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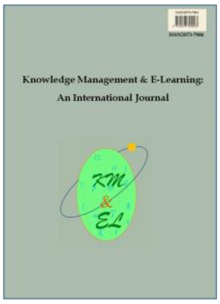

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Product selection by consumers utilizing the membership values

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Abstract: Assessment and assortment of the most favorable, competent, and trustworthy products is getting to be more and more important for businesses in the current scenario, so the majority of the production industries do their utmost to achieve the requirements, standards, and expenditure patterns of the consumer to attain their expectations. Approaches are made to fulfill the needs of the consumer, as shown by the creation of diverse technologies for improving the capability to manage complex optimizations regarding decision attributes that involve uncertainty. The procedure of detecting the best alternative from all the feasible options is the basis of the decision-making problem. This article recommended two approaches: (a) multi-objective decision making by utilizing fuzzy technique and (b) fuzzy TOPSIS (Technique for Ordering Preferences by Similarity to Ideal Solution) method for choosing the best product while many products with many attributes are present in the marketplace. As per the need of the customer priority and their satisfaction level, this paper used linguistic quantifiers to evaluate the weight of the objective that is represented in triangular or trapezoidal fuzzy numbers. To demonstrate the technique used for the suggested approaches a case study was conducted. The simulation result shows a specific path that fuzzy logic can evolve in the problem of decision making as well as in the planning process in the product selection practice. Solutions obtained by employing both techniques were compared, which urges that the fuzzy TOPSIS approach provides more accurate information than the fuzzy MODM (multi-objective decision making) approach to find the best product from among many available products. This work helps to assist the managers to adopt suitable techniques while planning to design the product according to the customer requirements so that the consumer can get the best product as per their objectives.

Keywords: MODM; TOPSIS; Linguistic variables; Triangular fuzzy; Frapezoidal fuzzy; Membership function; Fuzzy ranking; Fuzzy preference relation

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1. Introduction

For expanding the business, it is necessary to know the views of the customers because customers are the rulers in the business world; then the decision should be made by considering the needs of the customers (Drucker, 2007). Focusing on what the customer wants is important because they have innovative ideas that bring positive effects to the business. The activities carried out by the organization are aimed at satisfying the needs of customers. Consumer satisfaction is determined through an evaluation process that shows how the goods produced or the assistance provided by the organization meet the maximum needs of consumers. One of the most important measures of purchasing habits and customer loyalty is consumer satisfaction (Patel, 2018).

Consumers have specific needs; they evaluate products in order to find the one that can provide the greatest level of satisfaction. Nowadays, while social media plays a substantial role in making decisions, it's vital to keep an eye on the quality of customer service that the organization provides (Wang et al., 2018). As the product features are primarily contradictory, imprecise, and inadequate, it is extremely hard to fulfill all the customer's features and attributes at the same time. At this moment, the main attempt made by the customer is to be satisfied with the majority of the attributes instead of all the attributes, and this can be achieved properly with the utilization of the fuzzy logic concept. Fuzzy logic has the potential to illustrate such different degrees of satisfaction where each product shows a different level of satisfaction. Fuzzy logic uses linguistic quantifiers to calculate the objectives that are defined qualitatively as well as rank the products according to the buyer's priority level (Mohanty & Bhasker, 2005).

Lofti A. Zadeh introduced Fuzzy Logic (Zadeh, 1965), which is regarded as the numerical technique for defining ambiguity and vagueness in day-to-day activities. The fuzzy logic process allows for (a) the examination and application of multiple products attributes, and (b) the comparison of all products attributes with the customer's choices to determine the degree of similarity between the products available and the customer's needs. So the fuzzy approach helps to find the best prototype product. A large part of the market and customers follow the fuzzy approach to create customer-oriented, sustainable and innovative new product designs. The major initiative to develop fuzzy logic is that it provides an easy route to reach a specific solution based on information that is uncertain, inaccurate, and vague. The models of multi-level decision-making constructed through fuzzy logic are based on the weight and ranking method to solve fuzzy decision-making. Concerning MCDM (Multiple Criteria Decision Making), Bellman and Zadeh (1970) and Zimmermann (2011) proposed fuzzy sets to find a precise solution where information is uncertain. MCDM denotes the positioning, selection, and prioritization of alternatives in various conflicting situations (Fenton et al., 2006).

The techniques used for ranking alternatives can be represented in two steps. The former is based on evaluation of triangular and trapezoidal fuzzy numbers to get the degree of membership and then rank the alternatives according to the maximum degree of membership. That is the alternative having the highest degree of satisfaction is the best one. Bellman and Zadeh (1970) performed extensive research regarding the application of fuzzy numbers within the area of multi-objective decision making. The most important issue while resolving the problem of multi-objective decision making is to get relevant knowledge relating to the objectives satisfaction level along with ranking the alternatives of relative significance. This article presents techniques for multi-objective decision making where the objectives are different for different individuals as per their priority basis. These objectives help in decision making and permit business organizations to decide what their main focus should be. There are a number of objectives present while performing the decision-making process, and each objective will help to find the path and work towards achieving the target. The benefits of having clear business objectives are that it allows you to measure success and it can also improve the quality of decision making within an organization.

The main objective of the paper is to find the best product according to customer perception that satisfies them most. This paper presents an integrated methodology of obtaining customers' objectives, and the objectives are measured through the product attributes for the purpose of best product selection. The requirements of the methodology do not only include the techniques to select the best product but also represent the linguistic data that is obtained from the customers, which corresponds to their requirements. This incorporation poses a challenging problem of decision-making in an uncertain environment. In addition, there are numerous and conflicting objectives to be considered (Bose, 2009). By using fuzzy numbers, the Fuzzy MODM and Fuzzy TOPSIS are proposed in the decision-making process to rank the products with respect to a number of attributes. This will help in conducting the selection process and provide the decision maker with an optimal solution.

Any organization can greatly benefit from its people's skills for sharing, innovating, re-using, collaborating, and learning information. It is a good idea to strengthen knowledge management by implementing the right knowledge management techniques in your company so that you can fully reap the benefits. Improve the decision-making process (When making decisions, business collaboration tools facilitate access to the opinions and experiences of different people with different opinions and judgments.) (Cordes, 2016); increase customer satisfaction (knowledge sharing helps to increase the value of products and services presented to customers); support for innovation (stimulation of innovation required to develop the organization and meet changing business needs); acceleration of access to knowledge and information (Knowledge Management simplifies the operation of finding the information you need or people who have it); acceleration of delivery to customers (by sharing knowledge and innovation internally, you can significantly reduce the time it takes to provide a proposal, product, or customer service.), knowledge management makes it easier to find the information or people who have the information you need and increases efficiency. An improved knowledge management assists in discovering and reprocessing the vital information and reserves throughout the business. It helps in:

- Build improved products as well as services
- Build up superior strategies
- Increase gain

This knowledge management helps in decision making and a few questions are raised by it while selecting the best product in the market. These are given below:

- What is the information available on the market for the products?
- How to evaluate customers' objectives that they want to be present in a product?
- Are the customers satisfied with the available products?
- How could we improve the products that satisfy customer's needs?
- What features do customers want to see in the products they are interested in?
- What are the customer's requirements?
- Which product is the most popular one that is chosen by the majority of customers?
- Is the retailer known for which product is the best product that he launched on the market?

The novelty of the work is to find the best product which fulfills maximum objectives of the buyers wants. To achieve this, we have considered both the fuzzy representations of the numbers that are provided by the retailer and the buyers' preference ordering which are represented through triangular and trapezoidal fuzzy numbers that are revealed in the appendix. Then we get the membership value of each feature/attribute from both the fuzzy number representations. To find out the best product, we proposed fuzzy MODM method to evaluate this membership value to get the highest value. The optimal product is one having highest value is considered as the best product which fulfills maximum objectives of the buyer. But one difficulty arises while using this fuzzy MODM method that two different products occupy the same position, which makes it difficult for the retailer to identify which one is the better one. So, for this, we recommend fuzzy TOPSIS method, where we utilize the Shannon entropy method to obtain the weight of these membership values by intensely calculating each feature/ attribute for each product. Lastly, we utilize closeness coefficient to rank all the products uniquely by estimating the ideal solutions.

The process of fuzzy decision-making demonstrates and maintains the need to utilize the fuzzy technique to rank the alternatives (Coroiu, 2015). Several ranking techniques for fuzzy numbers have been built, but many of them ignore some key issues like ranking order, control, and preference of fuzzy numbers, along with the simplicity of using the algorithm for ranking. So, this paper contributes to building an innovative, perfect, useful, and competent approach that is efficient enough to handle fuzzy numbers and their membership values. The technique used in the ranking process for the fuzzy decision-making method presented in this paper uses the MODM model, which estimates both triangular and trapezoidal fuzzy numbers. The proposed method uses the membership values, which are generated by calculating both types of fuzzy numbers. A case study is provided which shows the performance of each technique in the product selection decision-making process.

This paper briefly reviews the earlier research investigations centered on the utilization of fuzzy MODM and TOPSIS processes in support of product selection as well as their ranking. The article is arranged like this: Section 2 provides a review of existing work. Section 3 describes the decision-making techniques used for product analysis. Section 4 gives details about the fuzzy-based techniques for product selection. Section 5 contributes a case study based on fuzzy decision-making approaches for product selection. Section 6 shows the result analysis based on a comparison of both techniques. Section 7 presents applications of the Fuzzy TOPSIS method, and Section 8 concludes the paper.

2. Review of existing work

To minimize the risk of loss, to increase profit of buyers along with to set up a prolonged and honest association amongst buyers and retailers is the ultimate goal for the development of product selection process (Monczka et al., 2015). Many of the models are designed for different types of the selection process, but the ranking method is mainly developed on a simple understanding of the decision-making process (De Boer et al., 1998; Lee et al., 2001). Many of the models appear to underestimate the complexity and unstructured nature of the daily needs-based decision making. The majority of the available decision-making models (Devi et al., 2020a) simply consider the quantitative criteria for the product selection process but ignore many useful features like partial information, objectives that are presented qualitatively, and priorities explained incorrectly, which are not usually considered for the process of decision-making.

The selection process generally includes a variety of steps like estimation, differentiation, and making the decision. During all these steps, application of fuzzy logic helps to achieve more accurate information regarding customer satisfaction. There are numerous strategies, techniques, and representations of various models that have come into existence to support the product selection process, such as the TOPSIS concept, which is used to get the result for solving the issues regarding product selection in a fuzzy environment (Chen, 2000), ELECTRE (Elimination and Choice Translating Reality) methods are used to discard some unacceptable alternatives to the problem and identify a set of solutions to a decision-making problem (Roy, 1991), Game theory methods used to analyze and solve decision problems to find the optimal solution (Devi et al., 2021; Peldschus & Zavadskas, 2005), the Multi-Objective Optimization on basis of Ratio Analysis (MOORA) technique computes the value of attributes criteria to assist decision-makers to make the right decision (Kalibatas & Turskis, 2008), and so on. Over the last few years, considerable possibilities have been developed for the applicability of these models to get benefits from fuzzy logic for the process of product selection, which suggests that product selection is the field having immense potential for the application of fuzzy logic.

