

The small strain stiffness of a railway ballast

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Abstract

The large particle sizes of railway ballast and rock fill have meant that conventional techniques used to measure the small strain stiffness of finer geomaterials have not been adopted, with the consequence that their stiffnesses are poorly defined. In a series of tests on a UK railway ballast, simple adaptations were made to existing local strain measuring systems to account for the larger particle sizes. The study showed that the small strain stiffnesses are different in second loading compared to virgin loading, but multiple cycles had little further effect on the stiffness. The large particle size was found rarely to have any detrimental effect on the quality of the strain measurements and the two independent measurements of axial strain taken at diametrically opposite locations were generally as consistent as for finer grained soils. As for other soils, the “external” measurements of strain across the apparatus platens were of little use in determining stiffness. The presence of water did not have a significant effect on the behaviour, and this was confirmed by inter-particle loading tests on single particle contacts. Despite the use of lubricated end platens there was a significant barrelling of the sample at large shear strains so that the internal measurement of the volumetric change diverged from the external measurement at large strains. The very small volumetric **strains** that occurred during isotropic loading meant that each sample could only be used to obtain one measurement of the virgin loading stiffness.

Keywords: Ballast, Stiffness, Shearing, Triaxial.

24 **Introduction**

25 Railway ballast testing has received growing interest due to the importance of this material in
26 the infrastructure of railway tracks. Laboratory based studies (Sun et al., 2019; Lackenby et
27 al., 2007; Indraratna et al., 1998) have often provided the validation for computer simulation
28 studies using discrete element modelling DEM (e.g. Quezada et al., 2012; Ngo et al., 2014;
29 Ferrellec et al., 2017), used then to predict the behaviour of ballast in situ. However, the
30 available data from testing these materials is limited and does not cover all aspects of the
31 mechanical behaviour when compared to other geomaterials with smaller particle sizes. There
32 is a lack of information on the behaviour of ballast at small strain levels due to the large size
33 of the particles, and neither the axial nor the volumetric strains have been well defined. The
34 measurement of volumetric strains and hence shear strains has been particularly problematic
35 due to very large membrane penetration (e.g. Knodel et al., 1992). Axial strains have typically
36 been measured across the sample platens with an “external” transducer mounted outside the
37 triaxial chamber and will be subject to the same bedding, seating, tilting and compliance errors
38 that are seen for finer grained soils (Jardine et al., 1985). Transducers mounted locally on the
39 sample as are common for finer grained soils (Burland & Symes, 1982; Clayton & Khatrush,
40 1986; Goto et al., 1991; Cuccovillo & Coop, 1997) have typically not been used.

41 Previous literature has shown different trials for the volumetric strain measurement of ballast
42 samples either by the use of a direct volume gauge to measure the change in sample pore water
43 volume for a saturated sample (e.g. Liu et al., 2008) or measuring the volume change within
44 the triaxial chamber for a dry sample, for example by monitoring the change in level of the
45 pressurised water surrounding the sample (Fair, 2003). Aursudkij et al. (2009) introduced the
46 use of differential pressure to obtain the volumetric change of a dry sample through the change
47 in volume of the pressurised fluid in a smaller chamber surrounding the sample. However, all
48 these methods cannot overcome the inaccuracy due to membrane penetration effect for this

49 material. A local measurement of sample circumference was advocated by Suiker et al. (2005),
50 attaching three cable-based devices around the sample to measure the change in its
51 circumference during the test. They then calculated the volumetric change of the dry sample
52 depending on both the circumferential deformation and an “external” axial deformation.

53 The small strain stiffness is important to understand the ballast behaviour under rail track.
54 Therefore, in this study a trial was made to measure the small strain Young’s modulus and
55 shear modulus of the material using high resolution local displacement LVDTs, scaling up
56 techniques developed by Cuccovillo & Coop (1997) and Ackerley et al. (2016). The volumetric
57 changes of the sample calculated from these measurements were also compared with the
58 volumetric changes measured using an external volume gauge which is based on the volume
59 of water entering or being expelled from a saturated sample. The aim of this work was primarily
60 to demonstrate how local strain measurement may be made accurately on ballast or rockfill
61 samples rather than make specific measurements for a particular rail track. The research formed
62 part of a large research project to develop an “Avatar” DEM model of a railway ballast, the
63 tests being used to validate the model. We therefore made no attempt to replicate the precise
64 void ratios of a particular in-situ location or the stress levels that would typically be somewhat
65 lower than the minimum used here. Triaxial testing at very low stresses requires other
66 techniques to ensure the accuracy of the stresses applied (see e.g. Jovicic et al., 2006).

67 Within the rail track industry measurements of the ballast stiffness are generally not made
68 directly but inferred from the in-situ measurements of the overall track stiffness, including the
69 rail, sleeper and subgrade. This may be by shear wave velocity measurements (e.g. Zhang et
70 al., 2020) or by back-calculation from the track displacements under a wheel load, with
71 associated assumptions about the load transmission and deflection profile (e.g. Priest & Powrie,
72 2009). The ballast stiffness is then inferred from the track stiffness assuming a suitable track
73 model and generally only the elastic stiffness is obtained. Apart from the various assumptions

74 necessary, the problem with such approaches is that the stiffness is measured under a stress
75 regime that is not completely known and which is highly anisotropic and variable within the
76 ballast. It is also measured only for the void ratio achieved in-situ by the placement method. In
77 triaxial testing the ballast fabric is unlikely to be exactly the same as that in-situ, but the
78 stiffness can be investigated under controlled stresses, strains and void ratios, giving the
79 degradation of stiffness from the very small strains to failure, not just the elastic value.

80 **Tested material**

81 The tests were carried out on a typical granite (granodiorite) ballast used in the UK railways,
82 from the Mountsorrel quarry in Leicestershire with a mineralogy mainly consisting of quartz,
83 potassium feldspar and plagioclase. It has a specific gravity of 2.68 (Scott and Rollinson 2015).
84 The original particle size distribution of this material as it is used as a ballast is shown in Figure
85 1 with a median particle size, D_{50} of 41mm. The limited size of the sample diameter of 150mm
86 used in this study meant that a parallel scaled grading of the same material had to be tested
87 with a D_{50} of 21mm. This kind of parallel grading in testing rockfills and ballast is quite a
88 common procedure (e.g. Cambio and Ge, 2007; Le Pen et al., 2013), to overcome the limitation
89 of triaxial cell size and its compatibility with the size of particles of the tested material.
90 Commonly a 1/3 scaling ratio is used for ballast (Aingaran et al., 2018) but a ratio of about 1/2
91 was preferred here, maximising the particle sizes that could reasonably be tested given the sample
92 dimensions. The literature is not clear about the effects of using a scaled ballast or rock fill
93 compared to the original grading, with often contradictory effects for different materials (e.g.
94 Varadarajan et al., 2003). Le Pen et al. (2013) emphasised the possible effect of differences in
95 shape between particles of different sizes, while McDowell & Li (2016) identified that
96 differences of particle strength could be responsible for a smaller influence of confining
97 pressure on the peak angle of shearing resistance for the scaled ballast than for the full sized
98 one. There may also be effects of sample size on the test data, given the relatively small ratio

99 of sample diameter to particle size, although Hu et al. (2011) have found that these are less
100 important in the pre-peak regime.

101

102 **Apparatuses and test procedures**

103 *Triaxial apparatus*

104 A hydraulic triaxial cell with computer-controlled pressure systems was used, which
105 accommodates a sample with 150mm diameter and 225mm in height. The limitation in sample
106 height gave a ratio of height/diameter less than two but was used because of the adoption of
107 lubricated ends. The top and lower platens were therefore made slightly larger than the sample
108 diameter (170mm) to allow sample lateral deformation freely at both ends over the lubricated
109 end platens (e.g. Goto and Tatsuoka, 1988; Ueng et al., 1988). Lubrication was achieved by
110 using two layers of greased membrane at each end, making radial cuts in the rubber discs to
111 reduce their resistance to radial strain. The membrane used in these tests both on the sample
112 and for the lubricated ends was 1.5mm thick and they were manufactured from latex, although
113 the sample membrane stiffness had a negligible radial restraint effect at the stress and strain
114 levels used in these tests.

115 External displacement measurements were made by using a spring guided LVDT, while both
116 cell pressure and back pressure, when applied, were monitored by pressure transducers with a
117 maximum capacity of 10 bar. The axial displacements for shearing were applied using a
118 constant rate of displacement system acting through the ram chamber at the lower part of the
119 hydraulic cell (Figure 2a). A submersible load cell with a capacity of 25kN was used to measure
120 the applied load through a half-ball system between the sample top platen and the load cell to
121 accommodate any misalignment of the sample and load cell.

122 The sample was prepared in three layers within a split mould, which were subjected to vibration
123 for 20 minutes each using an eccentric load vibrator while applying a deadweight surcharge
124 equivalent to 5kPa on top to obtain a relatively dense sample with a void ratio of 0.7 ± 0.01 .
125 This method was preferred over a direct compaction of the material to protect the contacts
126 between particles from breakage that might occur under direct compaction loads. The sample
127 was subjected to an effective stress of around 17kPa using suction through a Venturi system
128 while removing the mould and attaching the local strain instrumentation. During the first stage
129 of each test this suction was reduced gradually while increasing the cell pressure to minimise
130 any significant strains in the sample. Some samples were subsequently flooded while some
131 were tested dry. Two of the flooded samples were then saturated further under back pressure.
132 The strains for the saturated/wet samples were monitored and found to be minimal during
133 sample flooding and saturation stages.

134 Local strain measurements were made using four submersible RDP LVDTs with unguided
135 armatures and a total stroke of $\pm 12.5\text{mm}$ (model MD5/500WRA). Two were used for axial
136 strain measurements, two for radial strain measurements. The internal axial displacement
137 LVDTs adopted a scaled-up system similar to that of Cuccovillo & Coop (1997) but were
138 attached to the sample using relatively large mounts (65mm x 28mm) to span several particles,
139 based on the selected tested particle size (Figure 2a).

