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Contents

List of Figures

List of Tables

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Abstract

In the first chapter I evaluate the contribution of financial frictions in explaining the drop in aggregate TFP through misallocation during the Great Recession. I build a quantitative model with heterogeneous establishments; with the help of the model I compute the counterfactual drop in misallocation: by how much would aggregate TFP have decreased if the credit crunch had been absent. I find that a "real recession" would have caused a drop of only 0.16 percent, as opposed to 1.04 percent found in the data; therefore financial frictions account for a significant part of the drop in aggregate TFP. The key mechanism is the following: the increase in the cost of external finance affects negatively the reallocation of productive inputs from low to high productivity firms, by dampening the growth of small-highly productive firms.

In the second chapter I explore the hypothesis that the trend increase in firm-level risk, measured as the cross-sectional dispersion in growth rate of sales or TFP, is a main driver of the long-run increase in cash holdings by firms over the last decades. I document a novel stylized fact: the impact of firm level uncertainty on cash holdings is stronger for firms that belong to sectors with greater skweness and kurtosis in investment rates. To rationalize this empirical finding I build an heterogeneous dynamic model in which firms face a potentially binding collateral constraint and need to pay a fixed cost (on top of standard convex adjustment costs) when they wish to invest. The increase in the volatility of idiosyncratic TFP (or demand) shocks leads firms to hoard more cash holdings for precautionary reasons but this effect is quantitatively significant only when the fixed cost of adjusting the capital stock is calibrated to match a high degree of lumpiness. The economic intuition is that firms could finance investment opportunities with internal cash flow if there are only convex adjustment costs. However if firms have to pay a large fixed cost when investing, they are more likely to need external finance; furthermore if they become more uncertain about the time these investment opportunities will materialize, they are willing to increase their cushion of financial wealth for precautionary reasons.

How does a decline in house prices affect geographical reallocation and the labor market? In the third chapter, I focus on a financial friction: the down payment requirement in purchasing a home. When house prices fall, the amount of home equity declines, making it harder to afford the down payment on a new house after moving. To the extent that households care about owning a house, the decline in house prices affects their migration decisions. Some households that would normally move out of regions with low productivities may stay and look for jobs in distressed labor markets. Thus, the aggregate job finding rate decreases, which results in higher aggregate unemployment.

Chapter 1

Capital Misallocation during the Great Recession

1.1 Introduction

The Great Recession differs from other recessions that happened in the US during the post-war period in terms of both severity and persistence. It has been also characterized by a drop in aggregate TFP and reallocation of unprecedented amount (see Foster et al., 2014). What distinguishes the Great Recession is the disruption in financial markets; it is well understood that households were severely affected by the tightening in credit conditions, but also firms were badly hit: the credit spread between BB and AA corporate bonds, though typically countercyclical, increased much more in the 2007-2009 crisis than in past recessions. If we look at the heterogeneity in financing between firms, it is well documented that the corporate sector as a whole has become a net lender over the recent two decades. Two stylized facts are of paramount importance: the increasing trend in cash holdings by the US corporate sector and the fact that in the aggregate the average firm is able to finance its capital expenditures by internal cash flows. However by looking at disaggregated micro-level data one notices that small and highly productive firms are typically cash constrained: they need to raise external finance for growing up their scale of operations. It is palusible that credit tightening, of the amount seen in the credit recessions, affected more these small and young firms that are more profitable than others. Indeed the measured change in total factor productivity observed during the Great Recession can reflect an increased distortion in the allocation of capital between firms, where difficulty in obtaining credit affects more the growth of more productive but smaller firms.

In this work I document that a significant portion of the measured drop in TFP observed during the Great Recession can be attributed to financial conditions that disrupt the allocation of capital further from that implied by firm productivities. Following the approach of Olley and Pakes (1996), I decompose aggregate productivity in the economy into a technological component and into a second term defined as the covariance between firm size and firm productivity. This second component captures the allocative efficiency in the distribution of production factors between firms with heterogeneous productivity levels. I show that in the Compustat sample roughly 53 percent of the drop in TFP that occurred between the peak (2007:Q1) and the trough (2009:Q2) of the recession is accounted for by a decrease in allocative efficiency. Forthermore, exploiting the time coverage of the Compustat sample, I document that excluding the last two recessions the reallocation in output shares between firms, though still an important driver of TFP fluctuations, accounts for a much lower fraction of TFP changes. Therefore the Great Recession stands out because it witnessed an unprecedented decline in the covariance between firm size (measured by firm's output share) and firm productivity. It is therefore legitimate to consider recessions, and in particular the last episode, times of increased misallocation of resources between heterogeneous production units, rather than times of negative technological shocks.

The aim of this paper is to use an off-the-shelves model of heterogeneous plants with credit market imperfections to quantify the contribution of financial disruption to the dynamics of capital misallocation during the Great Recession. Financial market imperfections are introduced as an external cost function capturing the basic notion that external funds are more costly than internal cash flows. While most of the literature studying the impact of financial frictions on firms' investment focus on debt financing (see for example Kiyotaki and Moore, 1997 or Khan and Thomas, 2013), I explicitely allow for equity financing as well. Considering only debt can be a problem: if firms can avoid a tightening of frictions in debt financing by replacing debt with equity finance, then models that only allow for debt financing could overstate the importance of financial frictions. Moreover, Fama and French (2005) document that equity issuances are quantitatively important in the Compustat sample. A possible concern regarding my modelling choice is that I do not focus on debt and equity separately but I consider their sum, so that in my model debt and equity are perfect subsitutes. Covas and Haan (2011) document that at least for firms up to the 99th percentile of the size distribution both debt and equity issuances are procyclical, suggesting that during recessions firms find it more difficult to raise external finance in either form. Therefore focusing on external finance as the sum between equity and debt financing is not a relevant loss of generality for my purposes.

I use the model (calibrated to the pre-2007 period) to answer the following question: "By how much would misallocation have changed during the recession if borrowing costs had stayed constant to the pre-recession average?". In other words, with the help of the model, I can compute the counterfactual scenario of a recession driven by a productivity shock only, and assess the contribution of the change in financial frictions to the amount of allocative efficiency. To answer the question: "What is the contribution of costly external finance to the fall in misallocation"? I study the transitional path of the economy between two steady states (SS1 and SS2 from here on). The parameters in the initial steady state SS1 are calibrated to match cross-sectional moments of firm distribution in the pre-recession period 1980-2007. I use the Compustat dataset (a large panel of listed firms for the US) to calibrate my model so that is able to reproduce most of the fall in aggregate total factor productivity and misallocation observed from data. More precisely I perform two quantitative exercises: in the *baseline* I compute the transition between SS1 and SS2, using as inputs a deterministic path of aggregate TFP shocks (calibrated to match the observed drop in GDP) and the change in the parameters of the external cost function. In the *counterfactual*, I compute the transition between SS1 and SS2, using as inputs the deterministic path of aggregate TFP shocks but *keeping the parameters in the external cost function fixed at the pre-recession period values*. Using such counterfactual scenario I can answer the following question: "What would have been the fall in capital reallocation between firms if the degree of financial frictions had stayed constant at its pre-recession value?".

In order to highlight the mechanism though which financial frictions affect productivity, it is better to present the flow of funds equation for firms in my model:

$$d = \pi\left(k, z\right) - I - AC + e$$

where d denotes dividends, π is operating cash flow, I is investment, AC stands for physical adjustment costs and e stands for external financing. Dividends and external finance cannot be negative by definition. The flow of funds constraint simply states that if firm financing needs for capital expenditures (i.e. $I+AC$) exceed the cash flow generated internally then the firm has to raise additional funds by tapping the financial market (i.e. has to choose $e > 0$). Issuing new shares or borrowing however is costly: the firm pays a fixed cost λ_0 plus a variable cost λ_1. Financial frictions in the model therefore act through this channel: an increase in the cost of raising external finance reduces the share of firms raising external funds. But firms raising external funds are more productive than the rest, as I show in the following part.

Using the Compustat panel, I sort firms according to their finance regimes:

1. Dividends distribution regime ($d > 0$ & $e = 0$)
2. Financial inactivity regime ($d = 0$ & $e = 0$)
3. External finance regime[1] ($d = 0$ & $e > 0$)

[1] I consider that a firm is not raising external finance if the ratio between internal funds and capital expenditues is between 0.95 and 1.05. Perturbing this threshold does not change my results significantly.

1980-2007	**External Finance**	**Fin. Inactive**	**Div.distrib.**
Share of firms	0.23	0.297	0.474
Share of cap	0.028	0.059	0.913
Share of invest	0.039	0.057	0.904
Earnings/cap	0.567	0.275	0.355
Invest/cap	0.29	0.193	0.194
Tobin's q	3.76	1.78	2.83

Table 1.1: Distribution of Firms across Finance Regimes in the Data (Average over 1980-2007).

Notice, however, that about 20 percent of firms in this sample both raise external finance *and* distribute dividends. This behavior is puzzling given standard corporate finance theory, since it implies that there are profitable opportunities to reduce dividends and equity issuance or debt (remember that in my model, as it is standard in the literature, the cost of external funds is larger than internal funds). I decided to group these firms into the dividend distribution regime (for sure they are not liquidity constrained).

For any year (1980-2007) I compute some statistics for firms in each finance regime. Table 1.1 summarizes my findings.

Table 1.1 reveals that about half of the firms pay dividends. Firms paying dividends account for a large share of capital and investment in the sample, they are more productive than liquidity constrained firms but less productive than firms raising external finance. Firms raising external funds are much more productive than the rest, as measured by the earnings-capital ratio. These small firms (measured by capital) with high Tobin's q require external finance to finance investments. Higher costs of raising external funds during the crisis affected most these "growth firms".

1.2 Literature Review

The present work lies at the intersection between two strands of literature: the empirical literature about TFP growth and reallocation and the theoretical literature about dynamic general equilibrium models with heterogeneous firms and financial frictions.

On the empirical side the growing availability of longitudinal firm-level data has allowed the analysis of reallocation across individual producers and the connection of this reallocation to aggregate productivity growth. Representative work in this area includes Baily, Hulten and Campbell (1992) and Foster et al. (2001). A common theme of these studies is to decompose aggregate productivity growth into several parts to characterize the contributions of within plant productivity growth and reallocation, where the latter includes the contribution of reallocation among continuing establishments and the impact of entry and exit. Despite that their findings vary with the specific data sets and decomposition methodologies used, a uniform finding in these studies is an important role of reallocation in accounting for aggregate productivity growth in the U.S. manufacturing. For instance, Foster et al. (2001) document that reallocation accounts for about half of overall total factor productivity growth in U.S. manufacturing for the period 1977 to 1987. All these empirical studies use the sum of output (or employment) weighted firm/plant level TFP (or labor productivity) to measure the aggregate productivity of an industry. According to Baily, Hulten and Campbell (1992) the definition of aggregate productivity is as follows. Suppose the production function for plant i in period t is:

$$\begin{aligned} y_{it} &= f\left(k_{it}, l_{it}, m_{it}\right) \\ &= k_{it}^{\alpha_k} l_{it}^{\alpha_L} m_{it}^{\alpha_M} \end{aligned}$$

where k, l, m are capital, labor and intermediate inputs, respectively. Then establishment level TFP is computed as:

$$\log TFP_{it} = \log y_{it} - \alpha_k \log k_{it} - \alpha_l \log l_{it} - \alpha_m \log m_{it} \tag{1.1}$$

where α_k, α_l and α_m are return to scale factors for capital, labor and intermediate inputs. Finally aggregate productivity in period t (at the sector level) is defined as:

$$TFP_t = \sum_i \omega_{it} TFP_{it} \tag{1.2}$$

where ω_{it} is the output (or labor) weight of plant i in the sector. In this work, in order to measure reallocation of productive inputs I look at the time variation of a measure of allocative efficiency originally proposed by Olley and Pakes (1996), hereafter OP. OP noticed that aggregate productivity at a given point in time (as defined, for example, in 1.2) can be decomposed as follows:

$$TFP_t = \frac{1}{N_t} \sum_i TFP_{it} + \sum_i \left(TFP_{it} - \overline{TFP}_t\right)\left(\omega_{it} - \overline{\omega}_t\right) \tag{1.3}$$

where TFP_{it} is firm level productivity, ω_{it} is the share of output (or labor) of the firm, N_t is the total mass of active firms, and a bar over a variable indicates the unweighted average of the firm-level measure. This OP decomposition splits the aggregate productivity TFP_t, defined as the weighted average of firm-level productivity, into an *unweighted firm-level average* and a *covariance term*. The covariance term is a summary measure of the within-industry cross-sectional covariance between size and productivity: it is expected that in a well-functioning market economy such covariance is positive, i.e. firms with higher than average productivity have a larger than average size. A low covariance indicates then that aggregate productivity could improve by reallocating resources towards the most productive firms. This analysis of allocative efficiency by using OP decomposition in (1.3) has been performed in quite a few studies. In the seminal contribution of Olley and Pakes (1996), the authors found that the covariance term increased substantially in the US telecommunications equipment industry following the deregulation of the sector in the early 1980s. OP argued that this was because the deregulation permitted inputs to be reallocated more readily from less to more productive US firms. In a subsequent study, Bartelsman et al. (2013) found that the OP covariance term for labor productivity averages about 50 log points within US manufacturing industries: this implies that the industry index of labor productivity in the average US manufacturing industry is 50 percent higher than it would be if employment shares were randomly allocated within industries. Bartelsman and his coauthors found however that the OP covariance term reaches only 20-30 log points in Western Europe and it was close to zero, if not negative, in Central and Eastern European countries at the beginning of their transition to a market economy. They documented also that in Central and Eastern European countries the covariance term increased substantially in the 1990s as their transition to a market economy progressed.

On the theoretical side there are several studies that analyse an economy with heterogeneous production units, noting that aggregate TFP depends not only on the TFP's of the individual firms but also on how inputs are allocated across firms. These papers focus on distortions in product, labor or credit market and policies that can all slow down aggregate productivity growth by hindering the reallocation process among heterogeneous producers. A seminal contribution in this field is Hopenhayn and Rogerson (1993): using the Hopenhayn (1992) model of firm dynamics they quantify the aggregate TFP loss due to firing costs. A non-exhaustive list of more recent works comprises Buera and Shin (2013), Buera et al. (2013), Guner et al. (2008), Hsieh and Klenow (2009), Restuccia and Rogerson (2008) and Midrigan and Xu (2014). Much of this literature however emphasizes the role of frictions and policies in the cross-country difference in long-run TFP and, therefore, abstracts from the cyclical dynamics of misallocation. For example, Hsieh and Klenow (2009) build on the key insight that misallocation can result as lower aggregate TFP and using data on manufacturing try to measure the extent of misallocation in China and India compared to US (they need US as a control group that takes into account model misspecification and measurement error). They interpret the gap in marginal revenue product of capital between different establishments as evidence of misallocation; their calculations imply that if capital and labor were hypothetically reallocated to equalize marginal products to the extent observed in the US, manufacturing TFP would increase by 30-40% in China and by 40-60% in India. Restuccia and Rogerson (2008) explore the quantitative impact of *policy distortions* on aggregate productivity in a stationary equilibrium with heterogeneous plants. They show that policy distortions that create heterogeneity in the prices faced by individual producers lead to misallocation of resources across heterogeneous plants, and as a result can lead to sizable decreases in output and measured TFP. However, differently from my work, Restuccia and Rogerson (2008) focus their attention only on the steady-state distribution of firms and therefore are silent about the impact of policy distortions on reallocation during economic downturns. Midrigan and Xu (2014) also study the impact of financing frictions on misallocation and focus in particular on two distinct channels: borrowing costs distorting the entry decision of firms and borrowing costs distorting the allocation of capital among firms with different productivities. They find that only the first channel is quantitively relevant. Compared to my work, their main task is to explain cross-country differences in TFP whereas I focus my attention on the cyclical variation of TFP in the US economy during the recent recession.

My paper also contributes to the literature exploring the impact of financial shocks on business cycle fluctuations. Jermann and Quadrini (2012) document the behavior of debt and equity financing over the business cycle using aggregate data. They furthermore develop a *representative firm model* in which investment is financed using both debt and equity and costs of adjusting dividends prevent the avoidance of paying financial frictions. Jermann and Quadrini find that credit shocks have been an important source of business cycles. However the representative firm setting that they employ prevents them from studying the impact of financial shocks on resource misallocation.

Perhaps the work closest to mine is Khan and Thomas (2013) who study the cyclical implications of credit market imperfections in a quantitative dynamic general equilibrium model in which firms are subject to two frictions: collateralized borrowing and partial investment irreversibility. Collateral constraints limit the firm's investment behaviour and partial irreversibilities in investment lead firms to follow (S,s) rules with respect to their capital. The presence of these real and financial frictions slow down the reallocation of capital across firms. Since reallocation is essential in determining aggregate TFP, they show that a financial crisis (originating as a sudden shock to the firms' collateral constraint) can generate a large and protracted drop in aggregate TFP. Therefore the drop in TFP following a financial shock is endogenous because it is a consequence of the change in the distribution of firms. They study the behavior of aggregate quantities after a negative shock to borrowing conditions (in the spirit of Guerrieri and Lorenzoni (2011)) and find that their model predicts aggregate changes resembling those from the 2007 US recession. However they are not able to quantify the aggregate productivity loss due to the impact of financial frictions in the form of higher cost of external finance; indeed they assume that the only source of external funds is debt, subject to a collateral constraint. They hence rule the possibility of financing investment by issuing new equity; however it is important to include equity finance, Fama and French (2005) document that firms frequently issue equity, and equity issuances are quantitatively important. Finally, another recent paper investigating the link between credit market imperfections and misallocation is Azariadis and Kaas (2012), who propose a sectoral-shift theory of TFP. They build a model in which sectors are hit with different productivity shocks and limited enforcement in loans prevents reallocation of capital towards more productive sectors. The result is that the level and growth of aggregate TFP is negatively correlated with the dispersion of sectoral TFP growth rates.

The rest of the paper is organized as follows. The next section explains the empirical findings regarding misallocation. In section 2.4 I set out the model and characterize the optimal decision rules of the firms. In Section 1.5 I explain the calibration and simulation of the model and in Section 2.5 I analyze the results. Section 2.6 concludes the paper.

1.3 Measuring Misallocation over the Business Cycle

As discussed in the previous section, I look at the time variation of a cross-sectional measure of allocative efficiency (the Olley-Pakes gap) to assess the cyclical properties of capital reallocation. Some empirical studies tend to confirm the procyclical nature of reallocation (contrary to the *cleansing view* of recessions, in which more capital should be liquidated in recessions). Among these, Eisfeldt and Rampini (2006) document that *flows of capital* among firms decrease during downturns. In Figure 1.1 I plot the series for reallocation using updated data to 2012. The authors define capital reallocation as the sum of acquisitions plus sales of PP&E (property, plant and equipment). Their measure focuses hence only on instances when existing capital is sold or acquired but they are not able to tell whether such transfers of ownership are productivity enhancing or not. In other words, they are silent about the allocative efficiency of capital.

As I argued above, a more informative way to measure reallocation of capital and to assess whether it is productivity enhancing or not is to analyze the covariance of firm level multifactor productivity and firm size. If the creative destruction theory were true, this covariance should sharply increase during economic downturns, reflecting the fact that firms with productivity below the average become smaller since resources are shifted away from them.

Production function estimates. The first step in the computation of the OP covariance term, defined as the second term in the right-hand side of equation (1.3), is the estimation of the (log) firm-level total factor productivity, which requires the estimation of a production function. Using the Compustat panel, I estimate the production function given by:

$$y_{it} = \alpha_0 + \alpha_j^k k_{it} + \alpha_j^l l_{it} + z_{it} + \varepsilon_{it} \tag{1.4}$$

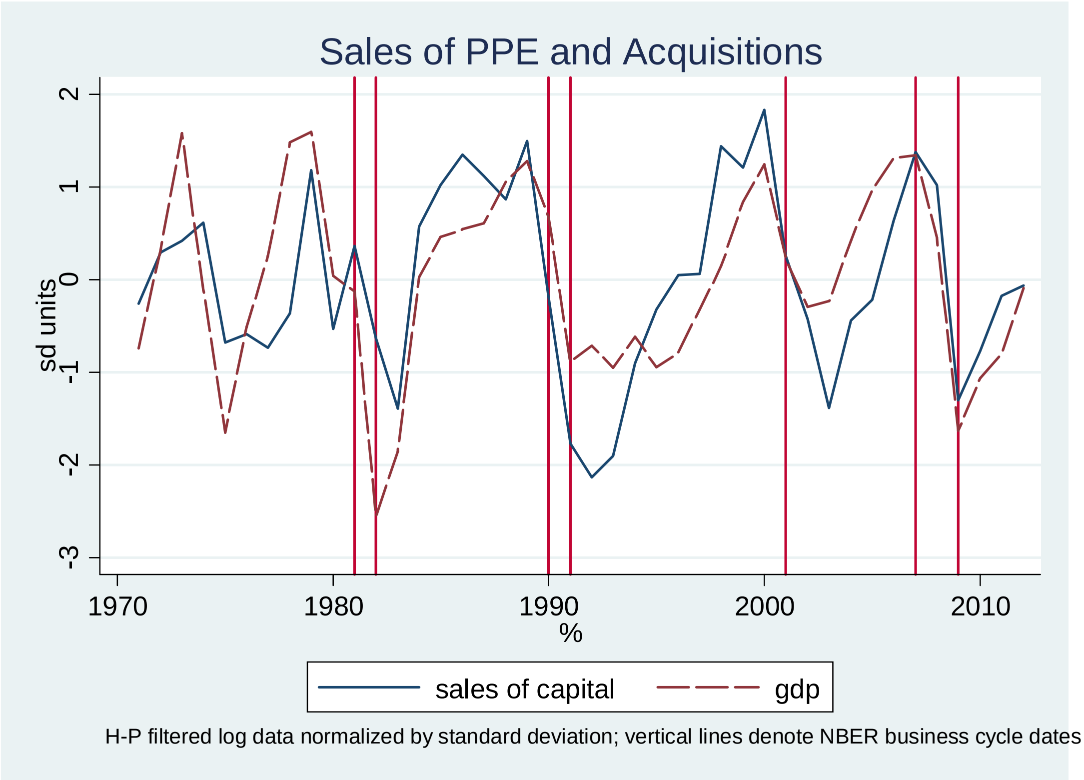


Figure 1.1: Reallocation over the cycle

where y_{it} is the log of value added[2] for firm i in period t, k_{it} is the log of capital and l_{it} is the log of labor inputs. In the baseline exercise the input elasticities $\{\alpha_k, \alpha_l\}$ are the same for all sectors; as a robustness check I allow them to vary across 2-digit sectors, indexed by j. The error term is composed by two parts: a pure shock ε_{it} that is not observed by the firm nor by the econometrician and a productivity shock observed by the firm but not by the econometrician. The most straightforward way to estimate (1.4) is by OLS. However the problem with estimating a production function using OLS is that firms that have a large productivity shock may respond by using more inputs, which would yield biased estimates of the input coefficients and hence biased measure of TFP (*simultaneity bias*). Since traditional estimators used to overcome endogeneity issues (fixed effects, instrumental variables) have not proven satisfactory for the case of production function, a number of semiparametric alternatives have been proposed. Both Olley and Pakes (1996) and Levinsohn and Petrin (2003) have developed a semiparametric estimator that addresses the simultaneity bias. The key difference between the two methods is that Olley and Pakes (1996) use investment whereas Levinsohn and Petrin (2003) use materials used in production as a proxy for TFP. Since data on investment is readily available and often non-zero at the firm level but data on materials is not, I follow Olley and Pakes (1996) to estimate the production function. Once I estimate the production function parameters I obtain the level TFP by

$$TFP_{it} = \exp\left(y_{it} - \widehat{\alpha}_0 - \widehat{\alpha}_k k_{it} - \widehat{\alpha}_l l_{it}\right) \tag{1.5}$$

In the estimation of (1.4) I use industry specific time dummies, hence my measure of tfp is free of the effect of industry or aggregate growth in any year.

