flow from each input, one can change the system for having minimum phase or non-minimum phase dynamics [10]. The configuration used in this paper and indicated in Figure 1 is a non-minimum phase one, with a greater flow coming from Pump2, via Tank1, into Tank2, in comparison with the flow coming directly from Pump1. This is due to the fact that the outlet diameter Out1 is bigger than the diameter Out2, while the outgoing orifices from each tank Do1, Do2, Do3 and Do4 have the same diameter. A similar situation occurs in the case of Tank4. Then, the dominant flow in Tank2 and Tank4 comes from the manner in which the physical coupling is implemented via the choice of the setup [26,27].

The model transfer function matrix has been previously determined experimentally to be [10]:

$$G(s) = \begin{bmatrix} \frac{1.64}{18.43s+1} & \frac{2.49}{178.8s^2+26.74s+1} \\ \frac{2.56}{172.2s^2+27.6s+1} & \frac{1.28}{15.92s+1} \end{bmatrix} \quad (9)$$

The transmission zeros for the quadruple water tanks system are: $z_1 = -0.26$; $z_2 = 0.07$; $z_3 = -0.06$; $z_4 = -0.05$. Due to the positive zero $z_2 = 0.07$, the system is non-minimum phase.

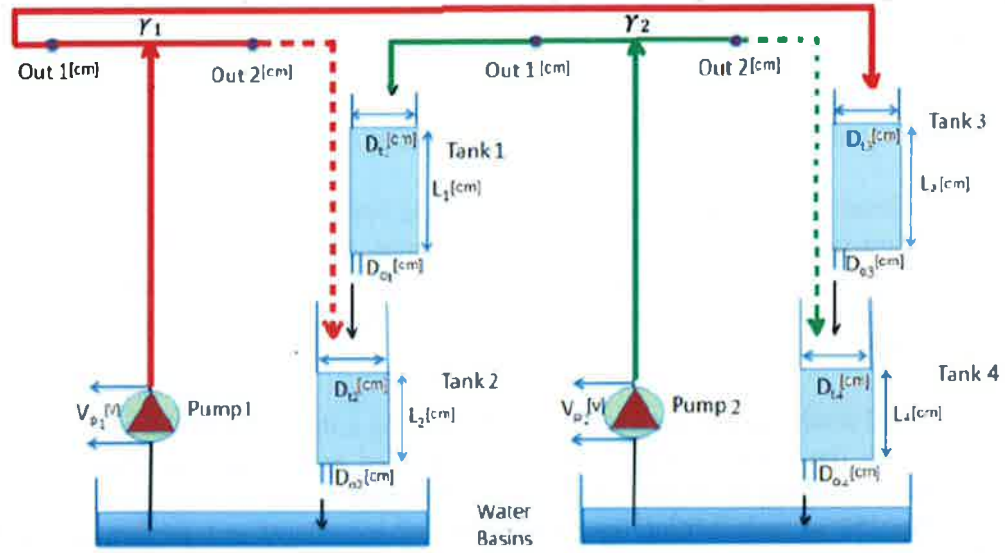


Fig. 1. Schematic diagram of the quadruple water tanks system

A simple RGA analysis show that for the configuration previously described the following RGA values are obtained [10]:

$$\Lambda = \begin{bmatrix} -0.49 & 1.49 \\ 1.49 & -0.49 \end{bmatrix} \quad (10)$$

According to (10), t1-2/2-1 pairing is selected and two FO-PI controllers are then computed. The following performance specifications are imposed for the two loops: $\omega_{gc1} = 0.03$, $\varphi_{m1} = 70^\circ$ and $\omega_{gc2} = 0.03$, $\varphi_{m2} = 70^\circ$. The resulting fractional order PI controllers, to be used in the decentralised approach are:

$$\begin{cases} H_{FO-PI_1}(s) = 0.3 \left(1 + \frac{0.024}{s^{1.16}} \right) \\ H_{FO-PI_2}(s) = 0.29 \left(1 + \frac{0.025}{s^{1.16}} \right) \end{cases} \quad (11)$$

To tune the fractional order controllers for the decoupling control strategy, the decoupler was first computed as:

$$G_m^\# = \begin{pmatrix} -0.3 & 0.58 \\ 0.6 & -0.38 \end{pmatrix} \quad (12)$$

Similar performance specifications were imposed to design the fractional order controllers for the decoupling strategy, $\omega_{gc1} = 0.02$, $\varphi_{m1} = 70^\circ$ and $\omega_{gc2} = 0.02$, $\varphi_{m2} = 70^\circ$, in order to obtain similar closed loop performance in terms of overshoot and settling time. The two fractional order controllers are:

$$H_{FO-PI_1}(s) = 0.53 \left(1 + \frac{0.022}{s^{1.16}} \right) \quad (13)$$

$$H_{FO-PI_2}(s) = 0.6 \left(1 + \frac{0.018}{s^{1.17}} \right) \quad (14)$$

with the final multivariable FO-PI controller determined using (8).

