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Research article

Multi-objective optimization to the transportation problem considering non-linear fuzzy membership functions

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Abstract: Considering the uncertainty of transporting goods from numerous origins to diverse destinations is a critical task for the decision-maker (DM). The ultimate goal of the DM is to make the right decisions that optimize the profit or loss of the organization under the vagueness of the uncontrollable effects. In this paper, mathematical models are proposed using fuzzy non-linear membership functions for the transportation problem considering the parameters' uncertainty that can help the DM to optimize the multi-objective transportation problems (MOTP) and to achieve the desired goals by choosing a confidence level of the uncertain parameters. Based on DM's selection of the confidence level, a compromise solution of the uncertain multi-objective transportation (UMOTP) is obtained along with the satisfaction level in percent for the DM. Two non-linear fuzzy membership functions are considered: the exponential and the hyperbolic functions. Using both membership functions, the sensitivity analysis was implemented by considering different confidence levels. According to the experimental results, the hyperbolic membership function gives 100% DM's satisfaction in many instances. Moreover, it shows stability against the exponential and linear functions.

Keywords: transportation problem; fuzzy multi-objective optimization; linear programming problem; non-linear fuzzy-membership functions; compromise solution
Mathematics Subject Classification: 03E72, 49Q22

1. Introduction

Transportation problem (TP) is a very distinct kind of cost-effective linear programming problem that helps the decision-maker (DM) to optimize the cost parameters. The ordinary transportation problem is associated with transporting goods from p deliverers to q receivers filling the obtainability of the supply equivalent to the demands. Sometimes the cost parameters such as supply, demand, and costs are not described preciously. In this situation, the question of uncertainty arises. Many researchers have shown their ingenuity in developing effective methods for the optimization of a TP. The fundamental concept of TP was developed by Hitchcock [1] in a mathematical form of the distribution of products from numerous sources to frequent warehouses and then [2] showed the best utilization of the system. Thie and Keough [3] have shown the models to solve the transportation problem as a linear programming problem, which is very efficient by using the simplex method that makes the problem very much calculative. On the other side, Zadeh [4] have given a brief description of a new technique for decision-making in a fuzzy environment. Lee and Moore [5] optimized transportation problems with multiple objectives applying the goal-programming concept. Zimmermann [6] applied the fuzzy linear programming concept with multi-objectives to solve MOTP. Isermann [7] have shown an algorithm for linear MOTP and Leberling [8] have given the idea of hyperbolic membership function for multi-objective linear programming problem. Ringuest and Rinks [9] suggested an interactive algorithm for the linear MOTP that makes it easy to find the compromise solution of the transportation problem for the set of non-dominated solutions. Arsham and Kahn [10] introduce a simple model for general transportation problem for stepping-stone method. Chanas et al. [11] and Delgado et al. [12] revealed a general model for fuzzy linear programming. Bit et al. [13] introduced an additive fuzzy programming model for the many criteria transportation problem considering the fuzzy membership function to construct a significant decision function. Yaghin and Darvishi [14] integrated decisions of the physical supply channel on the global scale including supplier selection, order allocation, and transportation planning under uncertainty. Verma et al. [15] apply fuzzy programming technique to solve multi-objective transportation problems with some nonlinear membership functions. El-Wahed and Abo-Sinna [16] discussed the mixture fuzzy-goal programming tactic to the optimization of multiple objective decision making problems. The values of the uncertain parameters are obtained by using uncertain normal distribution suggested by B. Liu [17–22] for uncertain measures theorem. Maity et al. [23] apply uncertain measures theorem to measure the problem with cost reliability. Baky [24] have drown a brief description of multi-level use of fuzzy goal programming where Ojha et al. [25] and Kundu et al. [26] show the applicability of fuzzy model in both single and multiple objective TP. In 2015, Guo et al. [27] applied the TP model in uncertain environment for e-Navigation and Maritime Economy. Uddin et al. [28] and Umarusman [29] have demonstrated the use of fuzzy goal programming with genetic algorithm in min-max approach for MOTP. Recently, Singh et al. [30] presented the concept of interactive fuzzy goal programming to solve the MOTP. Moreover, Uddin et al. [31] described TP considering all parameters uncertain with goal programming applying linear membership function. Recently, Darvishi et al. [32], Yaghin and Darvishi [33] have integrated inbound logistics decisions and multi-site aggregate production planning (APP) over the tactical planning horizon in textile industry using fuzzy logic. Fuzzy logic was considered in the planning of textile production management [33]. Ali et al. [34] apply the LR-type fuzzy in a multi-objective supply chain problem. Khan et al. [35] and Kaliyaperumal and Das [36] use non-linear programming to optimize multi-objective problems. Muhammad et al. [37], Kamal et al. [38], Mahajan and Gupta [39] and Khan et al. [40] are also apply the fuzzy goal programming technique for sustainable development. Kacher and Singh [41] developed a fully fuzzy MOTP with the consideration of fuzzy harmonic mean. A recent literature review can be found in [42] with a full description of the models of TP.

Based on the aforementioned literature, the uncertain multi-objective TP is still attracting the attention of the recent researches. However, most of the resent works considered the linear membership function for representing uncertainty. Only a few attempts were found considering the hyperbolic membership function or the exponential membership function. This shortage motivates this study to consider the uncertain multi-objective TP with nonlinear membership functions. Besides, it attempts to answer the question of what is the best membership function to be used amongst hyperbolic, exponential, or linear? Moreover, it proposes to develop a decision support tool by which the DM can interpret the MOTP. Another question was highlighted: Is there any tool that can help the DM to interpret the obtained solution rapidly? Considering these research interests, there is a need for a method that will help the DM to calculate in percent the desired goal with uncertain parameters using non-linear membership function corresponding to his confidence level.

In the present study, diverse attempts were taken to predict the parameters of the MOTP in an uncertain environment. Accordingly, this model will be very helpful for the decision-maker in a very complicated situation to take the right decisions. The key contributions of the present research can be summarized as follows:

- All parameters such as the supply, demand, and cost are considered uncertain.
- Different objectives are considered: beside the benefits, the penalty cost for transportation and damage cost due to delay or early transportation are considered as uncertain.
- The decision maker can choose a confidence level for each of the uncertain parameter to make it a certain one.
- Non-linear membership functions of fuzzy programming approach were considered to model the situation.
- The DM can find out his overall satisfaction in percent (%) corresponding to the chosen confidence level which will be a tremendous job to take right decision and as a whole for the desired profit of the organization.

The rest of this paper is furnished as in Section 2, the proposed mathematical model for non-linear membership function is designed and discussed. In Section 3, an illustrative example for the applicability of the proposed model is discussed. Then Section 4 presents a comparative analysis with discussions of the different membership functions. Finally, the conclusions and perspectives are presented.

2. Mathematical model

Nomenclature:

a_j: Supply parameter (in number of units)
b_j: Demand parameter (in number of units)
C_{ij}: Cost parameter for i^{th} origin to j^{th} destination (in monetary units)
λ: Uncertain variable
$\Re$: Uncertain distribution
$\emptyset$: Normal uncertain distribution
Ω: Uncertain measure function
e: Expected value of the parameter
σ: Standard deviation
ω: Confidence level
ξ: Satisfaction level of the DM
$\mu^E(Z_k)$: Exponential membership function for the k^{th} objective
$\mu^H(Z_k)$: Hyperbolic membership function for the k^{th} objective
X_{ij}: Amount to be transported from i^{th} origin to j^{th} destination
$\Re^{-1}(x)$: Inverse normal uncertain distribution.