Table 1.2 reports the estimates for the production function parameters and their standard errors using the entire sample period for manufacturing and non-manufacturing firms. The results for all the firms combined, presented in the second column of the table indicate a labor share of 0.74 and a capital share of 0.29. The estimates for the persistence and conditional volatility of TFP (not reported in the table) are 0.69 and 0.30 respectively.

Validation of my TFP estimates. In order to gauge the sensibility of my TFP measure, I contrast some of its properties with those obtained from studies that use longitudinal micro-level datasets different from Compustat.

[2] Value added is defined as sales - materials, or, equivalentely, as operating income before depreciation and amortization plus labor expenses. Unfortunately in COMPUSTAT only information about the number of employees is available, therefore I approximate labor expenses by multiplying the number of employees by average wages from Social Security Administration. See appendix for more on data construction.

Table 1.2: Production function parameters

lnreal_va VARIABLES	(1) All sample	(2) Manuf	(3) Non-manuf
lnlabor	0.736***	0.824***	0.699***
	(0.00147)	(0.00239)	(0.00196)
lnreal_capital	0.292***	0.213***	0.330***
	(0.00131)	(0.00207)	(0.00178)
Constant	-2.653***	-2.827***	-2.582***
	(0.0205)	(0.0210)	(0.0363)
Observations	204,158	108,199	95,959
R-squared	0.932	0.944	0.921

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Total Factor Productivity		
	75th/25th	90th/10th
US Census	1.56	2.68
Compustat	1.47	2.41

Table 1.3: TFP Dispersion - Comparison

Please notice that labor has a higher coefficient in the manufacturing subsample and the opposite is true for the capital coefficient. This is consistent with the findings of Foster et al. (2001) who examine manufacturing data and service sector data. There is significant dispersion in firm level TFPs. In an important contribution investigating the productivity distribution in the US manufacturing sector, Syverson (2004) finds out that the interquartile range (i.e. ratio between 75th and 25th percentile) of tfp is around 1.56; moreover including more of the tails amplifies the heterogeneity: the ratio between the 90th and the 10th percentile is as much high as 2.68. Using Compustat, I find similar results, as Table 1.3 summarizes. Another robust finding is that labor productivity is more dispersed than total factor productivity. The interquartile range is 2.53 whereas the 90th/10th ratio is 7.28.

TFP dispersion and allocation efficiency over time.

In Figure 1.2 I plot firm-level TFP dispersion, computed as follows. First I compute total factor productivity by estimating the production function and taking the exponent of the predicted residuals (see equation (1.5)).

$$y_{it} = \alpha_0 + \alpha_k k_{it} + \alpha_l l_{it} + z_{it} + \varepsilon_{it}$$

$$TFP_{it} = \exp\left(y_{it} - \widehat{\alpha}_0 - \widehat{\alpha}_k k_{it} - \widehat{\alpha}_l l_{it}\right)$$

Then I define TFP shocks (e_{it}) as the residual from the following first-order autoregressive equation for firm-level log TFP:

$$\log TFP_{it} = \rho \log TFP_{it-1} + \delta_t + e_{it}$$

where δ_t is a year fixed effect (to control for cyclical shocks). Since this residual will also contain firm-level demand shocks that are not controlled for by 2-digit price deflators, my measure will combine both demand and technological shocks[3].

Then I compute the cross-sectional standard deviation and the interquartile range (IQR) of tfp shocks for each year across firms. Finally, I take a simple average across all years in the sample. In Figure 1.2, I report the IQR since it is more robust to outliers, however the results change little if I use the standard deviation.

In Figure 1.2, the blue shaded columns represent the share of quarters in recession within a year. It is apparent the negative correlation between cross-sectional tfp dispersion and gdp growth: interquartile range of TFP (IQR) spikes up during recessions, displaying a clearly countercyclical behavior. The findings delivered by Figure 1.2 confirm that during recessions an increase in cross-sectional heterogeneity is observed, which means that, ceteris paribus, there are more benefits to reallocate resources to more productive firms. The thesis of this paper is that instead during recessions frictions to capital liquidity increase and this dampens a reallocation process between firms that would be otherwise productivity enhancing. This is even more apparent from the Great Recession which has been characterized by a huge financial turmoil which greatly increased the cost of borrowing for firms; this reduced access to market for equity to more productive and small firms, dampening their growth and contributing to the fall in the covariance between firm productivity and firm size. From Figure 1.3 it is possible to notice that such covariance (Olley-Pakes gap) is generally procyclical and dropped the most in the 2007-2009 recession.

After estimating the total factor productivity series as I explained above, I compute the output-weighted TFP for the firms in the Compustat panel:

$$TFP_t = \sum_i \omega_{it} z_{it}$$

[3]This is a common feature of TFP estimates, since firm-level prices are very difficult to obtain. A relevant exception is Foster, Haltiwanger and Syverson (2008).

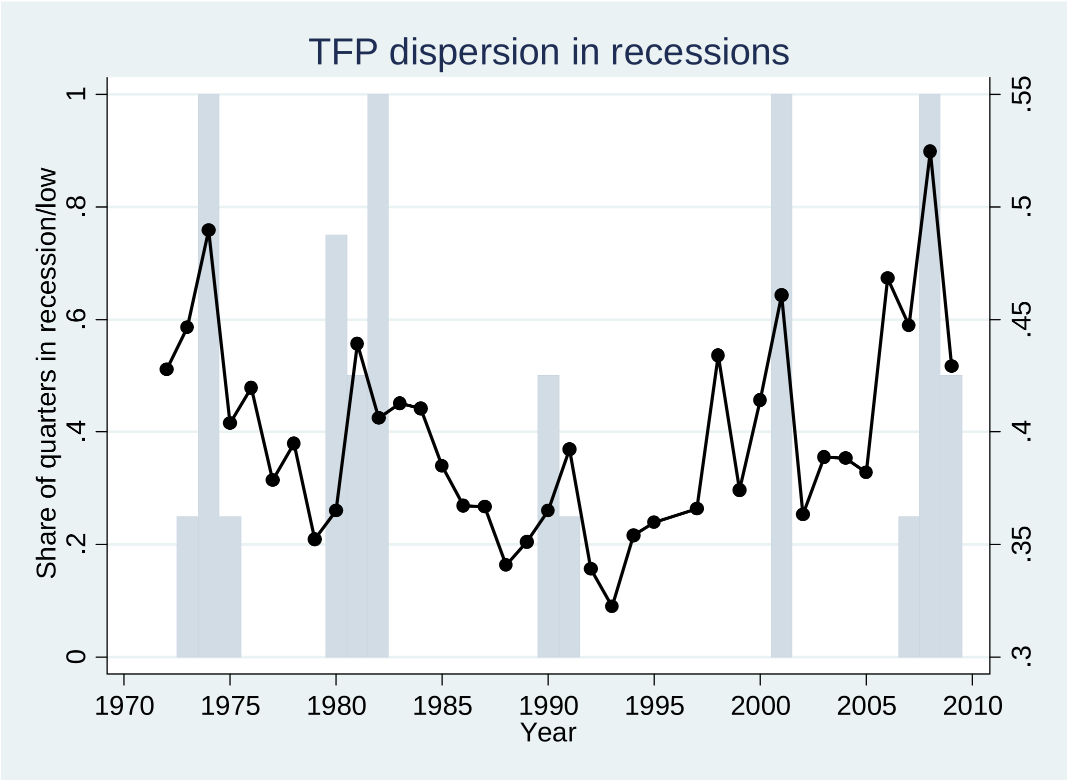


Figure 1.2: TFP dispersion in recessions

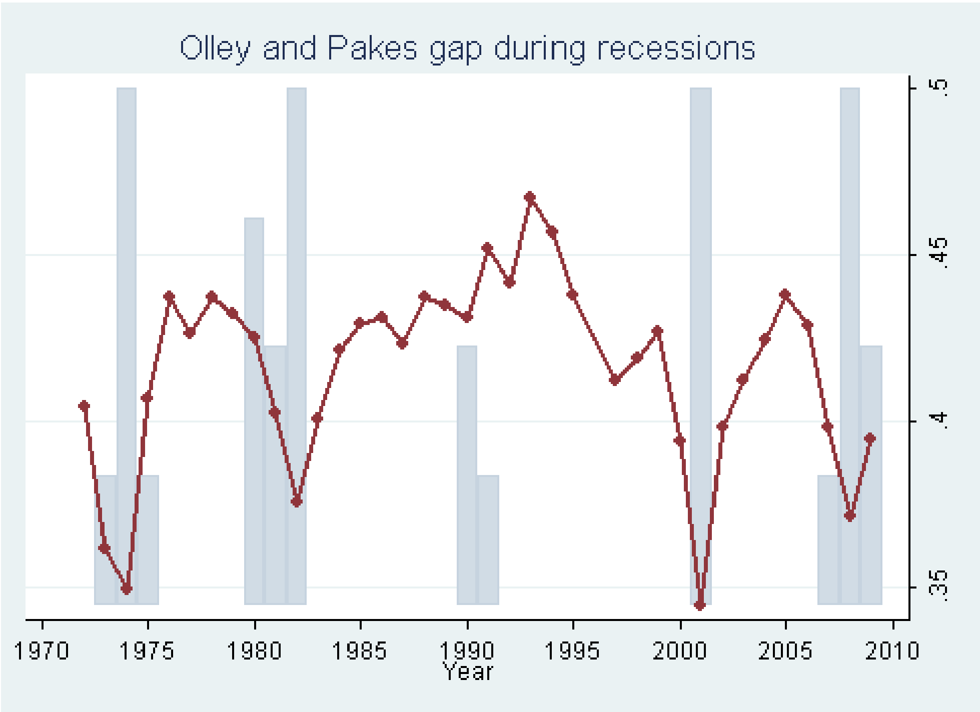


Figure 1.3: Covariance between size and productivity

I decompose the change in TFP during the Great Recession following the methodology of Olley and Pakes (1996) as the sum of unweighted component and a covariance term between size and total factor productivity:

$$\Delta TFP_{2009:2007} = \Delta UNW_{2009:2007} + \Delta COV_{2009:2007}$$

As I detailed above, the covariance between size and productivity is typically procyclical; it is striking however that in the Great Recession an unprecedented drop in this covariance term was observed. Since the Great Recession has been characterized by an unprecedented contraction in financing conditions, it is relevant to quantify the impact of financial frictions on the covariance term. In the real scenario I simulate the model by feeding only the aggregate shock A. Notice that the unweighted term in the model is given by

$$UNW = A \int \exp(z)\, \mu(dk, dz) = A \int \exp(z)\, \mu^z(dz)$$

hence I calibrate the shock A so to reproduce exactly the drop in UNW. Result: the aggregate shock A alone generates a small drop in COV. In the second scenario, on top of the aggregate shock A, I add the financial shock (modelled as a sudden and unexpected increase in the cost of raising external finance). By construction the drop in UNW is the same as before (and the same as what is observed in the data) but the drop in the COV is much higher than in the "real shock only" case. Hence I claim that the contribution of the financial crisis to reallocation is given by:

$$\Delta COV(\text{real} + fin \text{ shock}) - \Delta COV(\text{real shock}).$$

Compustat vs Census of Manufacturers. A key advantage of using the Compustat database is that I can decompose the change in allocative efficiency over time across both manufacturing and service, whereas previous studies that rely on the Longitudinal Research Database, LRD (from the US Census of Manufacturers) were limited on the manufacturing sector. Figure 1.4 and 1.5 plots the evolution of the share of manufacturing in terms of sales and employment over time. Given the decreasing importance of the manufacturing sector in terms of both output and employment, the advantage of a dataset like Compustat that covers all the sectors (though only for listed firms) is clear.

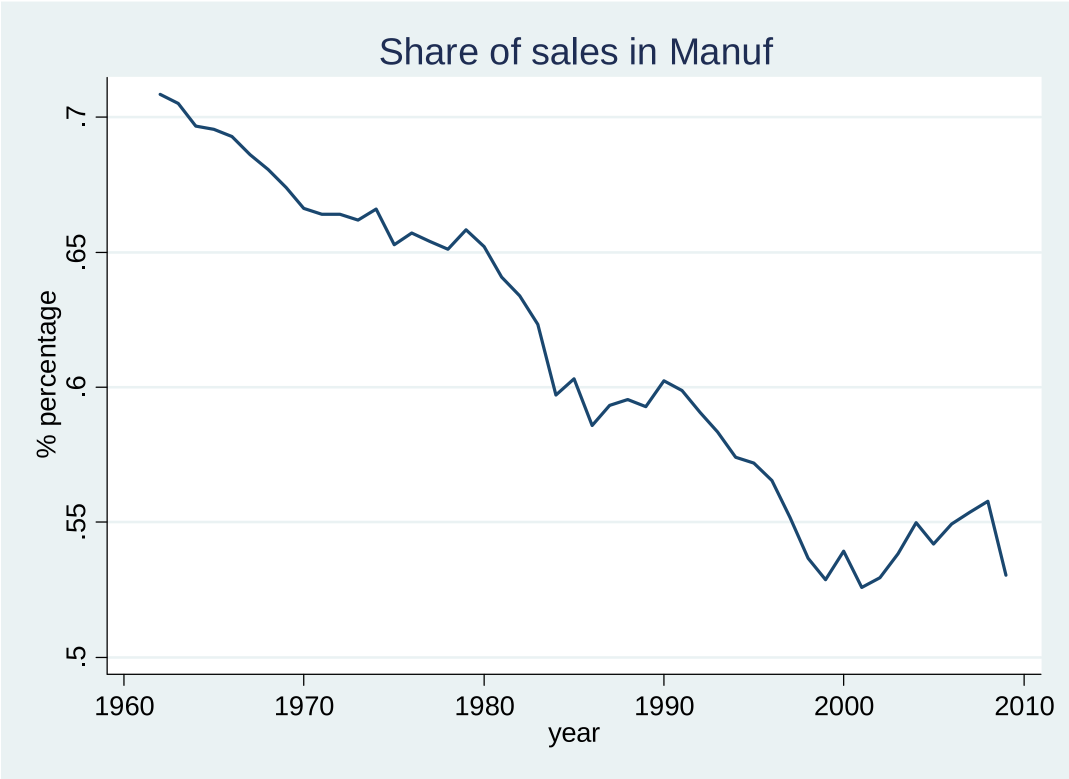


Figure 1.4: Share of sales in manufacturing over all sectors

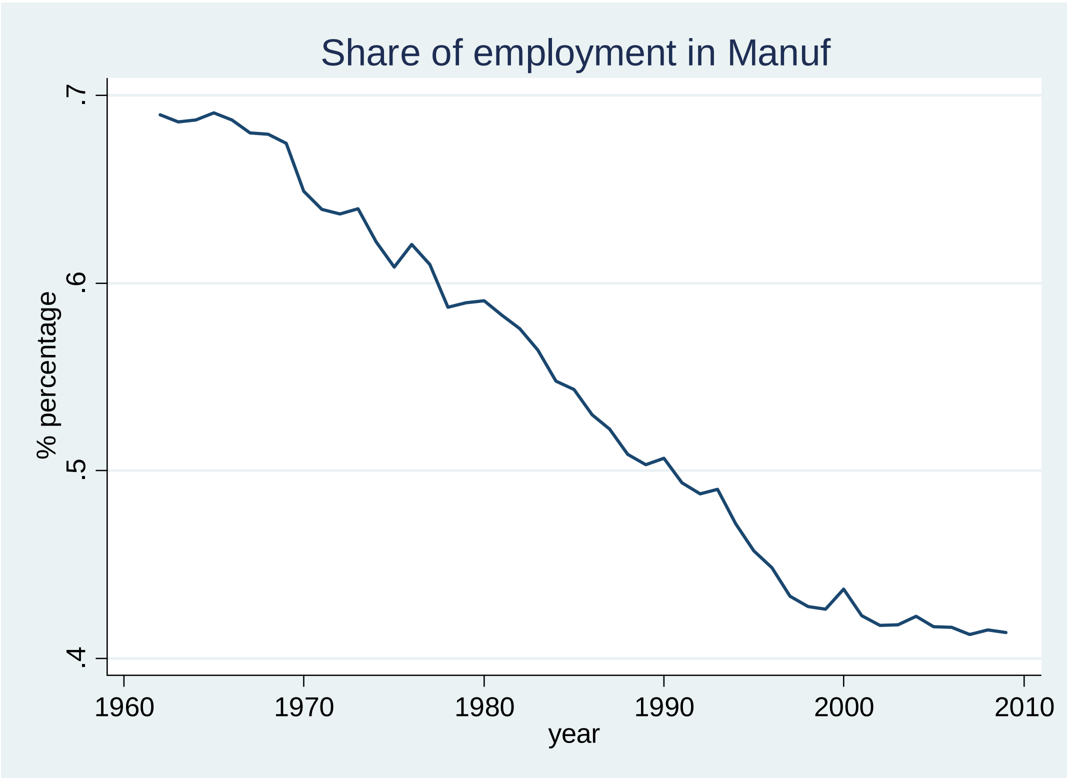


Figure 1.5: Share of employment in manufacturing sector over all sectors

1.4 The Model

The aim is to use my model to infer the impact of financial frictions on TFP through the reallocation channel. I therefore need a model that contains the following features:

- Firms with heterogeneous productivity levels, to make reallocation of capital among firms meaningful.
- Imperfect credit markets: I assume that in case firms want to raise additional funds (in excess over operating cash flow) they need to pay an additional cost.
- Shock to aggregate TFP to generate the recession; for the sake of tractability I model the negative aggregate shock to technology as a deterministic sequence that is unforeseen by economic agents[4].

1.4.1 Firms

I begin with describing the economic problem of the firms. Firms are ex-ante identical and are subject to an exogenous TFP shock A_t that is common across all firms and to an idiosyncratic productivity shocks denoted by z_{it}[5]. Since in the data firm-level productivity shocks show a high degree of persistence (as documented, among others, by Foster et al. (2001) and Bloom et al. (2012)) I assume that these shocks are generated by an autoregressive process with persistence ρ:

$$\log z_{it} = \rho \log z_{it-1} + \varepsilon_{it} \tag{1.6}$$

where ε_{it} is distributed as a $N\left(0, \sigma_\varepsilon^2\right)$. As it is standard in the literature I discretize the continuous time process described in (2.1) as a first-order Markov chain with transition matrix Q using Tauchen (1986) procedure. I assume $\Pr\left\{z' = z_j | z = z_i\right\} = Q_{ij} \geq 0$ and $\sum_j Q_{ij} = 1$ for each $i = 1, \ldots, N_z$. The sequence of aggregate shocks A_t is known with perfect foresight. Even though firms are ex-ante identical they differ ex-post since they experience different histories of idiosyncratic productivity shocks.

Firms use capital and labor as factor inputs and produce output by operating a decreasing returns to scale production function; the operating profit function (whose counterpart in the data is cash flow from operations) is:

$$\pi\left(A_t, k_{it}, z_{it}\right) = \max_{l_{it} \geq 0} \left\{A_t z_{it} F\left(k_{it}, l_{it}\right) - w l_{it}\right\} \tag{1.7}$$

[4] The alternative way would be to introduce explicitely aggregate uncertainty in the model, along the lines of Krusell and Smith (1996).

[5] The shocks z_{it} could in principle capture any shock affecting firm's revenues, hence not only shocks to technical efficiency but also (idiosyncratic) demand shocks.

Notice that $\pi(\cdot)$ is the operating profit function that is obtained after solving for the static labor choice, therefore it is a function of k and the shocks only. Denoting by I_{it} the investment made by firm i in year t, capital obeys the following law of motion

$$k_{i,t+1} = (1-\delta)\, k_{it} + I_{it},$$

where $\delta \in (0,1)$ denotes the depreciation rate. It is well-known that a model in which it is costless to adjust the capital stock delivers a time series for investment rates that is far too volatile; I therefore assume that the firm incurs quadratic adjustment costs when investing. It is also well-know, since at least Caballero et al. (1995), that plant-level investment is characterized by periods of inactivity followed by large spikes in investment; while it is hard to match this type of evidence with a quadratic cost of adjustment, I chose to adopt the quadratic specification for his computational tractability.

Firms can finance investment either with internal funds or borrowing from the financial market (by raising new equity or issuing debt). By raising external finance the firm incurs a variety of additional costs going from flotation costs to adverse selection premia. As in Gomes (2001) and Hennessy and Whited (2007) I do not model explicitly a setting with asymmetric information but I attempt to capture the simple fact that external funds are more costly than internal funds in a reduced form way. In particular, I assume that the additional cost of raising external finance is given by

$$c(e) = \lambda_0 + \lambda_1 \cdot \text{amount of external funds}$$

In other words there is a fixed cost λ_0 and a per unit cost λ_1 associated with external finance. A large body of empirical research provides detailed evidence regarding underwriting fees (see, among others, Altinkilic and Hansen (2000)) finding that there are significant economies of scale: this is why a cost function with decreasing average cost seems most appropriate. Cooley and Quadrini (2001) use a slightly different formulation which omits the fixed cost. However the fixed cost formulation is needed in my framework in order (i) to rationalize the presence of economies of scale and (ii) to match the degree of financial inaction that I documented in the Compustat sample (see Table 1.1).