To compare the results, a multivariable IMC strategy has been designed according to [10], to yield similar closed loop performance in terms of settling time, as compared to the decentralised and decoupling fractional order control algorithms given by (11) and (13)-(14), respectively. The closed loop simulation results, considering step changes in the reference signals for the levels L_2 and L_4 , are given in Fig. 2 and 3.

Since the simplified model in (9) was obtained by linearizing a nonlinear model around the operating point of 10cm [10], the results in Fig. 2 and 3 are regarded as nominal operating conditions. The decentralised and decoupling fractional order control strategies ensure no overshoot and 150 seconds settling time. The 150 seconds settling time will be further considered as a performance criteria. The MIMO IMC algorithm ensures the same settling time, but with an overshoot of 25%. It must be noted here that zero overshoot for the MIMO IMC strategy is possible to be obtained at the expense of a major increase in the settling time. In terms of interaction, the MIMO IMC offers the best results, however this is valid under the assumption of a perfect model. Among the fractional order control strategies, the decoupling approach provides better interaction responses than the decentralised control algorithm.

To test the robustness of the designed controller, similar step changes in the reference signals were considered, but with a variation of 30% of the gains and time constants of the process in (9):

$$G(s) = \begin{bmatrix} \frac{2.14}{23.96s+1} & \frac{3.24}{232s^2+26.74s+1} \\ \frac{3.33}{230s^2+27.6s+1} & \frac{1.66}{21s+1} \end{bmatrix} \quad (15)$$

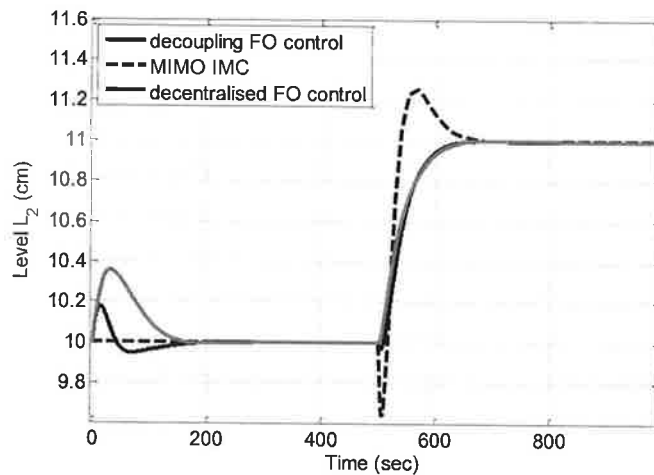


Fig. 2. Comparative nominal closed loop simulation results considering a step change in the reference signal for L_2

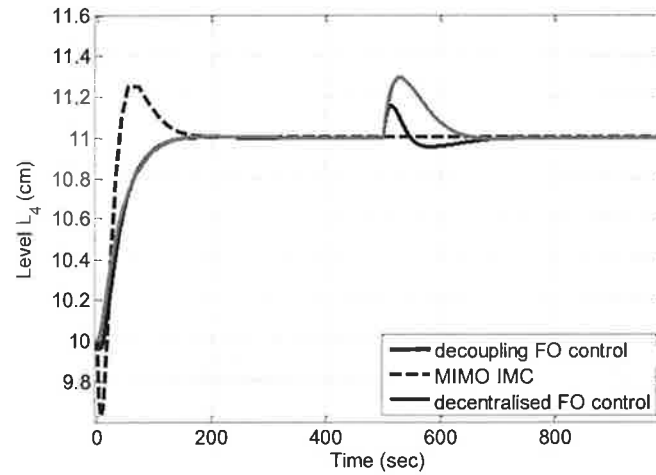


Fig. 3. Comparative nominal closed loop simulation results considering a step change in the reference signal for L_4