2.1. Multi-objective transportation problem

Transportation problems are generally designed to transport a number of products from some stocks (or sources) to different ends. Let us consider there are$p$sources and q demands. The sources can be factory or production facilities, warehouse etc. and these are denoted by the symbols $a_1, a_2, \ldots \ldots a_p$ and the destinations can be warehouse, outlets etc. and are generally denoted the symbols $b_1, b_2, \ldots \ldots b_q$. Let the transportation costs C_{ij} are related to transport a number of units from i^{th} origin to j^{th} destination and $x_{ij}$is the unknown quantity to be transported from the i^{th} origin to j^{th} destination. In these circumstances, the conventional transportation model is written as:

$$\left.\begin{array}{l}\text{Minimize} \quad : \quad Z(x) = \sum_{i=1}^{p}\sum_{j=1}^{q} C_{ij}x_{ij} \\ \text{Subject to the constraints:} \\ \quad \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1, 2, 3, \ldots\ldots\ldots\ldots, p \\ \quad \sum_{i=1}^{m} x_{ij} \ge b_j, \quad j = 1, 2, 3, \ldots\ldots\ldots\ldots, q \\ \quad \forall\ x_{ij} \ge 0, \quad i = 1,2,\ldots\ldots p; \quad j = 1,2,\ldots\ldots, q \end{array}\right\} . \tag{1}$$

Under the feasibility condition of $\sum_{i=1}^{m} a_i \ge \sum_{j=i}^{n} b_j$.

In the present challenging market condition, various objectives such as minimizing the transportation costs, minimize the transportation time, maximize the profit, minimize the damage cost due to delay or advanced transport of the products are very rational with transportation. On the other hand, these cost parameters are independent to each other and that is why they are considered as conflicting. In such environment, the MOTP can be demonstrated as follows:

$$\left.\begin{array}{l} \text{Minimize } : Z_k(x) = \sum_{i=1}^{p}\sum_{j=1}^{q} C_{ij}^{k} x_{ij} \\ \text{Subject to :} \\ \qquad \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1, 2, 3, \ldots\ldots\ldots\ldots, p \\ \qquad \sum_{i=1}^{m} x_{ij} \ge b_j, \quad j = 1, 2, 3, \ldots\ldots\ldots\ldots, q \\ \qquad \forall\ x_{ij} \ge 0, \quad i = 1,2,\ldots\ldots p; \quad j = 1,2,\ldots\ldots, q \end{array}\right\}. \tag{2}$$

Under the feasibility condition of $\sum_{i=1}^{m} a_i \ge \sum_{j=i}^{n} b_j$.

Where C_{ij}^{k} is the unit cost for transportation, a_i $(i = 1,2, \ldots\ldots, p)$ is the supply and $b_j(j = 1,2, \ldots\ldots, q)$ is the order parameter for the k^{th} $(k = 1,2, \ldots\ldots, K)$ objective function of the MOTP. In general, the number C_{ij}^{k}, a_i and b_j are considered as crisp. Incorporating the inverse measure theorem, Eq (2) can be written as follows:

$$\left.\begin{array}{l} \text{Max/Min } : Z_k(x) = \sum_{i=1}^{p}\sum_{j=1}^{q} \left[\Omega\left(C_{ij}^{k}\right) \ge \alpha_{ij}\right] x_{ij} \\ \text{Subject to :} \\ \qquad \Omega\left(\sum_{j=1}^{n} x_{ij} \le a_i\right) \ge \gamma_i \quad i = 1, 2, 3, \ldots\ldots\ldots\ldots, p \\ \qquad \Omega\left(\sum_{i=1}^{m} x_{ij} \ge b_j\right) \ge \delta_j, \quad j = 1, 2, 3, \ldots\ldots\ldots\ldots, q \\ \qquad \forall\ x_{ij} \ge 0, \quad i = 1,2,\ldots\ldots p; \quad j = 1,2,\ldots\ldots, q \end{array}\right\}. \tag{3}$$

We know that the inverse of the normal uncertain variable $N(e, \sigma)$ is as $\Re^{-1}(x) = e + \frac{\sqrt{3}\sigma}{\pi} ln\left(\frac{\omega}{1-\omega}\right)$ where ω is the level of confidence of the decision maker.

2.2. The proposed model for non-linear membership functions

The computational algorithm for fuzzy exponential membership function is constructed below for achieving the optimal solution and also the satisfaction level of the decision maker as follows:
Step 1: Obtain the crisp values of the parameters using inverse normal uncertain distribution.

Step 2: Construct the given MOTP for uncertain parameters to an ordinary TP using the crisp number obtained from step 1.
Step 3: Solve each single objective TP ignoring all others objectives.
Step 4: Using the result obtain from step 3, determine the corresponding values of each objective function and then construct the payoff matrix as shown in Table 1:

Table 1. The structure of the Payoff matrix.

	$Z_1(X)$	$Z_2(X)$	.	.	.	$Z_k(X)$
$X^{(1)}$	Z_{11}	Z_{12}	.	.	.	Z_{1K}
$X^{(2)}$	Z_{21}	Z_{22}	.	.	.	Z_{2K}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$X^{(k)}$	Z_{K1}	Z_{K2}	.	.	.	Z_{KK}

Where $X^{(1)}, X^{(2)}, \ldots\ldots\ldots, X^{(K)}$ are the optimal solutions and $Z_{1K}, Z_{1K}, \ldots\ldots\ldots, Z_{KK}$ are the corresponding values of the single objective functions.
Step 5: From step 4, for each single objective, find the M_k and the corresponding L_k for each solution set, where $M_k = \text{Maximum}(Z_{1K}, Z_{2K}, \ldots\ldots\ldots, Z_{KK})$ and $L_k = \text{Minimum}(Z_{1K}, Z_{2K}, \ldots\ldots\ldots, Z_{KK})$, $k = 1,2,\ldots.,K$. Then an initial fuzzy model can be written as follows:

$$\left.\begin{aligned} &\text{Find } X_{ij}, \quad i = 1,2,3,\ldots..,p, \quad j = 1,2,3,\ldots,q \\ &\qquad Z_K \le L_K, \ \ k = 1,2,\ldots.,K \\ &\text{Subject to:} \\ &\qquad \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1,\ 2,\ 3,\ldots\ldots\ldots\ldots..,p \\ &\qquad \sum_{i=1}^{m} x_{ij} \ge b_j, \quad j = 1,\ 2,\ 3,\ldots\ldots\ldots\ldots..,q \\ &\qquad \forall\ x_{ij} \ge 0, \quad i = 1,2,\ldots\ldots p; \quad j = 1,2,\ldots\ldots.,q \end{aligned}\right\}. \tag{4}$$

Assuming that any maximization objective can be transformed to a minimization one as pre-processing step.
Step 6: Describe exponential membership function $\mu^E(Z_k)$ for the k^{th} objective function.
Step 7: Convert model, in step 5, as a crisp model as Eq (5) for exponential fuzzy membership function and as Eq (6) for hyperbolic one:

$$
\left.\begin{array}{l}
\text{Maximize: } \xi \\
\text{Subject to:} \\
\qquad e^{-s\psi_k(X)} - (1 - e^{-s})\xi \geq e^{-s}\ , \quad k = 1,2,\ldots.,K \\
\qquad \sum_{j=1}^{q} X_{ij} = a_i\ , \quad i = 1,2,\ldots.,p \\
\qquad \sum_{i=1}^{p} X_{ij} = b_j\ , \quad j = 1,2,\ldots.,q \\
\qquad X_{ij} \geq 0, \quad \xi \geq 0,\ \forall i,j
\end{array}\right\}, \tag{5}
$$

where $\psi_k(X) = \frac{Z_k - L_k}{M_k - L_k}$.