Ordering fuzzy numbers into a single series is a very difficult task because their values are uncertain. So, a ranking process is used which produces a plausible ranking order of specified fuzzy numbers (Lee & You, 2003). Due to the significance and usefulness of the ranking process, numerous techniques have been evolved, which include the relevance of fuzzy ranking in multi-criteria decision making (Baas & Kwakernaak, 1977; Barajas & Agard, 2008). Also, for the fuzzy ranking method, fuzzy preference relations have been broadly applied (Delgado et al., 1988; Lee, 2000; Modarres & Sadi-Nezhad, 2001). Additional techniques include particular notions such as membership functions of triangular numbers (Chang, 1981), as well as set maximization and minimization techniques (Chen, 1985). The fuzzy ranking procedure contributed by Lee and You (2003) for fuzzy numbers includes a variety of exciting operations and indices like fuzzy assessment value, fuzzy satisfaction function, level of estimation, defuzzification relative index, and degree of defuzzification. (Ma & Li, 2008) offered a novel approach that comprises range reduction techniques and fuzzy preferences. For calculating fuzzy ranking techniques (Yuan, 1991) approved four criteria and also proposed a better ranking technique formulated on representations of fuzzy precedence relations, logical basis of fuzzy prioritization, and robustness, along with visibility.

By noting the environmental factors, Awasthi et al. (2010) proposed the fuzzy TOPSIS method to illustrate the problem of the selection process. To achieve more proficiency in steel manufacturing, Kumar et al. (2018) adapted the fuzzy TOPSIS process for the raw materials selection process. Sen et al. (2018) used intuitionistic MOORA, intuitionistic fuzzy grey relational analysis (IF-GRA), and intuitionistic fuzzy TOPSIS techniques to help out with the selection of suppliers (Sen et al., 2018). In a model, Zeydan et al. (2011) joined both the AHP (analytic hierarchy process) as well as the fuzzy TOPSIS technique, which first evaluates weights of criteria based on the fuzzy TOPSIS technique to rank a group of probable suppliers (Zeydan et al., 2011).

Rather than building a weighted decision matrix, Deng et al. (2000) adopted the modified TOPSIS technique to use weighted Euclidean distances for which positive ideal (PI) and negative ideal (NI) solutions are not correlated by a weighted decision matrix. To achieve the ideal solutions for the MCDM problems in a fuzzy environment, Mahdavi et al. (2008) recommended a TOPSIS model, for which they utilize a distance measure for fuzzy computational assessment in addition to the alternative lower bound through fuzzy triangular numbers. In the interval arithmetic fuzzy TOPSIS technique, Chu and Lin (2009) illustrated the rating of options against criteria along with criteria weights by linguistic expressions. These are specified through fuzzy numbers accompanied by interval arithmetic of fuzzy numbers to help in evaluating the membership function of all weighted fuzzy ratings.

For calculating the alternatives, Wang and Lee (2009) recommended the fuzzy TOPSIS approach, where they use an entropy-based objective weighting technique as well as a subjective weighting technique. Formulated by fuzzy TOPSIS and fuzzy LINMAP (Linear Programming Technique for Multidimensional Analysis of Preference) techniques, Ebrahimnejad et al. (2010) suggested a method for ranking where the ranking procedure is accomplished through fuzzy Euclidean distances and triangular fuzzy numbers. For the ranking and choosing of robots, Kumar and Garg (2010) proposed a predetermined quantitative model which followed a distance-based approach (DBA).

Based on a comprehensive literature review, considering the multi-objective structure of the product selection problem and the vagueness in the real environment, fuzzy TOPSIS is considered to be suitable and simple enough to select the best product. Although there are many papers on fuzzy multi-criteria supplier selection problems in the literature, few of them discuss the fuzzy multi-objective decision-making model. First, this paper solves a fuzzy multi-objective mathematical model for the product selection problem. The advantages of the fuzzy multi-objective mathematical model are the assignment of multiple objectives by considering multiple targets under multiple constraints. For example, companies can insert fuzzy multi-objective mathematical models with multiple objectives and additional constraints imposed on their importance for their practical applications. Proposed fuzzy multi-objective mathematics models in this study can allow goals and constraints according to their objectives. They can also achieve optimal solutions to achieve their goals by solving fuzzy multi-objective mathematical models.

The major gap in the literature is the difference between the customer's expectations of the product they are provided with and the business's organizational plans to provide the exact product. Another bottleneck is establishing customer specific target objectives by considering business missions and goals. So this paper establishes customers' objectives that they want in a product in a proper measurable structure and uses the Fuzzy MODM and Fuzzy TOPSIS methods to rank the products according to their preferences.

3. Decision-making techniques for product analysis

The fuzzy technique introduces a tool based on mathematics that holds uncertainties as well as indistinct words like "not very clear", "probably so", "very likely" and so on. Regular set theory shows that the membership value of an element held by the same set is formed on two-valued Boolean logic, whereas fuzzy set theory is established on multivariate fuzzy logic that interacts with membership degree. Here the valid unit interval, i.e., [0, 1], expresses the degree of membership of an element. Fuzzy numerals are a fuzzy subset of real numbers, usually depicted as triangular fuzzy numbers, trapezoidal fuzzy numbers, and Gaussian fuzzy numbers (Stević et al., 2019).

3.1. Multi criteria decision-making

For the selection of top possible alternative, a Multi-Criteria Decision Making (MCDM) technique is used for achieving the desired goal. This technique offers objectivity and measures the precedence value of the alternatives. According to the precedence value, the most favorable alternative is encountered, which can help in accomplishing the decision objectively. Several researchers divide this MCDM model into two parts: multi attribute decision making (or MADM) and multi objective decision making (or MODM). According to MODM analysis, decision problems contain continual decision space. Common example to represent this is the numerical coding problems of having multi-objective tasks. In contrast, MADM focuses on the decision problems having disconnected decision spaces. Within these decision problems, the alternative sets are predetermined. Even though MADM techniques may vary extensively, most of them have some common features (Chen & Hwang, 1992).

Until now, lots of practices have been developed to resolve the multi-level decision making issues. The decision-making method entails analyzing and selecting options based on the decision-makers principles and preferences (Devi et al., 2020b). Decision-making reveals that we not only find out as many of the feasible alternatives as possible but also select the one that better suits our aims, objectives, wants, and ethics.

3.2. Multiple objective decision-making

The majority of activities need decision making, which includes many feasible contradictory objectives. To overcome such situations, intelligent systems require designing and implementing algorithms that can competently find various paths to achieve the goal where such objectives are present.

Multi-objective decision problems generally involve the selection of a single alternative from the set X of probable alternatives that have a fixed set A of attributes. When X is limited, the following steps take place: (1) calculating the degree at which a variety of objectives fulfill every alternative; (2) gathering the objectives to create a global objective; and choosing the top alternative by the decision function D. But when X is extremely big or unbounded, then the difficulty of estimating the set of alternatives usually needs various kinds of computations or numerical programming elucidations.

The multi-objective decision-making technique is regarded as an efficient decision-making procedure. It permits a decision-maker to describe a decision, identify their objectives, outline the situation, and methodically estimate the decision. When decision alternatives and decision criteria are defined accurately, it is easy for the analyst to handle complex decisions. Once the framework has been established, the alternatives are then ranked against the decision objectives. The objective weights are then evaluated according to the decision maker's priority level, which results in the ultimate assignment of values to the decision alternatives.

3.3. TOPSIS (Technique for ordering preferences by similarity to ideal solution)

TOPSIS technique is recommended for providing a solution to the MCDM problems through "m" alternatives in a mathematical structure with "n" points in a matrix form. This technique formed on the idea that the alternatives that are selected must possess the least space to the positive ideal solution –PIS, in addition to also maintaining the largest space to the negative ideal solution, NIS (Jahanshahloo et al., 2006). Finally, such a technique selects the alternative having the highest resemblance to the PIS (Wang & Chang, 2007). In the conventional TOPSIS technique, every matrix column is multiplied by the relative criterion of importance to produce a weighted normalized decision matrix. At the end, the total priority of the alternative is measured through the Euclidean distance from PIS and NIS. These distances also rely on the respective weight of every criterion which must be present in the calculation. A PIS always increases the benefit criteria while diminishing the cost criteria, but NIS does the reverse (Wang & Lee, 2007).

Practically, the TOPSIS technique can be productive when the alternative set used is limited to resolving any selection issues. It is very simple to comprehend and apply. Also, it retains a strong logic which shows the foundation of an individual's preference along with it demonstrates one of the finest techniques by directing to the rank reversal problem.

4. Fuzzy based techniques for product selection

In many crucial circumstances, it is frequently advisable to more cautiously examine the alternatives with regard to their significance before selecting them and performing an analysis on them. This process includes methodically describing the objectives and recognizing measure attributes to specify the degree of satisfaction of the objectives for a variety of alternatives. Then an investigation of those alternatives can progress, quantitatively confirming the assumption and evaluation of the decision process.

The major attention of this paper is on:

- To generate an accurate set of objectives which are measured by the attributes?
- To find out the right attribute for calculating the degree to which each of those objectives can be attained by the alternatives,
- To achieve the mission proficiently and successfully, find the best alternative.

4.1. Multi objective decision making based on fuzzy techniques

This paper demonstrates a method that describes decision-based computation, which necessitates information regarding weights and the preference ranking (Yager, 1993). Suppose that the 'n' products accessible from the retailer are P1, P2, P3, ..., Pn and O_1, O_2, O_3, …., O_t which denotes "t" attributes. Determine O_i as the i^{th} attribute so for product P, the degree of membership for O_i is represented as $\mu_{Oi}(P)$ defines the level at which P fulfills the properties required for these attributes. In this situation, a certain decision function is used that executes all the decision objectives or attributes.

The intersection of all the attributes sets represented by the decision function as in Eq. (1)

$$D_f = \{O_1 \cap O_2 \cap \ldots\ldots \cap O_t\} \qquad (1)$$

For every product P, the decision function D_f possess the grade of membership shown by Eq. (2)

$$\mu\, D_f(P) = \mathrm{Min}\{\mu_{O1}(P), \mu_{O2}(P), \mu_{O3}(P), \ldots\ldots \mu_{Ot}(P)\} \qquad (2)$$

So * P the optimum decision is going to be as in Eq. (3),

$$\mu_D(P^*) = \underset{p\in p}{\mathrm{Max}}\ \{\mu_D(P)\} \qquad (3)$$

To measure the decision maker's attitude regarding the impact of every objective needed to be present on the selected product, a preference set R must be connected to every attribute.

Suppose an element r_i present on the preference set {R}, where i = 1, 2, 3……. t.

So, the Decision function D_f that includes both attributes with preference is as in Eq. (4)

$$D_f = \mathrm{Max}\,(O_1, r_1) \cap \mathrm{Max}\,(O_2, r_2) \cap \ldots \cap \mathrm{Max}\,(O_t, r_t) \qquad (4)$$

For a particular product P, the decision calculus can take the following format as in Eq. (5)

$$\mathrm{Max}\,(O_i(a), r_i) = r_i \rightarrow (O_i(P)) = \overline{r_i} \cup O_i(P) \qquad (5)$$

The combined intersection as in Eq. (6) of t decision calculus represents the decision model as

$$D_f = \bigcap_{i=1}^{t} (\overline{r_i} \cup O_i) \qquad (6)$$

The alternative which possesses the highest value of D_f is the optimum solution *P.

4.2. TOPSIS method using fuzzy technique

Fuzzy TOPSIS is a technique that facilitates objective as well as efficient ranking of alternatives when multiple criteria are present. At this point, the fundamental representation of the prescribed fuzzy theory has been explained to demonstrate the Fuzzy TOPSIS method.

