

# Ferrite Loss Measurement and Models in Half Bridge and Full Bridge Waveforms

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**Abstract—** The Steinmetz equation is applied in a pure hysteresis part and a remaining part following a dB/dt behavior. A double natural Steinmetz extension method for non-sinusoidal waveforms is proposed (DNSE). Tests have been done at 100kHz square waves for half bridges with variable duty ratio and full bridges with variable phase shift.

## I. INTRODUCTION

The classical Steinmetz equation (1), for the power loss/volume, corresponds to a linear fitting in a double logarithmic graph [1]. It is sufficiently accurate for a small frequency range or for two distinct frequencies.

$$P_v = k f^\alpha B^\beta \quad (1)$$

For ferrites, at low frequencies (20kHz), the power  $\alpha$  is close to 1 whereas at higher frequencies (100kHz-500kHz) it tends to be close to 2. The power  $\alpha=1$  corresponds to 'pure' hysteresis, where the losses are mainly dependent on the peak-peak induction. A power  $\alpha=2$  would correspond to a pure Foucault loss, which would correspond with a low frequency model of macroscopic eddy currents. As the resistivity of ferrites is quite high, these macroscopic eddy currents ( $\alpha=2$ ) are quite small and this type of model is not realistic. Also a model has been proposed which is exact for  $\alpha=1$  and  $\alpha=2$  which is called modified Steinmetz equation [2], [3]. It uses the rms value of the voltage to calculate an equivalent frequency. It also gives good results if the  $\alpha$  is fitted between the fundamental and the dominant harmonics [4],[5]. The drawback of the method is that measurements have to be made at the fundamental frequency and the dominant harmonics. The fundamental frequency might not be known in advance and the dominant harmonics are somewhat dependent on the waveform, for instance a half bridge or a full bridge type of waveform.

The usual waveforms in power electronics are square waves or a superposition of square waves rather than sine waves. Throughout the paper, we keep the same peak-peak induction but we vary the frequency and the waveform. As a rule, the losses are also dependent on the waveform and not only the peak-peak induction. A solution is to introduce a dependence of the losses on dB/dt. In the previous articles [4],[5] and [6], a Natural Steinmetz Extension (NSE) has been proposed, based on a given power  $\alpha$ .

$$P_v = P_r \frac{B}{B_r}^{\beta-\alpha} \frac{k_N T}{T} \left| \frac{dB}{dt} \right|^\alpha dt \quad (2)$$

$T$ : reference period (here 10  $\mu$ s)

$B_r$ : reference induction (here 0.1T)

$P_r$ : reference power/vol (loss at 100kHz, 0.1T)

In eq. (2)  $k_N$  is given by:

$$k_N = \frac{k}{(2\pi)^{\alpha-1} \int_0^{2\pi} |\cos \theta|^\alpha d\theta} \quad (3)$$

Since in non-linear magnetic materials, harmonic superposition is not allowed, the solution was to fit  $\alpha$  on a reference frequency (the fundamental) and on a frequency in the region of the most dominant harmonics. The disadvantage is that although the overall accuracy can be satisfactory, the modeling is somewhat dependent on the dominant harmonics.

## II. DOUBLE NATURAL STEINMETZ EXTENSION

It is clear that a higher number of parameters usually fits better, but it is valuable if the parameters can be determined and if it improves the modeling. We sum two Steinmetz equations one term with  $\alpha=1$ , which means pure hysteresis and one term with  $\alpha>1$ . For convenience we define a reference frequency, a reference induction and a reference power, we give them the index  $r$ :

$$P_{\sin} = P_r \gamma \frac{f}{f_r} \frac{B}{B_r}^{\beta_1} + P_r (1-\gamma) \frac{f}{f_r}^\alpha \frac{B}{B_r}^{\beta_2} \quad (4)$$

$f_r$ : reference frequency (here 100kHz)

$\gamma$ : fraction of hysteresis losses at the reference case

We expect that  $0 < \gamma < 1$ ,  $\alpha > 1$ ,  $\beta_1 > 2$  and  $\beta_2 > 2$

The reference power can be one of the measurement points, we did take it at 100kHz, 0.1T. The parameter  $\alpha$  and  $\gamma$  can be determined by fitting the experimental data or manufacturer data. We fit it with experimental data.

For  $\gamma = 1$  we are in the pure hysteresis situation; whereas  $\gamma = 0$  corresponds to a traditional Steinmetz with constant  $\alpha$ . For  $\alpha = 2$  we would have a loss type known as Foucault losses. In this article the parameter  $\alpha$  is only used as a curve

fitting, even an  $\alpha > 2$  is possible.