The closed loop comparative robustness simulation results are indicated in Fig. 4 and 5. As noted from the two figures, for the fractional order control strategies, the 30% change in the modeling parameters do not affect significantly the closed loop performance results, with no overshoot and a smaller settling time below 120 seconds for both outputs. Considering the performance criteria of 150 seconds maximum settling time, both the decentralised and the decoupling FO controllers meet this requirement. The robustness of the decentralized and decoupling control strategies are almost identical in terms of reference tracking. On the other hand, the MIMO IMC results show a degradation of the closed loop performance, with a slight increase in the settling time of 170 seconds, but a significant increase of the overshoot accounting to 50%. The maximum amplitudes of the interaction responses show that the MIMO IMC and the decoupling FO controllers have similar performance, with the decentralised FO controllers behaving the poorest. The settling time is however 50% larger with the MIMO IMC (150 seconds) compared to the decentralised and decoupling FO controllers. Overall, the proposed fractional order decentralised and decoupling strategies offer an increased robustness as compared to the previously proposed MIMO IMC algorithm.

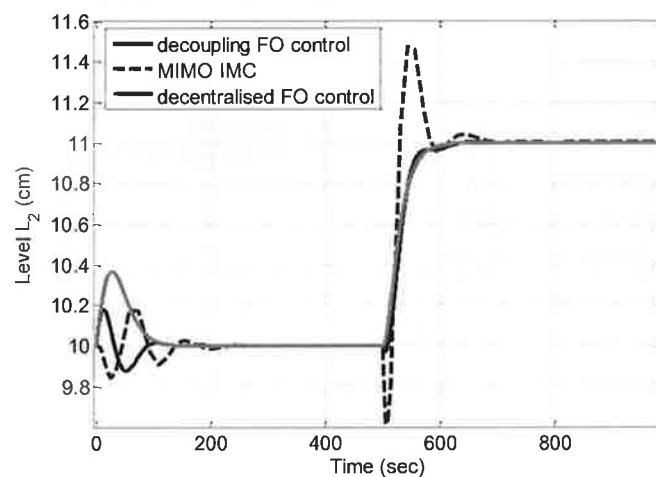


Fig. 4. Comparative robust closed loop simulation results considering a step change in the reference signal for L_2

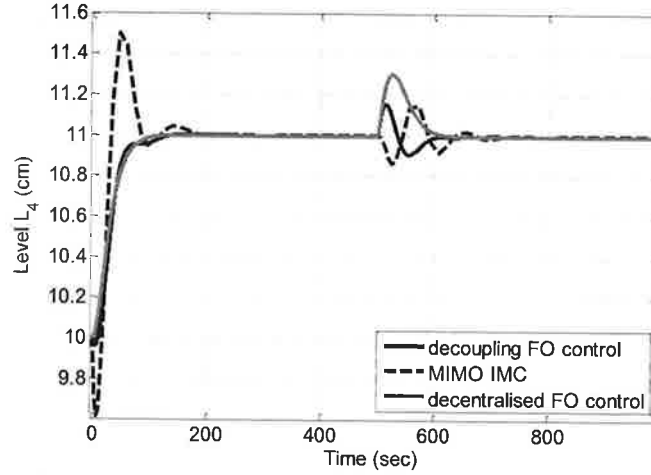


Fig. 5. Comparative robust closed loop simulation results considering a step change in the reference signal for L_4

Previous results [10] showed that poor disturbance rejection performance was achieved when using classical integer order PID controllers in a decentralised or decoupling approach, which justified the application of the more advanced MIMO IMC control. Fig. 6 and 7 present the disturbance rejection tests, considering the nominal conditions, while Fig. 8 and 9 present the same disturbance rejection tests in the case of the modeling errors in (15). The simulation results show that the MIMO IMC and the decoupling fractional order controller are outperformed in terms of settling times by the decentralised fractional order controller. Also, the MIMO IMC is more oscillating with increased amplitudes compared to the decoupling fractional order controller.

To evaluate the disturbance rejection tests, the following performance index was used:

$$J = \sum_{t=0}^{\infty} (r_i(t) - y_i(t))^2, \text{ with } i=1,2 \quad (16)$$

where r_i is the setpoint for the corresponding output signal y_i (either tank levels L_2 or L_4).