140 The radial strains were measured using the Imperial College system of Ackerley et al. (2016).
141 This consists of an LVDT holder and a rocker arm, which translates the radial displacement of
142 the sample into a vertical one. These are mounted on the base platen (see Figure 2b), so that
143 the transducers move with the platen. In the original system the rocker arm rests against the
144 sample membrane, but because of the irregular sample shape targets were made, again with a
145 relatively large size (37mm x 62mm) that were glued to the sample membrane using a flexible
146 silicon sealant.

147 Four amplifiers (RDP S7AC) were used, one for each local LVDT, and following the
148 techniques developed by Cuccovillo & Coop (1997) immediately prior to each small strain
149 shearing probe the potentiometers within the amplifiers were adjusted to set the output voltages
150 of each LVDT to zero, thereby optimising their resolution. Minimum noise in the LVDT
151 signals was ensured by shielding the cables and earthing the shields. A Datascan 7220 was used
152 to log all the transducers, this being a datalogger that is particularly stable.

153 The testing procedure for the dry tests started by applying cell pressure with the pressure
154 controller before carrying out shearing probes, typically over about 45 mins. Prior to the probes
155 it was ensured that the rate of creep strains was less than 1% of the applied shearing rate for
156 the probe. When multiple shearing probes were planned on the same sample, for all shearing
157 events before the final one, care was taken to reach an axial strain that was just sufficient to
158 show the stiffness degradation curve while minimising the loading history effect on any
159 subsequent probe (Table 1). For Test T2 relatively large strains of 0.84-1% were applied, but
160 these were reduced to of the order of 0.05-0.2% for most of the tests. Nevertheless, as will be
161 discussed later, even these small strains would be too large to allow multiple probes on one
162 triaxial sample. Some tests were also sheared to medium strains to observe the agreement of
163 the two local strain measurements beyond the small strain region. For the wet or saturated tests,
164 the sample was flushed with water at a cell pressure of 50kPa before increasing the cell pressure
165 to the desired value for the test. The sample pore pressure was left open to the atmosphere in
166 all tests except tests T9 and T9R in which the sample was saturated by increasing the cell and
167 the back pressures with the same rate to 150 and 100kPa respectively. The isotropic
168 compression stage followed without checking the B value as this procedure will not be
169 meaningful for a very large membrane penetration (Knodel et al., 1992). Details of the tests are
170 listed in Table 1. The test ID is followed by the confining pressure and condition of the test,

171 such that T2-50Dry means that the data are from test T2 at a confining pressure of 50kPa in a
172 dry condition.

173

174 *Inter-particle apparatus*

175 The Inter-Particle Loading Apparatus loads two particles at their contact and is described in
176 detail by Wong & Coop (2020). It consists of 3-axis control, in the vertical and two orthogonal
177 horizontal directions (Figure 3a). Linear actuators, load cells and displacement transducers on
178 the three loading arms control and measure the forces and displacements applied in each
179 direction. The actuators could control either the force or the displacement in any of the three
180 directions by the means of the stepper motor controllers. The loads were measured by high
181 accuracy stud load cells, which have a capacity of 1000N in the vertical direction and 500N in
182 the horizontal direction. Very high-resolution non-contact displacement transducers were used
183 for the displacements (Micro-Epsilon Model CSE2). These are of a capacitive type and are
184 highly stable. A similar datalogger to that used in the triaxial apparatus, a Datascan 7250, was
185 again employed because of its low noise and high signal stability. The apparatus stability and
186 compliance were all checked by Wong and Coop (2020) who showed that the apparatus has
187 very low flexibility under load while the bearing systems had very low friction.

188 The tested ballast particles were cut with a smooth flat surface that was attached to the top and
189 lower platens with a high stiffness epoxy resin, using a minimal thickness of the glue to
190 minimise compliance. Two tests were conducted using the IP apparatus to investigate the effect
191 of water on both the normal and tangential loading stiffness (Figure 3c). The particles were
192 prepared so that a nominal point was contacting a nominal flat. Tests details are given in Table
193 2. Test GB-N was carried out to investigate the effect of water on normal loading by flooding

194 the sample during normal loading, while Test GB-CS was done by applying small cyclic
195 tangential displacements with an amplitude of 50 microns to the contact which was normally
196 loaded to 100N. The first four cycles were done on a dry contact then the contact was flooded
197 and another six cycles on the flooded contact followed.

198 **The measurement of small strain stiffness**

199 The axial strain data for Test T2 are shown in Figure 4a. This was conducted in dry conditions
200 and showed an excellent agreement of the two local measurements, which was typical of most
201 tests. The figure also shows that vertical strains measured by the external LVDT are much
202 higher than the locally measured values and cannot be used for small strain stiffness
203 measurements any more than they can for soils of smaller particle size (Jardine et al., 1985). In
204 Test T2, three probes at three different confining pressures were carried out at 50 kPa, 100 kPa
205 and 150kPa respectively. The accuracy of local strain measurements at very low strains meant
206 that a calculation of modulus of elasticity down to mean internal strains of around 0.0005% is
207 possible. This is not as low as was achieved by Cuccovillo & Coop (1997) for finer grained
208 soils but is adequate for most purposes.

209 The tangent moduli of elasticity, E , measured during the three probes in Test T2 are shown in
210 Figure 4b. These are calculated as regressions to the stress-strain curves, taken over long
211 enough intervals to reduce noise in the stiffnesses but not so long that the shape of the decay
212 curve is altered, again techniques used by Cuccovillo & Coop (1997). Typical regression
213 intervals were about 11-21 data points at the start of shearing below about 0.01% strain where
214 there are fewer data, rising to around 51 points later in the test, the total number of data points
215 being around 1000. The stiffnesses for the externally measured strains are clearly very much
216 lower than those measured internally (Figure 2b). There is a clear change in behaviour from

217 the first probe to all other consecutive probes. The E value during the first probe degraded with
218 the progress of strain continuously, while the second and third shearing showed a persistent
219 stable and higher value at the start of shearing followed by a distinct knee or a gross yield point
220 before a much faster degradation. However, the volumetric strains calculated from the local
221 displacement measurements showed only very low volumetric strains of about 0.2-0.3% for
222 each 50kPa increase in confining pressure, so that it is likely there is only a minimal
223 rearrangement of particles and few new particle contacts are probably produced. This low
224 volumetric strain has been insufficient to erase the influence of the previous shearing and the
225 probes at 100kPa and 150kPa essentially give reloading stiffnesses, in all likelihood re-loading
226 the same contact chains through the sample established in the first shear probe. This means that
227 unlike finer grained clays and even sands (e.g. Jovicic & Coop, 1997), multiple shearing probes
228 on the same sample cannot be done on the same sample and a new sample is needed to
229 investigate the behaviour at each confining pressure. In this case the probes were terminated at
230 low axial strains (Table 1), but even had they been stopped at the minimum strain needed to
231 define the decay curve, the strains during isotropic compression would still be insufficient to
232 erase the influence of one probe on the next, which from experience on other soils would need
233 to be much larger than the strains applied in the shear probes (see Jovicic & Coop, 1997).

234 The effect on sample void ratio of the strains applied during the probes was very small. After
235 each shearing probe, sample relaxation was allowed overnight after unloading before carrying
236 out the next shearing probe. Figure 5b shows the volumetric strain during the first shearing of
237 Test T2 at 50kPa confining pressure. After the sample relaxation stage the void ratio had only
238 changed from 0.690 to 0.691. In this probe, higher axial strains were applied during the virgin
239 shearing stage than for other tests and for Test T6 the volumetric strains are very small indeed
240 (Fig.5c). It is therefore not the volume change during the probes that influences the stiffness in
241 the next probe.

242 The influence of one probe on subsequent probes is also consistent with the contact mechanics
243 measured for this ballast by Altuhafi et al. (2023) and Wong & Coop (2023) where both the
244 normal and lateral contact stiffnesses increased between first contact and the second loading
245 but for subsequent loading cycles remained almost constant. This explanation is not
246 incompatible with the effects of loading history on the force chains within the sample discussed
247 above as the stiffening of the contacts that will have been deformed can explain why large
248 volumetric strains are required between probes to disrupt those chains.

249 **The influence of confining pressure**

250 Figure 5a shows the degradation of tangent E with the mean internal strain during the first
251 shearing of dry samples at various stress levels, calculated and plotted using the mean of the
252 internal local axial strain readings. Figure 5d shows the tangent shear moduli, G for the same
253 probes. The shear strains and hence shear stiffnesses can only be calculated from the local axial
254 and radial strains as $\epsilon_s = 2/3 (\epsilon_a - \epsilon_r)$ since there is no externally measured volumetric strain
255 unless a technique such a double wall cell is used (Bishop & Donald, 1961). In all these
256 shearing probes yielding must have taken place at a very early stage at strains that were lower
257 even than what was possible to measure using the high accuracy local strain LVDTs. In some
258 tests, such as in the data of Test T6-100Dry in Figure 5a, some unexpected drops or jumps can
259 be seen, which might be due to discrete effects such as individual particle movements, but
260 generally the data are similar in nature and stability to those seen for finer grained soils. Since
261 the shear modulus calculation in Figure 5d requires radial strains as well as axial strains to be
262 considered, the stiffness can only be defined at slightly higher strains compared to E, typically
263 around 0.001%.

264 Figure 6 shows the influence of the mean effective stress p' on the stiffness at various strain
265 levels for all of the probes carried out. In first loading (Figure 6a) the exponent $n=0.60$ at the
266 smallest resolved strains is similar to those measured for smaller grained clastic soils (sands),
267 for example by Jovicic & Coop (1997). Membrane puncture at higher stresses restricted the
268 stress range to a maximum of 150kPa. For most soils the contours for increasing strain levels
269 define a series of lines, which gently converge at higher stresses, (Jovicic & Coop, 1997;
270 Viggiani & Atkinson, 1995), but the stress range that could be applied here was too small to
271 define this accurately, although the convergence is not very apparent. The patterns of behaviour
272 seen in second loading are rather different (Figure 6b) and are discussed below.