Since the criteria or parameters in multi-criteria decision problems are in an inappropriate format, it might generate troubles while calculating. Thus, to prevent such issues, fuzzy techniques are required. By utilizing fuzzy in TOPSIS for analysis of the criteria, the calculation process is more straightforward. Therefore, Fuzzy TOPSIS is a remedial technique that has an easy and feasible structure to include or exclude alternative solutions.

In 1981, Yoon and Hwang (1981) designed the TOPSIS technique, which is a multi-criteria decision analysis process. The normal TOPSIS procedure was developed on the idea where the preferred alternative must possess the lowest geometric distance from positive ideal solution, i.e., PIS, while showing more distance against the negative ideal solution, i.e., NIS. It is a technique that shows a compensatory aggregation technique that determines the weights of each criterion to compare a set of alternatives. To cope with fuzzy numbers, this technique has been adopted by many researchers from diverse fields (Kore et al., 2017; Wang et al., 2003; Chatterjee & Stević, 2019).

According to the features of Fuzzy TOPSIS, the alternative which is closest to Fuzzy PIS and most distant from Fuzzy NIS is selected as an optimal result. The finest performance values for every alternative are possessed by FPIS, while the poorest performance values are shown by FNIS.

Now the essential phases of the fuzzy TOPSIS approach are mentioned down.

Step 1: Build the fuzzy decision matrix.

Consider as 'm' primary decision making alternatives (Products) i.e. Ai = {A_1, A_2, A_3, ... Am} to be calculated against 'n' attributes Oj = {O_1, O_2, O_3, ... O_n}). Evaluations are performed by the decision maker to find out

(i) W_j (weighting vector) = $\{W_1, W_2, W_3, \ldots, W_n\}$ where $Wj \geq 0$, set as

$$\sum_{j=1}^{n} w_j = 1. \qquad \text{along with}$$

(ii) D (decision matrix) = $\{d_{ij}: i = 1,2, \ldots, m \text{ and } j = 1,2, \ldots, n\}$.

'*W*' known as weighting vector corresponds to comparative significance of 'n' decision attributes and 'D' the decision matrix represents the degree of membership of alternative Ai for attribute Oj. If W and D are obtained then it is easy to find the goal of the problem, to rank all the alternatives.

This can be represented in Eq. (7) in matrix form as follows:

$$D = \begin{array}{cc} & \begin{array}{cccc} O_1 & O_2 & \dots & O_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \ddots & \dots \\ d_{m1} & d_{m2} & \vdots & d_{mn} \end{bmatrix} \end{array}$$

$$W = \{W_1,\ W_2,\ W_3,\ \dots ., W_n\ \} \tag{7}$$

Step 2: Normalizing the decision matrix by applying the entropy method.

To measure the significance of each objective requires a weight. To find the objective weights, the most useful way for estimating the weight is the entropy method. Here the decision matrix must be normalized for each attribute to get attributes performance value i. e. $p_{ij} = \frac{d_{ij}}{\sum_{i=1}^{m} d_{ij}}$ Then the new decision matrix as shown in Eq. (8) is as follows:

$$D^{*} = \begin{array}{cc} & \begin{array}{cccc} O_1 & O_2 & \dots & O_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_5 \end{array} & \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{17} \\ P_{21} & P_{22} & \cdots & P_{27} \\ \vdots & \vdots & \ddots & \dots \\ P_{51} & P_{52} & \vdots & P_{57} \end{bmatrix} \end{array} \tag{8}$$

Step 3: Calculate the entropy of data for each attribute and construct the weighted fuzzy decision matrix.

Shannon information entropy calculates the expected value of the information enclosed in a message. This entropy method is considered the standard impulsiveness in a random variable and is identical to its information content. The famous technique for entropy (Yoon & Hwang, 1981; Zeleny, 1982) which helps to obtain the attribute weights is known as entropy weights. Lower entropy values indicate that all of the alternatives Ai = (1, 2, 3, ... m) have less similar criteria values that can be achieved.

After getting the normalized form of the decision matrix, then the entropy value E_j computed like as in Eq. (9)

$$E_j = -K \sum_{i=1}^{m} \left[P_{ij} . ln\left(P_{ij} j\right) \right] ; j = 1, 2.., n; i = 1, 2, ., m; \tag{9}$$

Where the constant is k which is considered as in Eq. (10)

$$K = (\ln(\mathrm{m}))^{-1} \tag{10}$$

The major feature is that for a criterion if all the normalized values turn out to be equal then p_{ij} =1/m, as a result, E_j = 1. The attribute weight (j = 1, …, n) denoted as w_j in Eq. (11)

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n} \left(1 - E_j\right)}. \tag{11}$$

The technique mentioned above is for the objective attributes weighting technique which employs the Shannon information entropy method to suggest the relative strength of attribute weights as well as divergence among the attributes.

Step 4: Building the decision matrix by multiplying with weights which is normalized.

Weighted normalized decision matrix obtained through $u_{ij} = w_j * p_{ij}$, is explained in Eq. (12):

$$u = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} O_1 & O_2 & \dots & O_3 \end{array} \\ \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ u_{21} & u_{22} & \dots & u_{2n} \\ \vdots & \vdots & \ddots & \dots \\ u_{m1} & u_{m2} & \vdots & u_{mn} \end{bmatrix} \end{array} \tag{12}$$

Step 5: Determine FPIS (A+) and FNIS (A−) which are respectively fuzzy positive ideal solution and fuzzy negative ideal solution. Then, we get positive ideal (PI) as well as negative ideal (NI) solutions as in Eq. (13) and (14) as follows:

$$A^+ = \left(\bar{u}_1^+,\ \bar{u}_2^+, \dots \dots \dots \dots, \bar{u}_{k,}^+\right) \tag{13}$$

$$A^- = \left(\bar{u}_1^-,\ \bar{u}_2^-, \dots \dots \dots \dots, \bar{u}_{k,}^-\right) \tag{14}$$

Step 6: Calculate distance measures for each alternative Ai, from FPIS as well as FNIS.

Evaluate Euclidean distance, from PIS and NIS. Distance measurements for positive along with negative ideal solutions for each alternative (Bojadziev & Bojadziev, 1995) are developed on the normalized Euclidean distance are given by the Eq. (15) and (16)

$$d_i^+ = \sqrt{\sum_{j=1}^{n}(\mathrm{uij} - \mathrm{uj}^{\,+})^2} \tag{15}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n}(\mathrm{uij} - \mathrm{uj}^{\,-})^2} \tag{16}$$

*Step 7***:** For each alternative compute the relative closeness coefficient (CC), and then rank the preference ordering for every alternative. Each alternative relative closeness coefficient (CC) in addition to fuzzy ideal solutions estimated like as in Eq. (17)

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \tag{17}$$

The higher value of CC signifies that the alternative is nearer to FPIS along with distant from FNIS at the same time. So, an order of ranking of all the alternatives may be resolute as per the *CC* values declining order. The alternative having the maximum CC value is the most preferred one.

5. A case study based on fuzzy decision-making approaches for product selection

This work provides any retailer who wants to know which product is the best product that he/she launched in the market according to the customer or buyer preference or which product best fulfill the objectives of the customer. To encounter this challenge, we go through the process as follows: The main objective of the retailer is to label the product which satisfies the objectives of the buyer at the maximum level; the retailer has four products in the market and the products are measured by taking into consideration the six features; the features of the products are presented through normal fuzzy numbers which are ranked from 0-10; the preference information is provided to the buyer to acknowledge the ordering process how to rank each feature of each product with the fuzzy numbers; this is offered to the customer for the easy ranking of each feature accurately as per their preference level; ranking too much numbers buyers get confused how to order by ending up not ordering the product and with fewer number retailer do not achieve any satisfactory result. So, for this, triangular and trapezoidal fuzzy numbers are used to represent the customer's objectives.

A motorcycle retailer wants to know what the best product for a buyer according to their preferences. For giving an offer to the buyer the retailer possesses four alternative products and the products are evaluated based on the six specifications, i.e., A1 – Mileage, A2 –Price, A3 –CC, A4 – Brand, A5 – Color, A6- Fuel Capacity. The main objective of the retailer is to identify the product which is most convenient according to the buyer's priority. The specifications are assigned with a preference value according to the buyers' objectives that are mentioned in Table 1 below. This survey was conducted on a motorbike company.

Table 1
Preference property of buyer specifications

	B1	B2	B3	B4
A1(Mileage)	EI	VI	EI	MI
A2(Price)	VI	EI	SI	LI
A3(CC)	SI	SI	MI	EI
A4(Brand)	MI	MI	VI	VI
A5(Color)	EQ	LI	EQ	VI
A6(Fuel Capacity)	LI	LI	LI	EQ

Note. B1, B2, B3, and B4 represent Buyers 1, 2, 3, and 4 respectively; EI denotes Extremely Important: the value of preference is 1.0: Fuzzy expression (7 9 10 10); VI denotes Very strongly Important: the value of preference is 0.8: Fuzzy expression (6 7 8 9); SI denotes Strongly Important: the value of preference is 0.6: Fuzzy expression (4 6 7 8); MI denotes Moderately Important: the value of preference is 0.3: Fuzzy expression (2 3 5 6); EQ denotes Equally Important: the value of preference is 0.1: Fuzzy expression (1 1 3 4); LI denotes Very Less Important: the value of preference is 0.0: Fuzzy expression (0 1 1 3);

Table 2
Property assessment of the products specifications

	Mileage	Price	CC	Brand	Color	Fuel Capacity
P1	(2 4 8)	(1 2 7 8)	(2 4 7 9)	(2 7 9)	(2 5 8 9)	(2 5 8)
P2	(3 5 7 9)	(4 6 9)	(5 5 8 9)	(4 5 6)	(1 3 5 5)	(1 3 6 7)
P3	(0 2 5 6)	(4 6 8 8)	(4 5 7 9)	(5 8 9)	(3 6 9)	(4 6 6 8)
P4	(4 7 9 9)	(0 4 6)	(3 5 8 10)	(4 6 7 9)	(4 5 8)	(2 4 7 9)

Table 2 shows the ranking order of the product's property.

The values of membership mentioned in Table 3 are generated by the intersection point of the curves for buyer preference value with the product property, which is present in Appendix I, which shows the grade of membership of all the given products for buyer 1. The membership values of Table 3 are represented in Fig. 1.

Table 3
Grade of membership for buyer 1

	P1	P2	P3	P4
Mileage	0.17	0.5	0.0	1.0
Price	1.0	0.75	1.0	0.0
CC	1.0	1.0	1.0	1.0
Brand	0.67	1.0	0.25	0.67
Color	0.5	1.0	0.25	0.0
Fuel Capacity	0.2	0.5	0.0	0.25

As per the figures given in Appendix 1, we conclude that according to buyer 1's preference, products 2 and 4 fulfill the needed criteria, as well as product 4 dominated by-products 2 in terms of price, brand, and color. Therefore, product 2 is the final selection of buyer 1. When the number of attributes and alternatives increases, the result occurs from the diagram is not definite. Thus, this manuscript recommended an approach to MODM (multiple objective decision making), which is easy to implement and helps to reach a distinct solution.