The used material was 3F3, the shape ETD44, the induction 0.1 T peak (0.2 T peak to peak). We performed the measurements in sine wave, shown in Table 1.

TABLE I  
PERFORMED THE MEASUREMENTS IN SINE WAVE

f, [kHz]	20	50	100	250	500	700
P [W]	0.136	0.410	1.18	6.25	25.6	50

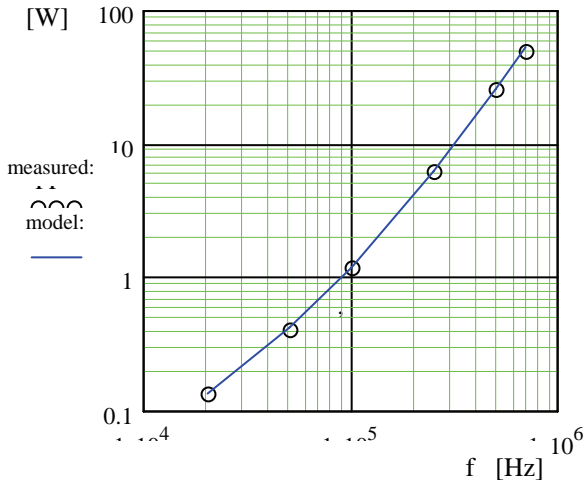


Fig.1. Measuring points: sinusoidal losses at 0.1T and 100°C, curve: double Steinmetz with  $\alpha=2.26$  and  $\gamma=0.50$ ,  $P_r = 1.18W$ .

All the measurements are done at 100°C as this corresponds to a minimum loss for the material. in this way stable and repeatable measurements can be done.

A good match is obtained for  $\alpha=2.26$  and  $\gamma=0.5$  as shown in Fig. 1.

We attribute the power  $\alpha$  part to a dependency on dB/dt. This part is similar to the proposed Natural Steinmetz Extension NSE [4],[5],[6]. The purpose is to match also non-sinusoidal cases.

We will refer to the following equation as the Double Natural Steinmetz Extension (DNSE). The word double refers that the Steinmetz extension is applied two times, once with  $\alpha=1$  in the first term where only the peak value influences the losses and once for  $\alpha>1$  in the second term:

$$P_v = \gamma P_r \frac{f}{f_r} \frac{B}{B_r}^{\beta_1} + (1-\gamma) P_r \frac{B}{B_r}^{\beta_2-\alpha} \frac{\kappa(\alpha)}{T} \int_0^T \left| \frac{dB}{dt} \right|^\alpha dt \quad (5)$$

The parameter  $\gamma$  is the part of the losses at the reference frequency, which follows the hysteresis losses.

The function  $\kappa(\alpha)$  is defined in such way that it satisfies the sine wave solution:

$$\kappa(\alpha) = \frac{1}{(2\pi)^{\alpha-1} \int_0^{2\pi} |\cos \theta|^\alpha d\theta} \quad (6)$$

Note that  $\kappa(\alpha)$  is only depending on  $\alpha$ , this is shown in fig.2. Corresponding to hysteresis losses, for  $\alpha=1$ , the function  $\kappa(\alpha) = 1/4$ . Corresponding to Foucault losses, for  $\alpha=2$ , the function  $\kappa(\alpha) = 1/(2\pi^2) = 0.0507$ .

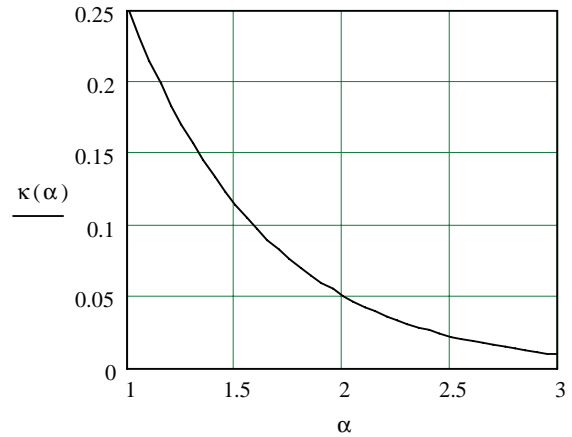


Fig.2: Factor  $\kappa(\alpha)$  as function of  $\alpha$ .

Note also that in this article we keep the peak inductance constant, so that the values of  $\beta_1$  and  $\beta_2$  are not important.