273 **Effect of water on behaviour**

274 The effect of water on the behaviour was investigated by carrying out some tests in which the
275 sample was flooded with water after an initial dry isotropic compression stage to 50kPa. Water
276 flooding was done by allowing the water to enter the sample through the lower drainage port
277 with a small pressure to ensure minimal disturbance of the sample, while the top drainage was
278 open to atmosphere. The sample was kept open to atmosphere after flooding and during further
279 isotropic compression and shearing in these tests, while in the two saturated Tests T9 and T9R
280 a saturation stage was applied after flooding and a back pressure applied using 100kPa back
281 pressure control system.

282 Figure 7 shows three shearing probes carried out at an effective confining pressure of 150kPa
283 for three different sample conditions, dry, wet and saturated. In the range of data which can be
284 measured in this study no effect of water on E can be seen. Wong & Coop (2020) had found
285 that there is no significant effect of water on the inter-particle friction coefficient of this
286 material when tested using the inter-particle loading apparatus. This stimulated the need to

287 carry out some inter-particle tests on the effect of water on the particle contact at smaller
288 displacements, particularly the stiffness during both normal loading and tangential shearing. In
289 both Tests GB-N and GB-CS the sample was flooded half-way through the test. This is the
290 only way to see the effect of water on inter-particle stiffness, because the differences between
291 similar tests on different particle pairs are generally too large to see any potential effect of
292 water. The same pair of particles could not be tested twice, once dry and once wet, because the
293 second test would necessarily be a reloading test which has a significant effect on the stiffness
294 (Altuhafi et al., 2023).

295 Figure 8a shows normal load against normal displacement during test GB-N. Flooding of
296 sample took place at 50N normal load. No clear effect of flooding is seen on normal stiffness
297 except the sudden temporary drop in stiffness value which coincided with the flooding process
298 in Figure 8b, after which the stiffness value stabilizes to more or less the same value before
299 flooding. The effect of water on the lateral stiffness of the contact is shown in Figure 9. Figure
300 9a shows the lateral displacement against the lateral load recorded during four lateral loading
301 cycles in dry condition and six cycles in wet condition. The loop size reduces with cycle
302 number and that continues after flooding, but no significant change of stiffness between dry
303 and wet cycles can be seen. The stiffness of each cycle is calculated from the gradient of the
304 secant passing through the two ends of each cycle (e.g. Richards et al., 2020). Figure 9b shows
305 the change in lateral stiffness values with cycle number. The main change in stiffness occurs
306 in the first few cycles before the flooding takes place, and there is no clear effect of flooding.
307 The inter-particle tests therefore confirm at the particle scale what is observed at the sample
308 size, that the presence of water has no effect on stiffness for this ballast.

309 **Effect of cyclic shearing on small strain stiffness**

310 Figure 10 shows the degradation curves for E obtained from four probes at 150kPa effective
311 confining pressure, some under different conditions. Tests T3-150Dry and T7-150Dry are two
312 virgin shearing probes on dry samples. The data shows the reasonably good repeatability of
313 tests results between different samples. Probe T9-150Sat was carried out on virgin saturated
314 sample with a back pressure of 100kPa, also shows strikingly similar results confirming again
315 the repeatability and the lack of effect of water on the behaviour of this material. Probe T2-
316 150Dry was a second shearing probe at the same pressure on a dry sample that had previously
317 been sheared at 50kPa and 100kPa. It showed a distinctly different behaviour represented by
318 the late yielding and the clear knee of the curve prior to the yielding point as was also seen in
319 Figure 4. This is again seen in Figure 11 which shows four consecutive probes on dry sample
320 T6 at an effective confining pressure of 100kPa. The difference between the second, third and
321 the fourth probes in this test was not significant, showing that most changes in behaviour take
322 place between the first and the second shearing probes. Figure 11b presents the volumetric
323 strains experienced by the sample during these four probes. In general, the volumetric strains
324 are very low and their effect on the void ratio is insignificant. It is also noted that the volumetric
325 strains during the virgin loading event are higher than those experienced during the subsequent
326 loading events (Figure 11b), which each give similar strains.

327 To investigate the effect of cyclic loading after large numbers of loading cycles, 49 cycles of
328 loading were applied after the three initial consecutive shear probes on dry sample in Test T8,
329 followed by a final probe at Cycle 53. The period of the 49 cycles was two hours, but small
330 strain data were not monitored during these cycles. The effective confining pressure in this test
331 was 150kPa and the axial load was alternated between 100N and 2100N using a control of the
332 axial pressure via the axial hydraulic chamber rather than axial displacement to ensure that
333 each cycle had the same magnitude in terms of axial stress. A minimum load of 100N was
334 specified to avoid possible loss of contact between the sample and the load cell had a minimum

335 of zero been used. Figure 12 shows the modulus of elasticity E in the first three shearing probes
336 and also the final probe at cycle No. 53. The stiffnesses obtained after the cyclic loading stage
337 were not very different to those obtained in the second and third shearing probes which again
338 supports the conclusion that the main change in stiffness takes place between the first and
339 second loading. There is perhaps a more pronounced gross yield point or knee in the data for
340 Cycle 53. It is interesting that while the triaxial stiffnesses are essentially stable after the first
341 loading cycle, it takes several cycles for the inter-particle shear stiffnesses to stabilise (Figure
342 9b).

343 In Figure 6b, which summarise the second loading stiffnesses for all tests and compares them
344 with the contours defined by first loading, the stiffnesses do not change much until after 0.03%
345 axial strain, when they drop very rapidly, the inconsistency of the data for 0.1% arising from
346 the fact they are taken from very steep decay curves.

347 **Volumetric strain measurements**

348 The local strain measurements using the LVDTs showed good consistency in the range of small
349 strains (Figure 4). To examine their consistency at moderate to high strains, loading was
350 continued to more than 10% axial strain in Test T9 and repeated in Test T9R. Significant
351 barrelling of the sample was however noted when higher strains were reached in spite of the
352 lubrication of the end platens, perhaps because of the coarse nature of the ballast and hence
353 high contact forces at local points with the platens, as could be seen from indentation of the
354 lubrication rubber after the test. In these two saturated tests the main purpose was to compare
355 the volumetric strains of the sample as calculated from local measurement with the volumetric
356 strains measured by the means of an external Imperial College type volume gauge which
357 monitored the amount of water entering or leaving the sample. Saturation of the sample was

358 required in this test, but a B value was not calculated after the saturation stage as the effect of
359 membrane penetration in such a material makes it meaningless. Instead, the saturation stage
360 under 100kPa back pressure was continued until there was no further negative change in
361 volume arising from air entering solution.

362 Figure 13 shows the radial and axial strains measured during Test T9R. The radial strains can
363 only be calculated as one measurement despite using two LVDTs, unlike the local axial strains
364 where the two LVDTs give two independent measurements. The two radial LVDT
365 measurements cannot be regarded as being independent as both are needed to obtain one
366 measurement of the change of sample diameter. At these larger strains than in Figure 4 there is
367 better agreement of the external axial strain with the internal local measurements.

368 Figure 14 shows the volumetric strains measured using the external volume gauge compared
369 to those values calculated from local measurements ($\epsilon_v = \epsilon_a + 2\epsilon_r$). Although there is very
370 significant membrane penetration for such a coarse material, at small strains the volumetric
371 amount of that penetration is unlikely to change significantly under a constant effective radial
372 stress, so it is doubtful that this is a significant factor in the divergence of the externally and
373 internally measured volumetric strains that starts to emerge even after about 1% axial strain
374 and barrelling is likely to be the sole cause. The agreement between the internal and external
375 volumetric strains up to 1% is however very good. More accurate volume change measurements
376 at larger strains would require either a more effective lubricated end platen, which seems
377 doubtful, or measurement of the radial strain at multiple points on the sample profile.

378 **Conclusions**

379 Using modified designs of local strain LVDT instrumentation, measurements of tangent
380 Young's modulus and shear modulus have been made at small strains for ballast samples down

381 to strains of around 0.0005%. The ballast tested in this study showed distinctly different
382 behaviour between first shearing events for a sample and consecutive shearing events, which
383 had a later yielding point and higher values of E and G than were measured in the virgin, initial
384 shearing event. The distinct increase of stiffness on second loading, together with only very
385 small strains during isotropic loading meant that each sample could only be used to measure
386 one virgin loading stiffness degradation curve. Water did not seem to be a significant factor in
387 the stiffness behaviour of the tested ballast in the triaxial tests and the contact mechanics tests
388 revealed that this is because the presence of water does not affect the micro-scale contact
389 stiffnesses in either normal or lateral loading. Despite the use of lubricated end platens, the
390 samples tended to barrel at large strains, so that the volumetric strains measured by the local
391 LVDTs were only similar to the externally made measurements at axial strains less than about
392 1%.

393 For the shear moduli, the involvement of radial strain in the calculation added an additional
394 source of error which resulted in more scatter in the data. This is in part because the radial
395 strain is calculated from the summation of two transducer readings while the axial strain is the
396 mean of two values, but it could also be due to greater sensitivity of the radial strain
397 measurement to local particle movements, which might be improved by adopting a larger
398 sample to particle size ratio or adding additional locations of radial strain measurement.

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492

493 **Notation**

- 494 D_{50} Particle size diameter corresponding to 50% passing of the total grading.
- 495 E Modulus of elasticity of the material.
- 496 G Shear stiffness modulus.
- 497 N Normal inter-particle force.
- 498 p' Mean effective stress.
- 499 q Deviator stress.

500	T	Tangential inter-particle force
501	ϵ_a	Axial strain
502	ϵ_r	Radial strain.
503	ϵ_s	Shear strain.
504	ϵ_v	Volumetric strain.