The firm problem is to choose investment and financial policy to maximize net payments to its shareholders[6], taking as given the real interest rate and the wage rate:

$$V(k,z) = \max_{d,e,I,k'} \left\{ d - e - c(e) + \frac{1}{1+r} \sum_{z'} V(k',z')\, Q(z'|z) \right\} \tag{1.8}$$

s.t.

$$d + I + \frac{\psi}{2}\frac{I^2}{k} = \pi(k,z,A) + e, \tag{1.9}$$

[6]See appendix D for a derivation of the optimal value maximization problem of the firm.

$$k' = (1-\delta)\,k + I, \tag{1.10}$$

$$c\,(e) = (\lambda_0 + \lambda_1 e)\,1_{\{e>0\}}, \tag{1.11}$$

$$d \geq 0, \tag{1.12}$$

$$e \geq 0. \tag{1.13}$$

Equation (2.4) describes the flow of funds condition for the firm. The sources of funds (on the right-hand side) consists of operating cash-flows π, and external funds, e. The uses of funds (on the left-hand side) consist of capital expenditures, adjustment costs and dividend payments. Please notice that in this setting the only way firms can save is by accumulating capital: I choose to rule out firm savings in cash holdings or other financial assets. Adding financial savings or debt would make the problem more realistic but would also increase considerably the computational burden: with the additional debt choice there is a cross-sectional distribution of firms over three states capital, debt and idiosyncratic productivity (k, b, z) that is more difficult to handle with[7].

Equation (2.6) describes the external finance cost function: these costs are positive and increasing if the firm uses external funds. If no external funds are required, these costs are zero. This formulation is consistent with the Pecking Order Hypothesis (Myers and Majluf (1984)): firms first use internal finance and if they do not have enough, then issue debt, and as a last resort equity. The pecking order hypothesis can account for the stylized facts that retentions and then debt are the primary sources of finance. Notice furthermore that it is never optimal to raise external finance and at the same time distribute dividends. Indeed

Lemma 1.4.1 *It is never optimal for the firm to choose $e > 0$ and $d > 0$.*
Proof. *Suppose by contradiction that the firms chooses $e > 0$ and $d > 0$. Then the firm can decrease both e and d by a small amount $\varepsilon > 0$, which induce a change in profits given by $\lambda_1 \varepsilon > 0$* ■

1.4.2 Household

Since I am mainly interested in reallocation of capital among firms with heterogeneous productivities, on the household's side I can focus on a representative agent formulation. The representative agent has preferences over consumption and labor that are summarized by the following utility function

$$\sum_{t=0}^{\infty} \beta^t U\,(C_t, N_t) \tag{1.14}$$

[7] However, I'm currently working on an extension to incorporate debt into the firm's problem.

Household's income comes from wages and dividends. In order to write its budget constraint, I must aggregate all firm-level quantities. To this end, let me define μ_t as the cross sectional distribution of firms over the individual state (k, z) in period t. The budget constraint can then be stated as:

$$C_t + \int P_t \theta_{t+1}(k, z)\, \mu_t(dk, dz) + b_{t+1} - (1 + r_t)\, b_t = w_t N_t + \int (d_t + P_t - e_t - c(e_t))\, \theta_t(k, z)\, \mu_t(dk, dz) \tag{1.15}$$

where θ_t denotes the shares owned by household, b_t denotes bond holdings. In equilibrium $\theta_t = 1$ and $b_t = 0$ for all t since all households are equal.

The representative household's problem is to maximize (2.9) subject to (2.10). The first-order conditions with respect to labor supply N_t and bond holdings b_{t+1} are respectively:

$$-\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = w_t,$$

$$U_c(C_t, N_t) = \beta U_c(C_{t+1}, N_{t+1})(1 + r_{t+1}).$$

Since I consider the model in the stationary equilibrium with interest rate r_t, wage rate w_t and aggregate quantities constant over time, the household's problem can be simplified in this following *static* problem:

$$\max_{C,N} U(C, N)$$

s.t.

$$C = wN + \int d(k, z; w)\, d\mu(k, z) - \int e(k, z; w)\, d\mu(k, z) - \int c(e(k, z; w))\, d\mu(k, z) \tag{1.16}$$

This is a standard concave problem with interior solutions. In the steady state the Euler equation pins down the interest rate as

$$r = \frac{1}{\beta} - 1 \tag{1.17}$$

and optimality condition with respect to labor supply becomes:

$$-\frac{U_n(C, N)}{U_c(C, N)} = w \tag{1.18}$$

Solving the household's problem I get the household's decision rules for consumption $C(w; \mu)$ and labor supply $L^s(w; \mu)$.

1.4.3 Stationary distribution and Aggregation

Assuming A_t constant, the solution to the firm's optimization problem (2.3) delivers the *policy functions*

$$k' = g(k, z),\ I(k, z),\ l(k, z),\ y(k, z),\ e(k, z)$$

mapping the firm's state variables k and z into the firm's current choices (please notice that for simplicity I omit the dependence of the policy functions upon the wage w). The vector of individual state variables $x = (k, z)$ lies in $X = [0, \infty) \times Z$, where Z is the discrete set for productivity shocks z, i.e. $Z = \{z_1, z_1, \ldots, z_{nz}\}$. Let $\mathbb{B}$ be the associated Borel σ algebra. For any set $B \in \mathbb{B}$, $\mu(B)$ is the mass of firms whose individual states lie in the set B. The transition function $T(x, B)$ defines the probability that a firm in state $x = (k, z)$ will have a state lying in B in the next period, given the decision rule g for next-period capital. I can define each set B as the Cartesian product $B_K \times B_Z$; then the transition function $T : X \times \mathbb{B} \rightarrow [0, 1]$ can be written as:

$$T((k, z), B_K \times B_Z) = \begin{cases} \sum_{z' \in B_Z} Q(z, z') & \text{if } g(k, z) \in B_K \\ 0 & \text{otherwise} \end{cases}$$

where $g(k, z)$ is the policy function for next-period capital. Given the transition function, I can define the probability measure μ as

$$\mu'(B) = \int_X T(x, B)\, \mu(dx) \tag{1.19}$$

Given the invariant distribution $\mu^*(k, z) = \mu = \mu'$, I can compute the aggregate variables:

- Aggregate investment:

$$I(w; \mu^*) = \int I(k, k'(k, z); w)\, \mu^*(dk, dz)$$

- Aggregate labor demand:

$$L^d(w; \mu^*) = \int l(k, z; w)\, \mu^*(dk, dz)$$

- Aggregate output supply:

$$Y(w; \mu^*) = \int y(k, z; w)\, \mu^*(dk, dz)$$

- Aggregate adjustment costs:

$$AC(w; \mu^*) = \int \frac{\gamma}{2} \frac{I(k, z; w)^2}{k} \mu^*(dk, dz)$$

- Aggregate external finance costs:

$$E(w; \mu^*) = \int c(e(k, z))\, \mu^*(dk, dz)$$

Now I give the definition of equilibrium in my model, focusing for simplicity on the steady-state.

Definition 1 *A stationary recursive competitive equilibrium is a list of value function V, policy functions, invariant measure μ and prices r, w such that:*

(1) Given the prices $\{r, w\}$, the policy functions $d(k,z)$, $e(k,z)$, $I(k,z)$, $k'(k,z)$ solve the optimization problem of the firm in (2.3)

(2) Factor prices (r, w) are determined by equations (2.12) and (2.13)

(3) Markets clear; in particular in the labor market supply equals demand:

$$N^s(w;\mu) = \int_{x=(k,z)} l(k,z)\,\mu(dk,dz) \tag{1.20}$$

and the good market clears:

$$C(w;\mu) + I(w;\mu^*) + AC(w;\mu^*) + E(w;\mu^*) = Y(w;\mu^*)$$

where the term $E(w;\mu^*)$ represents aggregate costs of raising external finance. Of course by Walras' law this last resource constraint is redundant[8]: it is implied by combining the firm's flow of funds constraint (2.4) with the household's budget constraint (2.11).

1.4.4 Economic mechanism

Before reporting the results from the simulation, it is useful to look at the steady state distribution. Firms can be in three different finance regimes (this why heterogeneity is important)

1. $d = 0$, $e > 0$: external finance regime
2. $d = 0$, $e = 0$: financial inactivity regime
3. $d > 0$, $e = 0$: dividend distribution regime

Figure 1.6 illustrates these regimes for the baseline model and reveals a few interesting features. First, firms that are either very small or very productive tap the financial market and do not distribute dividends (top-left region: high z and low k; remember that z and k are the firm's state variables). These firms are in the external finance regime. Second, firms that are either very large or less productive use internal funds to finance investment and also distribute dividends (bottom-right region: low z and high k). They are in the dividend distribution regime. Finally the remaining firms do not distribute dividends and

[8] Indeed I don't use it when computing the equilibrium but I verify ex-post that it is satisfied.

do not raise external finance. They are in the financial inactivity regime. Figure 1.9 confirms this.

Figure 1.7 depicts the policy function for external finance, $e(k, z)$. It confirms that large firms, with a high capital stock, generate enough internal cash flow and do not need to raise additional resources from banks or from the equity market. In particular, there exists a capital threshold $\overline{k}(z)$ such that only firms with $k < \overline{k}(z)$ choose a strictly positive value of e; interestingly, such threshold is increasing with respect to productivity: holding the size of the firm fixed, firms that are hit by higher productivity shocks are more likely to raise external finance. This policy rule for external finance shows an inverted U-shape form when external finance is positive. In this model, external finance is a double-edged sword. For a given productivity, a firm needs to borrow to invest and this increases their expected profits. On the other hand, this also increases the cost related to external finance. The policy function reflects these two opposing tendencies creating the inverted U-shape that we observe. Higher values of z increase the future profits, allowing the firm to borrow larger amounts and shift the policy rule up.

Figure 1.8 plots instead the policy function for dividends $d(k, z)$: small firms tend not to distribute dividends since they need to use all their internal cash flow to finance investment, and this effect is of course more pronounced for higher productivity firms.

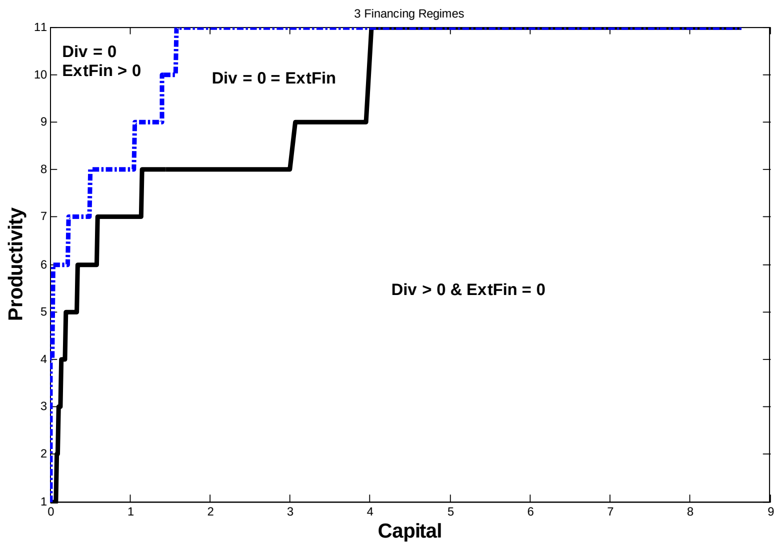


Figure 1.6: Finance Regimes

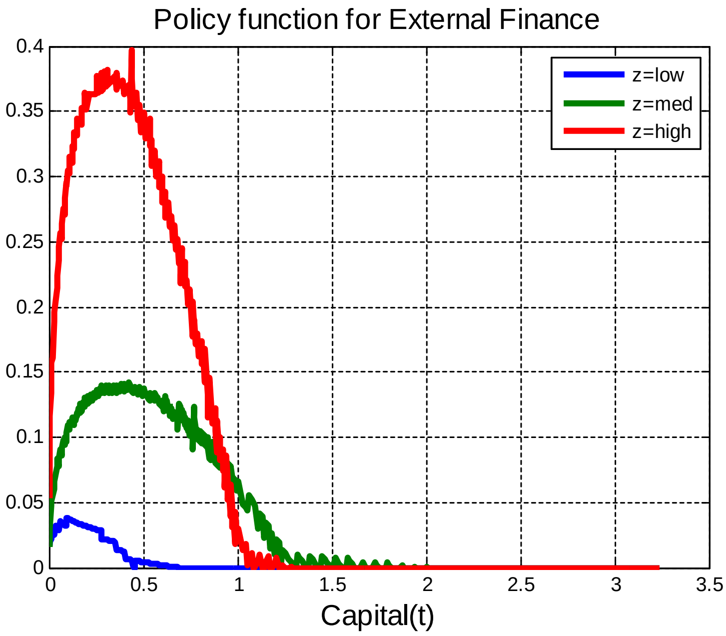


Figure 1.7: External Finance

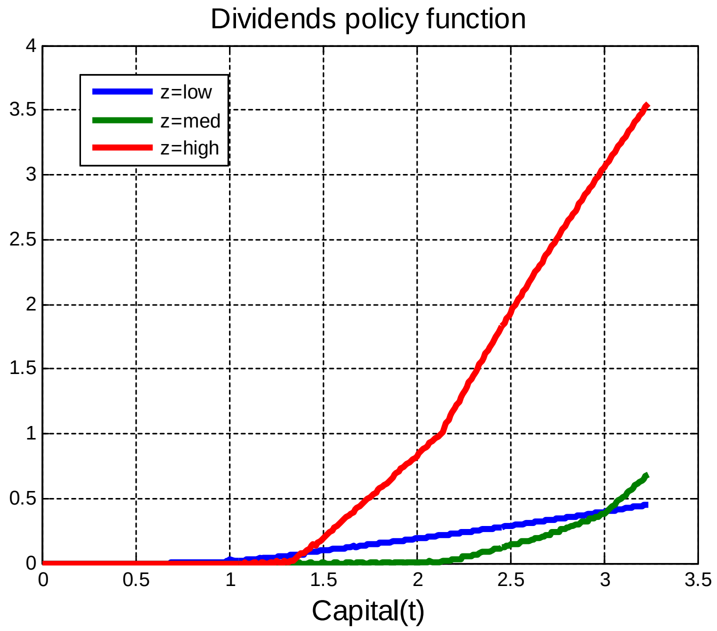


Figure 1.8: Policy Function for Dividends

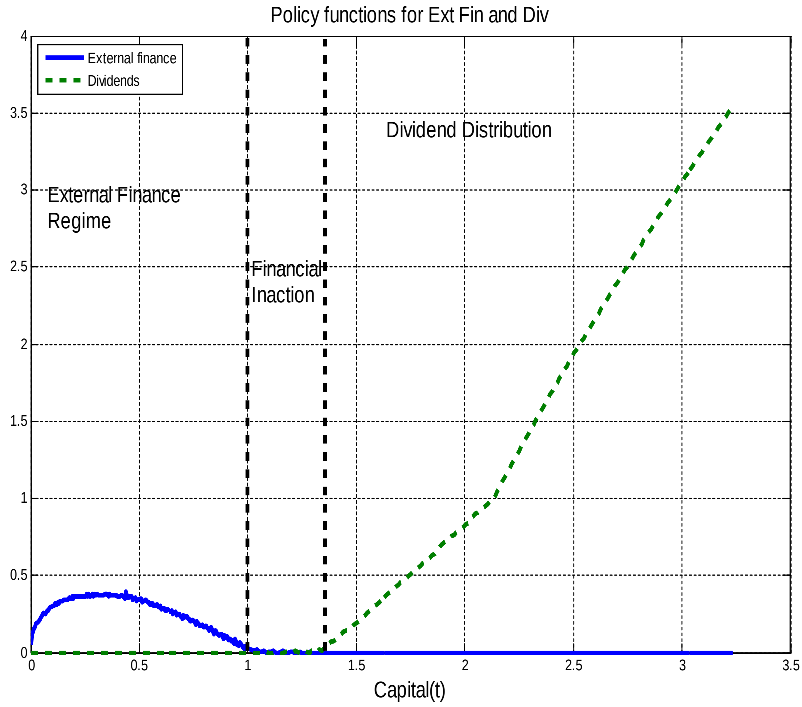


Figure 1.9: External finance, inaction and dividend distribution

1.5 Calibration and Quantitative Results

I assume that a time period corresponds to one year. I calibrate the baseline model to match some moments obtained from Compustat. The Longitudinal Research Database (LRD), a large panel dataset of U.S. manufacturing plants developed by the U.S. Bureau of the Census, is another dataset that is widely used in productivity and reallocation studies. One major shortcoming of the LRD for my purposes is that it lacks detailed data on firm's financing choices such as equity issuances, debt, interest expenses, etc. Another shortcoming of the LRD is that it is strictly limited to manufacturing establishments; hence the non-manufacturing sector, which is getting more important over time, is not represented at the LRD (see also discussion in section 3). Consequently I consider Compustat a better choice.

The sample period ranges from 1980 to 2006 which corresponds roughly to the Great Moderation period, before the 2007 recession started. The table below describes the calibration in the baseline scenario (i.e. steady state before the recession)

Preferences. Regarding the consumer's side of the economy, as I explained above it is highly stylized (representative agent) since I am mainly interested in firm dynamics. As

Parameter	Symbol	Calibration target
Exponent on capital	α_k	TFP process
Exponent on labor	α_l	TFP process
Depreciation rate	δ	Average I/K
Discount factor	β	Interest rate 4%
Weight on leisure	h	time spent on market work
Adjustment cost	ψ	std I/K
Shock persistence	ρ	TFP process
Shock standard deviation	σ_ε	TFP process
Fixed cost	λ_0	share firms $e > 0$
Marginal cost	λ_1	share firms $e > 0$

Table 1.4: CALIBRATION

per-period utility I choose the following functional form:

$$u(C, N) = \log C - \frac{h}{2} N^2$$

where h is the weight on leisure. This utility function has a unit Frisch elasticity of labor supply, which is reasonable for macro models as argued by Hall (2005). I choose the discount factor β such that the interest rate is equal to 4% using equation (2.12). I choose the parameter h to match the equilibrium labor supply of 0.3, which is the average fraction of time spent on market work.

Technology. I assume firms operate a Cobb-Douglas production function with decreasing returns to scale

$$y_{it} = A_t F(z_{it}, k_{it}, n_{it}) = A_t z_{it} k_{it}^{\alpha_k} n_{it}^{\alpha_l}$$

with $0 < \alpha_k + \alpha_l < 1$. Productivity shock follows the process

$$\log z_{it} = \rho \log z_{it-1} + \varepsilon_{it}$$

where ε_{it} is independently and identically distributed and normally distributed with mean 0 and variance σ_ε^2. The procedure for calibrating the parameter values α_k, α_l, ρ and σ exploits the micro level information on firm's technology provided by Compustat. As I explained in section 1.3 of my paper I estimate the following Cobb-Douglas production function in logarithms:

$$y_{it} = \alpha_0 + \alpha_{jk} k_{it} + \alpha_{jl} l_{it} + z_{it} + \varepsilon_{it}$$

allowing the factor elasticities to vary across 2-digit industries (as usual the index i refers to the firms whereas the index j refers to the sector). Then I consider the median across sectors of the α_{jk} and I set the capital coefficient in the model equal to this value. I do the same for the labor coefficient α_l. This procedure delivers a coefficient for capital $\alpha_k = 0.311$ and a coefficient for labor $\alpha_l = 0.65$. Interestingly the micro data do not reject the hypothesis of decreasing returns to scale.

To calibrate the persistence and the standard deviation of the stochastic process for idiosyncratic TFP shocks I first compute TFP in levels from the residual of the estimated equation:

$$\log TFP_{it} = \exp\left(y_{it} - \widehat{\alpha}_0 - \widehat{\alpha}_k k_{it} - \widehat{\alpha}_l l_{it}\right)$$

Then I fit a first-order autoregressive process to $\log TFP_{it}$

$$\log TFP_{it} = \rho \log TFP_{it-1} + \sigma e_{it},$$

where e_{it} is independently and identically distributed across i and t, and drawn from a standard normal distribution. These estimates imply that the parameters of the shock process z in the model are

$$\begin{aligned} \widehat{\rho} &= 0.742 \\ \widehat{\sigma} &= 0.275 \end{aligned}$$

It is useful to contrast these estimates for the productivity process with the study of Abraham and White (2006) who employ Census data. Their results imply that the persistence of firm-level shocks is surprisingly low: $\widehat{\rho}$ is only 0.37, whereas the standard deviation of the shock is 0.397 (Table 1 in their paper). This striking difference can be partly due to the different size of firms (firms in Compustat are typically bigger than firms in the Census) and to the fact that I consider all sectors in the economy (excluding only financial and government) whereas they can analyze only the manufacturing.

Another possible concern regarding my calibration is the non-standard choice for the Cobb-Douglas parameters α_k and α_l. Typically in the macro literature these parameters are calibrated to match the average labor share in aggregate data; however I find more reasonable to use micro-level estimates, since I do not have a representative firm with an aggregate production function in my model. The average labor share in my model implied by my calibration is 0.54 which is not too far from what reported in the real business cycle literature.

The final parameter to be calibrated is the adjustment cost parameter ψ. Because the volatility of the investment rate is very sensitive to this parameter, I choose a value to match the cross-sectional volatility of the investment rate in my data, which is around 0.16. More specifically, for any given value of ψ, I solve the model numerically and obtain the stationary distribution of firms. Using this stationary distribution, I compute the cross-sectional standard deviation of the investment rate in the model. Without adjustment cost, my model would imply excessive sensitivity of investment to variations in productivity shocks, which is inconsistent with empirical evidence. My calibrated value of ψ is close to the value reported by Cooper and Haltiwanger (2006), who estimate it using indirect inference.

Financing costs. The external cost function

$$c(e) = (\lambda_0 + \lambda_1 e)\, 1_{\{e>0\}}$$

is meant to capture the basic notion that external funds are more costly than internal ones. Broadly speaking, there are two types of costs associated with external finance: (i) informational costs and (ii) transaction costs. Informational costs are related to the bad signal the firm may transmit to the market when trying to raise funds (see agency cost theories, Myers and Majluf (1984)) but these are very hard to quantify. Transaction costs are given by compensation to intermediaries, legal and accounting costs associated to debt or equity issuance.

In order to calibrate the parameters of this external cost function, I need to construct an empirical measure of a firm's external financing needs. The aim is to choose λ_0 and λ_1 so that the model moments referring to external finance closely match the corresponding statistic computed from the data.

Consider the flow budget constraint of a firm in my model:

$$d_{it} - e_{it} = \pi(k_{it}, z_{it}) - I_{it} - \frac{\psi}{2}\frac{I_{it}^2}{k}, \tag{1.21}$$

where the left hand side represents the net financial flow out of the firm (if positive) or into the firm (if negative). If the right-hand side of (1.21) is positive, so that the firms's capital expenditure is less than the cash-flow generated by the firm in t, then funds flow *out* of the firm. In this case the firm is distributing dividends to its shareholders. Conversely, if the right-hand side of (1.21) is negative, then the firm's investment needs exceeds the available cash-flow, which means that funds flow *into* the firm. Then the firm is raising external funds, i.e. e_{it} is positive. Let me define the following two statistics:

- X_{it}: capital expenditures.
- AF_{it}: available funds. These are cash flow from operations net from interest payments.

Here I follow the standard approach in the literature on external finance dependence (see, among others, Rajan and Zingales (1998)). Since my model does not distinguish between investment in existing assets or acquisition of new assets, I compute the measure of investment as: X_{it}= capital expenditures + acquisition - sale of PPE (property, plant and equipment).