5.1. Fuzzy MODM approach

P* = {P1, P2, P3, P4}

A = {Mileage, Price, CC, Color, Brand, Fuel Capacity}

$R = \{r_1, r_2, r_3, r_4, r_5, r_6\}$

Here all the products are rated in connection with the objectives and these ratings are represented as fuzzy sets that are stated in Zadeh's notation (1965).

As per Table 3, these are the Membership Function for each product with respect to the objective:

$O_1 = \{0.17/P_1 + 0.5/p_2 + 0.0/p_3 + 0.1/p_4\}$

$O_2 = \{1.0/P_1 + 0.75/P_2 + 1.0/P_3 + 0.0/P_4\}$

$O_3 = \{1.0/P_1 + 1.0/P_2 + 1.0/P_3 + 1.0/P_4\}$

$O_4 = \{0.67/P_1 + 1.0/P_2 + 0.25/P_3 + 0.67/P_4\}$

$O_5 = \{0.5/P_1 + 1.0/P_2 + 0.25/P_3 + 0.0/P_4\}$

$O_6 = \{0.2/P_1 + 0.5/P_2 + 0.0/P_3 + 0.25/P_4\}$

The preference ordering of buyer1 is as provided in table 1 with respect to the six objectives are Mileage, Price, CC, Color, Brand, and Fuel Capacity. So, the preference ordering of the six objectives are:

$r_1 = 1.0$, $r_2 = 0.8$, $r_3 = 0.6$, $r_4 = 0.3$, $r_5 = 0.1$, $r_6 = 0.0$

Each of these actions carried out on fuzzy set is based on standard fuzzy operation and Zadeh's notation. Now

$\overline{r_1} = 0.0$, $\overline{r_2} = 0.2$, $\overline{r_3} = 0.4$, $\overline{r_4} = 0.7$, $\overline{r_5} = 0.9$, $\overline{r_6} = 1.0$

For each alternative decision measures are evaluated as per Eq. 6 which is expressed in Table 4.

Then
$D(P^{*}) = \mathrm{Max}\{D\ (P\ _{1}) \ldots \ldots \ldots \ldots \ldots . .\ D(P_{4})\}$

Table 4
Decision measures of buyer 1

Product	D_f
D(P1)	0.17
D(P2)	0.5
D(P3)	0.0
D(P4)	0.2

Table 5
Ultimate products ranking according to buyer 1

Alternatives	P1	P2	P3	P4
Rank	III	I	IV	II

Table 5 confers that buyer 1 gives the highest preference to product P2, which is based on the decision function as mentioned in Eq. 6.

Table 6
Grade of membership for buyer 2

	P1	P2	P3	P4
Mileage	0.4	1.0	0.0	1.0
Price	0.33	0.4	0.67	0.0
CC	1.0	1.0	1.0	1.0
Brand	0.67	1.0	0.25	0.67
Color	0.2	0.5	0.0	0.0
Fuel Capacity	0.2	0.5	0.0	0.25

The membership values provided in Table 6 is the representation of the curves from the figures given in Appendix II for buyer 2. The values of membership of Table 6 are shown in Fig. 2.

Table 7
Decision measures of buyer 2

Product	D_f
D(P1)	0.33
D(P2)	0.4
D(P3)	0.2
D(P4)	0.0

Table 7 shows the decision measures of buyer 2 for different products.

Table 8
Ultimate products ranking according to buyer 2

Alternatives	P1	P2	P3	P4
Rank	II	I	III	IV

Table 8 presents that buyer 2 has the maximum chance to choose the product P2 because it possesses the highest value of the decision function.

As indicated by the diagram the conclusion drawn as per the preference level of buyer2 is that only products 2 and 1 meet the requisite criteria. Here also product 2 is more powerful than product 1 in all specifications. Therefore, the ultimate choice for buyer 2 is product 2.

Table 9
Grade of membership for buyer 3

	P1	P2	P3	P4
Mileage	0.17	0.5	0.0	1.0
Price	1.0	0.5	0.5	0.5
CC	1.0	1.0	1.0	1.0
Brand	1.0	0.0	0.5	1.0
Color	0.5	1.0	0.25	0.0
Fuel Capacity	0.2	0.5	0.0	0.25

Table 9 is the representation of the membership values that are given in Appendix III for buyer 3. The grade of membership values of Table 9 is shown in Fig. 3.

Table 10
Decision measures of buyer 3

Product	D_f
D(P1)	0.17
D(P2)	0.2
D(P3)	0.0
D(P4)	0.5

Table10 shows the decision measures of buyer 3 for different products.

Table 11
Ultimate products ranking according to buyer 3

Alternatives	P1	P2	P3	P4
Rank	III	II	IV	I

Table 11 shows that buyer 3 selects the product P4 as it has the highest value based on the decision function.

According to the preference ordering of buyer 3 that is indicated in the diagram is that only products 4 and 2 meet the necessary criteria and product 4 is stronger than product 2. So, the final selection of buyer 3 is product 4.

Table 12
Grade of membership for buyer 4

	P1	P2	P3	P4
Mileage	1.0	1.0	1.0	0.5
Price	0.67	0.0	0.0	0.5
CC	0.5	0.67	0.5	0.75
Brand	1.0	0.0	1.0	1.0
Color	1.0	0.0	0.75	0.5
Fuel Capacity	0.5	1.0	0.0	0.67

Likewise, Table 12 represents the membership values for buyer 4 which is given in Appendix IV. The values of membership of Table 12 are shown in Fig. 4.

Table 13
Decision measures of buyer 4

Product	D_f
D(P1)	0.5
D(P2)	0.0
D(P3)	0.0
D(P4)	0.5

Table13 shows above the decision measures of buyer 4 for different products.

Table 14
Ultimate products ranking according to buyer 4

Alternatives	P1	P2	P3	P4
Rank	I	II	II	I

Table 14 represents that buyer 4 shows top preference to both products P1 and P4. Finally, the preference ordering of buyer 4 is the same for both the products i.e., product 1 and 4, so buyer 4 can choose either product 1 or product 4.

Table 15
Products ranking of all buyers

	B1	B2	B3	B4
$D(P_1)$	0.17	0.33	0.17	0.5
$D(P_2)$	0.5	0.4	0.2	0.0
$D(P_3)$	0.0	0.2	0.0	0.0
$D(P_4)$	0.2	0.0	0.5	0.5

According to the proposed method, each product is chosen as per the buyer's preference. The products are also ranked wherever the decision functions are equivalent for more than one product. Furthermore, the consequence is influenced by the fuzzy indifference degree.

Table 16
Product selection for each buyer

Buyers	Best product alternatives
B1	P_2
B2	P_2
B3	P_4
B4	P_1, P_4

The top products chosen by each buyer are listed in Table 16. Buyers B1, B2, and B3 receive one product as their best product in accordance with the decision function, however buyer 4 receives two items, which is rather problematic.

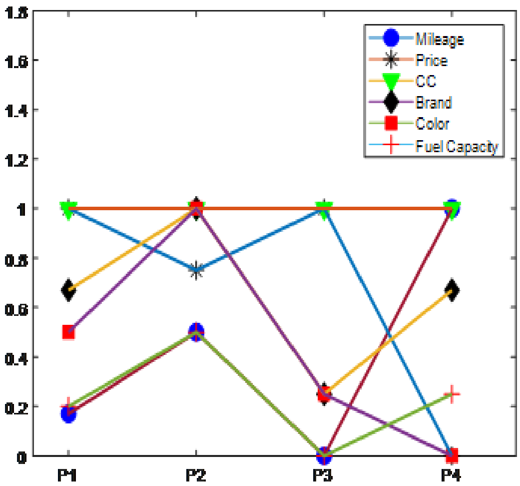

Fig. 1. Weight of all products for the buyer 1

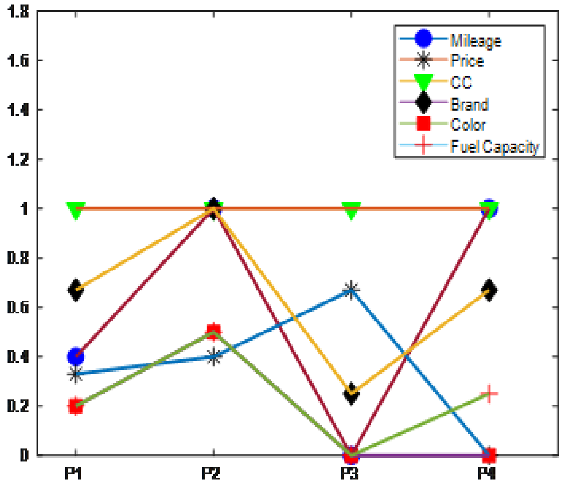

Fig. 2. Weight of all products for buyer 2

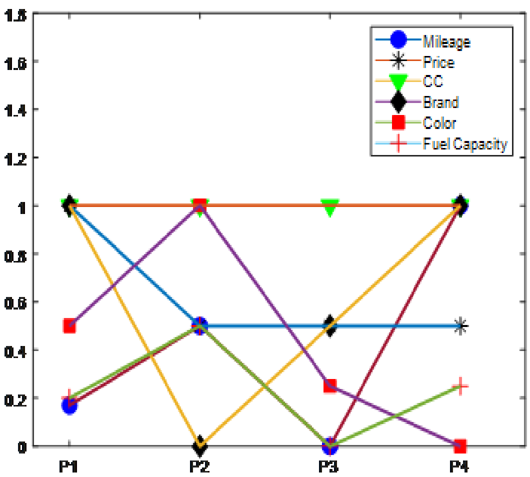

Fig. 3. Weight of all products for buyer 3

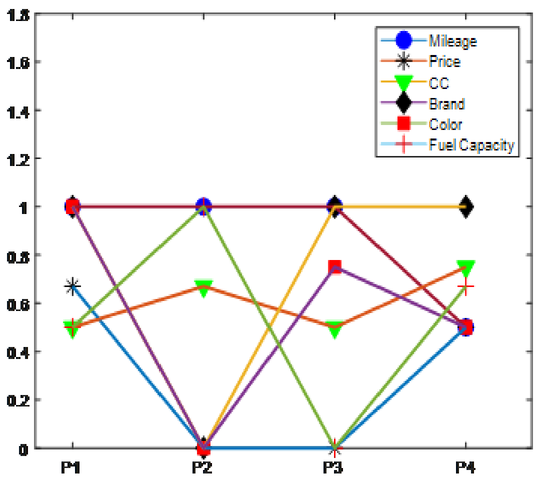

Fig. 4. Weight of all products for buyer 4

In this proposed methodology, one of the conflicting points that arise is in the selection of product 4, where the decision function of these four products shows similar values, that is, P1=P4 and P2=P3 show values of 0.0 and 0.5, respectively. So, buyer 4 gets confused with the products with higher values, i.e., P1 and P4, and has to choose which one is better. To resolve this issue, we have considered another technique, which is the Fuzzy TOPSIS method. This technique ranks the alternatives with their exact values.

5.2. Fuzzy TOPSIS approach

In this paper, we propose a fuzzy TOPSIS approach which is developed on the degree of membership values of triangular and trapezoidal fuzzy numbers, which have been determined previously through the Venn diagrams. The major significance regarding the proposed methodology is that it not just takes advantage of decision makers' proficiency but too involves end users in the entire decision-making process.

5.2.1. Ranking of products as per buyer 1

The following calculations are based on Table 3.

Step 1: Creating decision matrix.