Table 1. The triaxial tests

Test ID	Condition	Effective stress for the shear probes (kPa)	Axial strains reached	Notes
T2	Dry	50, 100, 150	0.84%, 0.99%, 1.97%	
T3	Dry /wet	150_Dry, 150_Wet	1.2%, 1.6%	1 st probe on dry sample then sample flooded and 2 nd probe after stabilising stage
T4	Wet	150	1.8%	Isotropic compression to 150kPa dry then flooded and left to stabilise before probe
T5	Dry	100	1.27%	
T6	Dry	100	0.19%, 0.14%, 0.14%, 0.12%	Four probes at 100kPa
T7	Dry	150	0.17%	
T8	Dry	150	0.1%, 0.05%, 0.05%	3 probes, to reach a maximum load of 2088N then cyclic loading for 49 cycles, then probe 53
T9, T9R	Saturated/ back pressure 100kPa	150	Taken to higher axial strains	Tests for volumetric strain comparison

Table 2. The tests using the Inter-Particle Loading Apparatus

Test ID	Normal Load (N)	Shear displacement amplitude (μm)	Details
GB-N	100	----	Contact was vertically loaded in dry condition until 50N then contact was flooded with water and loading was continued till 100N
GB-CS	100	50	Contact was vertically loaded to 100N normal load. Cyclic loading for 4 cycles in dry condition followed by 6 cycles after flooding

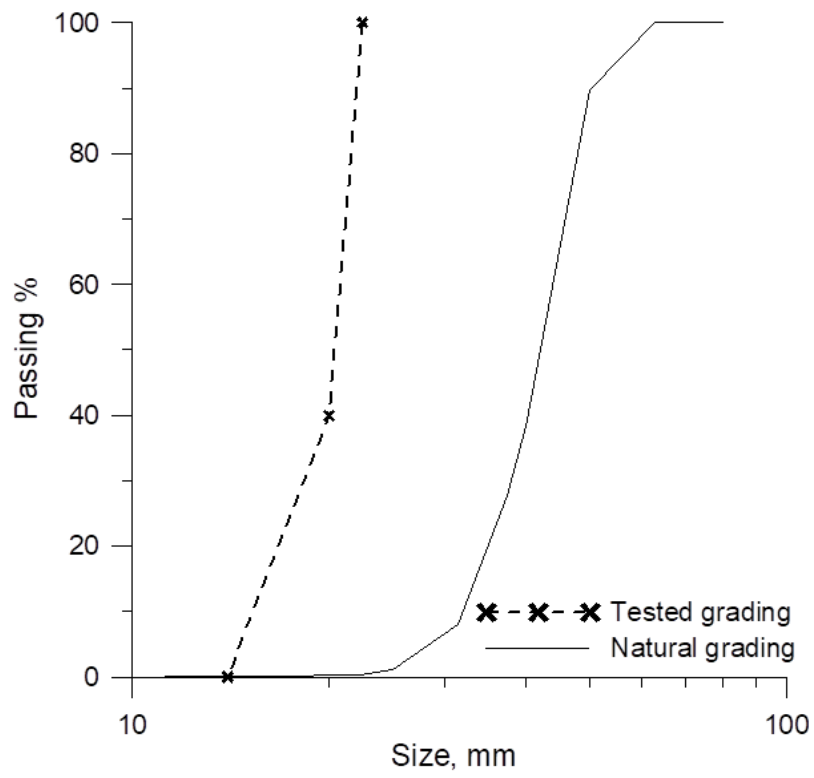
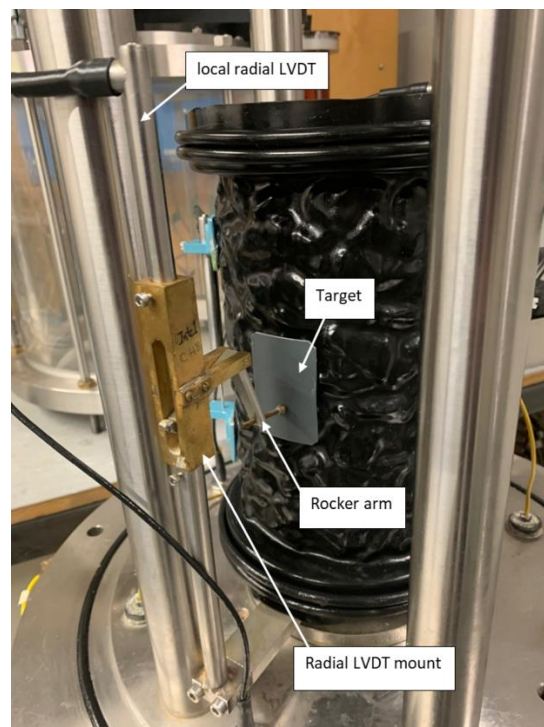


Figure 1. Gradings of the “natural” ballast and the selected scaled grading tested in this study.

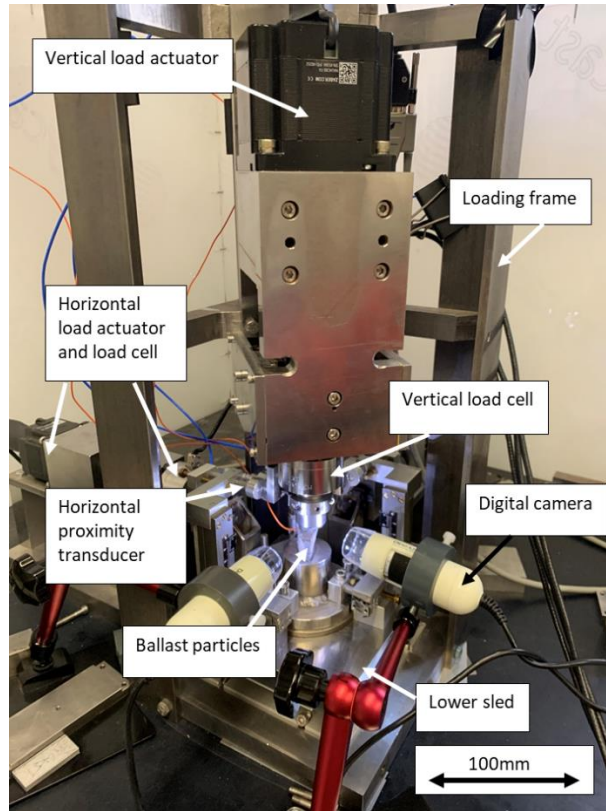


(a)

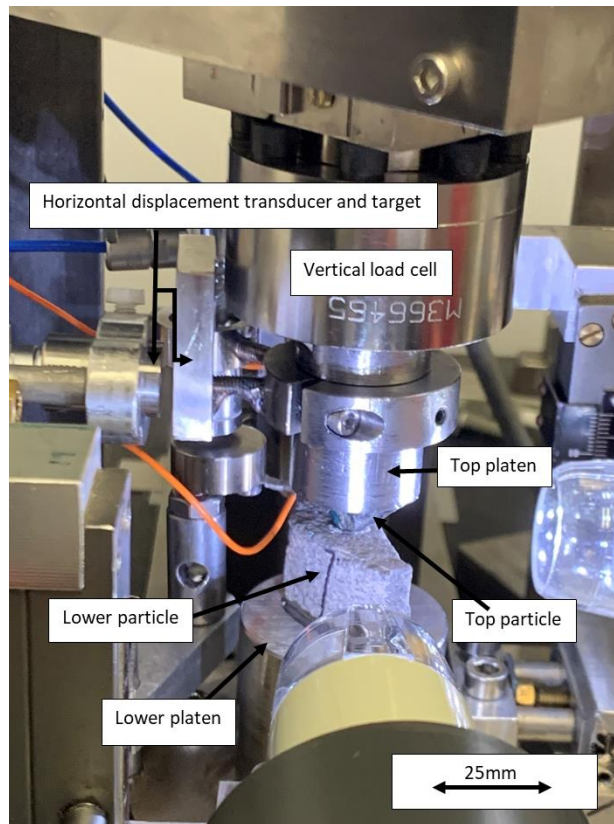


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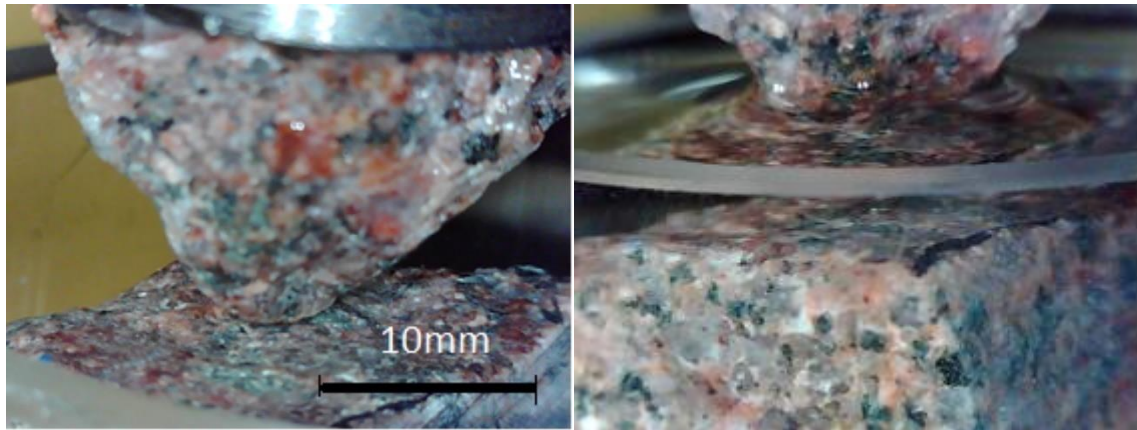
Figure 2. The triaxial apparatus, (a) local axial displacement measurement transducers, (b) local radial displacement measurement arrangement.