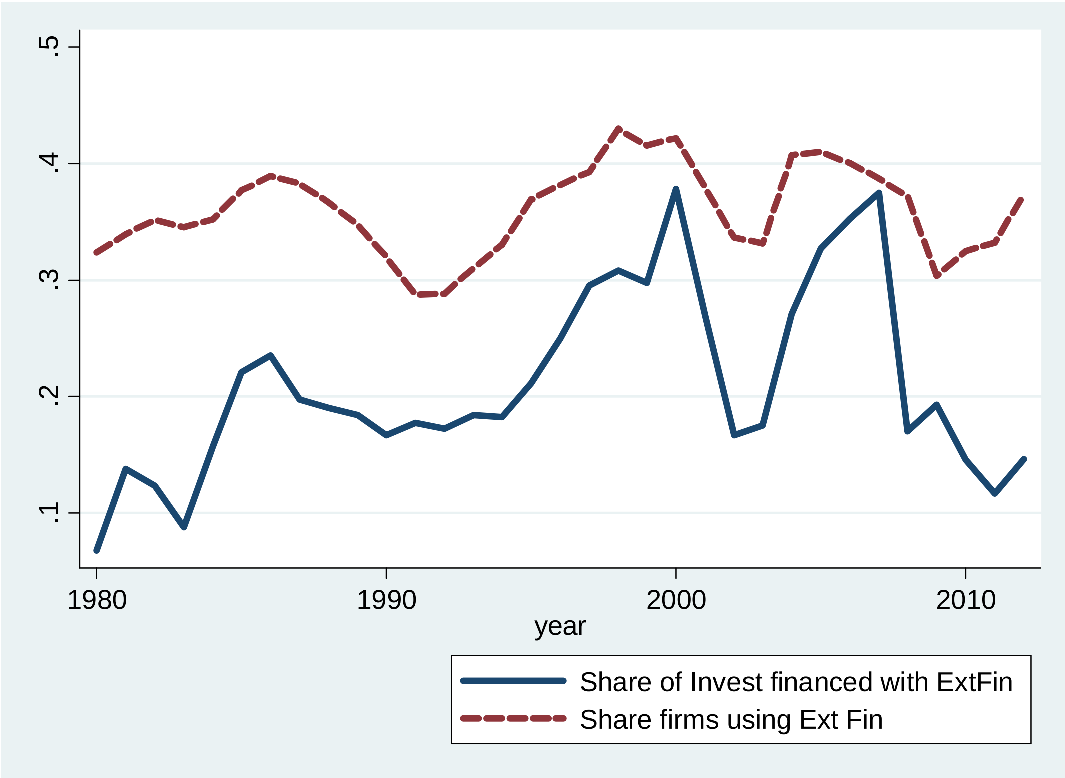


Figure 1.10: External Finance Flows

To compute available funds I have two possibilities: (1) Available Funds = Operating activities - net cash flow (OANCF) or Funds from operations (FOPT). (2) Available Funds = Income before extraordinary items (IBC) + depreciation and amortization (DPC). Both methods yield similar results. Then I can compute the share of firms raising external finance in year t as the number of firms whose investment is greater then their available funds in year t over the total number of firms in t:

$$\frac{\sum_{i=1}^{N_t} 1_{\{X_{it} > AF_{it}\}}}{N_t}. \tag{1.22}$$

I can also compute the fraction of investment that must be financed externally in year t as:

$$\frac{\sum_{i=1}^{N_t} (X_{it} - AF_{it})\, 1_{\{X_{it} > AF_{it}\}}}{\sum_{i=1}^{N_t} X_{it}}. \tag{1.23}$$

In Figure 1.10 I plot the evolution of (1.22) and (1.23) over time.

Calibration of cost function

I summarize the calibration in Table 1.5. The two moments reported in the table are computed taking the average of (1.22) and (1.23) across the years from 1980 to 2007; they

Parameter	value	Calibration target	Data moment
λ_0	0.10	Share of firms with $e > 0$	0.36
λ_1	0.28	Ext fin / investment	0.21

Table 1.5: Steady-state Calibration

Calibration target	Average 1980-2007	value in 2007	value in 2009
Share of firms with $e > 0$	0.36	0.39	0.30
Ext fin / investment	0.21	0.37	0.19

Table 1.6: Before and After the recession, Calibration

are meant to capture the average financing needs of firm in the steady state before the Great Recession broke out.

Note: when you read in the table that the external finance over investment ratio is 0.21 it means that on average 21% of investment undertaken by firms is financed externally (average across years). Please notice that this value is consistent with the empirical findings of Zetlin-Jones and Shourideh (2012).

As I discussed in the introduction, to simulate the impact of the Great Recession on firms financing environment I set the parameters of the cost function to match the share of firms raising external finance after the Great Recession hit the economy. As it is apparent from Figure (1.10) during the GR it became more difficult for firms to access credit: indeed the share of firms accessing outside financing dropped from 39% in 2007 to 30% in 2009. Moreover the fraction of investment financed with external funds dropped from 37% in 2007 to 19% in 2009 (see Figure 1.10 or Table 1.6).

1.6 Results

1.6.1 Steady State

In Table 1.7 I report the moments of the firm dynamics generated by the model and compare them with the corresponding data from Compustat. I report in italics the moments that are a calibration target, where the match is exact by construction. As I explained in the previous section, I chose the depreciation parameter δ to match the aggregate investment ratio and the adjustment cost parameter ψ to match the volatility of the investment rate. For the other quantities, one can see that my model matches most cross-sectional

Variable	Data	Model
Average I/K	*0.17*	*0.17*
std I/K	*0.156*	*0.156*
Autocorr. of I/K	0.596	0.64
Cov(ω,z)	0.438	0.534
External Finance	*0.36*	*0.36*
Financial Inactivity	0.213	0.147
Dividend distrib.	0.426	0.493

Table 1.7: Data vs Model, Results

moments reasonably well. In particular the model slightly overpredicts the autocorrelation of the investment rate that is observed in the data sample and slightly overpredicts the covariance between firm size and firm productivity.

Considering the financing regimes for the firms in the cross-section, the model by construction matches the shares of firms whose capital expenditures are larger than internal funds; however it generates more firms distributing dividends and less firms inactive than what is observed in the data.

1.6.2 Great Recession Simulation

As I discussed in section 3, I decompose the total factor productivity index for firms in Compustat as the sum of an unweighted component and a covariance component, following the methodology pioneered by Olley and Pakes (1996) and reprised by Bartelsman et al. (2013):

$$TFP_t = \sum_i \omega_{it} z_{it} = \overline{z}_t + \sum_i \left(\omega_{it} - \overline{\omega}_t\right)\left(z_{it} - \overline{z}_t\right) \tag{1.24}$$

where $\overline{z}_t$ and $\overline{\omega}_t$ represent unweighted mean productivity and unweighted mean share, respectively. This decomposition is useful to understand if the Great Recession impacted more on the productivity of the average firm or on the covariance between size and productivity. As documented in the first row of Table 1.8, the output-weighted total factor productivity decreased by 1.97% from 2007 to 2009; of such drop the unweighted term accounted for -0.93% and the covariance for -1.04%. Remember that the lower this covariance, the lower is the share of output that goes to more productive firms and the lower is the weighted productivity. But what is the contribution of the worsening in credit conditions on this covariance, which measures the allocative efficiency in the distribution of production factors across firms? I can evaluate this contribution by simulating the counterfactual scenario of a real recession only using my model.

	% ΔTFP	% Δunweighted	% ΔCov(k_{it},z_{it})
Data (Compustat)	-1.97	-0.93	-1.04
(I) Real shock	-1.095	-0.93	-0.165^{B}
(II) Real and fin shock	-1.79	-0.93	-0.86^{A}

Table 1.8: Data and Counterfactual Exercise

The production function equation in the model is given by:

$$y_{it} = A_t z_{it} k_{it}^{\alpha_k} l_{it}^{\alpha_l}$$

Hence total factor productivity in logs is equal to:

$$TFP_{it} = A_t z_{it}$$

and it is the product of an aggregate shock times a firm-level idiosyncratic shock. The output-weighted TFP, which is the model counterpart of (1.24) is:

$$TFP = A \int z \cdot \omega(k,z) \mu(dk,dz) = A \cdot \mathbf{E}(z) + A \cdot COV(z,\omega)$$

where $\omega(k,z)$ is the output weight of a firm with capital k and idiosyncratic productivity z:

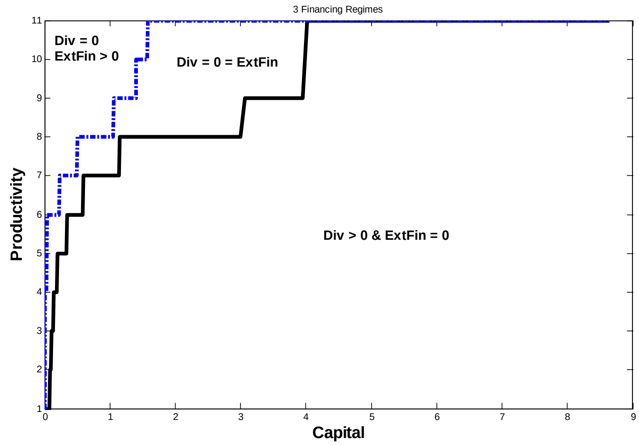
$$\omega(k,z) = \frac{y(k,z)}{\int y(k,z)\mu(dk,dz)}$$

This share is the model counterpart of ω_{it} in equation (1.24). The Great Recession had a negative impact both on the technological term and on the allocative efficiency term. I calibrate the aggregate shock to reproduce exactly the observed drop in the unweighted term.

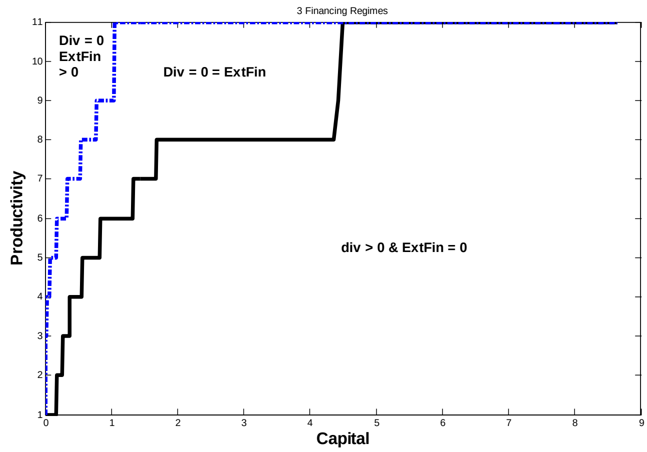
In the first scenario I hit the economy with a real shock only. From the table it is evident that a recession driven by a real aggregate shock only have a modest impact on the cross-sectional efficiency in the allocation of factors. A financial recession, as captured by the second exercise, instead, has a much larger impact on the covariance term. As in a diff-in-diff strategy, I can recover the contribution of financial frictions to the variation in the covariance by taking the difference between the two cells A and B in the table. In other words the impact of the financial shock on the cross-sectional efficiency of resources is

$$\begin{aligned} & \Delta Cov(\text{real+fin shock})\text{-}\Delta Cov(\text{real}) \\ = & -0.86-(-0.165)=-0.695 \end{aligned}$$

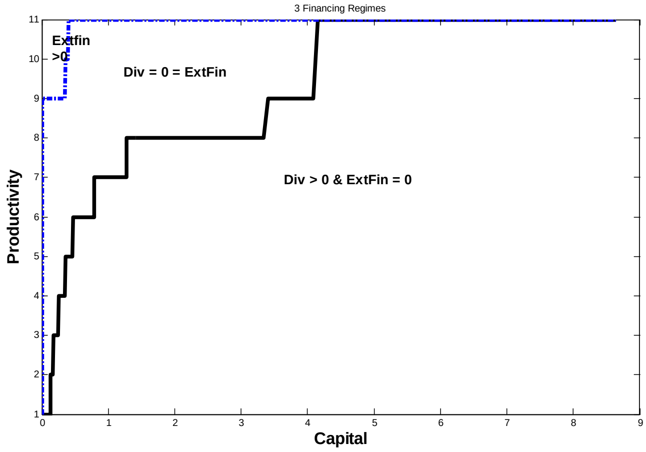
The main channel through which the increase in financial cost affects the covariance term is that it changes the distribution of firms across the three different financing regimes. The picture below show the partition of the state space into the 3 financing regimes



The financing costs are calibrated so that the share of firms in the external finance regime is roughly equal to the corresponding share in the data. In exercise I, when only the real aggregate shock hits the economy, the new distribution is



In exercise II, when also the financial shock hits the economy, the new distribution is:



Much less firms can access now external finance; but those firms who were accessing external finance were the most productive. Therefore the financial shock causes a reallocation of productive inputs from high to low productivity firms, decreasing the cross-sectional efficiency, as captured by the covariance term.

General Equilibrium Effect. Since my model is cast in general equilibrium, it is insightful to conduct the hypothetical experiment of shutting down the price feedback mechanism. Specifically, I fix the wage rate at its level in the steady state before the recession. At this wage I use labor demand to determine aggregate employment by ignoring the labor market-clearing condition (2.15). After solving the firm's problem, I derive aggregate investment and aggregate output. I then use the resource constraint to solve for aggregate consumption. The profit function of the firm, under the parametric assumptions described in section 1.5, is:

$$\pi\left(A,k,z;w\right)=\left(1-\alpha_l\right)\left(\frac{\alpha_l}{w}\right)^{\frac{\alpha_l}{1-\alpha_l}}A\left(zk^{\alpha_k}\right)^{\frac{1}{1-\alpha_l}}$$

The above equation reveals that the lower wage increases the firms' profits and its return to investment. Moreover, since π also represents operating sales net of labor payments[9], a lower wage increases the firm's internal cash flows. This equilibrium price feedback effect dampens the decrease in investment among firms that are raising external finance and hence the drop in the covariance between size and productivity is smaller in general equilibrium. Table 1.9 reports the results from the simulations in the partial equilibrium, i.e. the changes in the total factor productivity that I would observe if the wage stayed constant at the level before the recession. In particular the decrease in the covariance between size and productivity is about 4/5 times larger than in general equilibrium.

[9]Indeed $\pi\left(k,z\right)=\max_l\left\{zy\left(k,l\right)-wl\right\}$.

	% ΔTFP	% Δunweighted	% ΔCov(k_{it},z_{it})
Data (Compustat)	-1.97	-0.93	-1.04
(I) Real shock	-1.67	-0.93	-0.74
(II) Real and fin shock	-5.19	-0.93	-4.26

Table 1.9: Data and Counterfactual Exercise - Partial Equilibrium

To sum up, my numerical experiment demonstrates that performing counterfactuals in partial equilibrium can have potentially misleading outcomes.

1.7 Conclusions

In this work I document that in the Compustat dataset (representative sample of listed firms in the US) a significant part of the drop in total factor productivity observed during the Great Recession can be attributed to a decrease in the allocative efficiency of capital among firms (rather than to a technological effect common to all firms). Indeed the decrease in the covariance between size and productivity (a measure of allocative efficiency) is 1.04 percent out of roughly 2 percent decline in total factor productivity. The use of Compustat improves upon previous studies for at least two reasons. First the service sector, extensively represented in Compustat, has become increasingly important in the recent years; second while Compustat does not cover *small* firms (as long as small firms are not listed firms) it offers a very thorough description of *large* firms that account of more than 50 percent of total GDP and more than 30 percent of total employment in the US economy.

The empirical finding that the allocative efficiency of resources among firms worsens during economic downturns sharply contrasts with the *cleansing* view of recessions: according to this theory, that dates back at least to Schumpeter[10], recessions should be times of enhanced reallocation. Since during economic downturns the dispersion in profit growth rates increases (as documented by Eisfeldt and Rampini 2006) there are more benefits of reallocating capital from less to more productive plants; moreover during recessions the opportunity cost of resources are typically low (plants are underutilized); these observations should imply that recessions are times of accelerated productivity enhancing reallocation. However in the Great Recession financial conditions worsened and hence the increase in credit market frictions could have had a negative impact on reallocation. In particular during the Great Recession, reallocation of productive inputs was driven more by frictions in credit markets than by economic fundamentals such as productivity.

With the help of a model with heterogeneous firms I find out that the distribution

[10] The first formalization of the creative destruction theory is by Aghion and Howitt (1992).

of firms among financing regime is a crucial determinant of this covariance. Since in the data I see that the 2007-2009 period witnessed a large drop in the firms raising funds from financial market, I relate the increase in the cost of external financing to the misallocation of resources among firms. In the model a reduced-form cost function captures the basic notion than external funds are more costly than internally generated cash-flow. The increase in the cost of external finance affects most firms that are small and highly productive; these firms are growing and are giving a positive contribution to the covariance term. In order to assess the contribution of financial conditions to the covariance I simulate a counterfactual recession where the economy is hit by a technological worsening only; by construction this shock to the average total factor productivity matches exactly the drop in the unweighted tfp.

To summarize the two critical implications from my study are the following:

(i) variations in measured total factor productivity are only to a small extent variation in the productivity of the average firm. The main part is attributable to a reallocation of market shares between firms with heterogeneous productivity levels.

(ii) The extent to which more productive firms also enjoy a larger market share critically depends on the easiness in accessing financial markets to get external funds for investment.

1.8 Data Appendix

How to construct investment rates

Since firms record capital stock at book value rather than the more useful economic concept which is replacement value, I use perpetual inventory model (as described in Salinger and Summers 1983 and Gomes 2001) to convert book value of capital into replacement value for every firm-year. First, I set the replacement value of the initial capital stock equal to the book value of gross PPE for the first year that the firm shows up in Compustat. Then I estimate the useful life of capital goods in any year using the formula

$$L_{i,t} = \frac{Bk_{i,t-1} + I_{i,t}}{Depr_{i,t}}$$

where $Depr_{i,t}$ is the reported value of depreciation and amortization, and take the time average of L_{it}, which I call L_i. Finally I compute the series for the capital stock k_{it} (in market value terms) iterating on the following recursive formula:

$$k_{i,t} = \left[k_{i,t-1}\frac{P_t}{P_{t-1}} + I_{i,t}\right]\left(1 - \frac{2}{L_i}\right)$$

for $t = 1, 2, ..$, where P_t is deflator for non-residential investment. and L_i is the time average of $L_{i,t}$.

Derivation of the Olley-Pakes decomposition

The decomposition proposed by Olley and Pakes (equation 1.3 in the main text) follows after some algebra:

$$\begin{aligned} TFP_t &= \sum_i \omega_{it} TFP_{it} \\ &= \sum_i \left(\overline{\omega}_t + \omega_{it} - \overline{\omega}_t\right)\left(\overline{TFP}_t + TFP_{it} - \overline{TFP}_t\right) \\ &= N_t \overline{\omega}_t \overline{TFP}_t + \sum_i \left(\omega_{it} - \overline{\omega}_t\right)\left(TFP_{it} - \overline{TFP}_t\right) \\ &= \overline{TFP}_t + \sum_i \left(\omega_{it} - \overline{\omega}_t\right)\left(TFP_{it} - \overline{TFP}_t\right) \end{aligned}$$

where $\overline{TFP}_t = \frac{1}{N_t}\sum_i TFP_{it}$, where N_t is the number of active firms in period t.

Output weighted TFP in the model

- Let $\mu(k, z)$ denote the stationary distribution of firms over capital and productivity
- The output-weighted productivity in the model is computed as:

$$TFP = \int_z \int_k \omega(k, z)\, e^z$$

where

$$\omega(k,z) \equiv \frac{y(k,z)\,\mu(k,z)}{\int_z \int_k y(k,z)\,d\mu(k,z)}$$

1.9 Computation - Steady State

The algorithm follows Aiyagari(1994) and Huggett(1993). I start by guessing a value for the wage w. For the given wage I solve the firm's decision problem by value function iteration on a discrete grid. Then I compute the invariant distribution of firms over capital and productivity. As a last step I check whether the labor market equilibrium condition holds. If not, I update the wage.

More in detail:

- Step 1 - Make a guess for equilibrium wage w.
- Step 2 - Given w, solve the firm's problem by value function iteration on a discrete grid. Even if slow, it is the most robust method (better to use this because policy functions are non linear due to the fixed equity cost). Get policy function $k' = g(k,z)$ and the other decision rules.
- Step 3 - Using the policy function $g(k,z)$ computed in step 2 and the exogenous Markov chain for productivity shocks, compute the invariant distribution $\mu^*(k,z)$ by iterating on (2.14)
- Step 4 - Using the stationary distribution $\mu^*(k,z)$ obtained in step 3, compute aggregate labor demand $N^d(w) = \sum_{k,z} n(k,z)\,\mu^*(k,z)$. Then check if equation

  $$-\frac{U_n\left(C, N^d(w)\right)}{U_c\left(C, N^d(w)\right)} = w$$

  is satisfied[11]. If it is, stop; otherwise update the wage and go back to step 2. An alternative way is to compute explicitly the excess demand function for labor: $N^d - N^s$.
- Iterate until convergence.

[11] The function $-\frac{U_n\left(C,N^d(w)\right)}{U_c\left(C,N^d(w)\right)} - w = 0$ is not perfectly continuous given the discretized nature of the algorithm, and it is therefore not always possible to compute a clearing wage level to an arbitrary level of precision. However the problem is generally well-behaved with a tolerance level of 10^{-7} in the baseline simulation.

1.10 Computation - Transition

I describe the algorithm for a transitory shock (i.e. the initial and the final steady state are equal).

1. Compute steady state

2. Make a guess for the wage and the interest rate path along the transition: $\left\{r_t^{old}, w_t^{old}\right\}_{t=1}^{T}$. A good guess is r^{ss}, w^{ss}

3. Solve the firm's problem by backward induction, starting with $V_T = V^{ss}$. Compute policy functions $\{g(k,z)\}_t$ for $t = T-1, T-2, .., 1$

4. Using the exogenous Markov chain and the time-varying policy functions computed in the previous step, iterate forward the distribution starting from $\mu_1 = \mu^{ss}$:

$$\mu_{t+1}(k', z') = \sum_k \sum_z \Pi(z, z')\, 1_{\{k: g(k,z)=k'\}} \mu_t(k, z)$$

5. Using $\{\mu\}_{t=1}^{T}$ and $\{g(k,z)\}_{t=1}^{T}$ compute aggregate variables C_t, N_t, Y_t for each time t

6. Get new sequence of wage and interest rates $\{w_t', r_t'\}$ from the household's first order conditions:

$$w_t' = -\frac{U_n\left(C_t, N_t^d\right)}{U_c\left(C_t, N_t^d\right)}$$

$$r_t' = \frac{U_c(C_t, N_t)}{\beta U_c(C_{t+1}, N_{t+1})}$$

If $\max_t \left\{|r_t^{old} - r_t'| + |w_t^{old} - w_t'|\right\}$ is less than a precision threshold, stop. Otherwise update the prices sequences in this way:

$$\begin{aligned} w_t^{new} &= \phi w_t' + (1-\phi)\, w_t^{old}, \\ r_t^{new} &= \phi r_t' + (1-\phi)\, r_t^{old} \end{aligned}$$

and go back to step 2.