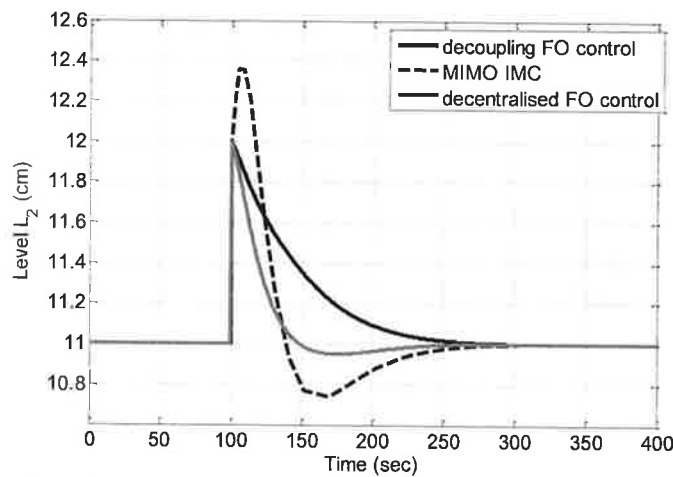


Fig. 6. Comparative disturbance rejection tests considering nominal conditions and a step change in the reference signal for L_2

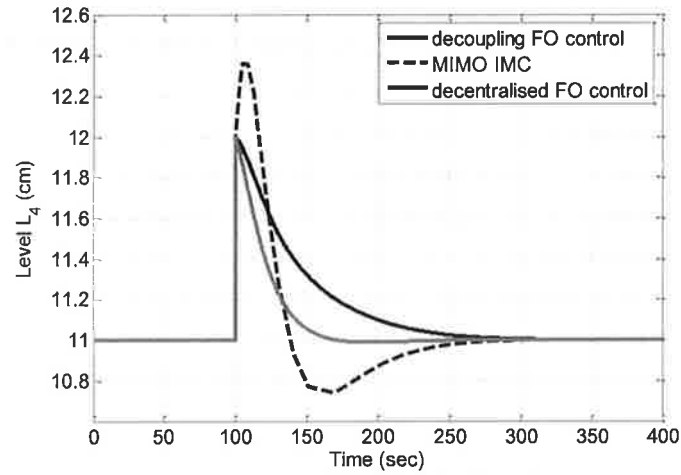


Fig. 7. Comparative disturbance rejection tests considering nominal conditions and a step change in the reference signal for L_4

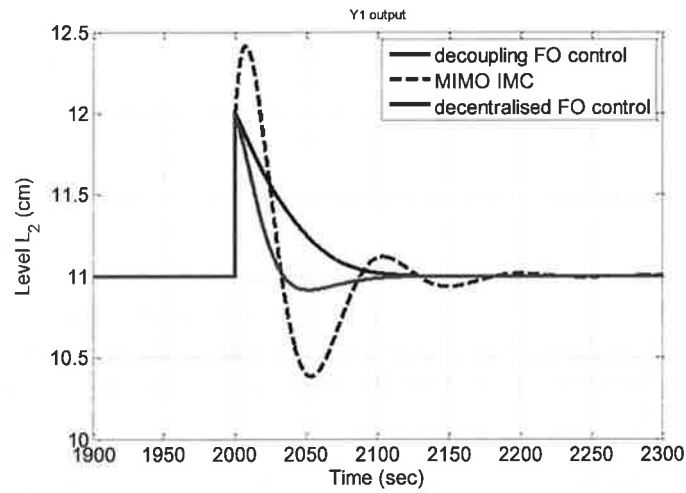


Fig. 8. Comparative disturbance rejection tests considering modelling errors and a step change in the reference signal for L_2

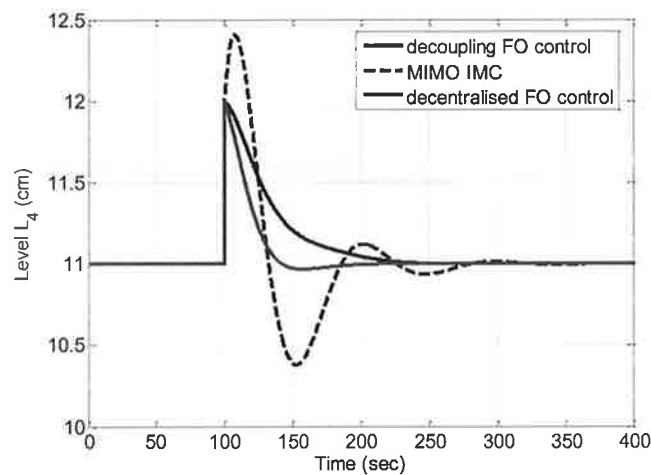


Fig. 9. Comparative disturbance rejection tests considering modelling errors and a step change in the reference signal for L_4

The computed values are given in Table 1 and show that the proposed fractional order control strategies outperform the MIMO IMC in terms of disturbance rejection, both under nominal as well as modelling errors.