$$
\left.\begin{array}{l}
\text{Maximize } \xi \\
\text{Subject to:} \\
\qquad Z_k(X)\alpha_k + \eta \leq \left(\frac{M_k + L_k}{2}\right)\alpha_k \\
\qquad \sum_{j=1}^{q} X_{ij} = a_i\ , \quad i = 1,2,\ldots.,p \\
\qquad \sum_{i=1}^{p} X_{ij} = b_j\ , \quad j = 1,2,\ldots.,q \\
\qquad \xi \geq 0,\ X_{ij} \geq 0, \quad \forall i,j
\end{array}\right\}, \tag{6}
$$

where $\eta = tanh^{-1}(2\xi - 1)$.

Step 8: Solving the model, required solution and the satisfaction level of the DM for the MOTP obtained.

To understand the computational algorithm straight forwardly, a flowchart is presented below. Moreover, an example is illustrated for ascertaining the usefulness of the mentioned transportation problem beside the pertinence of the computational system in the next segment.

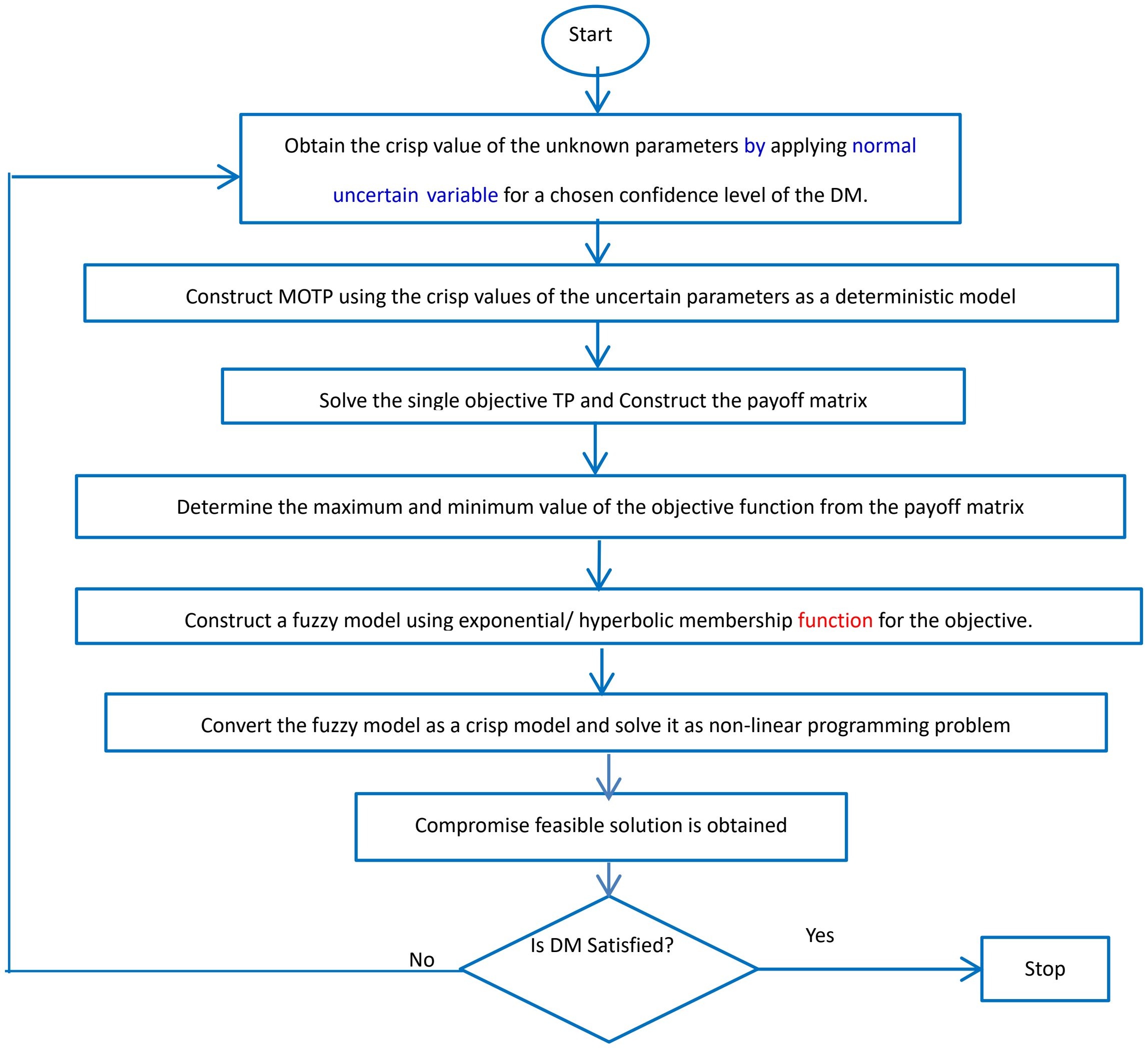

Figure 1. Flow chart of the proposed model using exponential/hyperbolic membership function.

3. Illustrations

To affirm the feasibility of the models, let us consider a multi objective TP with uncertainty in objective parameters. The decision maker desires to transport the merchandises from three sources S_1, S_2, S_3 to four destination points D_1, D_2, D_3, D_4 and wishes to optimize the objective functions as follows:

(i) Minimization of transportation cost (Z_1).
(ii) Maximization of profit (Z_2).
(iii) Minimization of damage cost (Z_3).

Note: First, we assumed that the stochastic profit matrix can be converted to a stochastic cost matrix as a prepressing step. We call it "transformed profit C_{ij}^2". This transformation can be performed by subtracting the stochastic profit matrix from a large number with zero variance. Accordingly, the proposed algorithm can be applied straight forwardly.

In the uncertain working environment, it is difficult for the DM to estimate his decision outcomes. But he/she can easily decide the level of confidence ω to achieve the desired goal. Then, the unpredictable parameters can be converted to a crisp number using the normal distribution $N(e,\sigma)$. Whereas, e stands for expected value of the uncertain parameter and σ for the deviation. To achieve this goal, the uncertainty is considered using the inverse normal uncertain distribution. In other words, the uncertain TP cost C_{ij}^1, transformed profit C_{ij}^2 and damage cost C_{ij}^3 for each unit of goods from i origins to j destinations for $i = 1,2,3$ and $j = 1,2,3,4$ are obtained using the inverse normal uncertain distribution for the confidence level ω chosen by the DM. Similarly, both the demands and supply are measured using inverse normal uncertain distribution.

$\Re^{-1}(x) = e + \frac{\sqrt{3}\sigma}{\pi} ln\left(\frac{\omega}{1-\omega}\right)$ and the supply are measured using inverse uncertain normal distribution. $\Re^{-1}(x) = e + \frac{\sqrt{3}\sigma}{\pi} ln\left(\frac{1-\omega}{\omega}\right)$.