(a)

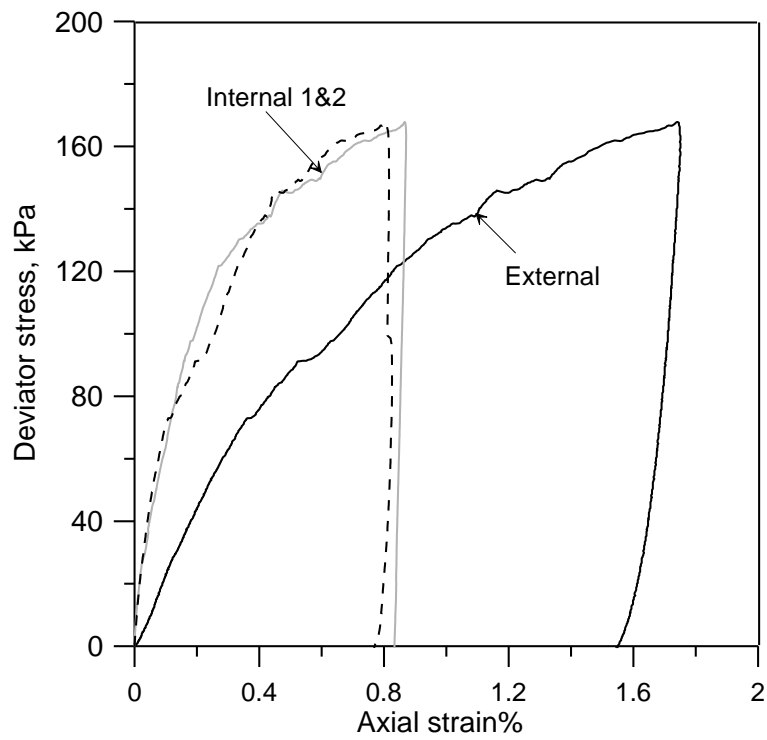


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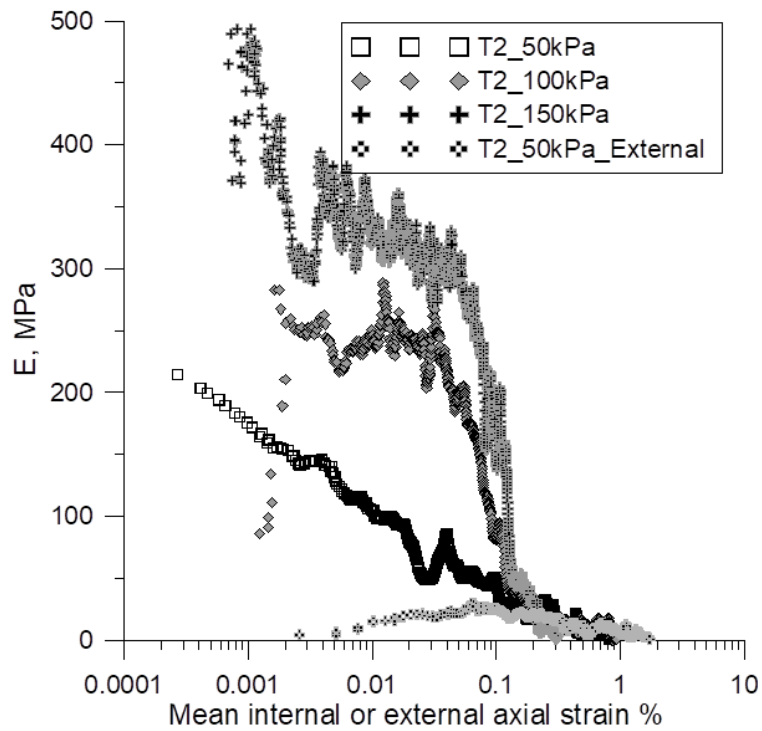


(c)

Figure 3. The Inter-Particle Loading Apparatus, (a) overall design, (b) details of the apparatus, (c) tested particles during dry and wet loading stages.

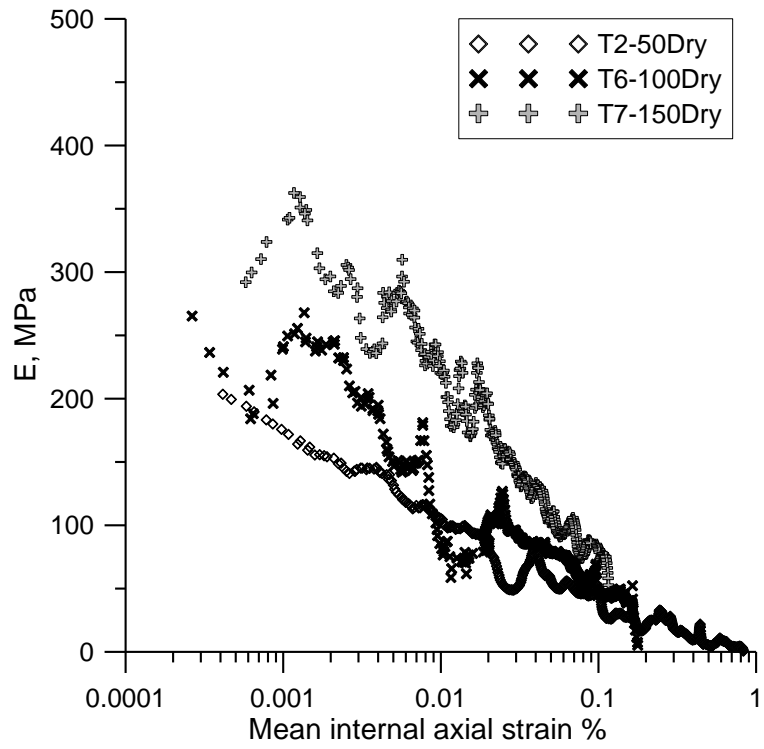


(a)

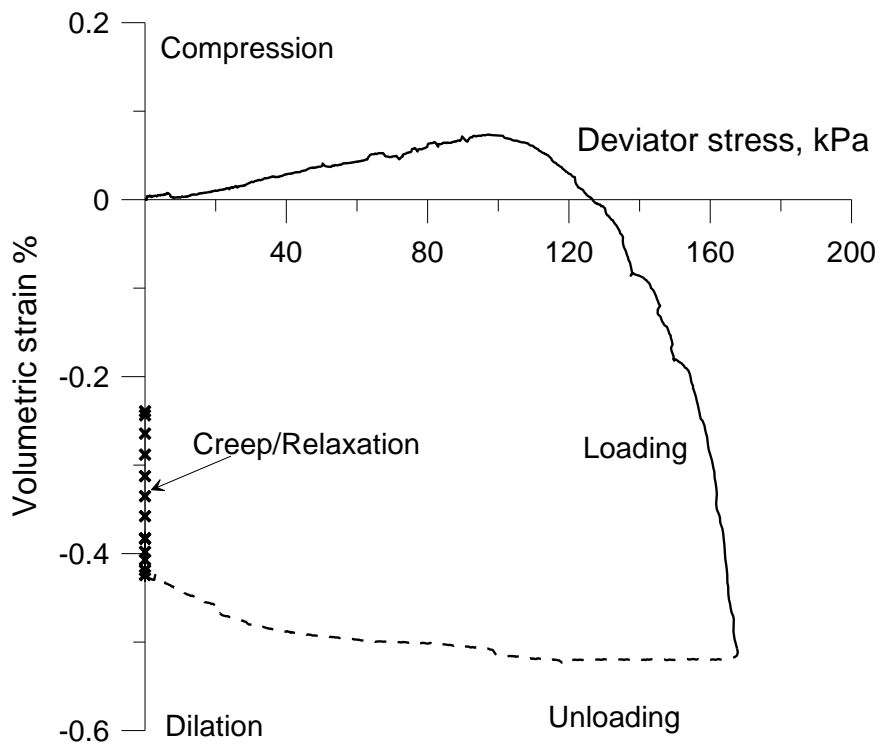


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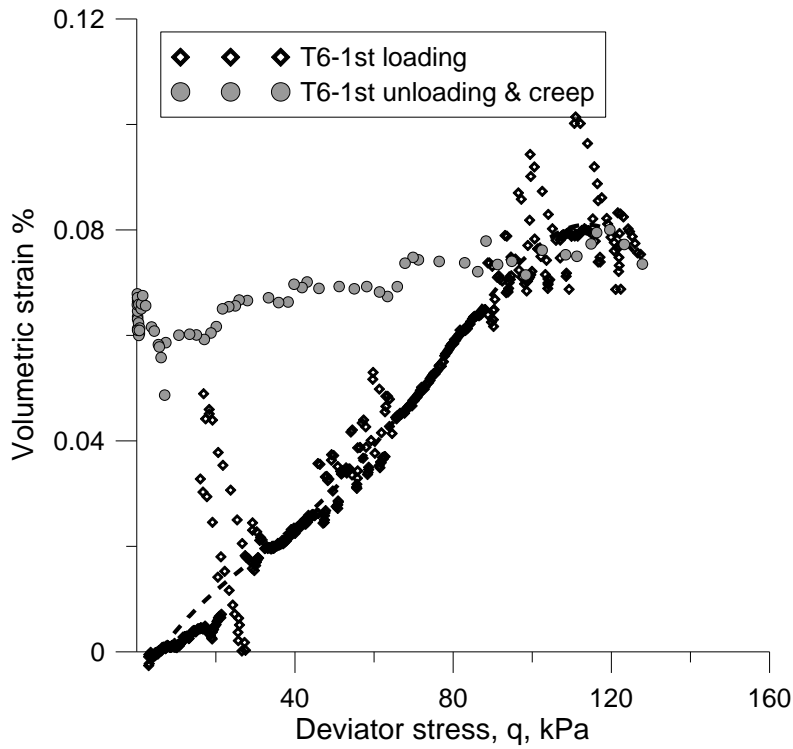
Figure 4. Data from dry Test T2 (a) comparison between external and internal LVDT readings at 50kPa, (b) tangent Young's moduli for probes at 50, 100 and 150kPa.