One decision matrix is constructed which includes four alternatives along with six attributes. The representation of the decision matrix is as follows:

Table 17
Decision matrix for buyer 1

	M	P	C	B	Co	F
P1	0.17	1.0	1.0	0.67	0.5	0.2
P2	0.5	0.75	1.0	1.0	1.0	0.5
P3	0.0	1.0	1.0	0.25	0.25	0.0
P4	1.0	0.0	1.0	0.67	0.0	0.25
$\sum_{i=1}^{m} d_{ij}$	1.67	2.75	4.0	2.59	1.75	0.95

Note. In table 17, where P1, P2, P3, and P4 represent the Products and M, P, C, B, Co, F denotes the attributes for Mileage, Price, Cubic Capacity, Brand, Color, and Fuel Capacity respectively

Step 2: By applying the entropy method normalizing the decision matrix.

This step includes the recent decision matrix that is dependent on the normalized performance
pij = $\frac{d_{ij}}{\sum_{i=1}^{m} d_{ij}}$ is as follows:

Table 18
Normalizing the decision matrix for buyer 1

	M	P	C	B	Co	F
P1	0.1018	0.3636	0.2500	0.2587	0.2857	0.2105
P2	0.2994	0.2727	0.2500	0.3861	0.5714	0.5263
P3	0.0	0.3636	0.2500	0.0965	0.1429	0.0
P4	0.5988	0.0	0.2500	0.2587	0.0	0.2632

Table 18 shows the normalized decision matrix for buyer 1, which is obtained by utilizing the entropy method.

Step 3: Calculating the entropy of data for each attribute and evaluating the weights.

Measure the weights of the attributes

$$E_j = -K \sum_{i=1}^{m} \left[P_{ij} \, .ln(P_{ij} j)\right] ; j = 1, 2.., n; i = 1, 2, ., m;$$

Weights of attribute (j = 1, . . . , n) be presented as

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n} (1 - E_j)}.$$

For buyer1 the resultant weights comprise of

w1=0.2898, w2=0.1767, w3=0.0002, w4=0.0559, w5=0.2570, w6=0.2204

Step 4: Construct the decision matrix by multiplying with weights.

Weighted normalized decision matrix is uij = wj * pij, can be expressed as follows:

Table 19
Normalized decision matrix with weights for buyer 1

	M (0.2898)	P (0.1767)	C (0.0002)	B (0.0559)	Co (0.2570)	F (0.2204)
P1	0.1018	0.3636	0.2500	0.2587	0.2857	0.2105
P2	0.2994	0.2727	0.2500	0.3861	0.5714	0.5263
P3	0.0	0.3636	0.2500	0.0965	0.1429	0.0
P4	0.5988	0.0	0.2500	0.2587	0.0	0.2632

Table 19 shows the normalized decision matrix with weights for each attribute for buyer 1.

Table 20
Normalized weighted decision matrix for buyer 1

	M	P	C	B	Co	F
P1	0.0295	0.0642	0.0000	0.0145	0.0734	0.0464
P2	0.0868	0.0482	0.0000	0.0216	0.1468	0.1160
P3	0.0000	0.0642	0.0000	0.0054	0.0367	0.0000
P4	0.1735	0.0000	0.0000	0.0145	0.0000	0.0580

Table 20 shows the weighted normalized decision matrix for buyer1.

Step 5: Get the PIS and the NIS.

Now we can obtain the PIS and the NIS solutions as

$$A^+ = \left(\bar{u}_1^+, \ \bar{u}_2^+, \dots \dots \dots \dots, \bar{u}_{k,}^+\right)$$

A^+ = {0.1735, 0.0642, 0.0000, 0.0216, 0.1468, 0.1160}

$$A^- = \left(\bar{u}_1^-, \ \bar{u}_2^-, \dots \dots \dots \dots, \bar{u}_{k,}^-\right)$$

A^- = {0.0000, 0.0000, 0.0000, 0.0054, 0.0000, 0.0000}

Step 6: From PIS and NIS calculate the Euclidean distance.

$$d_i^+ = \sqrt{\sum_{j=1}^{n}(uij - uj^+)^2}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n}(uij - uj^-)^2}$$

From the positive-ideal solution (PIS) the Euclidean distance calculus is as follows

d_i^+ P1= 0.1761 P2=0.0882 P3=0.2365 P4=0.1705

From the negative-ideal solution (NIS) the Euclidean distance calculus is as follows

d_i^- P1=0.1123 P2=0.2124 P3=0.0740 P4=0.1832

Step 7: Calculate the closeness coefficient (CC).

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}$$

Closeness coefficient (CC) of the alternatives

P1=0.3894, P2=0.7066, P3=0.2383, P4=0.5180

Preference ordering of buyer 1: P2 > P4 > P1 > P3

Table 21
Ranking of the products for buyer 1

Alternatives	P1	P2	P3	P4
Rank	III	I	IV	II

Table 21 presents, that buyer 1 selects product P2 which has the highest preference value obtained from the closeness coefficient.

Similarly for buyer 2 similar steps and procedures are followed as per buyer 1.

5.2.2. Ranking of products as per buyer 2

The following calculations are based on membership values of Table 6 for buyer 2.

Table 22
Decision matrix for buyer 2

	M	P	C	B	Co	F
P1	0.4	0.33	1.0	0.67	0.2	0.2
P2	1.0	0.4	1.0	1.0	0.5	0.5
P3	0.0	0.67	1.0	0.25	0.0	0.0
P4	1.0	0.0	1.0	0.67	0.0	0.25
$\sum_{i=1}^{m} d_{ij}$	2.4	1.4	4.0	2.59	0.7	0.95

Table 22 shows the decision matrix for buyer 2.

For buyer 2 weights are

w1=0.1842, w2=0.1724, w3=0.0001, w4=0.0482, w5=0.4052, w6=0.1899.

Table 23
Normalizing the decision matrix for buyer 2

	M	P	C	B	Co	F
P1	0.1667	0.2357	0.2500	0.2587	0.2857	0.2105
P2	0.4167	0.2857	0.2500	0.3861	0.7143	0.5263
P3	0.0000	0.4786	0.2500	0.0965	0.0000	0.0000
P4	0.4167	0.0000	0.2500	0.2587	0.0000	0.2632

Table 23 shows the normalized decision matrix for buyer 2 by applying entropy method.

Table 24
Normalized decision matrix with weights for buyer 2

	M (0.1842)	P (0.1724)	C (0.0001)	B (0.0482)	Co (0.4052)	F (0.1899)
P1	0.1667	0.2357	0.2500	0.2587	0.2857	0.2105
P2	0.4167	0.2857	0.2500	0.3861	0.7143	0.5263
P3	0.0000	0.4786	0.2500	0.0965	0.0000	0.0000
P4	0.4167	0.0000	0.2500	0.2587	0.0000	0.2632

Table 24 shows the normalized decision matrix with weights for each attribute of buyer 2.

Table 25
Normalized weighted decision matrix for buyer 2

	M	P	C	B	Co	F
P1	0.0307	0.0406	0.0000	0.0125	0.1158	0.0400
P2	0.0768	0.0493	0.0000	0.0186	0.2894	0.0999
P3	0.0000	0.0825	0.0000	0.0047	0.0000	0.0000
P4	0.0768	0.0000	0.0000	0.0125	0.0000	0.0500

Table 25 shows the normalized weighted decision matrix for buyer 2.

A^+ = {0.0768, 0.0825, 0.0000, 0.0186, 0.2894, 0.0999}

A^- = {0.0000, 0.0000, 0.0000, 0.0047, 0.0000, 0.0000}

d_i^+ P1=0.1940 P2=0.0332 P3=0.3159 P4=0.3051

d_i^- P1=0.1329 P2=0.3198 P3=0.0825 P4=0.0920

Closeness coefficient (CC) of the alternatives

P1=0.4065 P2=0.9059 P3=0.2071 P4=0.2317

Preference ordering of buyer 2: P2 > P1 > P4 > P3

Table 26
Ranking of the products for buyer 2

Alternatives	P1	P2	P3	P4
Rank	II	I	IV	III

Table 26 shows that buyer 2 has the highest priority of selecting the product P2 as per the closeness coefficients results.

5.2.3. Ranking of products as per buyer 3

The following calculations are based on Table 9.

Table 27
Decision matrix for buyer 3

	M	P	C	B	Co	F
P1	0.17	1.0	1.0	1.0	0.5	0.2
P2	0.5	0.5	1.0	0.0	1.0	0.5
P3	0.0	0.5	1.0	0.5	0.25	0.0
P4	1.0	0.5	1.0	1.0	0.0	0.25
$\sum_{i=1}^{m} d_{ij}$	1.67	2.5	4.0	2.5	1.75	0.95

Table 27 shows the decision matrix for buyer 3.

Table 28
Normalizing the decision matrix for buyer 3

	M	P	C	B	Co	F
P1	0.1018	0.4000	0.2500	0.4000	0.2857	0.2105
P2	0.2994	0.2000	0.2500	0.0000	0.5714	0.5263
P3	0.0000	0.2000	0.2500	0.2000	0.1429	0.0000
P4	0.5988	0.2000	0.2500	0.4000	0.0000	0.2632

Table 28 shows the normalized decision matrix for buyer 3.

For buyer 3 weights are
w1=0.2905, w2=0.0324, w3=0.0001, w4=0.1983, w5=0.2577, w6=0.2210.

Table 29
Normalized decision matrix with weights for buyer 3

	M (0.2905)	P (0.0324)	C (0.0001)	B (0.1983)	Co (0.2577)	F (0.2210)
P1	0.1018	0.4000	0.2500	0.4000	0.2857	0.2105
P2	0.2994	0.2000	0.2500	0.0000	0.5714	0.5263
P3	0.0000	0.2000	0.2500	0.2000	0.1429	0.0000
P4	0.5988	0.2000	0.2500	0.4000	0.0000	0.2632

Table 29 shows the normalized decision matrix with weights for buyer 3.

Table 30
Normalized weighted decision matrix for buyer 3

	M	P	C	B	Co	F
P1	0.0296	0.0130	0.0000	0.0793	0.0736	0.0465
P2	0.0870	0.0065	0.0000	0.0000	0.1472	0.1163
P3	0.0000	0.0065	0.0000	0.0397	0.0368	0.0000
P4	0.1740	0.0065	0.0000	0.0793	0.0000	0.0582

Table 30 shows the normalized weighted decision matrix for buyer 3.

A^+ = {0.1740, 0.0130, 0.0000, 0.0793, 0.1472, 0.1163}

A^- = {0.0000, 0.0065, 0.0000, 0.0000, 0.0000, 0.0000}

d_i^+ P1=0.1765 P2=0.1179 P3=0.2400 P4=0.1584

d_i^- P1=0.1216 P2=0.2068 P3=0.0541 P4=0.1999

Closeness coefficient (CC) of the alternatives

P1=0.4079, P2=0.6369, P3=0.1840, P4=0.5579

Preference ordering of buyer 3: P2 > P4 > P1 > P3

Table 31
Ranking of the products for buyer 3

Alternatives	P1	P2	P3	P4
Rank	III	I	IV	II

From Table 31, we get that buyer 3 has the maximum chance to choose the product P2.

5.2.4. Ranking of products as per buyer 4

The following calculations are based on Table 12.