In each of the case, the DM uses the same confidence level to determine all the uncertain parameters. The data used to illustrate the proposed approach is taken from the work presented in Uddin et al. [29], as listed in Table 2.

3.1. Using exponential membership function

For confidence level $\omega = 0.75$ Table 2 can be changed to Table 3 respectively using confidence level $\omega = 0.75$ and the inverse normal uncertain distribution.

Table 2. Data to illustrate the example.

Entity	Data matrix				
Uncertain transportation cost C_{ij}^1:		D_1	D_2	D_3	D_4
	S_1	(20, 2)	(18, 2)	(22, 3)	(24, 3)
	S_2	(10, 1)	(12, 2)	(15, 3)	(13, 1)
	S_3	(22, 3)	(20, 3)	(24, 2)	(23, 2)
Uncertain transformed profit C_{ij}^2:		D_1	D_2	D_3	D_4
	S_1	(5, 1)	(6, 1.5)	(4, 1)	(3, 0.5)
	S_2	(6, 1)	(5, 1.5)	(5, 0.5)	(4, 1)
	S_3	(9, 1)	(8, 1.5)	(8, 2)	(10, 2)
Uncertain damage cost C_{ij}^3:		D_1	D_2	D_3	D_4
	S_1	(4, 1)	(4, 1)	(3, 1)	(5, 2)
	S_2	(3, 1)	(6, 1)	(4, 1)	(4, 1)
	S_3	(4, 1.5)	(3, 1)	(4, 1)	(5, 1.5)
Uncertain demand:		b_1	b_2	b_3	b_4
		(40, 3)	(36, 4)	(35, 5)	(40, 3)
Uncertain supply:		a_1	a_2	a_3	
		(55, 4)	(60, 5)	(70, 4)	

Now, the solution of each single objective of the TP are presented using "TORA".

$$X^2 = (11,\ 0,\ 0,\ 42,\ 19,\ 0,\ 19,\ 0,\ 31,\ 32,\ 0,\ 0)$$
$$X^1 = (0,\ 38,\ 15, 0, 42, 0, 0, 15, 0, 0, 23, 27)$$
$$X^3 = (15,\ 0,\ 38,\ 0,\ 27,\ 0,\ 0,\ 30,\ 0,\ 38,\ 0,\ 12)$$

Table 3. Crisp values for the uncertain transportation data for confidence level $\omega = 0.75$.

Entity	Data matrix				
Crisp value for cost C_{ij}^1:		D_1	D_2	D_3	D_4
	S_1	21.22	19.22	23.83	25.83
	S_2	10.61	13.22	16.83	13.61
	S_3	23.83	21.83	25.22	24.22
Crisp value for the transformed profit C_{ij}^2:		D_1	D_2	D_3	D_4
	S_1	5.61	6.92	4.61	3.31
	S_2	6.61	5.92	5.31	4.61
	S_3	9.61	8.92	9.92	11.22
Crisp value for damage cost C_{ij}^3:		D_1	D_2	D_3	D_4
	S_1	4.61	4.61	3.61	6.22
	S_2	3.61	6.61	4.61	4.61
	S_3	4.92	3.61	4.61	5.92
Crisp value for demand:		b_1	b_2	b_3	b_4
		41.8	38.4	38.05	41.83
Crisp value for supply:		a_1	a_2	a_3	
		52.6	57	67.56	

The different values of Z_k corresponding to X^k are listed in the following pay-off matrix Table 4.

Table 4. The Payoff matrix of the illustrative example.

	$Z_1(X)$	$Z_2(X)$	$Z_3(X)$
$X^{(1)}$	2968.8	1193.69	715.84
$X^{(2)}$	3368.8	980.4	776.68
$X^{(3)}$	3038.79	1049.51	648.6
M_k	3368.8	1193.69	776.68
L_k	2968.8	980.4	648.6

From the pay-off matrix, one can find the maximum M_k and the minimum L_k thresholds for each objective, as listed in the last rows of the above matrix.

$$\left.\begin{array}{c} 2968.8 \le Z_1 \le 3368.8, \quad 980.4 \le Z_2 \le 1193.69 \\ \text{and } 648.6 \le Z_3 \le 776.68 \end{array}\right\}. \tag{7}$$

The exponential membership function is formulated as follows:

$$e^{-s\psi_k(x)} - (1 - e^{-s})\ \xi \ge e^{-s}, \text{ where } \psi_k(X) = \frac{Z_k - L_k}{M_k - L_k}. \tag{8}$$

Based on Eq (8), one can have the exponential membership function as follows:

$$\left.\begin{array}{l}\psi_1(x)=\dfrac{Z_1-L_1}{M_1-L_1}=\dfrac{Z_1-2968.6}{3368.8-2968.8}=\dfrac{Z_1-2968.6}{400}\\ \psi_2(x)=\dfrac{Z_2-L_2}{M_2-L_2}=\dfrac{Z_2-980.4}{1193.69-980.4}=\dfrac{Z_1-980.4}{213.29}\\ \text{and } \psi_3(x)=\dfrac{Z_3-L_3}{M_3-L_3}=\dfrac{Z_3-648.6}{776.68-648.6}=\dfrac{Z_3-648.6}{128.08}\end{array}\right\}. \tag{9}$$

Taking $s = 1$ in Eq (8) together with the Eq (9), the exponential membership function can be written as an equivalent crisp model as follows:

Maximize ξ

Subject to:

$$exp\left\{-\left(\frac{\begin{array}{l}21.22x_{11}+19.22x_{12}+23.83x_{13}+25.83x_{14}+10.61x_{21}+13.22x_{22}+16.83x_{23}\\+13.61x_{24}+23.83x_{31}+21.83x_{32}+25.22x_{33}+24.22x_{34}-2968.6\end{array}}{400}\right)\right\}$$

$$-0.63\xi\geq 0.37$$

$$exp\left\{\frac{-\left(\begin{array}{l}5.61x_{11}+6.92x_{12}+4.61x_{13}+3.31x_{14}+6.61x_{21}+5.92x_{22}+5.31x_{23}\\+4.61x_{24}+9.61x_{31}+8.92x_{32}+9.92x_{33}+11.22x_{34}-980.4\end{array}\right)}{213.29}\right\}-0.63\xi$$

$$\geq 0.37$$

$$exp\left\{-\left(\frac{\begin{array}{l}4.61x_{11}+4.61x_{12}+3.61x_{13}+6.22x_{14}+3.61x_{21}+6.61x_{22}+4.61x_{23}\\+4.61x_{24}+4.92x_{31}+3.61x_{32}+4.61x_{33}+5.92x_{34}-648.6\end{array}}{128.08}\right)\right\}-0.63\xi$$

$$\geq 0.37$$

$$\begin{array}{l}x_{11}+x_{12}+x_{13}+x_{14}\leq 52.6\\ x_{21}+x_{22}+x_{23}+x_{24}\leq 57\\ x_{31}+x_{32}+x_{33}+x_{34}\leq 67.56\\ x_{11}+x_{21}+x_{31}\geq 41.8\\ x_{12}+x_{22}+x_{32}\geq 38.4\\ x_{13}+x_{23}+x_{33}\geq 38.05\\ x_{14}+x_{24}+x_{34}\geq 41.83\\ \quad x_{ij}\geq 0\ \text{ for all intger } i,j.\end{array}$$

Using "LINGO" software, the optimal compromise solution is obtained as follows:

$X^* = (x_{11} = 26.6300,\quad x_{12} = 3.4887,\quad x_{13} = 22.4812,\quad x_{14} = 0.000,\quad x_{21} = 15.170,\quad x_{22} = 0.00,\quad x_{23} = 0.00,\quad x_{24} = 41.830,\quad x_{31} = 0.00,\quad x_{32} = 34.9112, x_{33} = 15.5687,\quad x_{34} = 0.00\ )$`

The values of the objective functions are $Z_1(X^*) = 3052.887$, $Z_2(X^*) = 1025.237$ and $Z_3(X^*) = 665.4063$. The overall satisfaction of the DM is $\xi = 0.6990$, that is, the overall satisfaction is 69.90%. The following Table 5 shows the different objective values and the satisfaction level of the DM for different confidence levels.