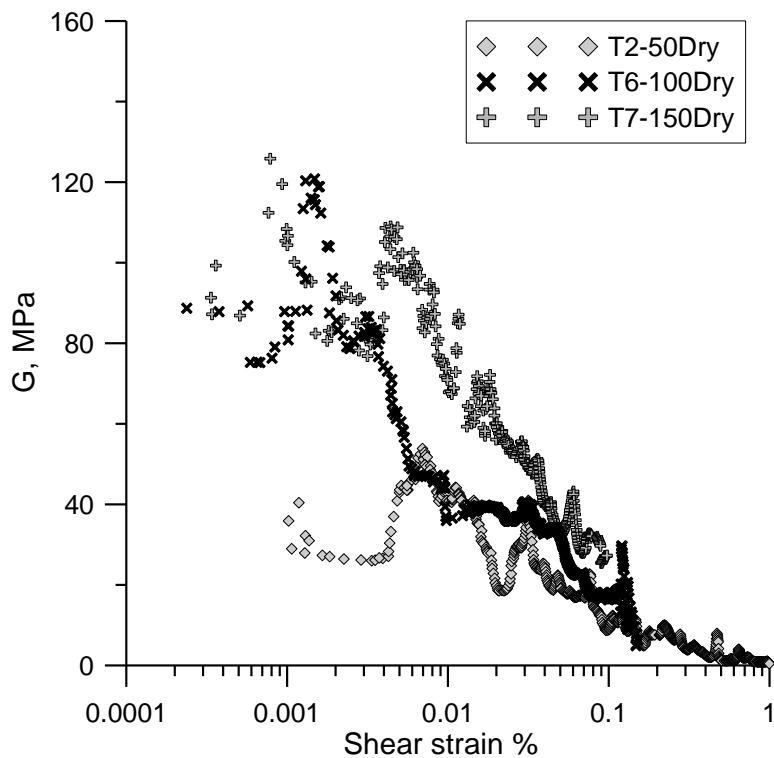
(a)



(b)

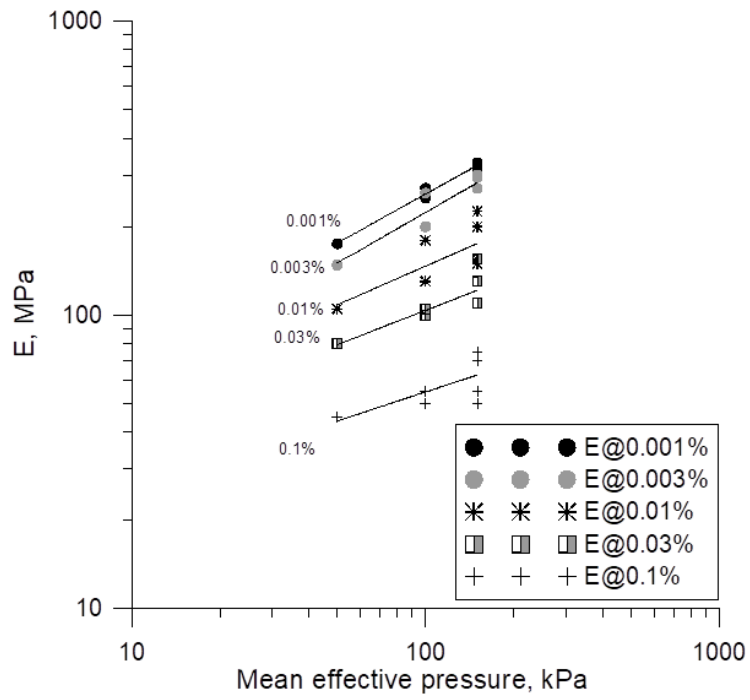


(c)

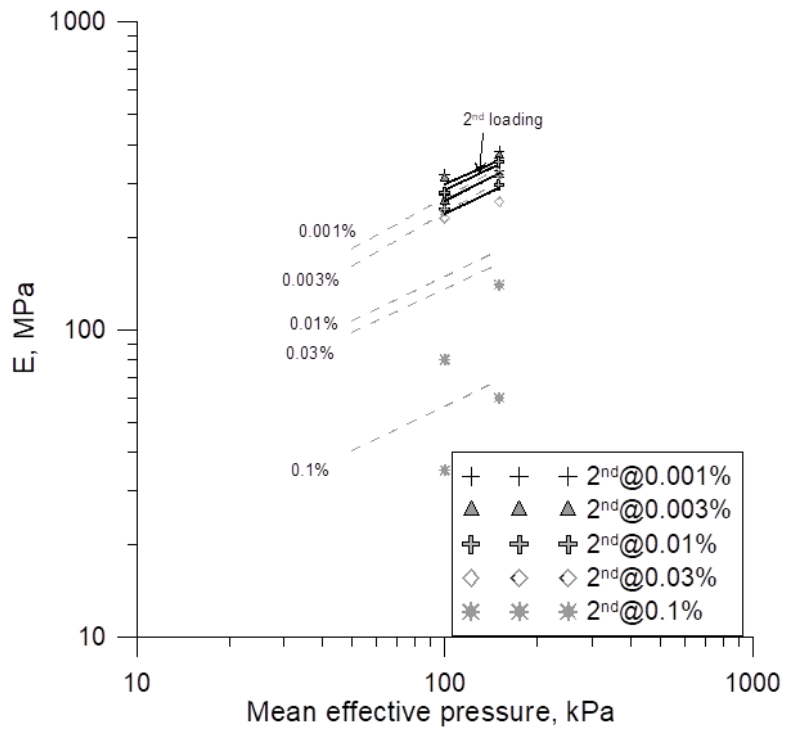


(d)

Figure 5. Data from virgin loading stage of Tests: T2, T6 and T7 during shearing at three different confining pressures (50, 100 and 150 kPa) in dry conditions (a) Degradation of tangent Young's moduli (b) volumetric change during loading cycle of virgin loading of T2 at 50kPa confining pressure (c) volumetric change during loading cycle of virgin loading of T6 at 100kPa confining pressure (d) tangent shear moduli.



(a)



(b)

Figure 6. The influence of effective confining pressure on Young's modulus (a) first loading tests (b) reloading tests with trends from 1st loading shown.

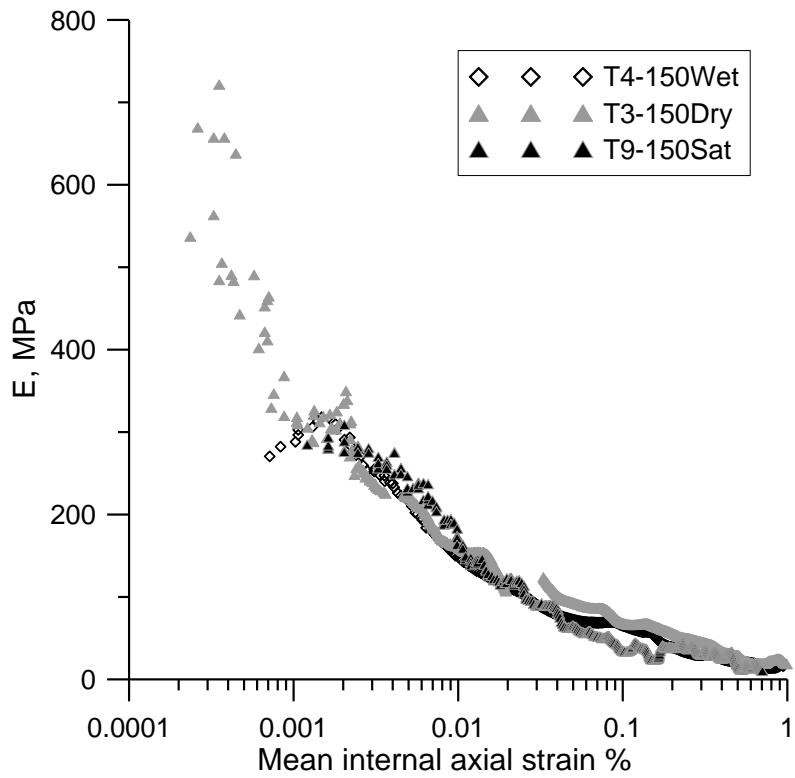
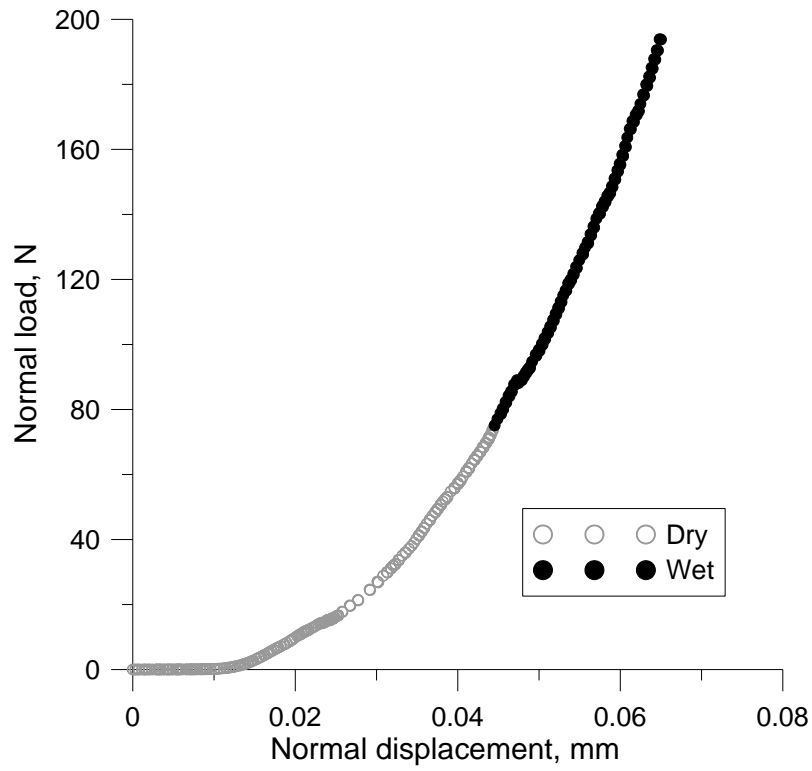
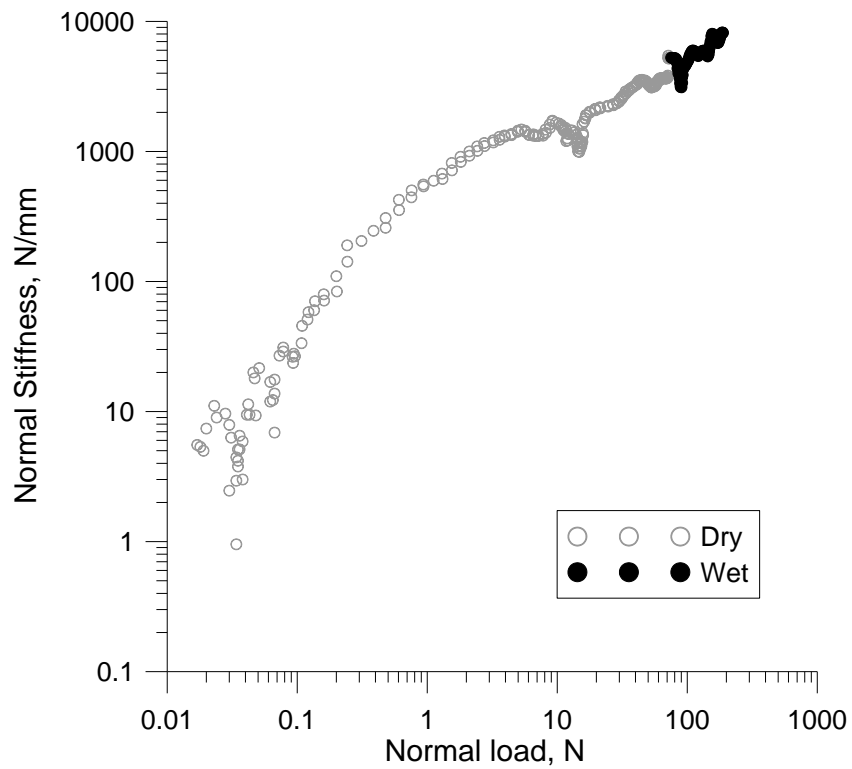