Table 32
Decision matrix for buyer 4

	M	P	C	B	Co	F
P1	1.0	0.67	0.5	1.0	1.0	0.5
P2	1.0	0.0	0.67	0.0	0.0	1.0
P3	1.0	0.0	0.5	1.0	0.75	0.0
P4	0.5	0.5	0.75	1.0	0.5	0.67
$\sum_{i=1}^{m} d_{ij}$	3.5	1.17	2.42	3.0	2.25	2.17

Table 32 shows the decision matrix for buyer 4.

Table 33
Normalizing the decision matrix for buyer 4

	M	P	C	B	Co	F
P1	0.2857	0.5726	0.2066	0.3333	0.4444	0.2304
P2	0.2857	0.0000	0.2769	0.0000	0.0000	0.4608
P3	0.2857	0.0000	0.2066	0.3333	0.3333	0.0000
P4	0.1429	0.4274	0.3099	0.3333	0.2222	0.3088

Table 33 shows the normalized decision matrix for buyer 4.

For buyer 4 weights are

w1=0.0205, w2=0.4151, w3=0.0095, w4=0.1695, w5=0.1919, w6=0.1935.

A^+ = {0.0059, 0.2377, 0.0029, 0.0565, 0.0853, 0.0892}

A^- = {0.0029, 0.0000, 0.0020, 0.0000, 0.0000, 0.0000}

d_i^+ P1=0.0446 P2=0.2588 P3=0.2548 P4=0.0796

d_i^- P1= 0.2626 P2=0.0893 P3= 0.0854 P4= 0.2001

Closeness coefficient (CC) of the alternatives

P1= 0.8548 P2 = 0.2565 P3 = 0.2510 P4 = 0.7154

Preference ordering of buyer 4: P1 > P4 > P2 > P3

Table 34
Normalized decision matrix with weights for buyer 4

	M (0.0205)	P (0.4151)	C (0.0095)	B (0.1695)	Co (0.1919)	F (0.1935)
P1	0.2857	0.5726	0.2066	0.3333	0.4444	0.2304
P2	0.2857	0.0000	0.2769	0.0000	0.0000	0.4608
P3	0.2857	0.0000	0.2066	0.3333	0.3333	0.0000
P4	0.1429	0.4274	0.3099	0.3333	0.2222	0.3088

Table 34 shows the normalized decision matrix with weights for buyer 4.

Table 35
Normalized weighted decision matrix for buyer 4

	M	P	C	B	Co	F
P1	0.0059	0.2377	0.0020	0.0565	0.0853	0.0446
P2	0.0059	0.0000	0.0026	0.0000	0.0000	0.0892
P3	0.0059	0.0000	0.0020	0.0565	0.0640	0.0000
P4	0.0029	0.1774	0.0029	0.0565	0.0426	0.0598

Table 35 shows the normalized weighted decision matrix for buyer 4.

Table 36
Ranking of the products for buyer 4

Alternatives	P1	P2	P3	P4
Rank	I	III	IV	II

From Table 36, we can conclude that buyer 4 has the highest preference for choosing the product P1, which is based on the closeness coefficients. Higher the closeness coefficients better the chance to select the product.

Table 37
Products ranking of all buyers

	B1	B2	B3	B4
$D(P_1)$	0.39	0.41	0.41	0.85
$D(P_2)$	0.71	0.91	0.64	0.26
$D(P_3)$	0.24	0.21	0.18	0.25
$D(P_4)$	0.52	0.23	0.56	0.72

According to the TOPSIS method, each product is chosen as per the buyer's preference, and the products are also ranked as shown in Table 38. Here, product P2 is chosen by the majority of buyers.

Table 38
Product selection for each buyer

Buyers	Best product alternatives
B1	P_2
B2	P_2
B3	P_2
B4	P_1

6. Result from analysis and discussion

The proposed framework in this paper is concentrated on improving and managing the decision-making procedures through customers' preferences while satisfying the objectives for the product selection process. It identifies the issues which are vital for the decision-makers who have to take decisions concerning product selection. The techniques used in this paper are contemplated through their implementation in the decision-making model.

In this work, we propose a new technique to rank the alternative products on the basis of buyer objectives. The significant contributions of this paper are summarized as follows.

First of all, this work solves the problem that the objectives of the customer preferences are not fully reserved in the existing work. In this paper, the preferences of customers' objectives are presented in linguistic form. Each customer has different objectives according to their tastes and social standards. Secondly, we propose a fuzzy technique to represent the objectives of the customers in triangular and trapezoidal fuzzy numbers. Membership value of each attribute of each product is obtained by evaluating the objectives of the customers with the corresponding product features. Next, the membership value of each attribute is calculated for each product by utilizing the fuzzy MODM method. Then the rankings of all products are obtained. But one of the disadvantages of this method is that more than one product ranks in the same position, for which it is difficult to identify the ordering of all products. In addition to this drawback, this paper proposes another method for ordering the product by means of the TOPSIS technique.

In this paper, the proposed methods i.e., fuzzy MODM and the fuzzy TOPSIS techniques, majorly select product P2, because it is a highly demanding one by the buyers. The outcomes demonstrate that the proposed fuzzy MODM method helps to tackle the fuzzy MADM issues having fully undetermined information about the objective weights. As an alternative to the fuzzy TOPSIS technique, which employs the entropy method to find out attribute weights through objective preferences proportional to the trustworthiness of the input data, we use the Shannon information entropy technique to state the relative strengths of attributes significance and also to measure the objective attribute weights. The modified TOPSIS uses a newly defined weighted Euclidean distance to evaluate competing products based on the overall evaluation results for multiple criteria.

Table 39 is based on the result obtained from Table 15 (MODM technique) and Table 37 (TOPSIS approach). We combined both the tables in Table 39 to do the analysis accurately. According to our result analysis, the fuzzy TOPSIS approach is better than the fuzzy MODM approach because in the fuzzy MODM method, while ordering the preference, Table 15 shows value zero in many places, which makes it difficult to analyze a product with zero values because each product has a certain value in the market. A problem arises when alternatives possess similar values, as we can see in Table 39. P2 and P3 of buyer 4 possess similar values as 0 and again, P1 and P4 have a similar value of 0.5. But in case of the fuzzy TOPSIS method, the ranking for each alternative is obtained because we get an independent value for each product. We verified the result in Table 39 and found that buyer 2 prefers to choose P2, as it has the highest closeness coefficient among all the other alternatives, and most buyers give the highest preference to product P2 because in most of the cases product P2 shows maximum value. So, the most favorable product for buyers is product P2. From this result analysis, the retailer can identify that among the products he launched in the market, P2 is the best product according to the buyer's perspective.

Table 39
Comparison of both the approaches

Buyers	Approaches	P1 (Rank)	P2 (Rank)	P3 (Rank)	P4 (Rank)
B1	MODM	0.17 (III)	0.5 (I)	0.0 (IV)	0.2 (II)
	TOPSIS	0.39 (III)	0.71 (I)	0.24 (IV)	0.52 (II)
B2	MODM	0.33 (II)	0.4 (I)	0.2 (III)	0.0 (IV)
	TOPSIS	0.41 (II)	0.91 (I)	0.21 (IV)	0.23 (III)
B3	MODM	0.17 (III)	0.2 (II)	0.0 (IV)	0.5 (I)
	TOPSIS	0.41 (III)	0.64 (I)	0.18 (IV)	0.56 (II)
B4	MODM	0.5 (I)	0.0 (II)	0.0 (II)	0.5 (I)
	TOPSIS	0.85 (I)	0.26 (III)	0.25 (IV)	0.72 (II)

7. Applications and future research

The methodologies used in this work have many applications, some of which are provided in this paper. These techniques are used for location selection under uncertain situation, resolving the issue regarding supplier selection, to rank the renewable energy supply networks, complicated project selection in associations to accomplish learning goals, supply chain management to choose the most efficient supplier regarding the essential components for the manufacturing system, to choose best weapon in defense industry, for oil field construction, performance evaluation of computers in an organization, personnel selection for employment, to handle environmental hazards, to locate the energy efficient network, to handle any kinds of risks in construction projects, to detect fault in manufacturing, for investment purpose in stock exchange, to resolve cloud problems, biomedical complications, and for prediction.

Though the study covers many aspects that affect the retailers' decision making in selecting the best product, some areas are still there where future research work can be addressed. These are stated below.

The current study identified only six features or attributes (i.e., Mileage, Price, CC, Brand, Color, and Fuel Capacity) that effectively help in decision making. However, when these features are more, then it needs a thorough investigation. In this context, analysis might be conducted to find out the constituent features of each product which can be taken into consideration in future study.

- The present study might be extended by considering the integration of AHP (Analytic Hierarchy Process) with TOPSIS, whereby various technical aspects can be identified to satisfy the customer requirements.
- This study can be broadened to use in material section process because materials used in a product have a huge impact on the product's success and can also be utilized for manufacturing process selection for the Product.
- Purchasing activity for different products runs along the manufacturer-retailer-customer range, where manufacturers are at the highest level of the system. The current work, however, has taken the retailer side only as the customer base. In this regard, if the future research helps in decision making at the manufacturer level, then the best products can be acquired by the customer as per their objectives.

8. Conclusion

Fuzzy MODM methods make it possible to obtain more realistic results in decision-making problems. Decision makers face the problem that different objectives are in conflict with each other and also that other information may be incomplete and/or vague. Fuzzy set approaches are suitable and helpful in solving uncertainty and conflicts in processes. In multi objective decision-making, application functions (membership functions or fuzzy sets) are introduced, which measure the degree of fulfillment of the decision-maker's requirements (achievement of goals, proximity to the ideal point, satisfaction, etc.) to objective functions and help in the process of searching for a "good compromise" solution. MODM is a modeling and methodological tool for solving complex engineering problems, production planning, logistics, environmental management, banking and financial planning and many other problems. In our paper, we proposed a fuzzy approach to solving the multi-objective problem; where we used membership functions prescribe the degree of satisfaction for solving buyers' problems. Fuzzy linguistic terms are used in all MODM techniques to eliminate uncertainty and ambiguity in results.

Among the decision-making models, the essential one is the multiple-objective decision-making approach for product selection. Hence, fuzzy techniques have been proposed that can hold the vagueness which is combined with the decision maker's subjective perception. This paper also evaluates the priority of the objectives, which are solved through simulation by using MATLAB to maximize their value. After receiving the priority of the objectives, Fuzzy MODM and Fuzzy TOPSIS techniques are utilized for the ranking of the alternative orders. Here, the fuzzy TOPSIS technique incorporates subjective and objective weight by getting the benefits from the theory of fuzzy sets so that decision making turns out to be realistic and effective. As the subjective values are dependent on the decision maker's priority basis, we used Shannon's entropy for evaluating the subjective weights.

Product selection is a vital part of any organization as it determines the influence and portrait of an organization on its customers. Product selection is directly connected to serving customers by interpreting what products are obtained and utilized in the market along with the variety of products that a customer can get. Product selection, as a result, brings on proficient usage of resources, decreases the chances for faults, and enhances customer-oriented results. A retailer can raise the number of customers by revising and developing the product selection process. This helps the customer to identify the product they were searching for or a suitably alike one that satisfies their objectives. This facilitates analyzing the buyer's preference orders regarding the products in the market. In addition, it would be beneficial to recognize the other alternatives that the customers might prefer if they are unsuccessful in getting the exact product.