We can separate the losses in:

$$P_{DNSE} = P_{hyst} + P_{NSE} \quad (7)$$

with:

$$P_{hyst} = \gamma P_r \frac{f}{f_r} \frac{B}{B_r}^{\beta_1} \quad (8)$$

$$P_{NSE} = (1-\gamma) P_r \frac{B}{B_r}^{\beta_2-\alpha} \frac{\kappa(\alpha)}{T} \int_0^T \left| \frac{dB}{dt} \right|^\alpha dt$$

For special waveforms we can fill in the integral. For a square wave with duty ratio D (**half bridge**), we have By using (10), (2) and (8) yield

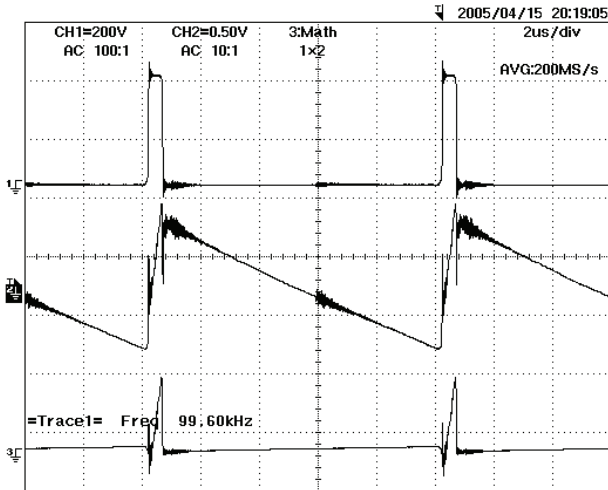


Fig.3. Measured Voltage, current and power for a half bridge at D=0.05, 200V/div 500mA/div, 100W/div



Fig.4. Measured Voltage, current and power for a full bridge at D=0.05, 200V/div 500mA/div, 100W/div

$$\frac{1}{T} \int_0^T \left| \frac{dB}{dt} \right|^\alpha dt = f^\alpha B^\alpha \left[ \frac{2}{D}^\alpha D + \frac{2}{1-D}^\alpha (1-D) \right] \quad (9)$$

$$P_{DNSE} = \gamma P_r \frac{f}{f_r} \frac{B}{B_r}^{\beta_1} + (1-\gamma) P_r \frac{f}{f_r}^\alpha \frac{B}{B_r}^{\beta_2} \kappa(\alpha) 2^\alpha (D^{1-\alpha} + (1-D)^{1-\alpha}) \quad (11)$$

This is also

$$\frac{1}{T} \int_0^T \left| \frac{dB}{dt} \right|^\alpha dt = f^\alpha B^\alpha 2^\alpha (D^{1-\alpha} + (1-D)^{1-\alpha}) \quad (10)$$

We show the current, voltage and power waveforms for 100 kHz and with a duty ratio of 5% in fig. 3 and 4.

At this extreme duty ratio, a notch in the current is clearly visible during the high voltage pulse.

and substituted in (7):

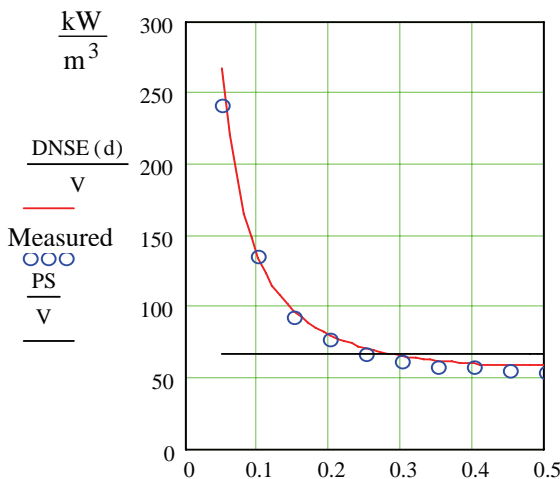


Fig.5. Ferrite losses/volume for half bridge measurement as function of D, 0.1T, 100kHz  
DNSE, measured and normal Steinmetz

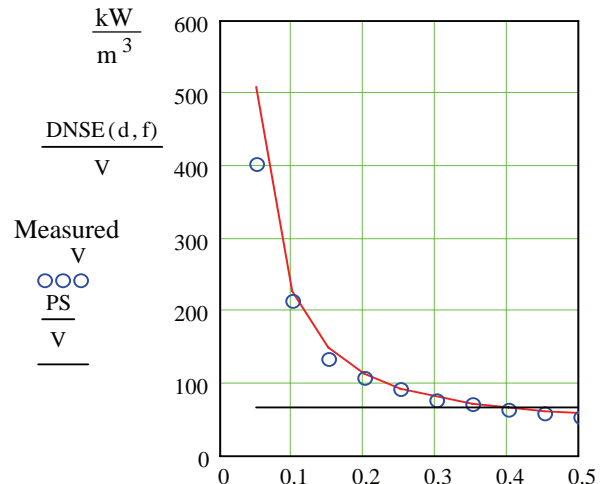


Fig.6. Ferrite losses/volume for full bridge measurement as function of D, 0.1T, 100kHz  
DNSE, measured and normal Steinmetz