Figure 7. The effect of water on the shearing probe data at 150kPa. Test T4-150Wet on a flooded sample (zero back pressure), T3-150Dry in dry condition and T9-150Sat on saturated sample with 100kPa back pressure.

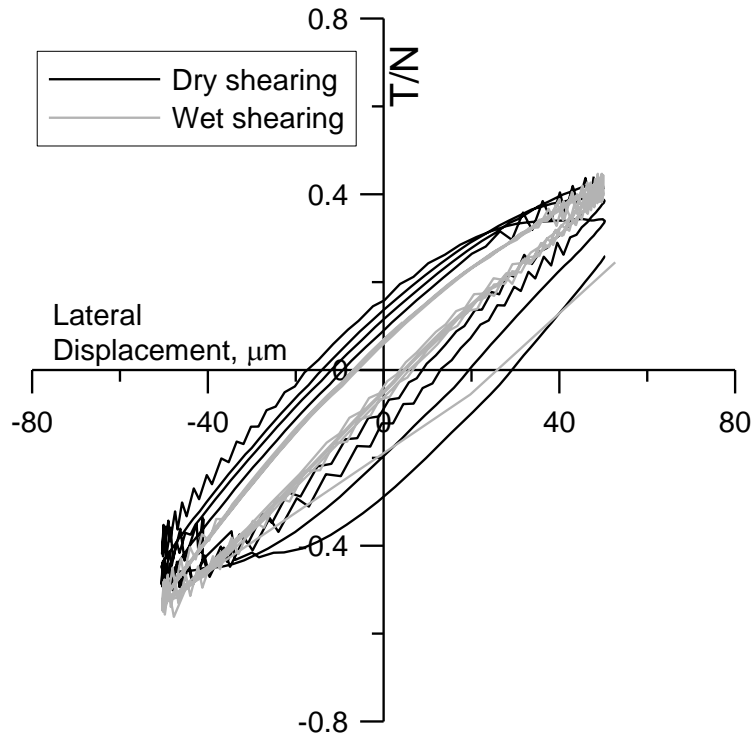


(a)

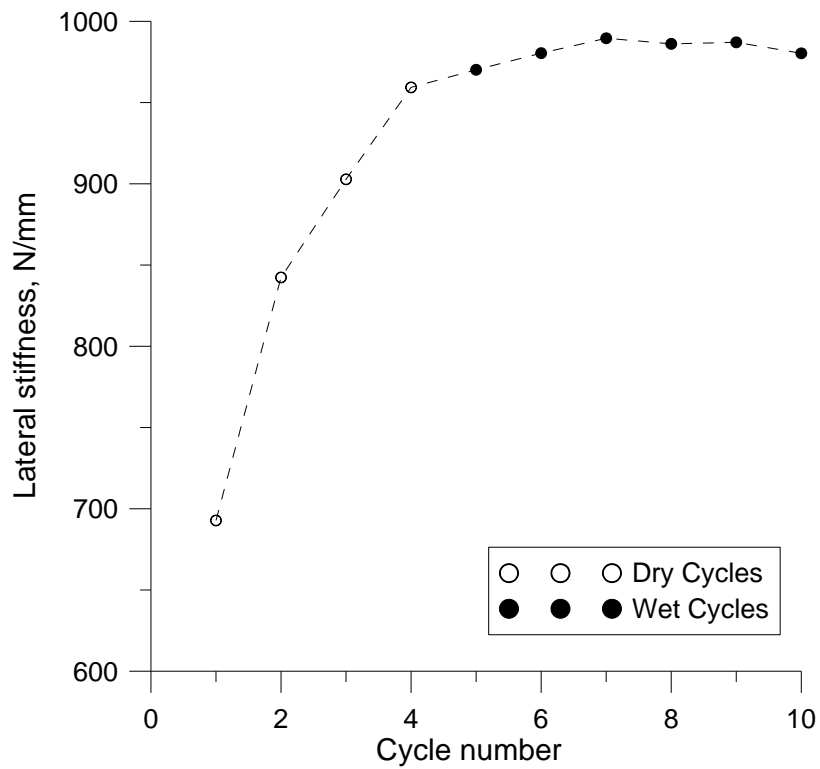


(b)

Figure 8. Effect of water during normal loading at a particle contact (a) force-displacement curve (b) force stiffness relationship.



(a)



(b)

Figure 9. Pre-sliding failure lateral loading cycles under both dry and wet conditions at a particle contact (a) shear to normal force ratio (T/N) change with lateral displacement (b) lateral stiffness change with cycle number.

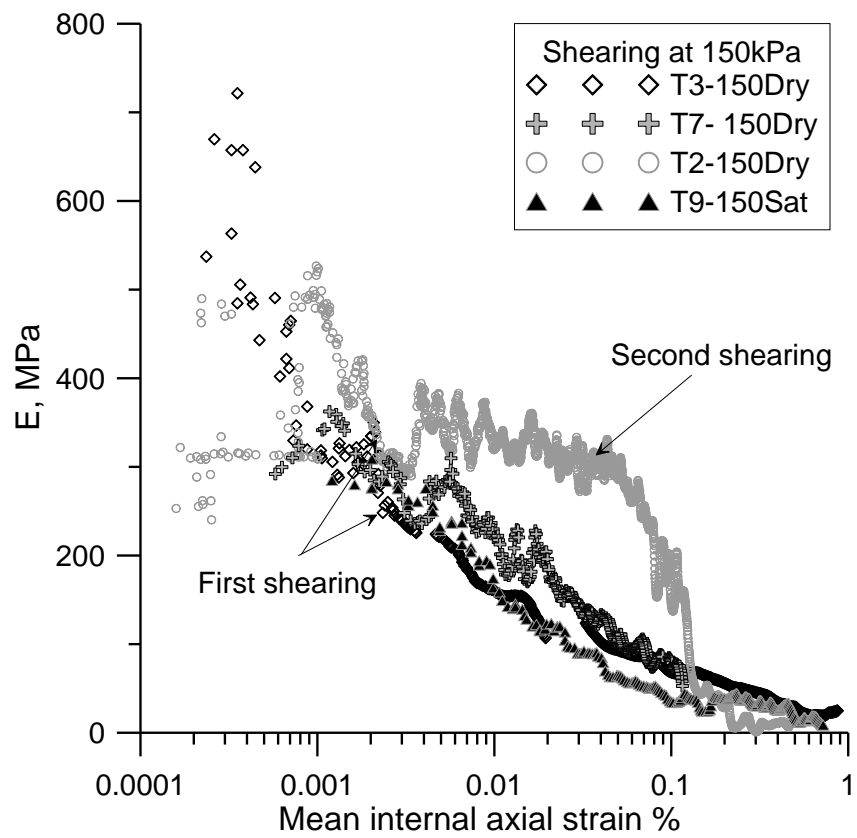
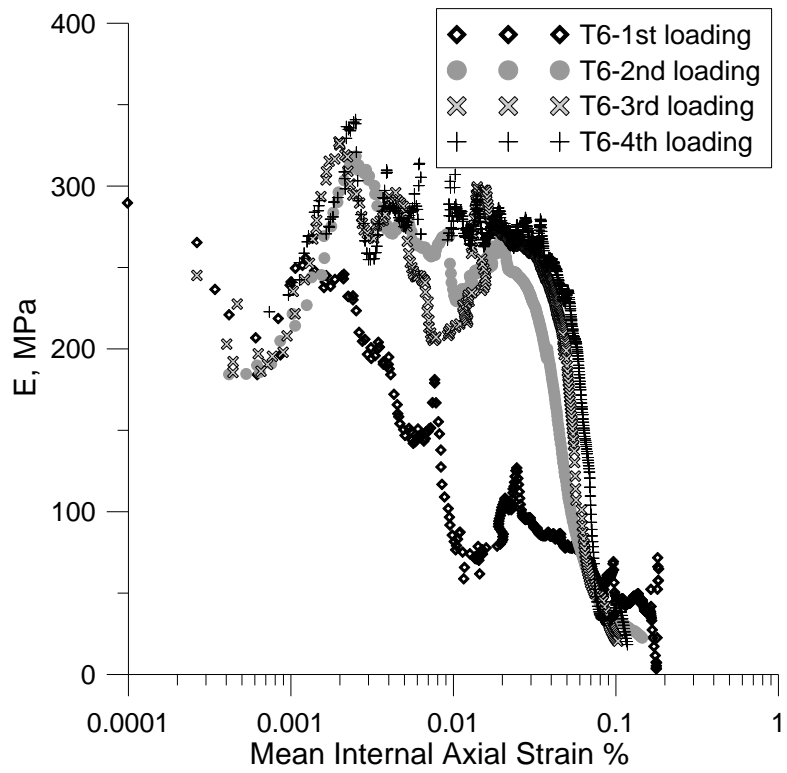
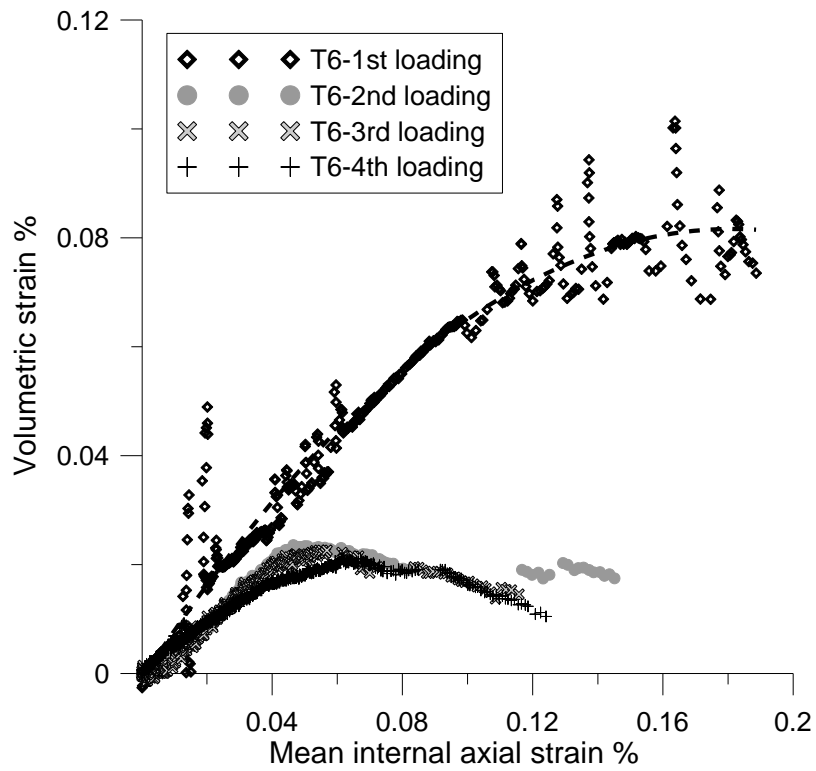