To resolve this issue, a mathematical illustration of choosing a bike is given to elucidate the process of the projected methods. Even though the main focus of the case studies is on choosing bikes, the proposed methodologies can be applied to many problem-solving practices where the alternatives and criteria are mentioned. The acquired result by applying the proposed approaches is easy and quick, and the said products are ranked in keeping with the preferences needed by the customers. This innovative technique is suitable for contributing to better widespread practice in the process of decision-making. The application areas of the proposed techniques mainly cover decision-making in the selection of goods or services, logistics and optimization of supply chains, and personnel selection.

Author Statement

The authors declare that there is no conflict of interest.

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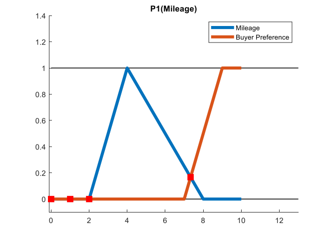
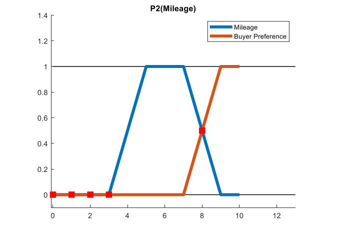
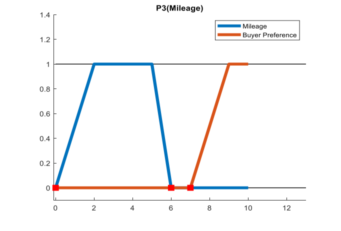
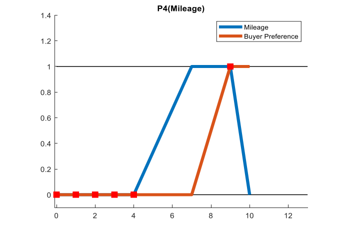
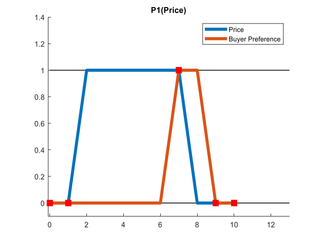
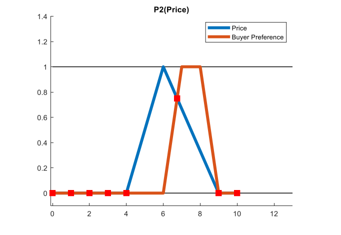
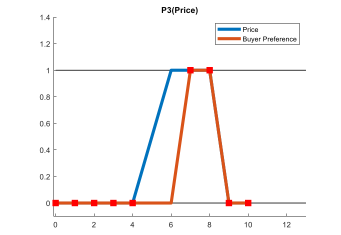
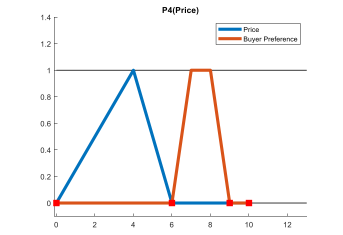
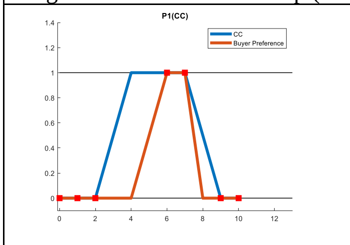
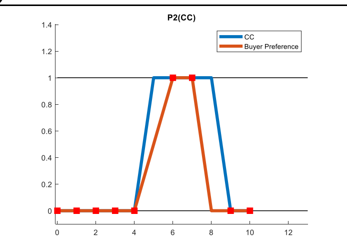
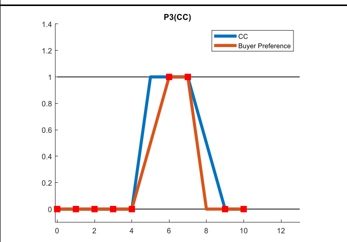
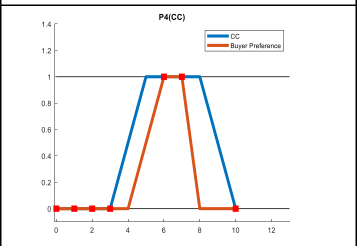
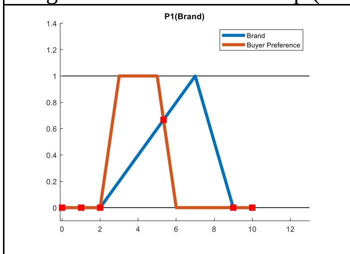
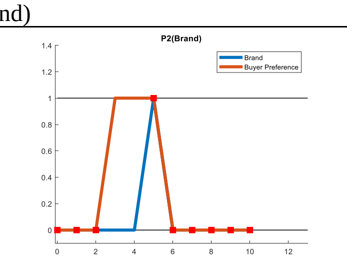
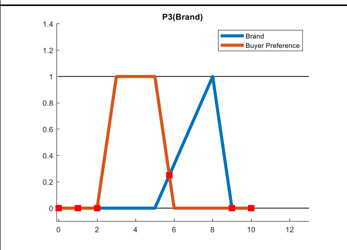
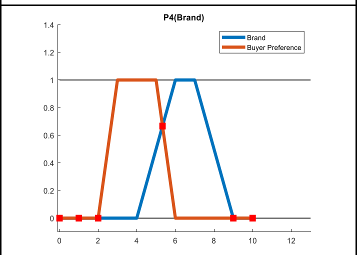
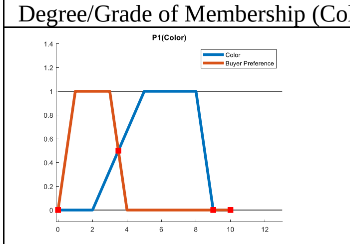
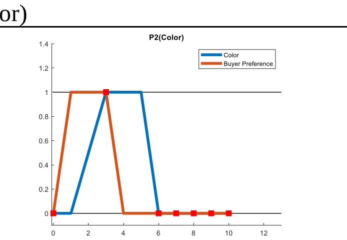
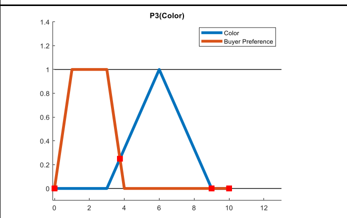
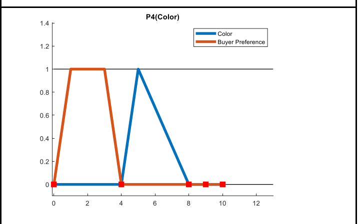
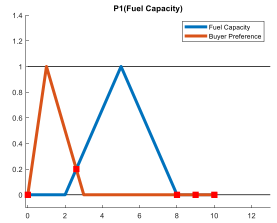
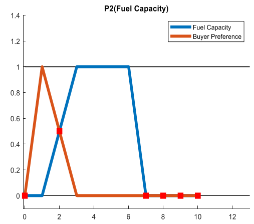
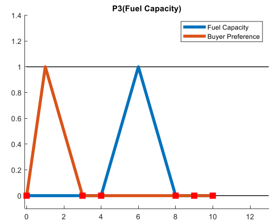
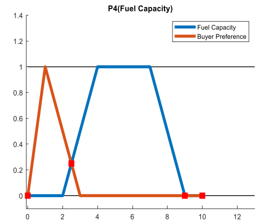
Appendix I

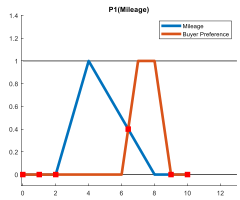
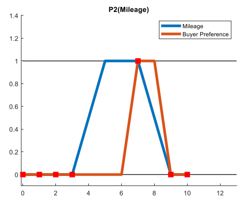
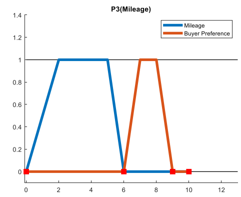
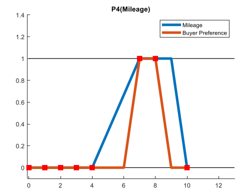
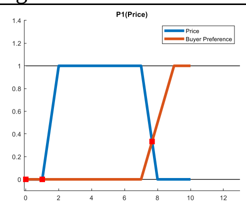
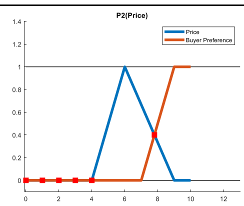
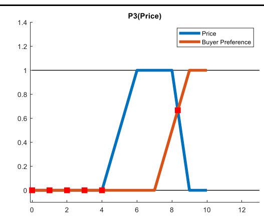
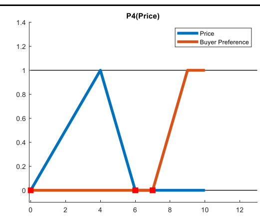
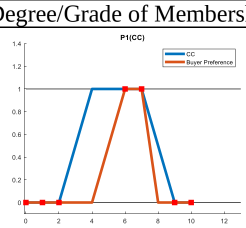
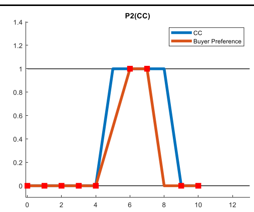
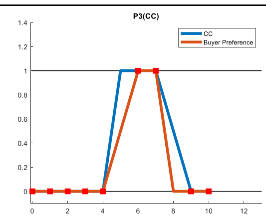
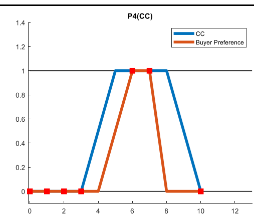
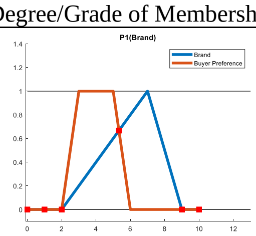
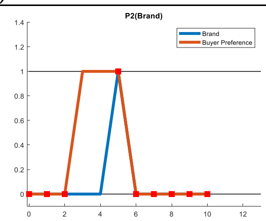
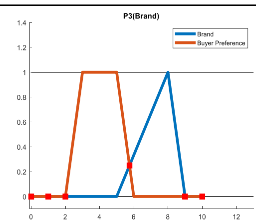
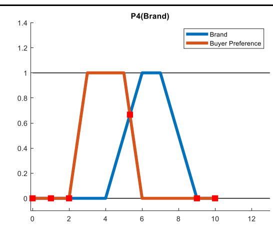
Membership values of all the products for buyer 1

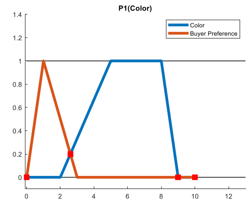
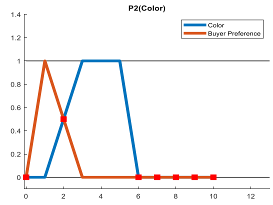
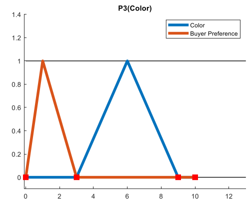
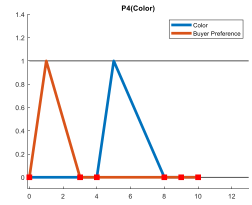
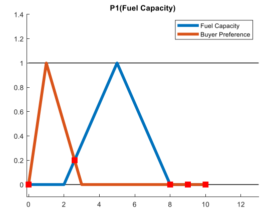
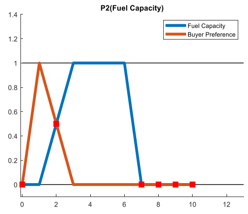
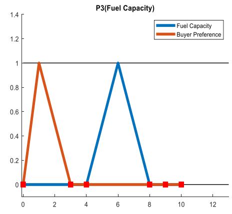
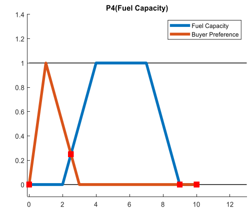
P 1	P 2	P 3	P 4
Degree/Grade of Membership (Mileage)			
			