Table 5. Results for exponential membership function for different confidence levels.

ω	ξ	Objective values			Feasibility	Satisfaction (%)
		Z_1	Z_2	Z_3		
00-0.49	---	---	---	---	Infeasible	---
0.50	Deviation vanishes and problem exists in certain parameters					
0.51	0.3264	2703.93	884.33	535.83	Feasible	32.649%
0.55	0.4111	2733.02	900.273	550.17	Feasible	41.11%
0.60	0.4462	2797.80	945.64	573.7	Feasible	44.62%
0.65	0.5158	2918.63	975.63	599.41	Feasible	51.58%
0.70	0.5365	2998.33	1009.23	630.55	Feasible	53.65 %
0.71	0.8995	2986.69	975.66	661.32	Feasible	89.95%
0.72	**0.9420**	**2975.95**	**970.74**	**660.97**	**Feasible**	**94.20%**
0.73	0.9038	2797.78	986.31	653.32	Feasible	90.38%
0.74	0.8629	2757.58	996.35	655.95	Feasible	86.29%
0.75	0.6990	3052.88	1025.23	665.4	Feasible	69.90 %
0.76	0.6329	3076.89	1038.30	674.18	Feasible	63.29%
0.77	0.5735	3101.43	1051.45	683.00	Feasible	57.35%
0.78	0.5549	3111.67	1057.35	690.65	Feasible	55.49%
0.79	0.5123	3133.11	1068.76	697.00	Feasible	51.23%
0.80	0.5091	3231.33	1113.65	719.65	Feasible	50.91 %
0.81-1.00	-	-	-	-	Infeasible	-

Note: '---'indicates not applicable.

Figure 2 exposes an explicit view of DM satisfaction over the confidence level using the exponential membership function (EMF) of the fuzzy programming. It shows the satisfaction of the DM is almost static for the confidence level from 0.51 to 0.70 and then dramatically increase for 0.71 and 0.72. After that the level of confidence of the DM gradually go down at the up to 0.85. However, the present transportation problem provides infeasible solution for confidence level 0.00 to 0.49 and for 0.81 to onwards. The reason behind of this result is that the demand becomes greater than supply for choosing the confidence level 0.86 to onwards. A salient observation in accordance with the performed numerical illustration is the decision maker achieves the highest satisfaction level (94.20%) when he selects the confidence level 0.72. Deviation of the parameter vanishes, and the problem becomes in certain environment when DM chooses the confidence level 0.50.

Again, Figure 3 shows the behaviour of the objectives of the TP for various confidence level of the DM using the exponential membership function (EMF). The graph shows the increasing pattern of the objective's values except the TP cost slightly decreasing from 0.72 to 0.74 and profit slightly decreasing at 0.74 while again static up to the end. The damage cost using exponential membership function for this present transportation problem showing increasing behaviour everywhere of the confidence level where the problem has feasible solution.

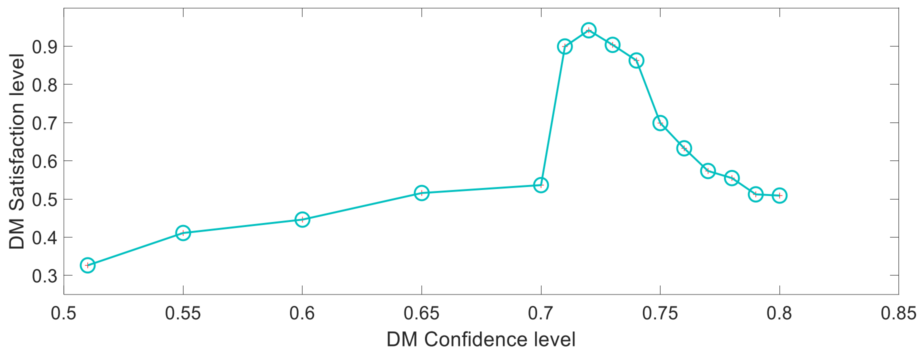

Figure 2. Satisfaction level versus Confidence level for exponential membership function.

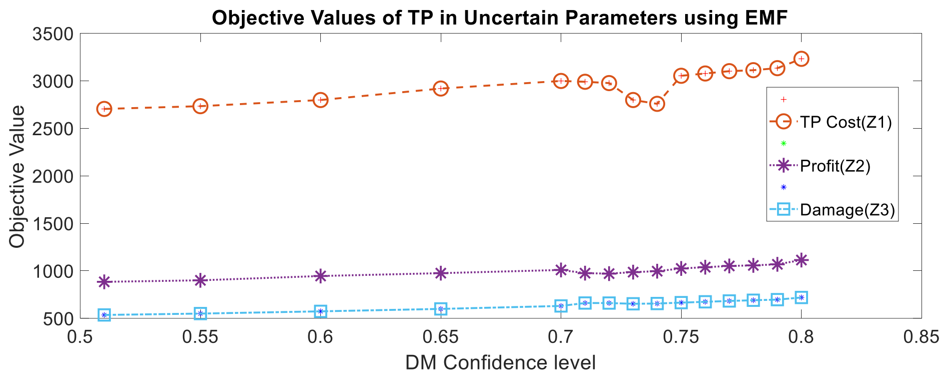

Figure 3. Confidence level versus objective value for exponential membership function.

3.2. Illustrative example using hyperbolic membership function

Table 6. Crisp values for the uncertain transportation data for confidence level $\omega = 0.80$.

Entity	Data matrix				
Crisp value for cost C_{ij}^1:		D_1	D_2	D_3	D_4
	S_1	21.52	19.52	24.28	26.28
	S_2	10.76	13.52	17.28	13.76
	S_3	24.28	22.28	25.52	24.52
Crisp value for the transformed profit C_{ij}^2:		D_1	D_2	D_3	D_4
	S_1	5.76	7.14	4.76	3.38
	S_2	6.76	6.14	5.38	4.76
	S_3	9.76	9.14	9.52	11.52
Crisp value for damage cost C_{ij}^3:		D_1	D_2	D_3	D_4
	S_1	4.76	4.76	3.76	6.52
	S_2	3.76	6.76	4.76	4.76
	S_3	5.14	3.76	4.76	6.14
Crisp value for demand:		b_1	b_2	b_3	b_4
		42.3	39.0	38.8	42.28
Crisp value for supply:		a_1	a_2	a_3	
		52.0	56.2	66.96	

For $\omega = 0.80$ Table 2 can be changed to Table 6 respectively.