We consider a **full bridge** waveform with a phase shift  $D$ ;  $D=0$  corresponds with no phase shift;  $D=1$  with a phase shift of a full period. For this condition, the double Steinmetz equation, becomes

$$P_{DNSE} = \gamma P_r \frac{f}{f_r} \frac{B}{B_r}^\beta + (1-\gamma) P_r \frac{f}{f_r}^\alpha \frac{B}{B_r}^\beta \kappa(\alpha) \frac{2}{D}^\alpha 2D \quad (12)$$

The following measurements were made with 3F3 material for a half bridge (fig.5) and a full bridge (fig. 6), using a test platform [7]. The test object is an ETD44 core with 5 primary turns (two Litz wires in parallel of 60 strands of 0.1mm) and 5 secondary turns for flux and power measurement of 0.3mm, see figure 7.

Fig.7. Test object

Care has been taken to obtain a low capacitance between primary and secondary windings, and a low leakage inductance.

Note that the power loss for  $D=50\%$  is in full and half bridge is almost equal as it concerns the same waveform. At  $D=5\%$ , the losses in the full bridge are almost twice the losses in the half bridge, although the peak-peak induction is the same!

A wide band current probe has been used (150Hz-50MHz) [4], together with an oscilloscope power measurement. A voltage probe was constructed with almost the same characteristic as the current probe, to obtain a phase shift close in the order of 1ns at 50MHz.

A comparison with calorimetric power measurements

similar to [6] shows a typical deviation in power measurement of 3% and 5% in extreme duty ratios.

### III. INTERPRETATION

At low frequency, the losses are quite independent of the waveform when no extreme  $dB/dt$  is present, the losses seem to be mainly determined by hysteresis effects [1].

If we look at the waveform of the current and the exponent  $\alpha$  on  $dB/dt$ , which is close to 2, it is likely to consider a macroscopic resistive current for high frequency.

The voltage of one turn is:

$$V = A_{\min} \frac{dB}{dt} \quad (13)$$

$A_{\min}$  : section of the mid-leg  $172\text{mm}^2$

The E field at the circumference of the mid-leg is

$$E = A_{\min} \frac{dB}{dt} \frac{1}{\mathcal{C}l_c} \quad (14)$$

$d_c$ : diameter of the center leg

The E-field in the ferrite increases in a linear way with the radius. This results in an average E-field loss/volume, which is 2 times lower than the loss/volume at the circumference.

This model is allowed as even for 1 MHz the penetration depth [8],[9] in the ferrite is still 11.3mm (bigger than the radius) if you consider a resistivity of  $2\Omega\text{m}$ , a relative permeability of 4000 and a frequency of 1MHz, which contains already a big part of the harmonics.

The expected losses/volume with this field pattern are

$$P_E = \frac{A_{\min} \frac{dB}{dt} \frac{1}{\mathcal{C}l_c}^2}{\rho} \frac{1}{2} \quad (15)$$

$\rho$ : resistivity:  $2\Omega\text{m}$  at DC and  $25^\circ\text{C}$  (data)

For the half bridge we can calculate

$$\left\langle \frac{dB}{dt} \right\rangle_{rms} = \frac{2B}{T} \sqrt{\frac{1}{D} + \frac{1}{1-D}} \quad (16)$$

If we consider a duty ratio of 0.05 for the half bridge, we get  $28\text{kW/m}^3$ , which is only 12% of the measured losses.

We know that the E-field in the yoke and side legs is smaller, so that the average eddy current losses/volume in the total core will be even less than the center leg.

So we can say that the bulk eddy currents caused by DC resistivity are not sufficient to explain the  $dB/dt$  losses.

#### IV. CONCLUSION

For a given peak induction, the sinusoidal losses of ferrites can be modeled with a double extended Steinmetz equation, for common frequencies in power electronics. The dependency of the waveform in half bridge and full bridge configurations can be well modeled using a hysteresis part and a part dependent on dB/dt. The losses at extreme duty ratio of the full bridge can be almost twice the losses of a half bridge, for the same peak-peak induction.

Although the dB/dt losses are close to a resistive effect, a simple model with using the DC resistivity does not explain the losses.

High losses are present at extreme duty ratios, but in practical applications, fortunately the peak-peak induction will be lower in such cases.

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