1.11 Firm's value problem

Consider the representative household's maximization problem which I re-write below for convenience:

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \tag{1.25}$$

s.t.

$$C_t + \int P_t \theta_{t+1} d\mu_t + b_{t+1} - \left(1 + r_t\right) b_t = w_t N_t + \int \left(d_t + P_t - e_t - c\left(e_t\right)\right) \theta_t d\mu_t \tag{1.26}$$

The first-order conditions with respect to bond b_{t+1} and share holdings θ_{t+1} are:

$$U_1\left(C_t, N_t\right) = \beta\left(1 + r_{t+1}\right) U_1\left(C_{t+1}, N_{t+1}\right)^{12}$$

and

$$U_1\left(C_t, N_t\right) P_t = \beta U_1\left(C_{t+1}, N_{t+1}\right) E_t\left\{d_{t+1} + P_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}$$

Hence combining the two equations I get the result that the required rate on return on equity must be equal to the real interest rate (in other words, there is no risk premium).

$$\left(1 + r_{t+1}\right) = \frac{E_t\left\{d_{t+1} + P_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}}{P_t}.$$

or,

$$P_t = \left(\frac{1}{1 + r_{t+1}}\right) E_t\left\{d_{t+1} + P_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}$$

Iterating forward yields

$$P_t = \sum_{n=1}^{\infty} \prod_{j=1}^{n} \left(\frac{1}{1 + r_{t+j}}\right) E_t\left\{d_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}$$

which corresponds to (**??**) or (2.3).

1.12 An alternative formulation of the firm's problem

An alternative formulation of the firm's problem is the following[13]:

$$V\left(k, z\right) = \max_{k' \geq 0, I} \left\{ \begin{array}{c} \pi\left(k, z\right) - I - \frac{\psi}{2}\frac{I^2}{k} - \lambda_0 1\left\{I + \frac{\psi}{2}\frac{I^2}{k} > \pi\left(k, z\right)\right\} - \lambda_1 \max\left\{I + \frac{\psi}{2}\frac{I^2}{k} - \pi\left(k, z\right), 0\right\} \\ + \frac{1}{1+r}\mathbb{E}_{z'|z} V\left(k', z'\right) \end{array} \right\}$$

[12] Along the transition aggregate variables and prices are deterministic sequences, hence I do not need the expectation operator.

[13] I would like to thank Matthias Messner for suggesting this equivalent formulation

s.t.

$$k' = (1 - \delta) k + I,$$

where $1\{\cdot\}$ is an indicator function. The term in brackets is the sum of current net cash flow and expected discounted continuation value; net cash flow is current profits minus investment spending and financing costs. If current profits are lower than desired capital expenditures then the firm has to pay an additional cost (both fixed and linear). To see the equivalence with (2.3) it is useful to define the auxiliary variable e:

$$e = \max\left\{I + \frac{\psi}{2}\frac{I^2}{k} - \pi(k, z), 0\right\}$$

The term $I + \frac{\psi}{2}\frac{I^2}{k} - \pi$, if positive, represents the amount of external finance raised by the firm. As it is explained in the main text, the cost function captures the basic fact that external funds are more costly than internal funds.

Chapter 2

Idiosyncratic firm risk, Cash Holdings and Lumpy Investment

2.1 Introduction

The US corporate sector is holding record-high amounts of cash. Understanding this phenomenon, many argue, may help us to shed light on the reasons for the slow recovery from the Great Recession.

A close look at balance sheets data of publicy traded US firms shows that their cash holdings have increased dramatically since the 1980s except for a slowdown around the financial crisis. One explanation most frequently given for the secular growth in cash relates to firm level uncertainty. It is well-documented that firm level risk has been trending up since the 1980s (see for example Comin and Philippon, 2005). The common story is that financially constrained firms accumulate more cash in order to protect themselves from negative profit shocks, and this effect can increase if the volatility of the underlying stochastic process increases. This precautionary behaviour has been extensively studied in the consumption literature: see among other Aiyagari (1994).

In this paper I document a novel empirical finding: the positive effect of firm level uncertainty on cash holdings is stronger for firms that belong to sector with higher investment lumpiness. In order to better understand the mechanisms behind these empirical stylized facts I build a structural model with heterogeneous firms, non-convex adjustment costs to capital and collateral constraints.

I start by documenting the stylized facts of the link between the rise in firm level risk and the secular trend in corporate cash holdings.

I build on Comin and Philippon approach to construct a new set of firm level measures of uncertainty for all non-financial firms in Compustat between 1970 and 2010. Robustness checks....

Using this firm-level measure of uncertainty, I document several new empirical regularities on the link between firm financing and

In particular, there is a strong positive relation between firm risk and corporate cash both in the cross-section and in the time-series, with firm level shocks volatility emerging as the most important determinant of cash holdings. The key empirical fact that I subsequntly discover is that the link between cash and firm level risk is especially strong for firms that are financially constrained and those that belong to sectors with greater investment inflexibility.

My explanation and key mechanism

2.2 Literature Review

The paper proceeds as follows. In section 2.3 I document the empirical evidence regarding the relation between corporate cash holdings, firm-level risk and investment lumpiness. In section 2.4 I set up a structural model of heterogeneous firms with debt and collateral constraints to rationalize the empirical findings detailed in the previous section. In section 2.5 I explain the calibration of the model and perform the comparative statics exercise. Section 2.6 concludes the paper.

2.3 Stylized facts and Empirical Evidence

- Review the empirical evidence regarding the secular increasing trend in cash holdings.
- Empirical evidence (relatively less-known) regarding the increase in firm-level uncertainty
- Document the positive link between cash holdings and risk, and the fact that such impact is stronger for firms that belong to sectors with higher investment lumpiness (and for firms that are more credit constrained).
  - My indicators of investment inaction and investment spikes are based on Cooper and Haltiwanger 2006

- I measure time series skewness of sector level investment following Cabellero (1999).
- I use a number of indicators for being financially constrained: dividend dummy, WW-index (based on Whited and Wu (2006)), size dummy, etc. In particular in every year I rank firms based on several ex-ante indicators of their financial status, which include firm size, dividend payer status, the WW-index based on Whited and Wu (2006), a measure of asset liquidation value[1], etc.

Empirical Fact 1: Firm cash holdings show a secular upward trend in 1970-2010, whereas net leverage show a decreasing trend over the same time span.

It is widely recognized that US firms have been accumulating large piles of cash during the last three decades; this increasing trend in cash comes together with a decreasing trend in net leverage.

Why is it important to consider NET leverage as opposed to leverage? Net leverage is computed at the firm level as (LT debt + current debt – Cash)/Book Assets, whereas leverage is computed as (LT debt + current debt – Cash)/Book Assets. It is more meaningful to consider net leverage as a measure of firm level financial leverage, and by considering net leverage instead of leverage we get dramatically different conclusions. The Net leverage series shows important business cycle fluctuations, but still the downward trend is apparent. Around 2004 net leverage becomes negative!

[What about plotting also the trend component using HP filter for annual observations?]

I estimate a simple regression of the cash ratio on a constant and time to see whether there is a statistically significant trend in the cash ratio. I report my result below (standard errors in parenthesis)

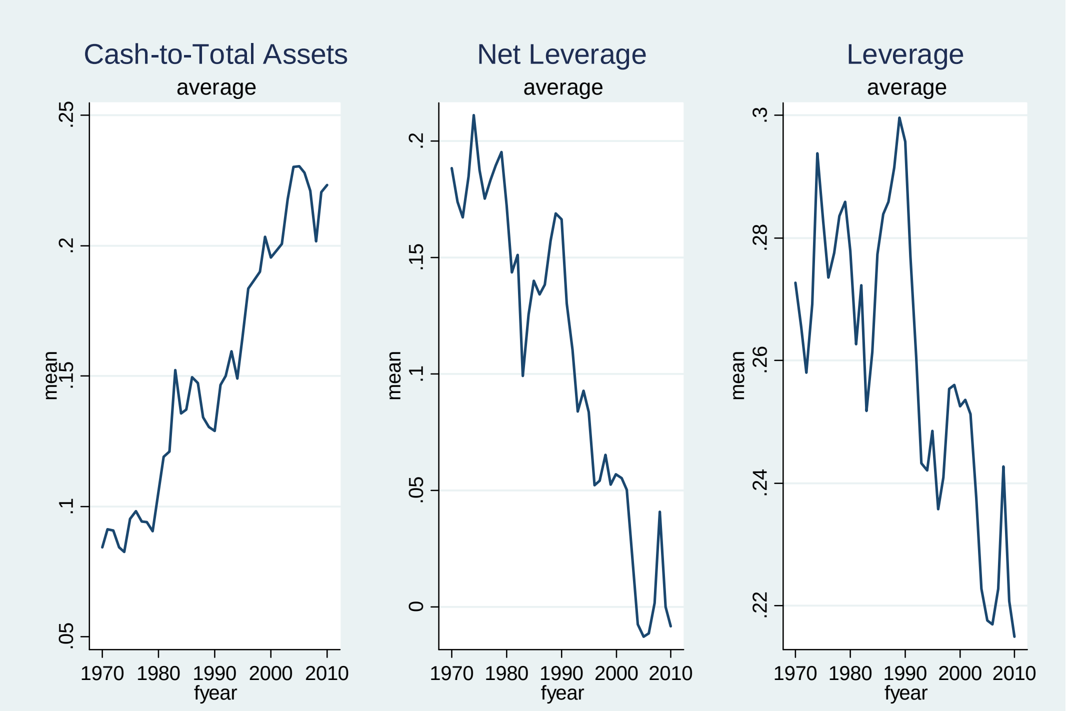
[INSERT TABLE WITH REG CASH OVER TIME (TO SEE THE TREND)]

The coefficient for the time trend for the average cash ratio corresponds to a yearly increase of 0.39% and is highly statistically significant.

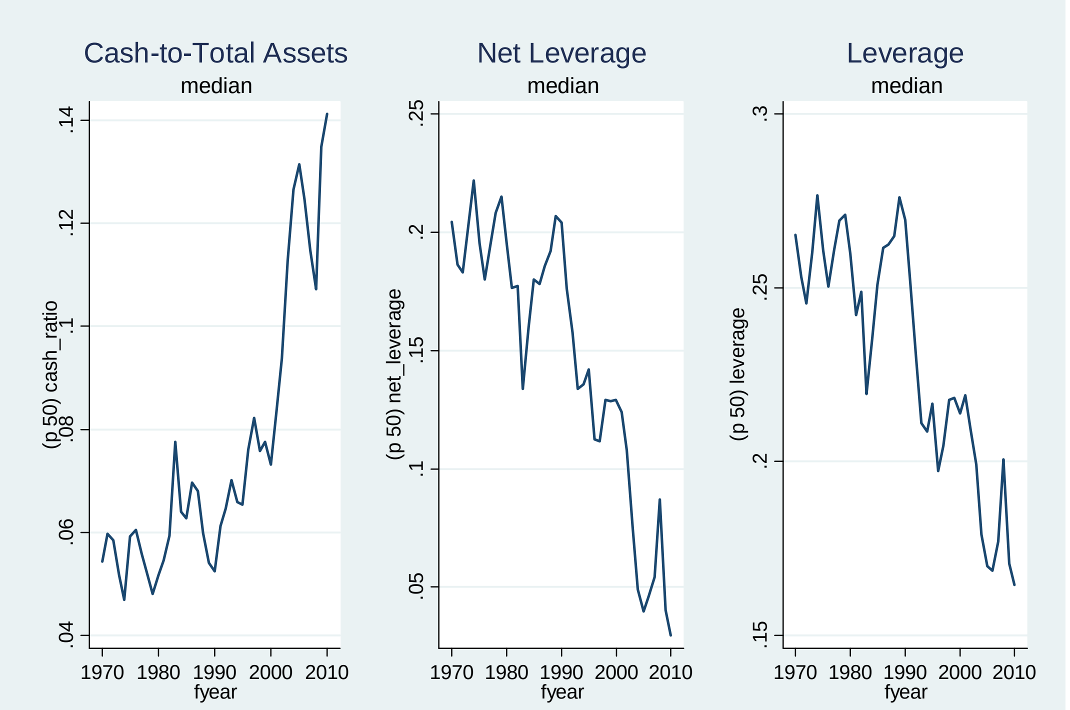
When I regress instead net leverage (mean) on a constant and a time trend I get a decrease of 0.54%, again significative at the 99 percent level.

Empirical Fact 2: *The increasing trend in cash is not driven by the largest firms or by a particular sizeclass of firms.*

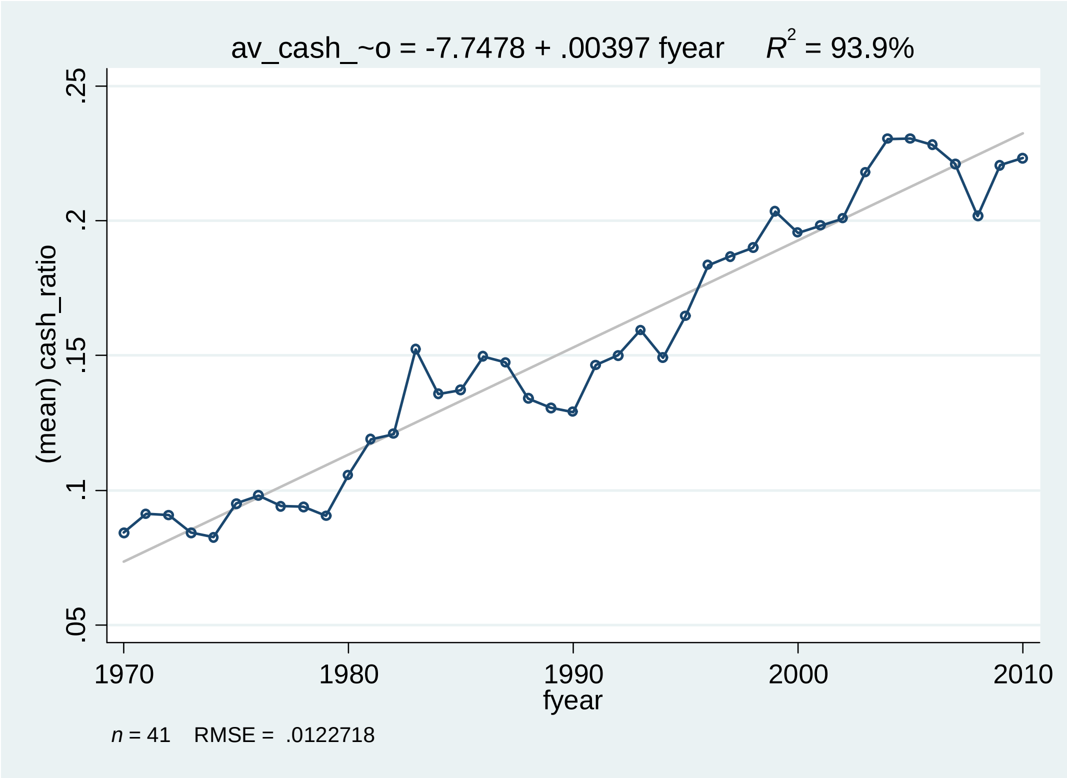
[1]The idea is that firms whose assets are more liquid have easier access to credit; indeed creditors anticipate that in case of default it would be easier to liquidate the firm and recover the loan.



Net leverage = (long-term debt + debt in current liabilities - cash and marketable securities)/Book assets

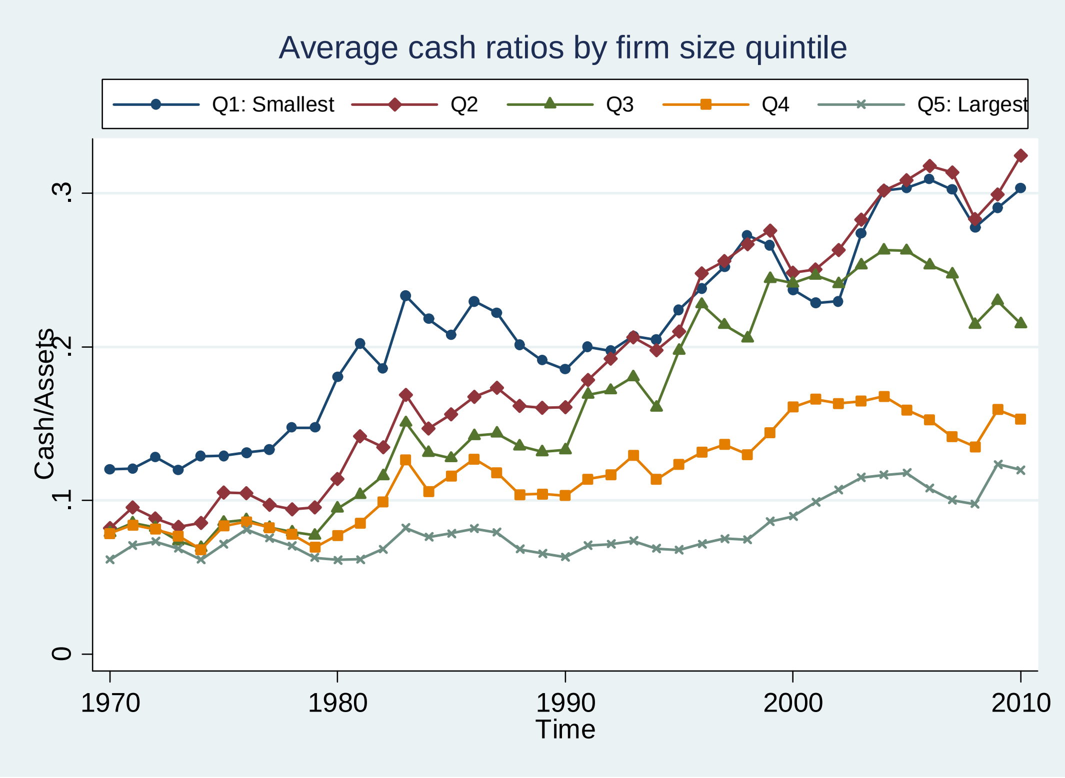


Net leverage = (long-term debt + debt in current liabilities - cash and marketable securities)/Book assets


av_cash_~o = -7.7478 + .00397 fyear R^2 = 93.9%
(mean) cash_ratio
.05
.1
.15
.2
.25
1970
1980
1990
2000
2010
fyear
n = 41 RMSE = .0122718


av_net_l~e = 10.937 - .00544 fyear R^2 = 87.7%
(mean) net_leverage
0
.05
.1
.15
.2
1970
1980
1990
2000
2010
fyear
n = 41 RMSE = .0247109

In the figure below I break the firms into quintiles of size (assets). Considering that small firms may find it harder to access credit markets, I would expect smaller firms to have higher cash-to-assets ratios. And indeed this intuition is confirmed in the data: smallest firms (those in the bottom quintiles of total assets, Q1/Q2) have on average higher cash-to-assets ratios. Moreover the increasing trend in cash ratios is more pronounced for smaller firms.

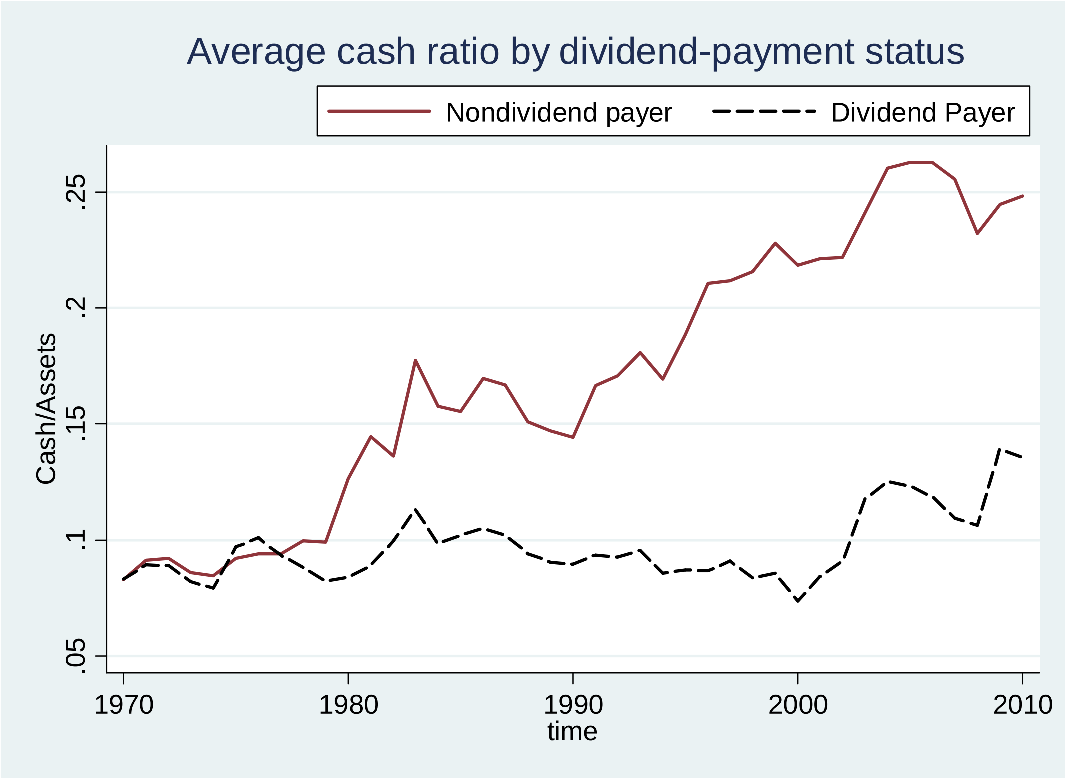


Still to do:Plot average cash ratio for firms with 11 or more years, in order to minimize the effects of sample composition changes.

Empirical Fact 3: *The increasing trend in cash is significant only for* ***non-paying dividend*** *firms.*

Many papers in the corporate finance literature [ADD REFERENCES] consider non-dividend paying firms as firms that are more likely to be credit constrained. The evidence above suggests that the increase in cash holdings occurred mostly in financially constrained firms.

From the above figure you can see that there is a dramatic increase in the cash ratio among the nondividend payers, but is much less so among the dividend payers. If



we buy the assumption that nondividend paying firms are more likely to be financially constrained, this evidence means that the increase in cash holdings occurred most in financially constrained firms.

Note: the above figure refers to the second table below (i.e. uses dvc series)

Empirical Fact 4: *Firm level risk (or uncertainty) is a major determinant of the increase in corporate cash holdings*

Key Question: is there an increasing secular trend in firm level uncertainty?

How do we measure firm level risk?