Degree/Grade of Membership (Price)			
			
Degree/Grade of Membership (CC)			
			
Degree/Grade of Membership (Brand)			
			
Degree/Grade of Membership (Color)			
			
Degree/Grade of Membership (Fuel Capacity)			

Appendix II

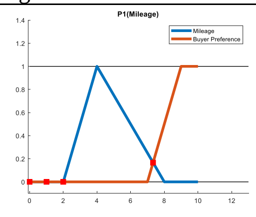
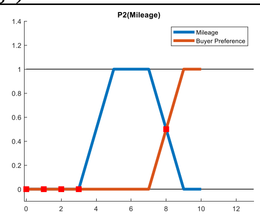
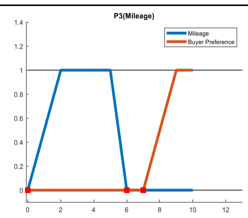
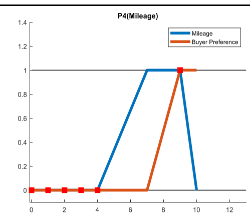
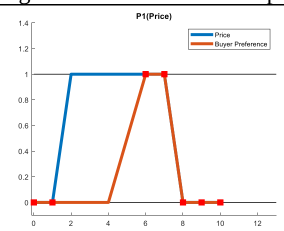
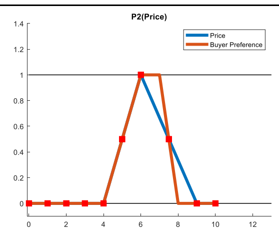
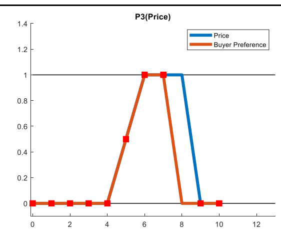
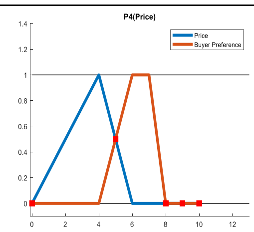
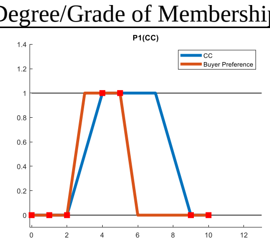
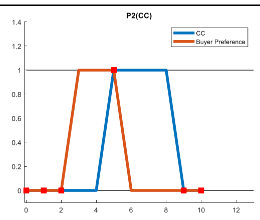
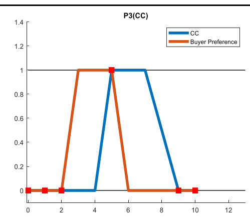
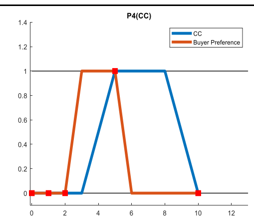
Membership values of all the products for buyer 2

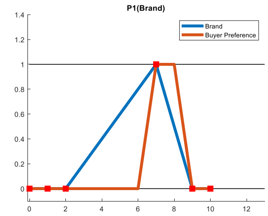
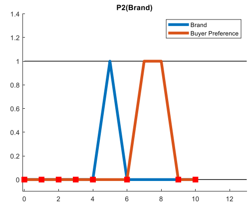
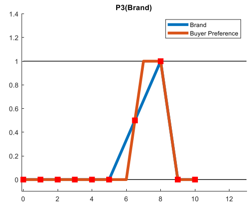
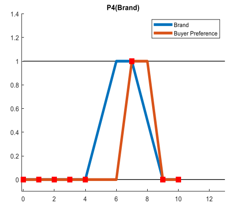
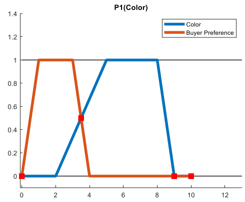
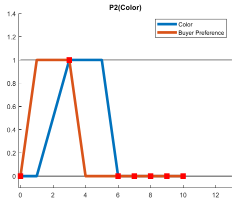
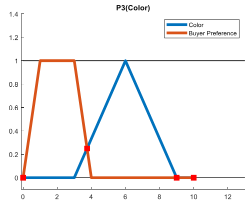
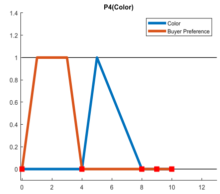
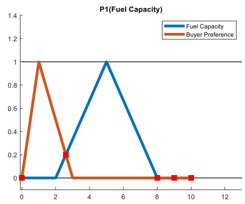
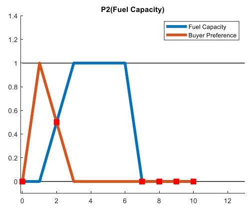
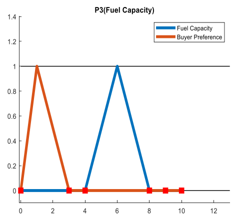
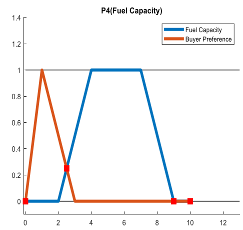
P 1	P 2	P 3	P 4
Degree/Grade of Membership (Mileage)			
			
Degree/Grade of Membership (Price)			
			
Degree/Grade of Membership (CC)			
			
Degree/Grade of Membership (Brand)			
			
Degree/Grade of Membership (Color)			

			
Degree/Grade of Membership (Fuel Capacity)			
			

Appendix III

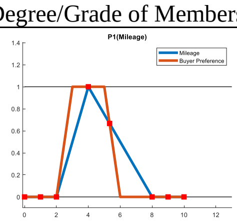
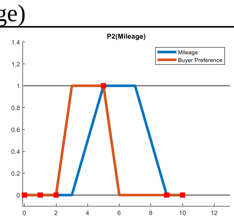
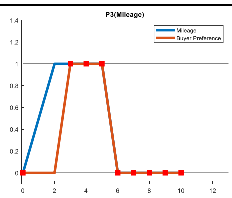
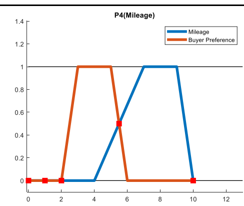
Membership values of all the products for buyer 3

P 1	P 2	P 3	P 4
Degree/Grade of Membership (Mileage)			
			
Degree/Grade of Membership (Price)			
			
Degree/Grade of Membership (CC)			
			
Degree/Grade of Membership (Brand)			

			
Degree/Grade of Membership (Color)			
			
Degree/Grade of Membership (Fuel Capacity)			
			

Appendix IV

Membership values of all the products for buyer 4

P 1	P 2	P 3	P 4
Degree/Grade of Membership (Mileage)			
			
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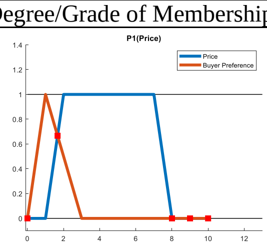	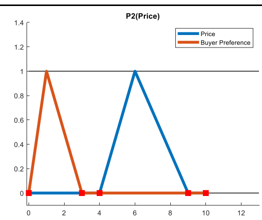	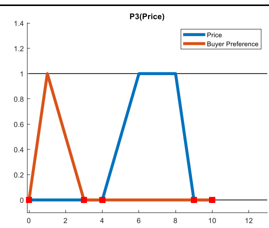	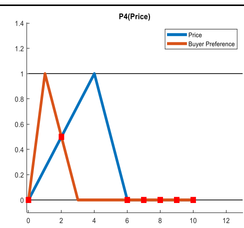
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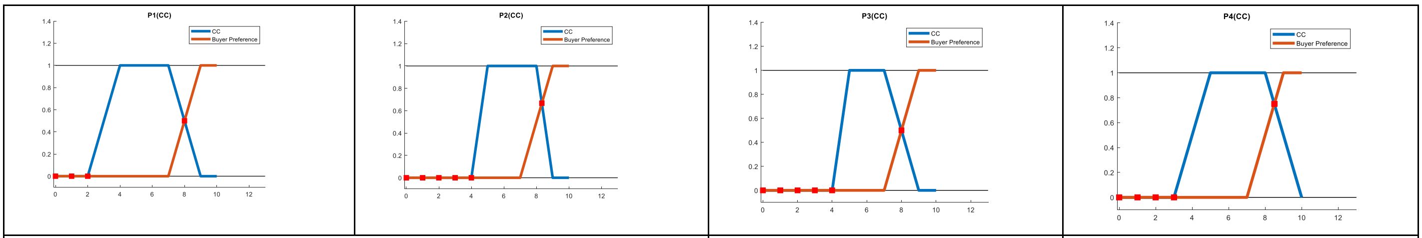

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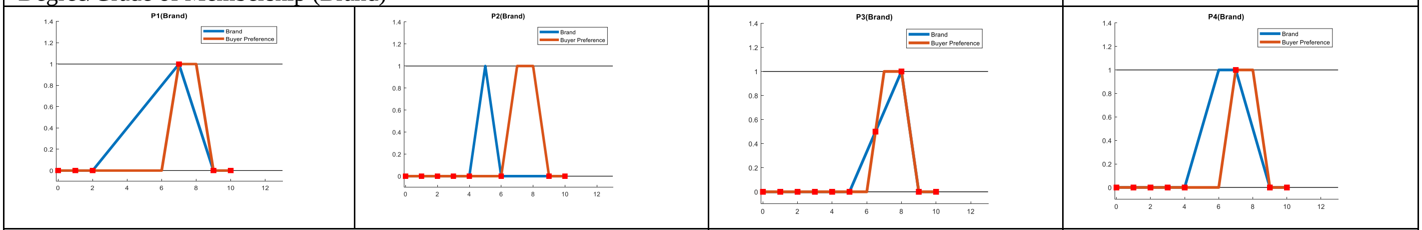

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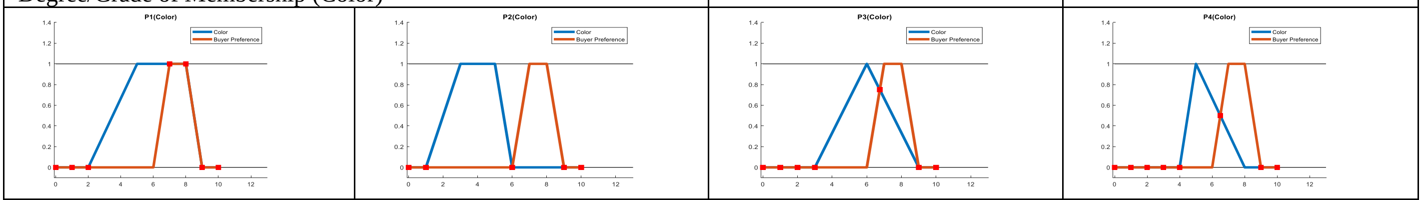

Degree/Grade of Membership (Fuel Capacity)