Now, the solution of each single objective of the TP using "TORA" are

$$X^1 = (0,\ 39,\ 13,\ 0,\ 42,\ 0,\ 14,\ 0,\ 0,\ 26,\ 28)$$
$$X^2 = (10,\ 0,\ 0,\ 42,\ 0,\ 17,\ 39,\ 0,\ 32,\ 22,\ 0,\ 0)$$
$$X^3 = (13,\ 0,\ 39,\ 0,\ 29,\ 0,\ 0,\ 27,\ 0,\ 39,\ 0,\ 15)$$

Using the results, we have the pay-off matrix as follows in Table 7:

Table 7. The Payoff matrix.

	$Z_1(X)$	$Z_2(X)$	$Z_3(X)$
$X^{(1)}$	3071.56	1260.98	754.76
$X^{(2)}$	3489.84	1027.16	869.2
$X^{(3)}$	3146.96	1114.34	984.82
M_k	3489.84	1260.98	984.82
L_k	3071.56	1027.16	754.76

From the pay-off matrix, one can identify the maximum M_k and the minimum L_k thresholds for each objective.

$$\left.\begin{array}{l}\text{i.e., } 3071.56 \le Z_1 \le 3489.84, \quad 1027.16 \le Z_2 \le 1260.98 \\ \text{and } 754.76 \le Z_3 \le 984.82\end{array}\right\}. \tag{10}$$

The hyperbolic membership function can be represented as follows:

$$Z_k(X)\alpha_k + \eta \le \left(\frac{M_k + L_k}{2}\right)\alpha_k \text{ where } \alpha_k = \frac{6}{M_k - L_k} \tag{11}$$

Based on Eq (11), one can have the hyperbolic membership function as follows:

$$Z_1(X)\left(\frac{6}{M_1 - L_1}\right) + \eta \le \left(\frac{M_1 + L_1}{2}\right)\left(\frac{6}{M_1 - L_1}\right)$$

$$Z_1(X)\left(\frac{6}{3489.84 - 3071.56}\right) + \eta \le \left(\frac{3489.84 + 3071.56}{2}\right)\left(\frac{6}{3489.84 - 3071.56_1}\right)$$

$$\left.Z_1(X)\left(\frac{6}{418.28}\right) + \eta \le \left(\frac{6561.4}{2}\right)\left(\frac{6}{418.28}\right)\right\} \tag{12}$$

$$Z_2(X)\left(\frac{6}{M_2 - L_2}\right) + \eta \le \left(\frac{M_2 + L_2}{2}\right)\left(\frac{6}{M_2 - L_2}\right)$$

$$Z_2(X)\left(\frac{6}{1260.98 - 1027.16}\right) + \eta \le \left(\frac{1260.98 + 1027.16}{2}\right)\left(\frac{6}{1260.98 - 1027.16}\right),$$

and

$$\left.Z_2(X)\left(\frac{6}{233.82}\right) + \eta \le \left(\frac{2288.14}{2}\right)\left(\frac{6}{233.82}\right)\right\} \tag{13}$$

$$Z_3(X)\left(\frac{6}{M_3 - L_3}\right) + \eta \le \left(\frac{M_3 + L_3}{2}\right)\left(\frac{6}{M_3 - L_3}\right) \tag{14}$$

$$Z_3(X)\left(\frac{6}{984.82-754.76}\right)+\eta \leq \left(\frac{984.82+754.76}{2}\right)\left(\frac{6}{984.82-754.76}\right)$$

$$Z_3(X)\left(\frac{6}{230.06}\right)+\eta \leq \left(\frac{1739.58}{2}\right)\left(\frac{6}{230.06}\right)\Big\}.$$

Using Eqs (12–14) the hyperbolic membership function can be written as an equivalent crisp model as follows:

Maximize ξ

Subject to:

$$\left(\begin{array}{l}21.52x_{11}+19.52x_{12}+24.28x_{13}+26.28x_{14}+10.76x_{21}+13.52x_{22}+17.28x_{23}\\+13.76x_{24}+24.28x_{31}+22.28x_{32}+25.52x_{33}+24.52x_{34}\end{array}\right)\left(\frac{6}{418.28}\right)$$

$$+\eta \leq \left(\frac{6561.4}{2}\right)\left(\frac{6}{418.28}\right)$$

$$\left(\begin{array}{l}5.76x_{11}+7.14x_{12}+4.76x_{13}+3.38x_{14}+6.76x_{21}+6.14x_{22}+5.38x_{23}\\+4.76x_{24}+9.76x_{31}+9.14x_{32}+9.52x_{33}+11.52x_{34}\end{array}\right)\left(\frac{6}{233.82}\right)$$

$$+\eta \leq \left(\frac{2288.14}{2}\right)\left(\frac{6}{233.82}\right)$$

$$\left(\begin{array}{l}4.76x_{11}+4.76x_{12}+3.76x_{13}+6.52x_{14}+3.76x_{21}+6.76x_{22}+4.76x_{23}\\+4.76x_{24}+5.14x_{31}+3.76x_{32}+4.76x_{33}+6.14x_{34}\end{array}\right)\left(\frac{6}{230.06}\right)$$

$$+\eta \leq \left(\frac{1739.58}{2}\right)\left(\frac{6}{230.06}\right)$$

$$x_{11}+x_{12}+x_{13}+x_{14} \leq 52$$
$$x_{21}+x_{22}+x_{23}+x_{24} \leq 56.2$$
$$x_{31}+x_{32}+x_{33}+x_{34} \leq 66.96$$
$$x_{11}+x_{21}+x_{31} \geq 42.3$$
$$x_{12}+x_{22}+x_{32} \geq 39$$
$$x_{13}+x_{23}+x_{33} \geq 38.8$$
$$x_{14}+x_{24}+x_{34} \geq 42.28$$
$$\eta = tanh^{-1}(2\xi-1)$$
$$\xi \geq 0, x_{ij} \geq 0 \text{ for all intger } i,j.$$

Using "LINGO" software, the optimal compromise solution is obtained as follows:

$X^* = (x_{11}=30.00,\ x_{12}=7.34,\ x_{13}=14.65,\ x_{14}=0.000,\ x_{21}=13.00,\ x_{22}=0.00,\ x_{23}=0.00,\ x_{24}=43.00,\ x_{31}=0.00,\ x_{32}=32.65, x_{33}=25.34,\ x_{34}=0.00\ )$`

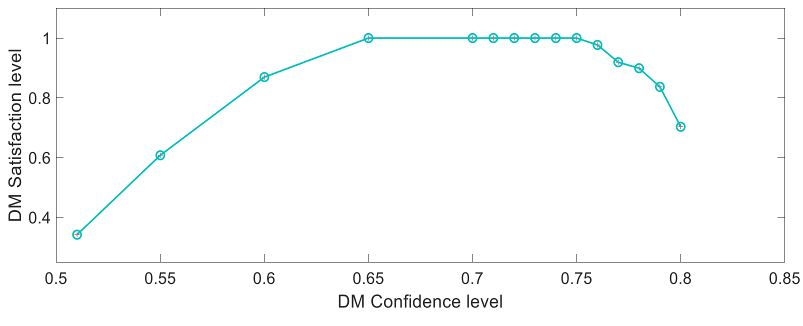

Figure 4. Satisfaction level versus Confidence level for hyperbolic membership function.