1. Balance sheet data such as sales, cash flow, profit (standard deviation of)

2. TFP shocks (standard deviation of)

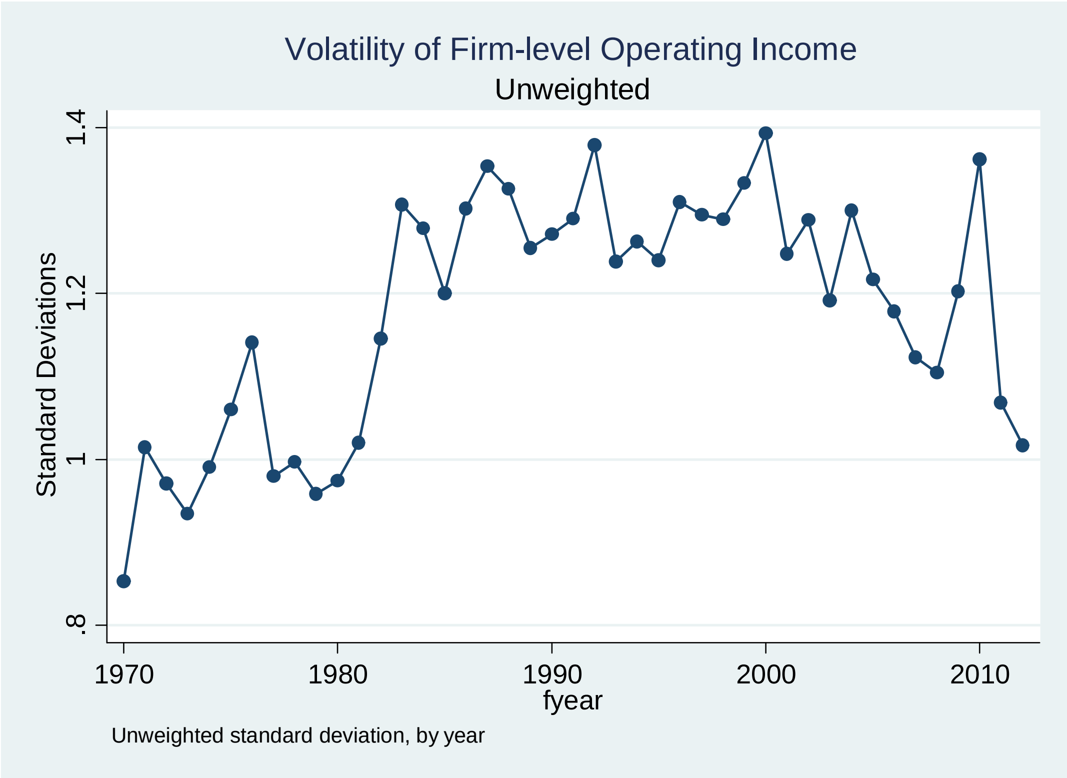
The several figures below report the time evolution of cross-sectional standard deviation, for different variables at the firm level. We use sales, pretax profit and operating income before depreciation; notice that all these variables are highly correlated (Actually the correlations are significant at the 5% but the values are low)

[INSERT TABLE ON CORRELATION BETWEEN SALES PROFIT AND OPER INCOME (XS standard deviation of)]

Firm level Uncertainty – CROSS SECTION

Unweighted

The figures below are the *unweighted* measures



Weighted

As weights we can use either assets or sales

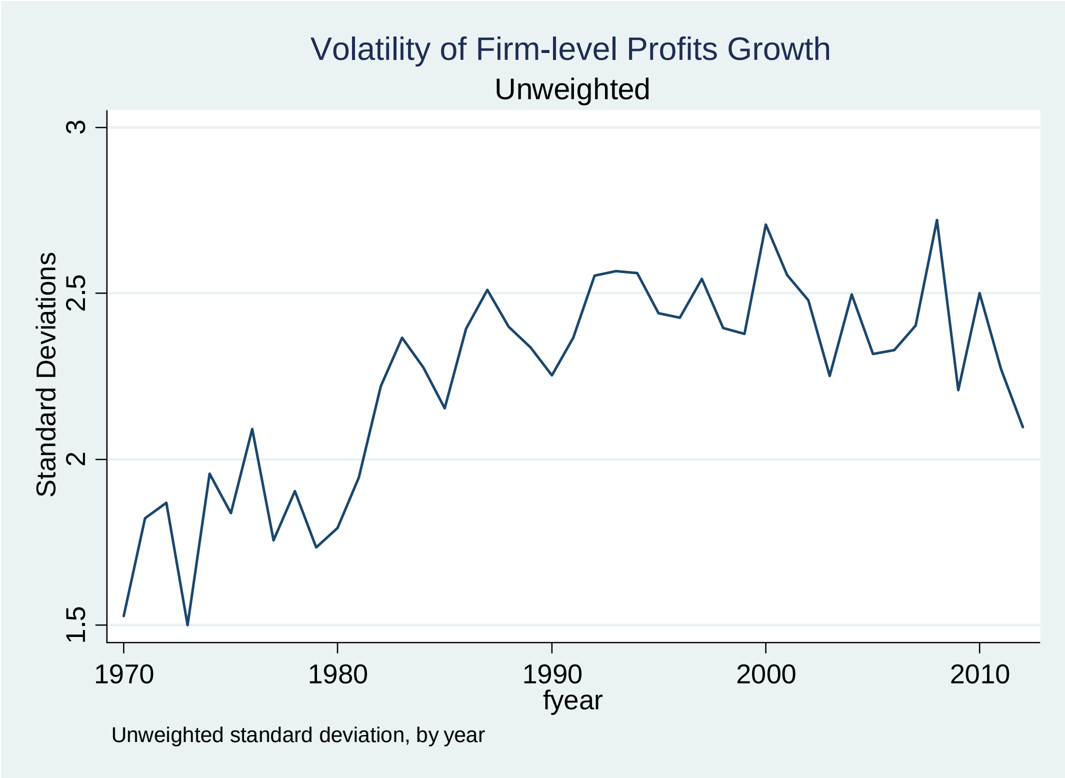
These instead are the *weighted*

See Bloom 2009 appendix

Same figure as before but dropping firms with gaps in annual data on sales or less than 11 years

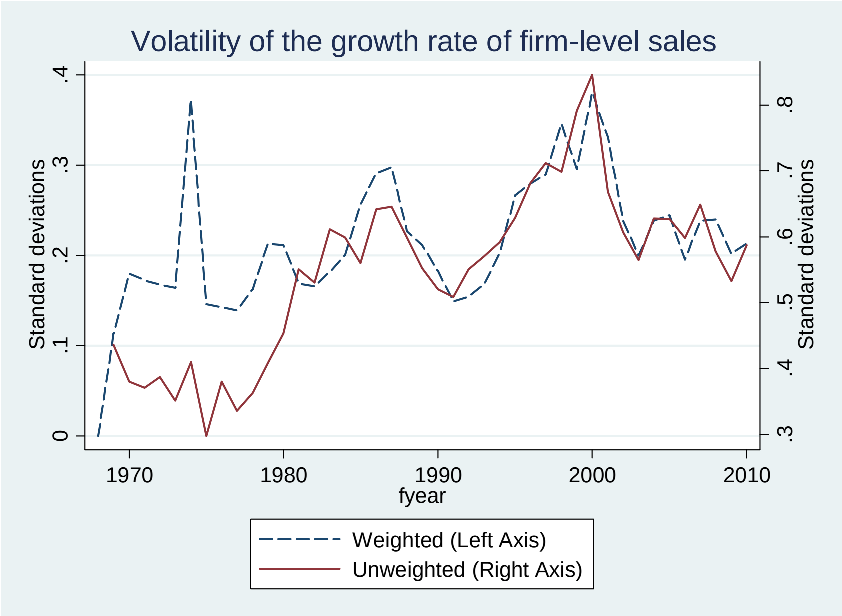
To discuss:

1. Real sales: use PPI aggregate producer price index STILL TO DO

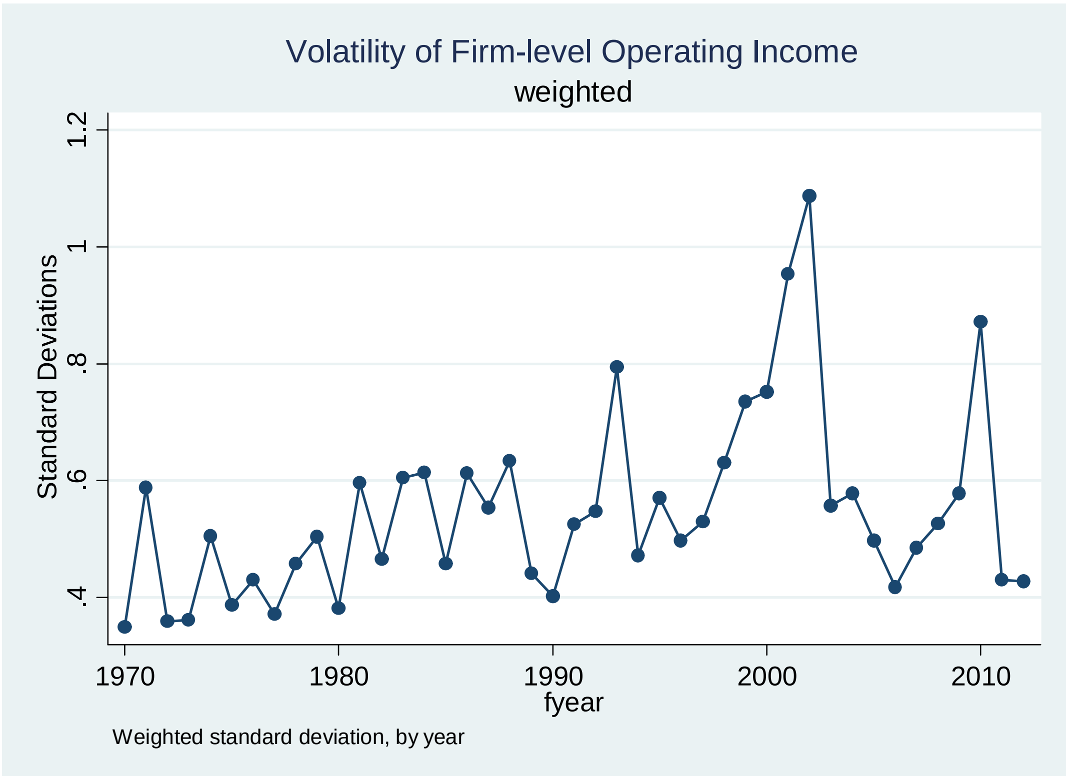


Volatility of Firm-level Profits Growth
Unweighted
Standard Deviations
1.5
2
2.5
3
1970
1980
1990
2000
2010
fyear
Unweighted standard deviation, by year

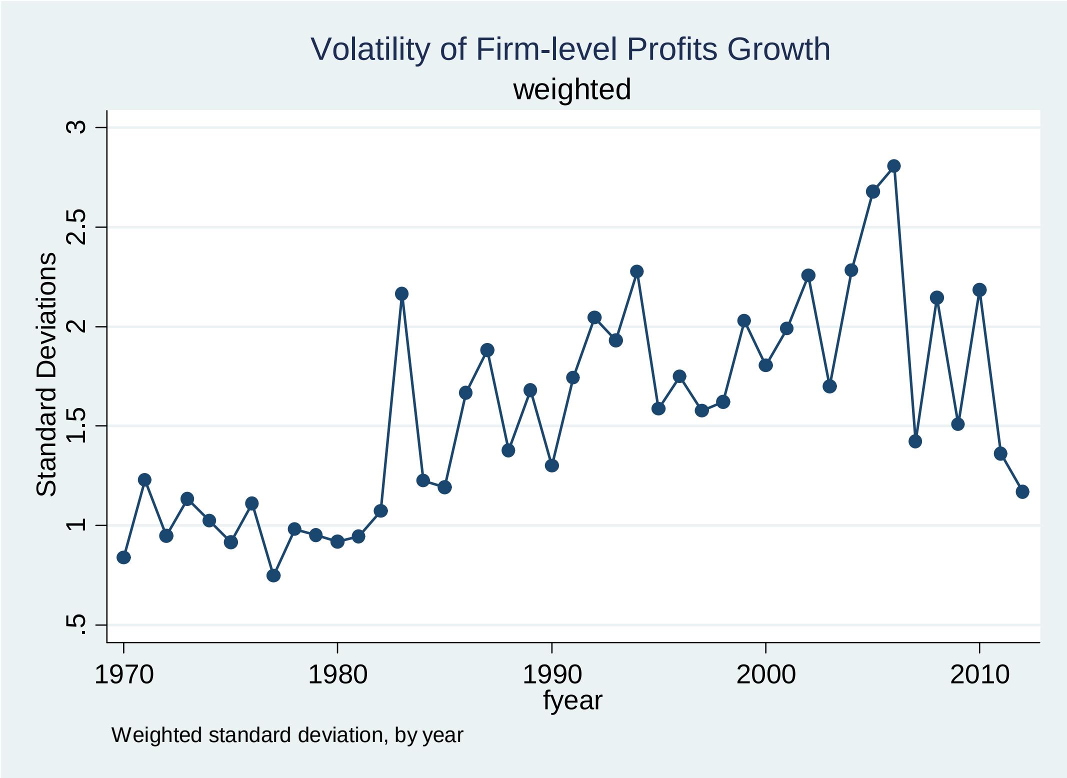
Volatility of Firm-level Sales Growth
Unweighted
Standard Deviations
.2
.25
.3
.35
.4
1970
1980
1990
2000
2010
fyear
Unweighted standard deviation, by year

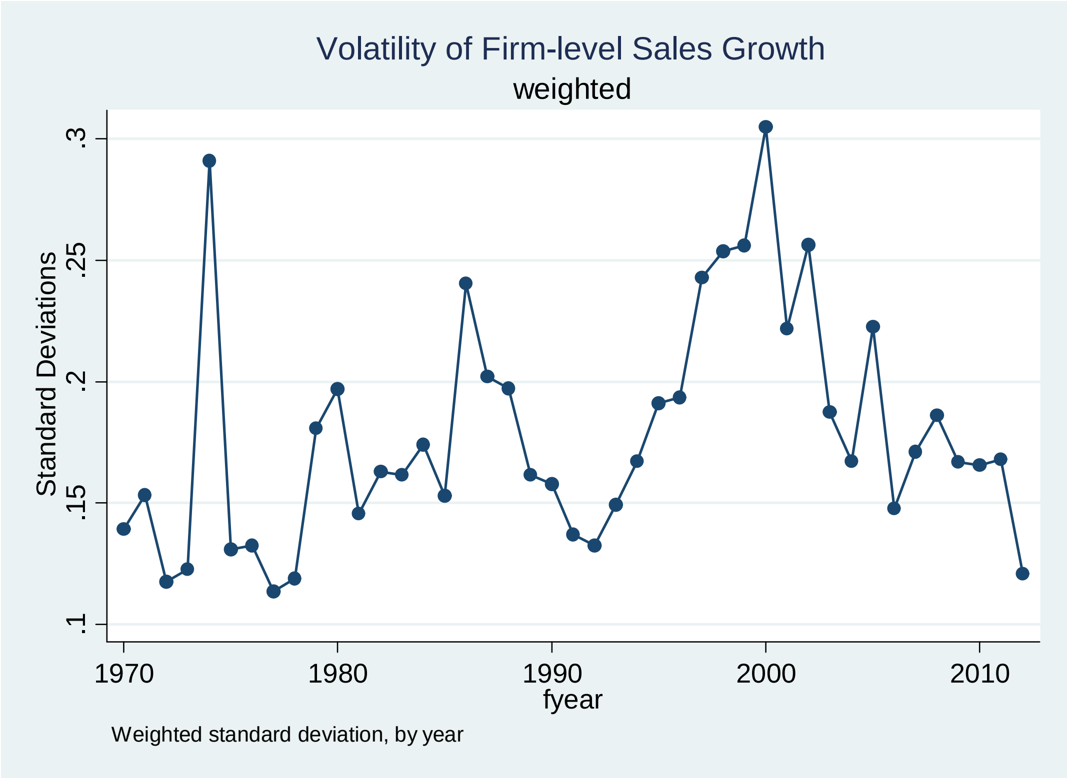


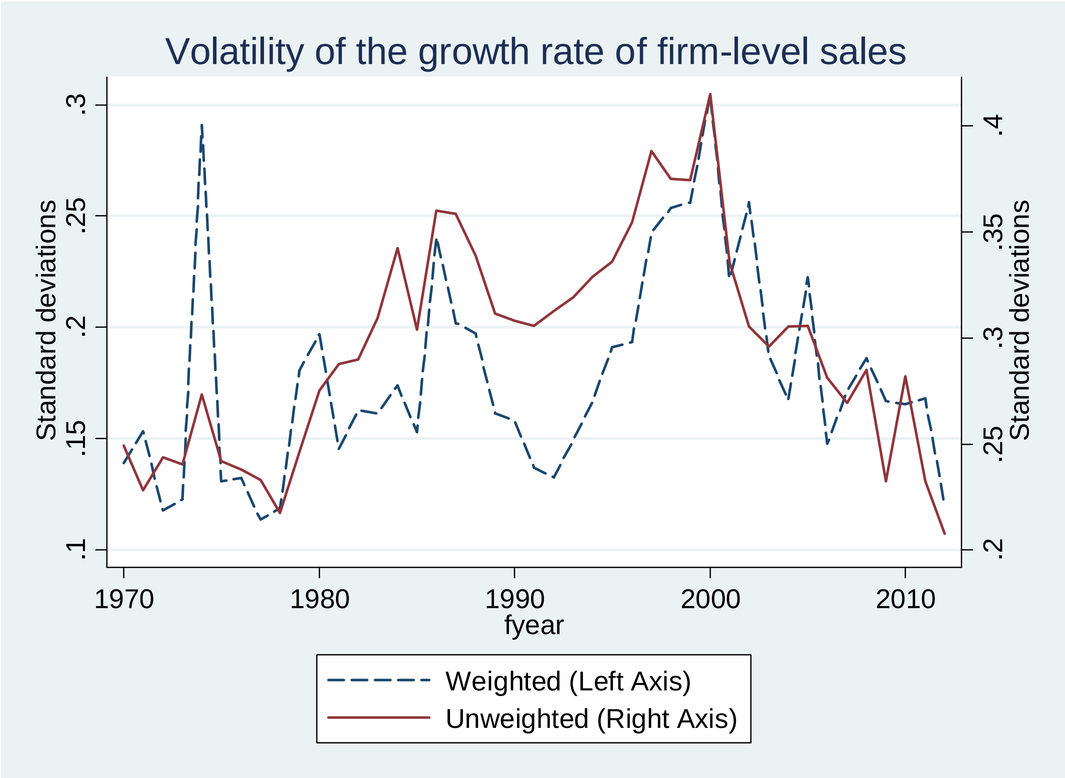
Volatility of the growth rate of firm-level sales
Standard deviations
0
.1
.2
.3
.4
.3
.4
.5
.6
.7
.8
1970
1980
1990
2000
2010
fyear
Weighted (Left Axis)
Unweighted (Right Axis)



Volatility of Firm-level Operating Income
weighted
Standard Deviations
.4
.6
.8
1
1.2
1970
1980
1990
2000
2010
fyear
Weighted standard deviation, by year


Volatility of Firm-level Profits Growth
weighted
Standard Deviations
.5
1
1.5
2
2.5
3
1970
1980
1990
2000
2010
fyear
Weighted standard deviation, by year


Volatility of Firm-level Sales Growth
weighted
Standard Deviations
.1
.15
.2
.25
.3
1970
1980
1990
2000
2010
fyear
Weighted standard deviation, by year



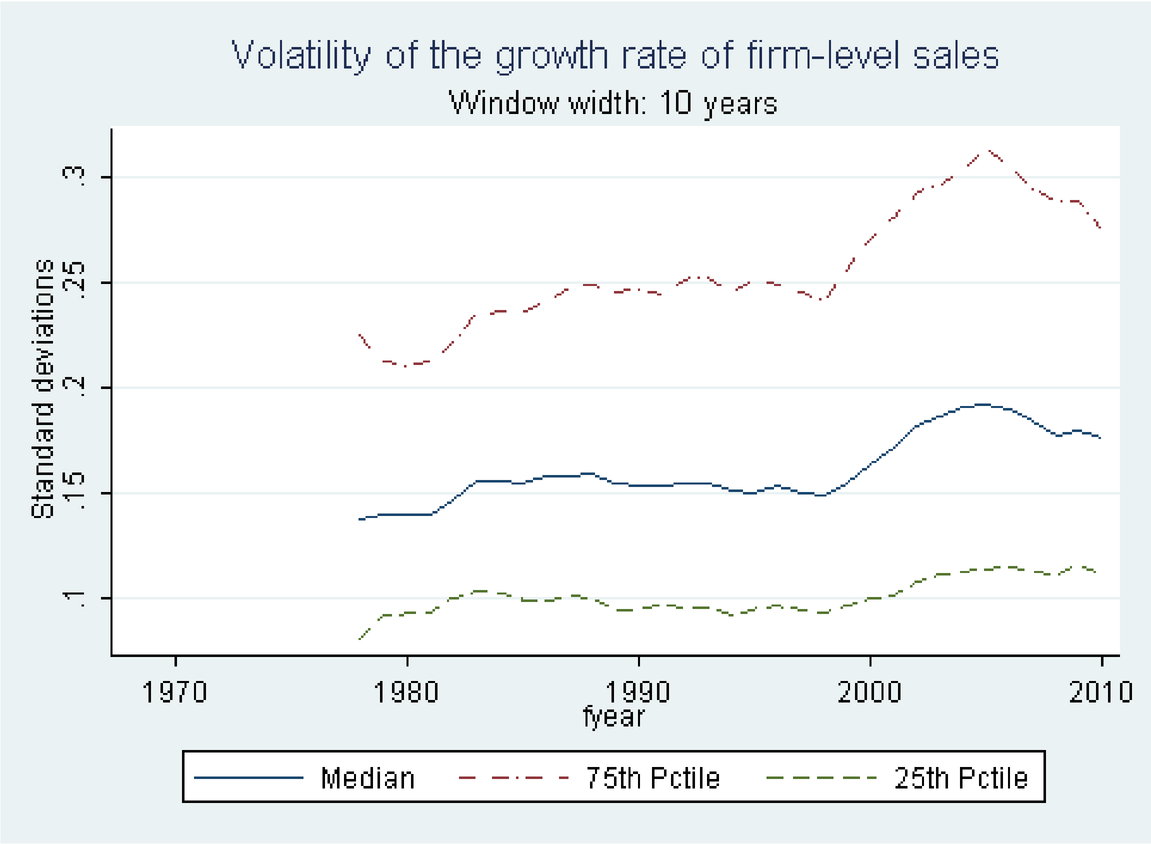
2. Net sales; item sale in compustat è gross o net?? STILL TO DO
3. Rolling standard deviation: how to do it in stata in the panel?? OK
4. Firm level price deflators? TOO DIFFICULT

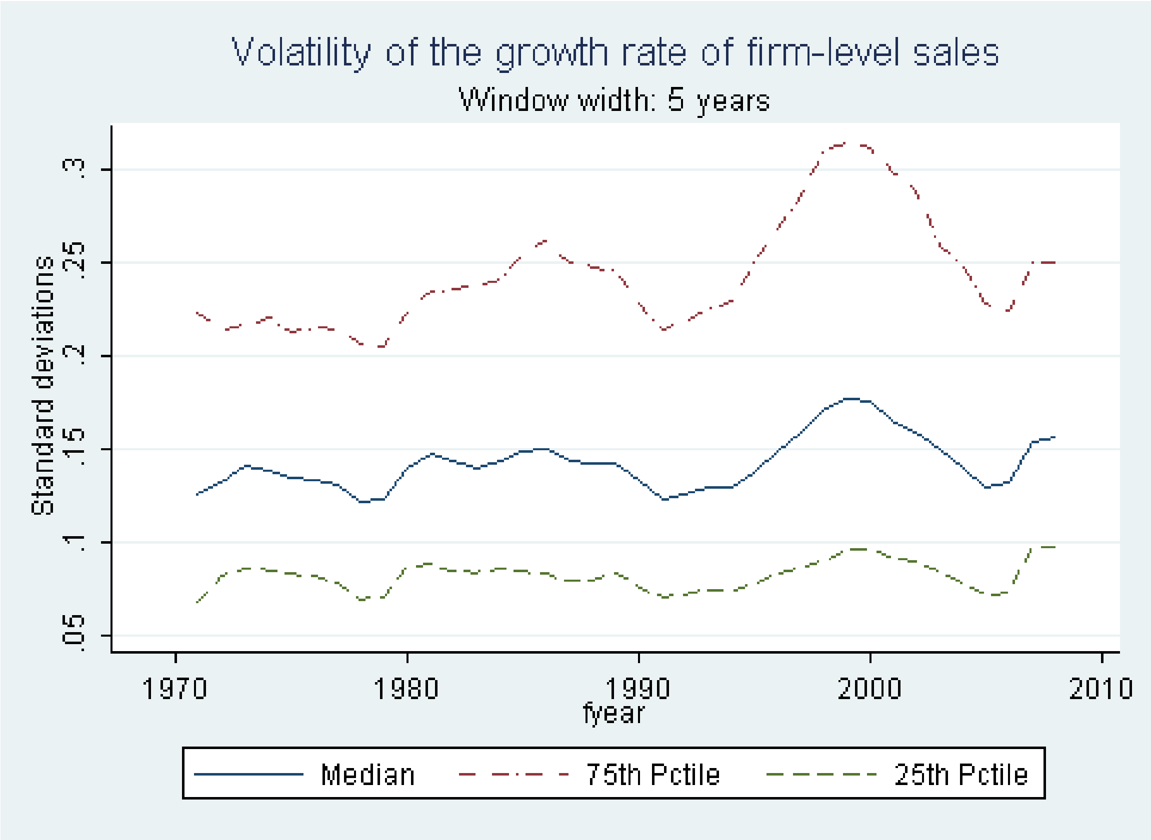
Sopra ho cercato di riprodurre per il nostro sample la tabella 3.1 nber chapter di **comin-philippon**. Diciamo che come numero di firms proprio non ci siamo, evidentemente hanno fatto una sample selection diversa. Invece i valori di median sales vol non sono troppo dissimili. Average sales è sballato perché non ho deflazionato ovviamente.