Control strategy	Output y1		Output y2	
	Nominal	Modelling errors	Nominal	Modelling errors
Decentralised fractional order control	J = 10.35	J = 9.37	J = 11.35	J = 10.03
Decoupling fractional order control	J = 23.6	J = 19.57	J = 23.6	J = 19.3
MIMO IMC	J = 34.23	J = 43.72	J = 34.22	J = 43.31

Table 1. Performance index for the disturbance rejection tests

EXPERIMENTAL RESULTS

The two fractional order control strategies described above have been tested on the coupled tanks system by Quanser. Figures 10 and 11 show the comparative closed loop tests for both the decentralised and the decoupling fractional order control strategies. The case study considered here consists in a step change for the L_2 reference from 9 cm to 11 cm, thus near the linearization point. The L_4 reference signal is kept at 10 cm.

In terms of interaction, the decoupling fractional order control strategy performs better than the decentralised fractional order control algorithm. Using the same performance index as in (16) for the interaction responses yields a value of $J=9.14$ for the decoupled version, while for the decentralised control strategy, $J=21.26$. This suggests that the decoupled control algorithm ensures a 50% reduction of the interaction responses.

In terms of reference tracking, the experimental results in figure 10 show that the two fractional order control strategies achieve similar performance. The settling time in both cases is 150 seconds, as obtained in the simulation results presented in the previous section, with a slight increase in the overshoot of 2%.

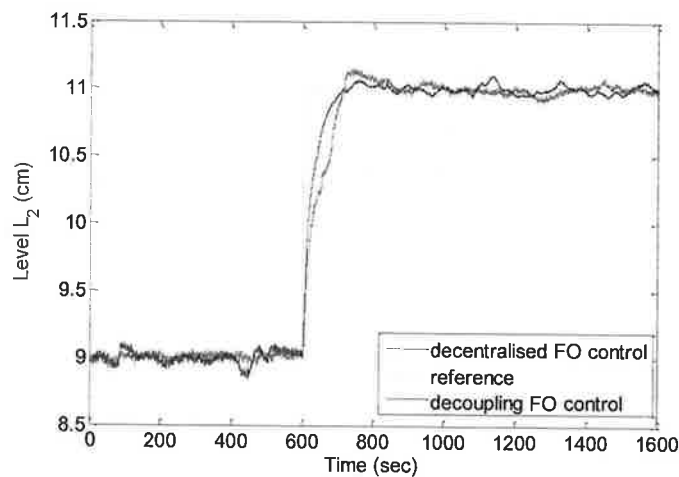


Fig. 10. Comparative closed loop experimental results for reference tracking

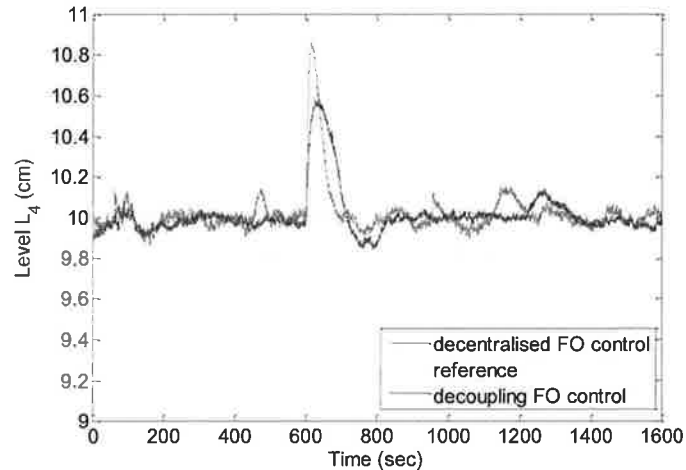


Fig. 11. Comparative closed loop experimental results for interaction response

CONCLUSIONS

This paper presented two alternative solutions for controlling non-minimum phase systems and significant coupling. The previous traditional decentralised and decoupling strategies applied for the presented case study, the quadruple tank system, have shown the necessity for an advanced control solution, such as the MIMO IMC. The alternative solutions proposed in this paper consist in decentralised and decoupling fractional order control strategies. The simulation results prove that the proposed multivariable fractional order control algorithms outperform the MIMO IMC solution previously proposed, in terms of closed loop performance, disturbance rejection, both under nominal conditions, as well as modelling errors.

The experimental results considering the decoupled and decentralised fractional order control strategies are in good agreement with the closed loop simulation results. The settling time and the overshoot obtained on the experimental quadruple tank system meet the performance criteria specified for the simulated closed loop system. The decoupled fractional order control algorithm achieves similar performance in terms of reference tracking when compared to the decentralised fractional order control strategy. The most important contribution of the decoupled fractional order control algorithm is the 50% reduction in the interaction response, compared to the decentralised approach.

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