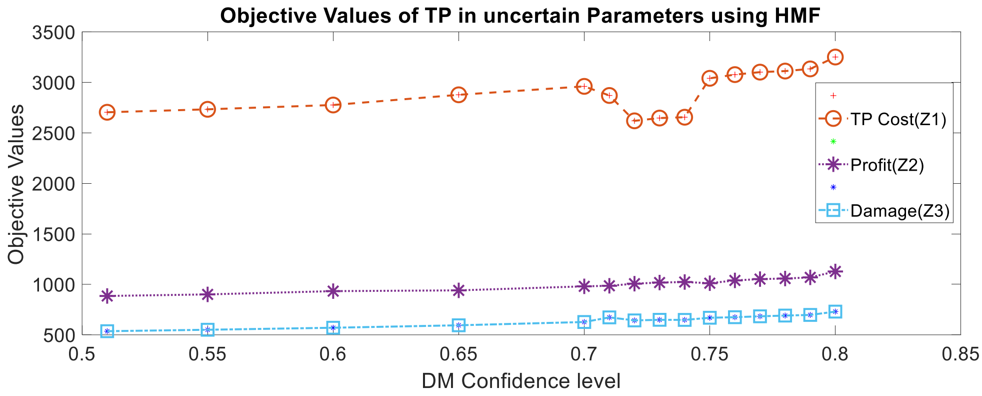

Figure 5. Confidence level versus objective value for hyperbolic membership function.

The values of the objective functions are $Z_1(X^*) = 3250.71$, $Z_2(X^*) = 1127.30$ and $Z_3(X^*) = 729.84$. The overall satisfaction of the DM is $\xi = 0.7030$, that is, the overall satisfaction is 70.30%. The following Table 8 shows the different objective values and the satisfaction level of the DM for different confidence levels.

Table 8. Results for hyperbolic membership function for different confidence levels.

ω	ξ	Objective values			Feasibility	Satisfaction (%)
		Z_1	Z_2	Z_3		
00-0.49	---	---	-----	---	Infeasible	---
0.50	Deviation vanishes and problem exists in certain parameters					
0.51	0.3416	2703.80	884.65	535.90	Feasible	34.16%
0.55	0.6076	2733.02	900.27	550.17	Feasible	60.76
0.60	0.8693	2775.79	932.65	570.11	Feasible	86.93%
0.65	1.00	2876.73	940.12	594.44	Feasible	100%
0.70	1.00	2960.22	980.15	626.90	Feasible	100%
0.71	1.00	2870.00	985.55	672.69	Feasible	100%
0.72	1.00	2618.98	1006.22	642.82	Feasible	100%
0.73	1.00	2646.48	1017.60	648.61	Feasible	100%
0.74	1.00	2654.76	1022.94	647.75	Feasible	100%
0.75	1.00	3040.13	1011.05	668.96	Feasible	100%
0.76	0.9770	3076.68	1038.73	674.50	Feasible	97.70%
0.77	0.9190	3101.21	1051.87	683.32	Feasible	91.90%
0.78	0.8990	3111.67	1057.36	690.65	Feasible	89.90%
0.79	0.8367	3133.11	1068.76	697.00	Feasible	83.67%
0.80	0.7030	3250.71	1127.30	729.84	Feasible	70.30 %
0.81-1.0	---	---	---	---	Infeasible	---

Note: '---'indicates not applicable.

Figure 4 exposes an explicit view of the DM's satisfaction level against the different confidence level to achieve the crisp value of the uncertain parameters using the hyperbolic membership function (HMF) of the fuzzy programming. It shows the satisfaction of the DM is dramatically increasing from confidence level 0.51 to 0.65 and then almost static 0.65 to 0.75 and then have a nose down from 0.76 to onward. However, the present transportation problem, solution is not feasible for confidence level 0.00 to 0.49 and for 0.81 to onwards. The reason behind of this result is that the demand becomes greater than supply for choosing the confidence level 0.81 to onwards. The DM shows 100% satisfaction for the confidence level 0.65 to 0.75 and for the rest it decreases in every cases. Deviation of the parameter vanishes and the problem becomes in certain environment when DM chooses the confidence level 0.50.

Again, Figure 5 implies all three objectives for the current problem using the hyperbolic membership function are increasing except for the TP cost and damage at point 0.70. In this point, TP cost goes down slightly and again goes up before going almost straight for the next three point. The damage cost using hyperbolic membership function for this present transportation problem showing increasing behaviour everywhere of the confidence level where the problem has feasible solution.

4. Comparative results and discussions

From the previous discussions, it is clear that there are slight differences in the objective values and the corresponding satisfaction level of the DM for applying different membership function of fuzzy approach. In Table 9, we will have an explicit overview of the objective values applying the fuzzy membership functions for various choice of the DM in between the confidence level of 0.70–0.85.

In Figures 6 to 9, we have a very concrete observations of various parameters of the uncertain transportation problem using fuzzy membership functions. Figure 6 reveals the satisfaction level of the DM for various confidence level using the fuzzy membership functions, linear, exponential and hyperbolic. It is clear from the graph that the satisfaction of DM always better for hyperbolic other than the exponential and linear. For hyperbolic membership function, DM satisfaction level is 100% through confidence level 0.65 to 0.75. On the other hand, for linear and exponential, the DM satisfaction pick their high at 0.72. Again, Hyperbolic and exponential membership function shows their infeasibility after 0.80 where linear is solvable up to 0.85.

Figure 7 unwrapped the transportation cost against the confidence level for the fuzzy membership functions. All the three functions have shown almost same pattern throughout the region except 0.70 to 0.75. The hyperbolic membership function has shown more fluctuation regarding the satisfaction of DM then other two in the area 0.72 to 0.74 whereas the others have same oscillation on that region. All the three membership functions have given equal TP cost from 0.75 to 0.80. The DM have highest satisfaction for transportation cost using the membership functions for confidence level 0.72.

Figure 8 represents the profit corresponding to the confidence level using the fuzzy membership functions. All the membership functions have shown increasing behaviour throughout the interval except at 0.70 where they have slight downward pattern and then again increasing continue towards the end. From the graph it is observed that linear membership function has shown maximum number of profit within our expectation.

From Figure 9, we observed the damage cost of the transportation problem when transporting using the mentioned fuzzy membership functions in uncertain parameters. The graphs presented have shown almost same objective values at all the points where solutions are feasible except have some more increment at 0.71 with immediate nose at 0.72 continue up to 0.74. The DM have the height satisfaction for the damage cot around 660.00 for the chosen confidence level 0.72.

Table 9. Results of the membership functions for different confidence level of decision maker.

Hyperbolic membership function					Exponential membership function					DM confidence level
Z_1	Z_2	Z_3	Satisfaction (%)	Feasibility	Z_1	Z_2	Z_3	Satisfaction (%)	Feasibility	
2960.22	980.15	626.90	100%	Feasible.	2998.33	1009.23	630.55	53.65%	Feasible.	0.70
2870.00	985.55	672.69	100%	Feasible.	2986.69	975.66	661.32	89.95%	Feasible.	0.71
2618.98	1006.22	642.82	100%	Feasible.	2975.95	970.74	660.97	94.20%	Feasible.	0.72
2646.48	1017.60	648.61	100%	Feasible.	2797.78	986.31	653.32	90.38%	Feasible.	0.73
2654.76	1022.94	647.75	100%	Feasible.	2757.58	996.35	655.95	86.29%	Feasible.	0.74
3040.13	1011.05	998.96	100%	Feasible.	3052.88	1025.23	665.4	69.90%	Feasible.	0.75
3076.68	1038.73	674.50	97.70%	Feasible.	3076.89	1038.30	674.18	63.29%	Feasible.	0.76
3101.21	1051.87	683.32	91.90%	Feasible.	3101.43	1051.45	683.00	57.35%	Feasible.	0.77
3111.67	1057.36	690.65	89.90%	Feasible.	3111.67	1057.35	690.65	55.49%	Feasible.	0.78
3133.11	1068.76	697.00	83.67%	Feasible.	3133.11	1068.76	697.00	51.25%	Feasible.	0.79
3250.71	1127.30	729.84	70.30%	Feasible.	3231.33	1113.65	719.65	5091%	Feasible.	0.80
-	-	-	-	Infeasible	-	-	-	-	Infeasible	0.81
-	-	-	-	Infeasible	-	-	-	-	Infeasible	0.82
-	-	-	-	Infeasible	-	-	-	-	Infeasible	0.83
-	-	-	-	Infeasible	-	-	-	-	Infeasible	0.84
-	-	-	-	Infeasible	-	-	-	-	Infeasible	0.85

Note: ‘ ---’ indicates not applicable.