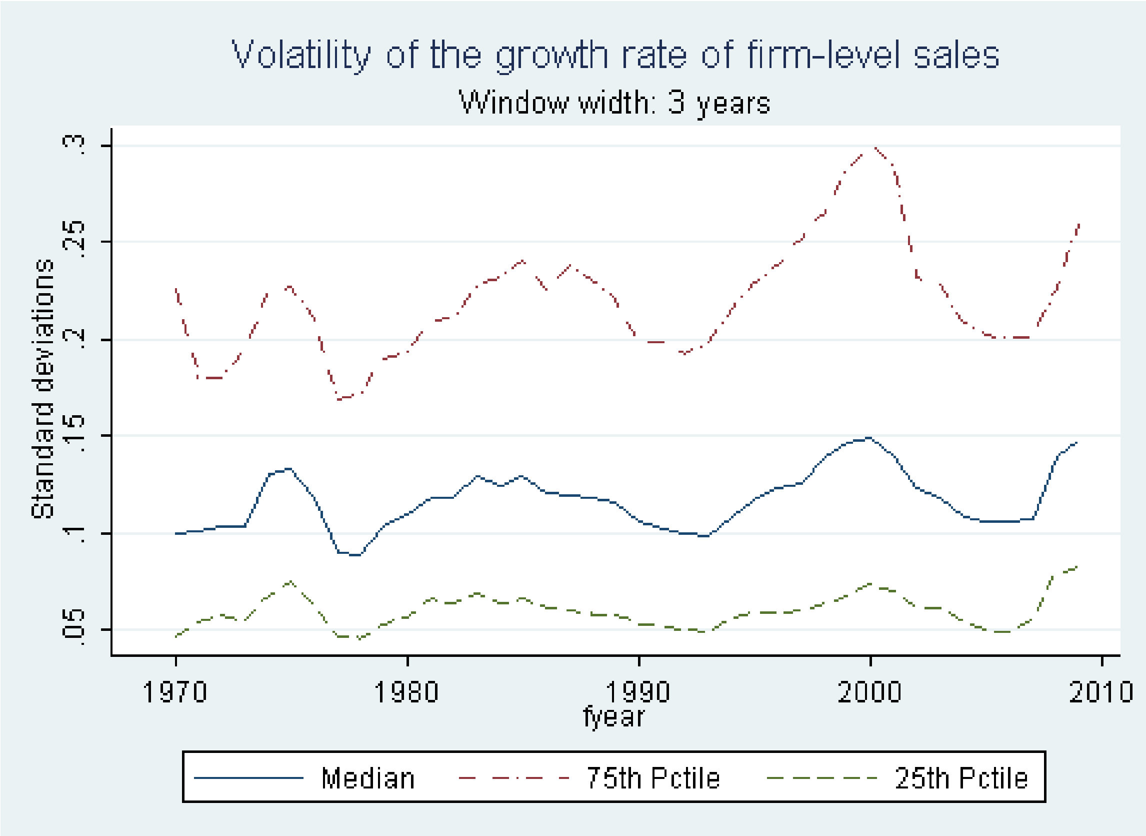
Firm level Uncertainty – ROLLING STANDARD DEVIATION

Dalle figure appare chiaramente quello che avevi trovato anche tu: considerando la 10 year MA che smussa di più mi dati si trova un drop nella volatilità dopo il 2000; considerando invece windows più corte si becca un aumento dopo il 2006 (per la recession?)

The figure below include also the subset of firms that have been continuously listed since 1970 (see Acemoglu's discussion in Nber chapter. It seems that the trend in the cross-sectional standard deviation is not affected by the changing composition of the sample, although it is true that older firms that are continuously listed are on average less risky than younger firms that become listed in the recent past.


Volatility of the growth rate of firm-level sales
Window width: 10 years
Standard deviations
.1
.15
.2
.25
.3
1970
1980
1990
2000
2010
fyear
Median
75th Pctile
25th Pctile


Volatility of the growth rate of firm-level sales
Window width: 5 years
Standard deviations
.05
.1
.15
.2
.25
.3
1970
1980
1990
2000
2010
fyear
Median
75th Pctile
25th Pctile



Da fare

1) Plot cash holdings per uncertainty quartile

2) Panel regression; vedi pag. 46 falato. Noi invece di intangible capital abbiamo volatility. Provare pooled ols e fixed effect

Domanda: cosa significa firm controls: Yes nella FE?? Con FE si elimina l'unobserved heterogeneity per la firm; cioè elimini tutti gli errori che sono costanti per gli id. Quindi con FE non puoi stimare i coefficienti sui repressori time invariant (invece con lsdv sì)

LSDV è pooled ols con firm dummies

Empirical Fact 5: *The positive impact of firm level uncertainty on corporate cash holdings is higher for financially constrained firms (and for firms belonging to sectors with higher lumpiness in investment?)*

Below I report correlation between cash holdings and volatility of sales growth (measured as the 5-year-window rolling standard deviation of sales growth)

The correlation is significative only for financially constrained firms!

Pooled ols with clustered standard errors

In the remainder of this section, I confirm the stylized facts I presented above using panel data analysis. To this end I regress cash holdings on my preferred measure of firm-level uncertainty, while controlling for a set of standard determinants of cash holdings [cite Bahle and Stultz, Opler and other papers]

[insert baseline regression]

$$Cash_{it} = \beta_0 + \beta_1 Risk_{it} + \beta_2' X_{it} + \delta_i + \lambda_t + \varepsilon_{it}$$

Why does a higher level of uncertainty lead firms to accumulate more cash? In the following part I highlight the role of financial and real frictions. As I summarized in the introduction, higher uncertainty increase the need to protect against negative shocks, but this effect is relevant as long as firm face credit constraints. Moreover, firms that are subject to large fixed costs when investing, are more likely to need external finance, since their cash flow from operations may not be enough to cover large capital expenditures. When the volatility of firm level shocks increases, firm become more uncertain about the timing of future investment opportunities, and anticipating the possibility of financial frictions, accumulate more cash and financial assets.

[insert regressions with the interactions and sample splitting]

2.4 The Model

In summary the empirical regularities that I focused on in the previous section are

The aim is to use my model to [...]. I therefore need a model that contains the following features:

- Firms with heterogeneous productivity levels,[...].
- *Financial frictions*: [...]
- Non-convex costs of capital adjustment to generate *real frictions*

2.4.1 Technology

I begin with describing the economic problem of the firms. Firms are ex-ante identical and are subject to an exogenous TFP shock A_t that is common across all firms and to an idiosyncratic productivity shocks denoted by z_{it}[2]. Since in the data firm-level

[2] The shocks z_{it} could in principle capture any shock affecting firm's revenues, hence not only shocks to technical efficiency but also (idiosyncratic) demand shocks.

productivity shocks show a high degree of persistence (as documented, among others, by Foster et al. (2001) and Bloom et al. (2012)) I assume that these shocks are generated by an autoregressive process with persistence ρ:

$$\log z_{it} = \rho \log z_{it-1} + \varepsilon_{it} \tag{2.1}$$

where ε_{it} is distributed as a $N\left(0, \sigma_{\varepsilon}^{2}\right)$. As it is standard in the literature I discretize the continuous time process described in (2.1) as a first-order Markov chain with transition matrix Q using Tauchen (1986) procedure. I assume $\Pr\left\{z' = z_j | z = z_i\right\} = Q_{ij} \geq 0$ and $\sum_j Q_{ij} = 1$ for each $i = 1, \ldots, N_z$. The sequence of aggregate shocks A_t is known with perfect foresight. Even though firms are ex-ante identical they differ ex-post since they experience different histories of idiosyncratic productivity shocks.

Firms use capital and labor as factor inputs and produce output by operating a decreasing returns to scale production function; the operating profit function (whose counterpart in the data is cash flow from operations) is:

$$\pi\left(A_t, k_{it}, z_{it}\right) = \max_{l_{it} \geq 0} \left\{A_t z_{it} F\left(k_{it}, l_{it}\right) - w l_{it}\right\} \tag{2.2}$$

Notice that $\pi\left(\cdot\right)$ is the operating profit function that is obtained after solving for the static labor choice, therefore it is a function of k and the shocks only. Denoting by I_{it} the investment made by firm i in year t, capital obeys the following law of motion

$$k_{i,t+1} = \left(1 - \delta\right) k_{it} + I_{it},$$

where $\delta \in (0, 1)$ denotes the depreciation rate. It is well-known that a model in which it is costless to adjust the capital stock delivers a time series for investment rates that is far too volatile; I therefore assume that the firm incurs quadratic adjustment costs when investing. It is also well-know, since at least Caballero et al. (1995), that plant-level investment is characterized by periods of inactivity followed by large spikes in investment; while it is hard to match this type of evidence with a quadratic cost of adjustment, I chose to adopt the quadratic specification for his computational tractability.

Firms can finance investment either with internal funds or borrowing from the financial market (by raising new equity or issuing debt). By raising external finance the firm incurs a variety of additional costs going from flotation costs to adverse selection premia. As in Gomes (2001) and Hennessy and Whited (2007) I do not model explicitly a setting with asymmetric information but I attempt to capture the simple fact that external funds are more costly than internal funds in a reduced form way. In particular, I assume that the additional cost of raising external finance is given by

$$c\left(e\right) = \lambda_0 + \lambda_1 \cdot \text{amount of external funds}$$

In other words there is a fixed cost λ_0 and a per unit cost λ_1 associated with external finance. A large body of empirical research provides detailed evidence regarding underwriting fees (see, among others, Altinkilic and Hansen (2000)) finding that there are significant economies of scale: this is why a cost function with decreasing average cost seems most appropriate. Cooley and Quadrini (2001) use a slightly different formulation which omits the fixed cost. However the fixed cost formulation is needed in my framework in order (i) to rationalize the presence of economies of scale and (ii) to match the degree of financial inaction that I documented in the Compustat sample (see Table 1.1).

The firm problem is to choose investment and financial policy to maximize net payments to its shareholders[3], taking as given the real interest rate and the wage rate:

$$V(k,z) = \max_{d,e,I,k'} \left\{ d - e - c(e) + \frac{1}{1+r} \sum_{z'} V(k',z') \, Q(z'|z) \right\} \tag{2.3}$$

s.t.

$$d + I + \frac{\psi}{2} \frac{I^2}{k} = \pi(k,z,A) + e, \tag{2.4}$$

$$k' = (1-\delta) k + I, \tag{2.5}$$

$$c(e) = (\lambda_0 + \lambda_1 e) \, 1_{\{e>0\}}, \tag{2.6}$$

$$d \geq 0, \tag{2.7}$$

$$e \geq 0. \tag{2.8}$$

Equation (2.4) describes the flow of funds condition for the firm. The sources of funds (on the right-hand side) consists of operating cash-flows π, and external funds, e. The uses of funds (on the left-hand side) consist of capital expenditures, adjustment costs and dividend payments. Please notice that in this setting the only way firms can save is by accumulating capital: I choose to rule out firm savings in cash holdings or other financial assets. Adding financial savings or debt would make the problem more realistic but would also increase considerably the computational burden: with the additional debt choice there is a cross-sectional distribution of firms over three states capital, debt and idiosyncratic productivity (k, b, z) that is more difficult to handle with[4].

Equation (2.6) describes the external finance cost function: these costs are positive and increasing if the firm uses external funds. If no external funds are required, these costs are zero. This formulation is consistent with the Pecking Order Hypothesis (Myers and Majluf (1984)): firms first use internal finance and if they do not have enough, then issue debt, and as a last resort equity. The pecking order hypothesis can account for the stylized facts that retentions and then debt are the primary sources of finance. Notice furthermore that it is never optimal to raise external finance and at the same time distribute dividends. Indeed

[3]See appendix D for a derivation of the optimal value maximization problem of the firm.

[4]However, I'm currently working on an extension to incorporate debt into the firm's problem.

Lemma 2.4.1 *It is never optimal for the firm to choose $e > 0$ and $d > 0$.*
Proof. *Suppose by contradiction that the firms chooses $e > 0$ and $d > 0$. Then the firm can decrease both e and d by a small amount $\varepsilon > 0$, which induce a change in profits given by $\lambda_1 \varepsilon > 0$* ■

2.4.2 Financing frictions

Since I am mainly interested in reallocation of capital among firms with heterogeneous productivities, on the household's side I can focus on a representative agent formulation. The representative agent has preferences over consumption and labor that are summarized by the following utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{2.9}$$

Household's income comes from wages and dividends. In order to write its budget constraint, I must aggregate all firm-level quantities. To this end, let me define μ_t as the cross sectional distribution of firms over the individual state (k, z) in period t. The budget constraint can then be stated as:

$$C_t + \int P_t \theta_{t+1}(k,z)\, \mu_t(dk,dz) + b_{t+1} - (1+r_t)\, b_t = w_t N_t + \int (d_t + P_t - e_t - c(e_t))\, \theta_t(k,z)\, \mu_t(dk,dz) \tag{2.10}$$

where θ_t denotes the shares owned by household, b_t denotes bond holdings. In equilibrium $\theta_t = 1$ and $b_t = 0$ for all t since all households are equal.

The representative household's problem is to maximize (2.9) subject to (2.10). The first-order conditions with respect to labor supply N_t and bond holdings b_{t+1} are respectively:

$$-\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = w_t,$$

$$U_c(C_t, N_t) = \beta U_c(C_{t+1}, N_{t+1})(1 + r_{t+1}).$$

Since I consider the model in the stationary equilibrium with interest rate r_t, wage rate w_t and aggregate quantities constant over time, the household's problem can be simplified in this following *static* problem:

$$\max_{C,N} U(C, N)$$

s.t.

$$C = wN + \int d(k,z;w)\, d\mu(k,z) - \int e(k,z;w)\, d\mu(k,z) - \int c(e(k,z;w))\, d\mu(k,z) \tag{2.11}$$

This is a standard concave problem with interior solutions. In the steady state the Euler equation pins down the interest rate as

$$r = \frac{1}{\beta} - 1 \tag{2.12}$$

and optimality condition with respect to labor supply becomes:

$$-\frac{U_n(C,N)}{U_c(C,N)} = w \tag{2.13}$$

Solving the household's problem I get the household's decision rules for consumption $C(w;\mu)$ and labor supply $L^s(w;\mu)$.

2.4.3 Stationary distribution and Aggregation

Assuming A_t constant, the solution to the firm's optimization problem (2.3) delivers the *policy functions*

$$k' = g(k,z),\ I(k,z),\ l(k,z),\ y(k,z),\ e(k,z)$$

mapping the firm's state variables k and z into the firm's current choices (please notice that for simplicity I omit the dependence of the policy functions upon the wage w). The vector of individual state variables $x = (k,z)$ lies in $X = [0,\infty) \times Z$, where Z is the discrete set for productivity shocks z, i.e. $Z = \{z_1, z_1, \ldots, z_{nz}\}$. Let $\mathbb{B}$ be the associated Borel σ algebra. For any set $B \in \mathbb{B}$, $\mu(B)$ is the mass of firms whose individual states lie in the set B. The transition function $T(x,B)$ defines the probability that a firm in state $x = (k,z)$ will have a state lying in B in the next period, given the decision rule g for next-period capital. I can define each set B as the Cartesian product $B_K \times B_Z$; then the transition function $T : X \times \mathbb{B} \rightarrow [0,1]$ can be written as:

$$T((k,z), B_K \times B_Z) = \begin{cases} \sum_{z' \in B_Z} Q(z,z') & \text{if } g(k,z) \in B_K \\ 0 & \text{otherwise} \end{cases}$$

where $g(k,z)$ is the policy function for next-period capital. Given the transition function, I can define the probability measure μ as

$$\mu'(B) = \int_X T(x,B)\,\mu(dx) \tag{2.14}$$

Given the invariant distribution $\mu^*(k,z) = \mu = \mu'$, I can compute the aggregate variables:

- Aggregate investment:

$$I(w;\mu^*) = \int I(k, k'(k,z); w)\,\mu^*(dk,dz)$$

- Aggregate labor demand:

$$L^d(w;\mu^*) = \int l(k,z;w)\,\mu^*(dk,dz)$$

- Aggregate output supply:

$$Y(w;\mu^*) = \int y(k,z;w)\,\mu^*(dk,dz)$$

- Aggregate adjustment costs:

$$AC(w;\mu^*) = \int \frac{\gamma}{2}\frac{I(k,z;w)^2}{k}\mu^*(dk,dz)$$

- Aggregate external finance costs:

$$E(w;\mu^*) = \int c(e(k,z))\,\mu^*(dk,dz)$$

Now I give the definition of equilibrium in my model, focusing for simplicity on the steady-state.

Definition 2 *A stationary recursive competitive equilibrium is a list of value function* V*, policy functions, invariant measure* μ *and prices* r, w *such that:*

(1) Given the prices $\{r, w\}$*, the policy functions* $d(k,z)$, $e(k,z)$, $I(k,z)$, $k'(k,z)$ *solve the optimization problem of the firm in (2.3)*

(2) Factor prices (r, w) *are determined by equations (2.12) and (2.13)*

(3) Markets clear; in particular in the labor market supply equals demand:

$$N^s(w;\mu) = \int_{x=(k,z)} l(k,z)\,\mu(dk,dz) \tag{2.15}$$

and the good market clears:

$$C(w;\mu) + I(w;\mu^*) + AC(w;\mu^*) + E(w;\mu^*) = Y(w;\mu^*)$$

where the term $E(w;\mu^*)$ *represents aggregate costs of raising external finance. Of course by Walras' law this last resource constraint is redundant*[5]*: it is implied by combining the firm's flow of funds constraint (2.4) with the household's budget constraint (2.11).*

[5] Indeed I don't use it when computing the equilibrium but I verify ex-post that it is satisfied.

2.4.4 Economic mechanism

Before reporting the results from the simulation, it is useful to look at the steady state distribution. Firms can be in three different finance regimes (this why heterogeneity is important)

1. $d = 0$, $e > 0$: external finance regime

2. $d = 0$, $e = 0$: financial inactivity regime

3. $d > 0$, $e = 0$: dividend distribution regime

Figure 1.6 illustrates these regimes for the baseline model and reveals a few interesting features. First, firms that are either very small or very productive tap the financial market and do not distribute dividends (top-left region: high z and low k; remember that z and k are the firm's state variables). These firms are in the external finance regime. Second, firms that are either very large or less productive use internal funds to finance investment and also distribute dividends (bottom-right region: low z and high k). They are in the dividend distribution regime. Finally the remaining firms do not distribute dividends and do not raise external finance. They are in the financial inactivity regime. Figure 1.9 confirms this.

Figure 1.7 depicts the policy function for external finance, $e(k, z)$. It confirms that large firms, with a high capital stock, generate enough internal cash flow and do not need to raise additional resources from banks or from the equity market. In particular, there exists a capital threshold $\overline{k}(z)$ such that only firms with $k < \overline{k}(z)$ choose a strictly positive value of e; interestingly, such threshold is increasing with respect to productivity: holding the size of the firm fixed, firms that are hit by higher productivity shocks are more likely to raise external finance. This policy rule for external finance shows an inverted U-shape form when external finance is positive. In this model, external finance is a double-edged sword. For a given productivity, a firm needs to borrow to invest and this increases their expected profits. On the other hand, this also increases the cost related to external finance. The policy function reflects these two opposing tendencies creating the inverted U-shape that we observe. Higher values of z increase the future profits, allowing the firm to borrow larger amounts and shift the policy rule up.

Figure 1.8 plots instead the policy function for dividends $d(k, z)$: small firms tend not to distribute dividends since they need to use all their internal cash flow to finance investment, and this effect is of course more pronounced for higher productivity firms.