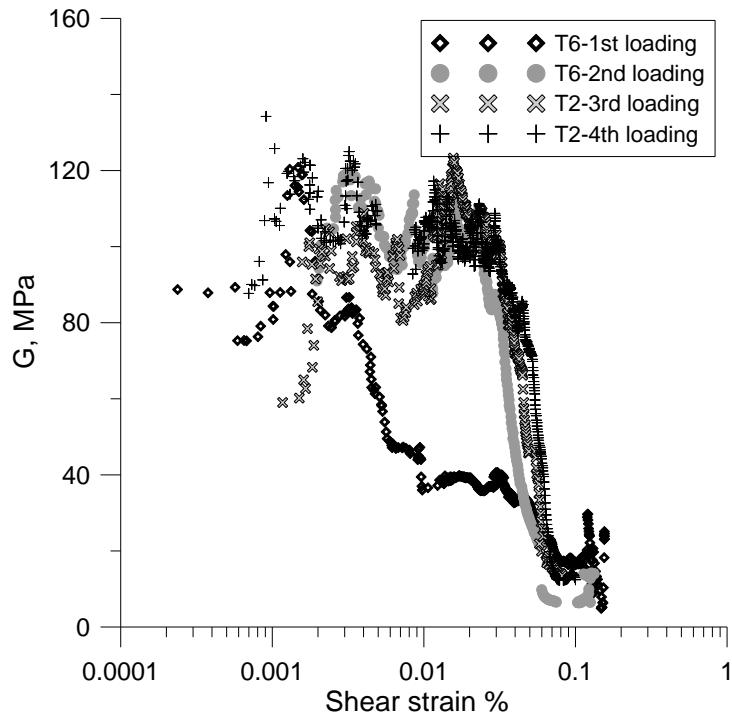
Figure 10. Comparison between initial tangent Young's moduli and second shearing stiffnesses during shearing probes at 150kPa confining pressure.



(a)



(b)



(c)

Figure 11. Effect of repeated loading for dry Test T6 at 100kPa (a) Young's moduli (b) volumetric strains (c) shear moduli.

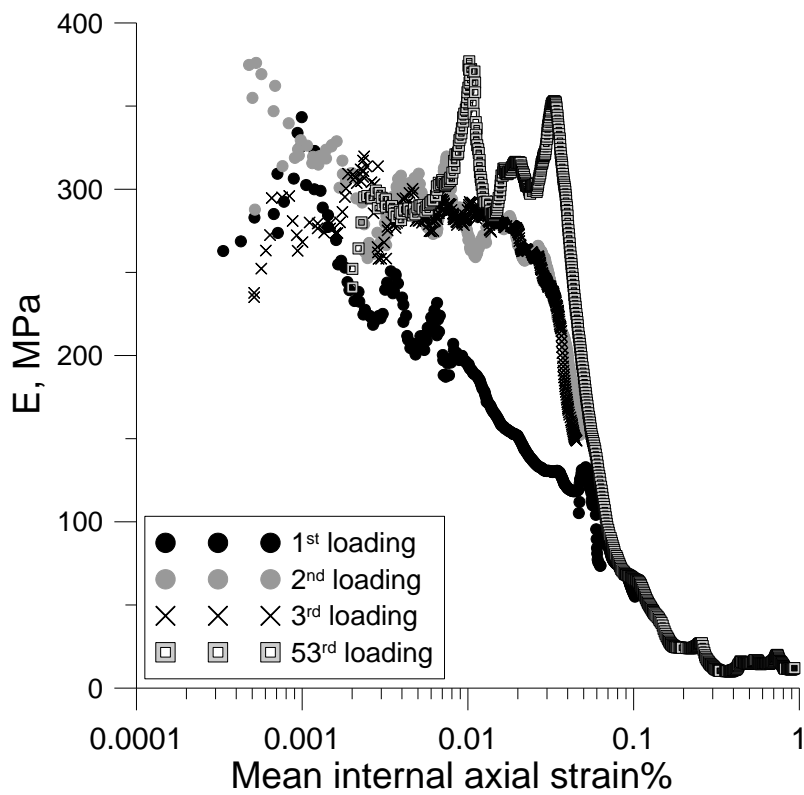


Figure 12. Tangent Young's moduli changes during the first, second and third shearing and after cyclic loading (53rd loading) in Test T8.

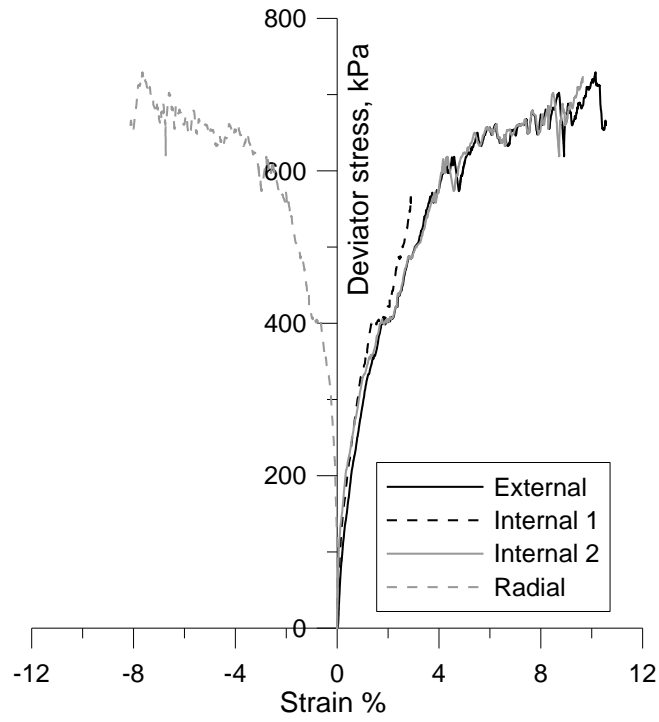


Figure 13. External and internal axial strain and internal radial strain changes during Test T9R.

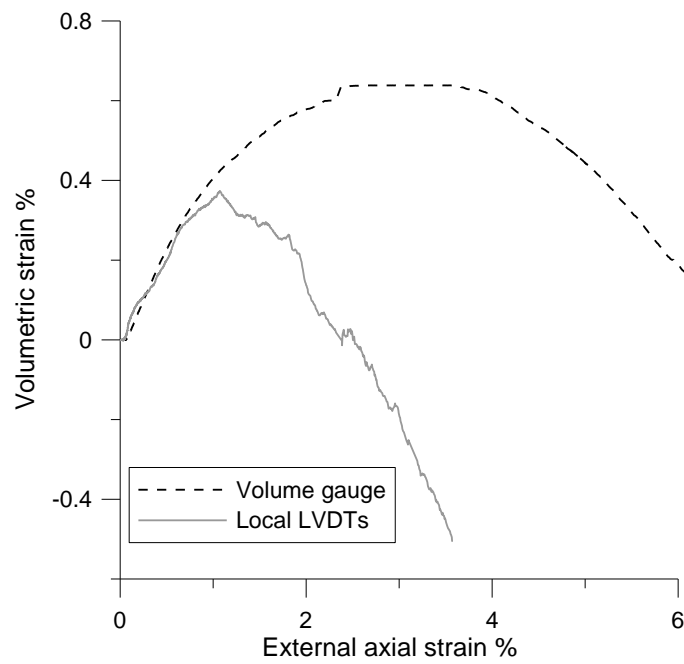


Figure 14. Volumetric strain measurement comparison for Test T9R.