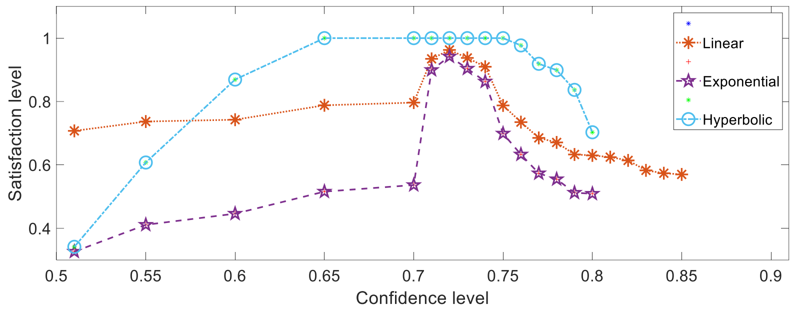

Figure 6. Satisfaction level versus Confidence level for the three membership functions.

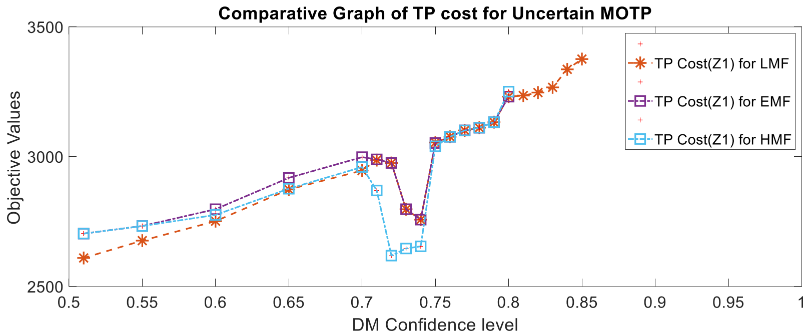

Figure 7. Confidence level versus Objective (Z1: TP Cost) for uncertain MOTP.

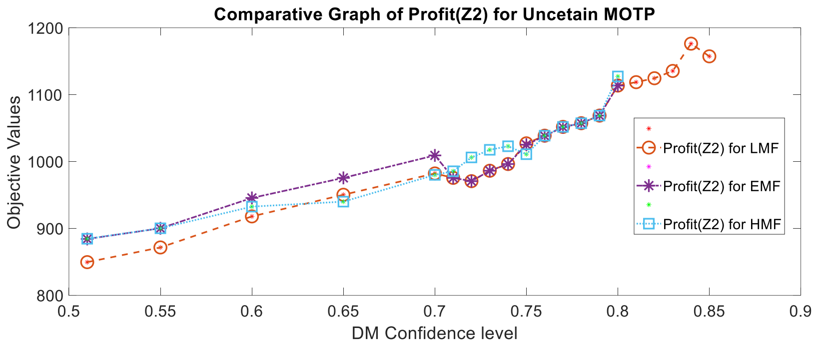

Figure 8. Confidence level versus Objective (Z2: Profit) for uncertain MOTP.

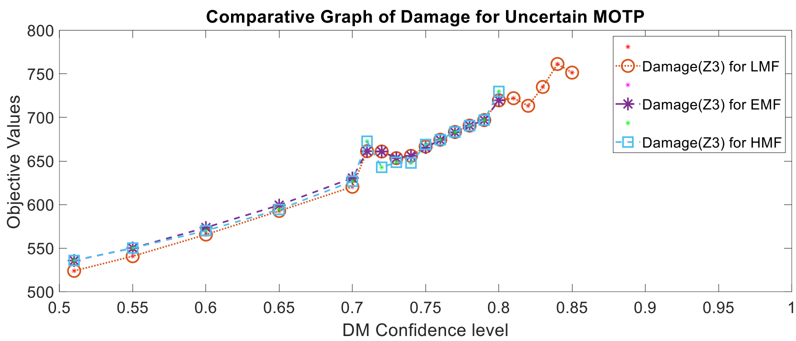

Figure 9: Confidence level versus Objective values (Z3: Damage) for uncertain MOTP.

The problem we have discussed can be adopted in many logistic applications including procurement, production, sales, recycling, or even consolidation of shipments. The implications of considering the problem uncertain parameters can be useful for reflecting the real situation while solving it optimally. For the current turbulent working environment, DMs/managers are seeking for tools that can guide them to take the right decision. Decision can be taken by applying this method according to the identified confidence level. The model permits the DMs to calculate the satisfaction level for different uncertain situations. Consequently, the DM can predict his/her desired goal in percent relying on his confidence level before taking decisions. This methodology can also be used by owners, contractors especially in the planning or risk analysis phases.

This study provides a significant contribution that achieves many managerial implications within the logistic sectors:

- It can be used by the DM as an optimization tool. In this case, the DM can select the solution according to the desired objective that could be transportation cost, damage cost or even the profit.
- It presents a decision support system for working with uncertain transportation environment. In which the DM can compute the expected satisfaction level based on his certainty of parameters estimation (i.e., confidence level).
- DM should estimate the confidence level properly in order to avoid the infeasibility of the transportation problem. Whereas, the confidence level affects the variance of the parameters that may lead to increase the demand against the supply.
- The hyperbolic membership function gives stable results against the exponential and linear functions, therefore, the DM could rely on the hyperbolic function.

5. Conclusions

In this study, uncertain MOTP has been studied throughout the proposed methods. The unknown uncertain parameters were resolved by uncertain normal distribution. Multiple number of models has been designed for MOTP in uncertain parameters using the fuzzy non-linear membership functions with their mathematical algorithm and we have shown the applicability of this algorithms by a heuristic example of same data table with a variety of confidence level of the DM for each case. Sometime the problem becomes infeasible for a chosen confidence level due to the violation of the feasibility condition of the transportation problem. The satisfaction in percent of DM were obtain for the chosen confidence level. Relying on the comparative result, Results show that the hyperbolic function has shown multiple time 100% of the DM in a region but solution is not feasible for a large scale of confidence level but exponential membership function has given more feasible and continuously suitable solution to the DM than hyperbolic. However, the hyperbolic function gives more satisfaction level than the other functions. Priority is one of the realistic features for the DM and hence priority/weight can be incorporated to goal as an uncertain parameter in the current MOTP. Fractional differential equation can be applied to estimate the uncertain MOTP with specific goal to each objectives converting the fuzzy function into fractional order differential equations.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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