2.5 Results

2.5.1 Calibration

2.5.2 Comparative statics: increase in firm level risk

2.6 Conclusions

2.7 Data Appendix

2.8 Computation - Steady State

The algorithm follows Aiyagari(1994) and Huggett(1993). I start by guessing a value for the wage w. For the given wage I solve the firm's decision problem by value function iteration on a discrete grid. Then I compute the invariant distribution of firms over capital and productivity. As a last step I check whether the labor market equilibrium condition holds. If not, I update the wage.

More in detail:

- Step 1 - Make a guess for equilibrium wage w.
- Step 2 - Given w, solve the firm's problem by value function iteration on a discrete grid. Even if slow, it is the most robust method (better to use this because policy functions are non linear due to the fixed equity cost). Get policy function $k' = g(k, z)$ and the other decision rules.
- Step 3 - Using the policy function $g(k, z)$ computed in step 2 and the exogenous Markov chain for productivity shocks, compute the invariant distribution $\mu^*(k, z)$ by iterating on (2.14)
- Step 4 - Using the stationary distribution $\mu^*(k, z)$ obtained in step 3, compute aggregate labor demand $N^d(w) = \sum_{k,z} n(k, z) \mu^*(k, z)$. Then check if equation

  $$-\frac{U_n\left(C, N^d(w)\right)}{U_c\left(C, N^d(w)\right)} = w$$

  is satisfied[6]. If it is, stop; otherwise update the wage and go back to step 2. An alternative way is to compute explicitly the excess demand function for labor: $N^d - N^s$.
- Iterate until convergence.

[6]The function $-\frac{U_n\left(C,N^d(w)\right)}{U_c\left(C,N^d(w)\right)} - w = 0$ is not perfectly continuous given the discretized nature of the algorithm, and it is therefore not always possible to compute a clearing wage level to an arbitrary level of precision. However the problem is generally well-behaved with a tolerance level of 10^{-7} in the baseline simulation.

2.9 Firm's value problem

Consider the representative household's maximization problem which I re-write below for convenience:

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \tag{2.16}$$

s.t.

$$C_t + \int P_t \theta_{t+1} d\mu_t + b_{t+1} - \left(1 + r_t\right) b_t = w_t N_t + \int \left(d_t + P_t - e_t - c\left(e_t\right)\right) \theta_t d\mu_t \tag{2.17}$$

The first-order conditions with respect to bond b_{t+1} and share holdings θ_{t+1} are:

$$U_1\left(C_t, N_t\right) = \beta\left(1 + r_{t+1}\right) U_1\left(C_{t+1}, N_{t+1}\right)$$ [7]

and

$$U_1\left(C_t, N_t\right) P_t = \beta U_1\left(C_{t+1}, N_{t+1}\right) E_t\left\{d_{t+1} + P_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}$$

Hence combining the two equations I get the result that the required rate on return on equity must be equal to the real interest rate (in other words, there is no risk premium).

$$\left(1 + r_{t+1}\right) = \frac{E_t\left\{d_{t+1} + P_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}}{P_t}.$$

or,

$$P_t = \left(\frac{1}{1 + r_{t+1}}\right) E_t\left\{d_{t+1} + P_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}$$

Iterating forward yields

$$P_t = \sum_{n=1}^{\infty} \prod_{j=1}^{n} \left(\frac{1}{1 + r_{t+j}}\right) E_t\left\{d_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}$$

which corresponds to (**??**) or (2.3).

[7] Along the transition aggregate variables and prices are deterministic sequences, hence I do not need the expectation operator.

Chapter 3

House prices and workers geographical mobility during the Great Recession

3.1 Introduction

The recent economic downturn in the U.S. was the most severe since the Great Depression. House prices plummeted in 2007, and the unemployment rate increased from 5% in January 2008 to $10,1\%$ in October 2009 (the largest level since the 1980s). At the same time, the dispersion of unemployment rates across metropolitan statistical areas (MSAs) doubled. In this work, I propose a mechanism through which a decline in house prices can reduce the extent of geographical reallocation and affect local and aggregate unemployment.

How does a decline in house prices affect geographical reallocation and the labor market? In this work, I focus on a financial friction: the down payment requirement in purchasing a home. When house prices fall, the amount of home equity declines, making it harder to afford the down payment on a new house after moving. To the extent that households care about owning a house, the decline in house prices affects their migration decisions. Some households that would normally move out of regions with low productivities may stay and look for jobs in distressed labor markets. Thus, the aggregate job finding rate decreases, which results in higher aggregate unemployment.

The decline in house prices during the housing bust is asymmetric across MSAs, ranging from 5% to 50%. Furthermore, the decline in local labor productivity across MSAs, measured as output per worker, is also asymmetric and ranges from 0% to 9%. Exploiting these variations, I document the following facts: first, controlling for the decline in labor productivity, the unemployment rate increased more in MSAs that experienced larger housing busts. Second, controlling for local labor productivity, MSAs with larger housing busts had smaller out-migration rates. These facts suggest that the housing bust in 2007 may have affected geographical migration and local unemployment rates.

3.1.1 A simple example

A very simple example coul help the reader understand the mechanism I propose.

Consider a stylized economy made of two regions: region A and region B.

- Imagine a household/family living in region A with a house initially worth 100,000$, an outstanding mortage of 85000$ and no other assets (assume furthermore that the family is credit constrained).
- Suppose region A is hit by a negative productivity shock: local wages and local job finding rates decline as firms find it less profitable to post vacancies. This asymmetric shock creates the benefit of migration from region A to region B.
- The purchase of a new house requires a minimum down-payment of 10 percent. In this simple economy there is no rental market: the only way a family can occupy a house is by owning it.
- Absent the housing bust, our family would sell its old house, pay off outstanding mortage and still have 15000$ to make a downpayment on a new house of comparable size after moving.
- But if house prices fall by 10 percent (ONLY in the house market of residence), the family will afford only a downpayment of 5000$. Rather than moving to a much smaller house, they may rationally choose to stay where they are.

Compared to a recession without a housing bust, more unemployed look for jobs in region A, with lower productivity (and thus fewer job openings). As a consequence, aggregate job finding rate is lower and aggregate unemployment is higher.

3.2 Literature Review

The possibility that the housing bust in 2007 exacerbated unemployment by reducing migration has been recognized both by academic experts and also by the popular media. From the New York Times website, "Slump creates lack of mobility for Americans" we can read the following:

"Experts said the lack of mobility was of concern on two fronts. It suggests that Americans were unable or unwilling to follow any job opportunities that may have existed

around the country, as they have in the past. And the lack of movement itself, they said, could have an impact on the economy, reducing the economic activity generated by moves".

In the academic world several authors argued that unemployment in the Great Recession is really different from previous recessions. According to Kocherlakota[1], for example, the dramatic increase in unemployment in 2008 and 2009 was not due to weak aggregate labor market conditions, but to 'structural' problems, which generate mismatch between available jobs and workers:

"Firms have jobs, but can't find appropriate workers. The workers want to work, but can't find appropriate jobs. There are many possible sources of mismatch - geography, skills, demography- and they are probably all at work."

Apart from this anectodical evidence, there is a growing empirical literature that investigates the effect of declining house prices on the mobility of workers. The idea that house price declines deter geographic mobility has been supported by several microeconometric studies: Chan (2001) and, more recently, Ferreira et al. (2010). The latter study the relationship between houshold leverage and household mobility in the context of the recent housing bust; in particular they find that negative equity reduces mobility up to 30%.

My work is also related to the literature that studies *mismatch* in the labor market. Mismatch in fact captures the idea that the characteristic of workers (either skills or geographical location) may not coincide with the needs of firms. Several authors pointed out at a kind of *jobless recovery*, investigating why the unemployment rate remained high even while job openings appeared to have increased during the recent recovery. In other words, starting from 2008, the negative relation between unemployment and vacancies (the so called Beveridge Curve) experienced a right-ward shift, meaning that for the same number of vacancies posted by firms, unemployment is higher. In other words, matching efficiency declined. This fact has been pointed out especially by Shimer (see Figure 3.1).

In the context of this debate, economists have recently paid close attention to mismatch and have investigated whether it is causing the currently high unemployment rate in the U.S. Some evidence suggests that mismatch might have increased since the recession started. Most new positions have been created in some sectors, while most job loss has been concentrated in others. Since these new jobs usually require different skills than what unemployed workers from different sectors have, firms and unemployed workers may take longer to find their best matches. For example, over 50 percent of the jobs lost between December 2007 and February 2011 were in manufacturing and construction, while more than 90 percent of new positions opened in other industries. The education and health sector has experienced steady employment growth since the recession started; 20 percent

[1]Speech in Marquette, MI, on August 17 2010 as president of the Federal Reserve Bank of Minneapolis.

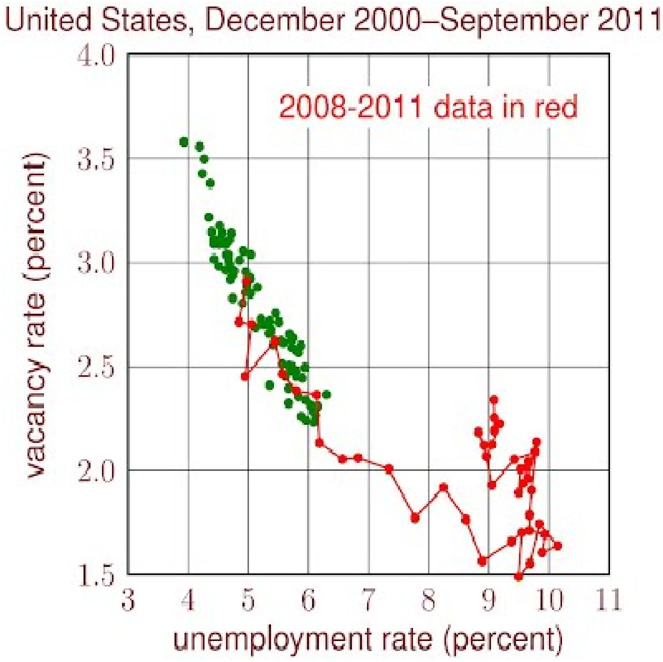


Figure 3.1: Source: Robert Shimer's website

of all job openings have occurred in this sector (see Figure 3.2).

Sahin et al. (2014) formalize the notion of mismatch and provide a framework to measure the extent to which mismatch has caused higher unemployment during the recent recession. In their framework, the aggregate labor market is comprised of many small labor markets, categorized by skill levels or working locations (e.g., industries and MSAs). Sahin et al. (2014) define mismatch as the distance between the observed allocation of unemployed workers across sectors and the "optimal" allocation. The optimal (constrained-efficient) allocation of unemployed workers is the allocation that, given the distribution of vacancies in the economy, would occur if there were free movement of workers across labor markets. In this way, Şahin and others can construct a *counterfactual* time series for unemployment (i.e. unemployment rate that would prevail in the economy if workers could freely relocate between sectors and geographical residences) that, compared with the actual time series gives a well-defined measure of mismatch. In their paper, Şahin and others focus mainly on *sectoral* mismatch and not geographical mismatch.

Using five industries as divisions of the aggregate labor market, Sahin et al. (2014) found that the fraction of unemployed workers misallocated increased by 10 percentage points during the 2007-09 recession; the fraction then dropped but remained at a level higher than its prerecession level. But this increase in mismatch can explain only between 0.4 and 0.7 percentage points of the total increase of five percentage points in the unemployment rate from the beginning of 2007 to the middle of 2009. Therefore Şahin

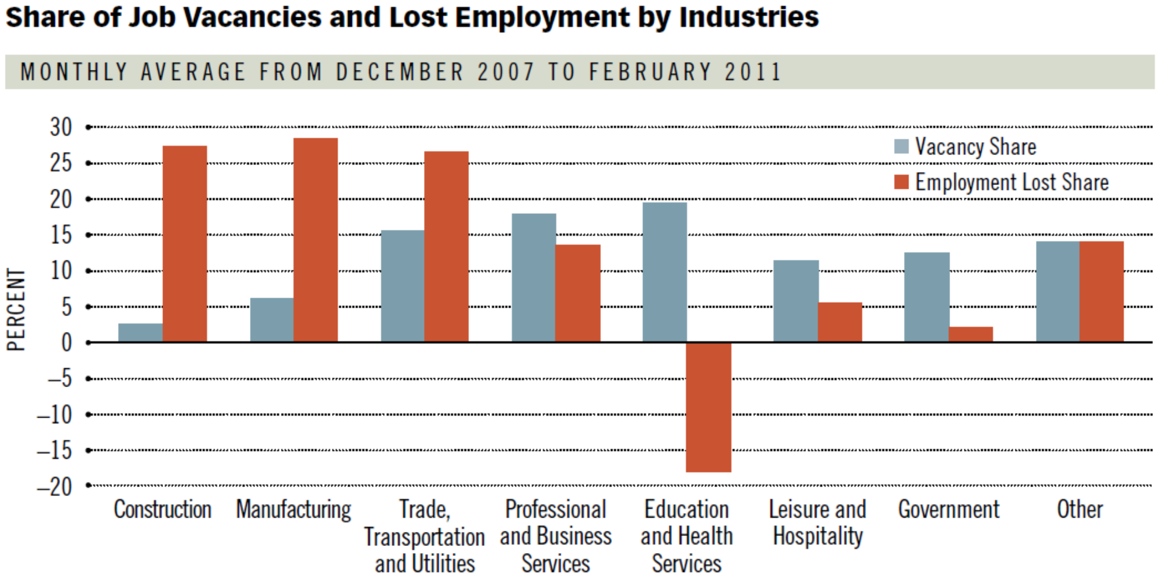


Figure 3.2: SOURCES: Job Openings and Labor Turnover Survey (JOLTS) and the Current Population Survey (CPS).

and others conclude stating that, although skill mismatch increased during the recession and influenced unemployment to some degree, it is not the main source of the increase in the unemployment rate. They also find that geographic mismatch (across census regions) does not have a significant effect on the labor market. However, by using census regions insted of MSAs (where the former are much larger than the latter) it is likely that they lose a lot of evidence about geographic mobility by workers.

3.3 Empirical Investigation

In this section my aim is to show the empirical evidence that supports the "geographic mobility channel" that I propose as the main driving force behind the sluggish recover of the US employment.

For the 'geographic mismatch explanation' to play a role, I need to find evidence of an asymmetric decline in local labor productivity (across islands) and an asymmetric decline in house prices (during housing bust). I want to document behaviour of labor productivity, unemployment, house prices, population flows across regions (MSAs) during the great recession. In order to do so I constructed an annual panel dataset of the 366 MSAs of US during the 2007-2009 recession[2], containing information on labor productivity, unemployment, house prices, population flows. Geographic characteristics can be measured

[2] Actually my dataset runs from 2000 to 2009.

at different levels, such as metropolitan statistical areas (MSAs), states and, at an even larger level, census regions. Why Metropolitan Statistical Areas?

A *metropolitan statistical area* is is a geographical region with a relatively high population density at its core and close economic ties throughout the area. Such regions are not legally incorporated as a city or town would be, nor are they legal administrative divisions like counties.

MSAs are defined by the U.S. Office of Management and Budget only, and used by the U.S. Census Bureau and other U.S. government agencies for statistical purposes. Below I show a figure of the Metropolitan Statistical Areas in the US (Figure 3.3).

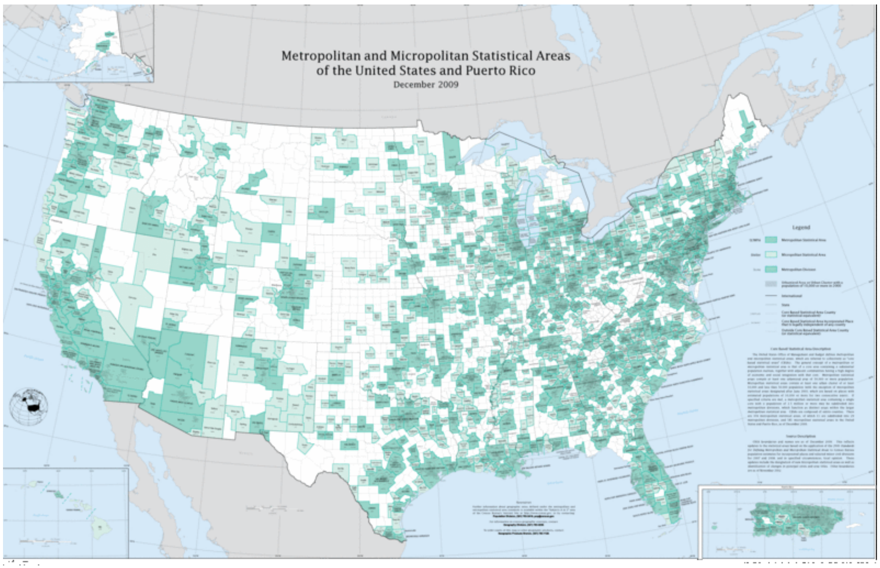


Figure 3.3: Legend of map: 942 Core Based Statistical Areas (CBSAs) of the United States. The 366 Metropolitan Statistical Areas (MSAs) are shown in medium green.

It is reasonable to use MSA's as the level of disaggreagtion for my analysis, preferable with respect to less disaggreagted levels like *states* or too much disaggregated levels like *counties*. To get just a feeling of the dispersion of the variation in unemployment rates between 2007 and 2009, it is useful to look at the following tables that display local labor market unemployment rates, pre and post-recession, for states and for Metropolitan Statistical Areas (MSAs), respectively. I also display a table summarizing the variation in house prices across states between the pre-recession and the post-recession levels.

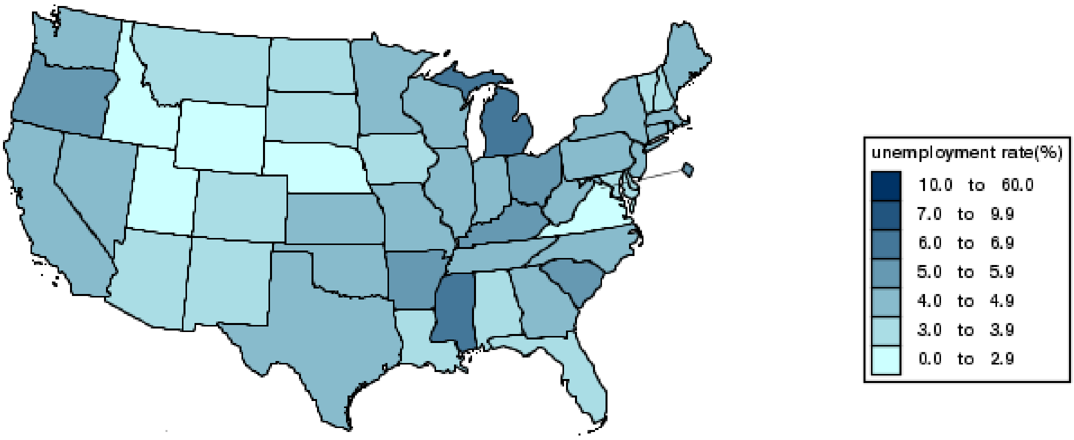


Figure 3.4: Unemployment Rates by State, January 2007.

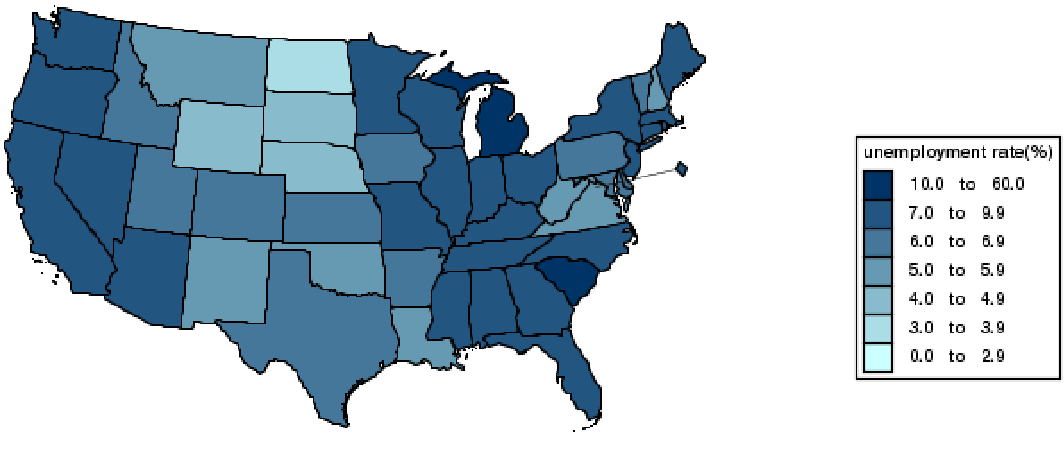


Figure 3.5: Unemployment Rates by State, January 2009.

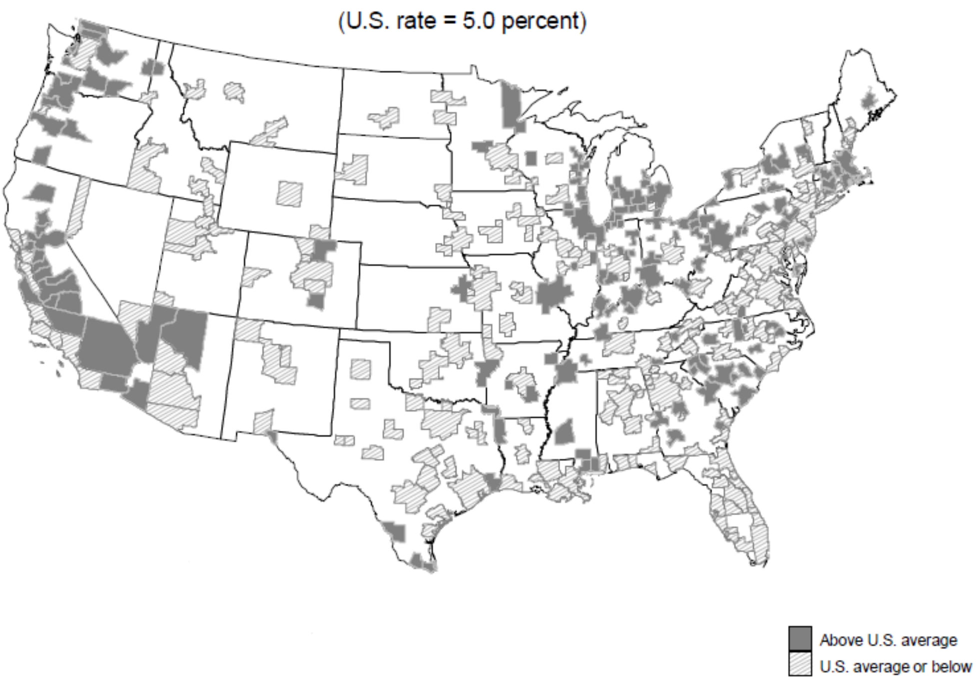


Figure 3.6: Unemployment Rates for MSA's in January 2007: differences to the US average.

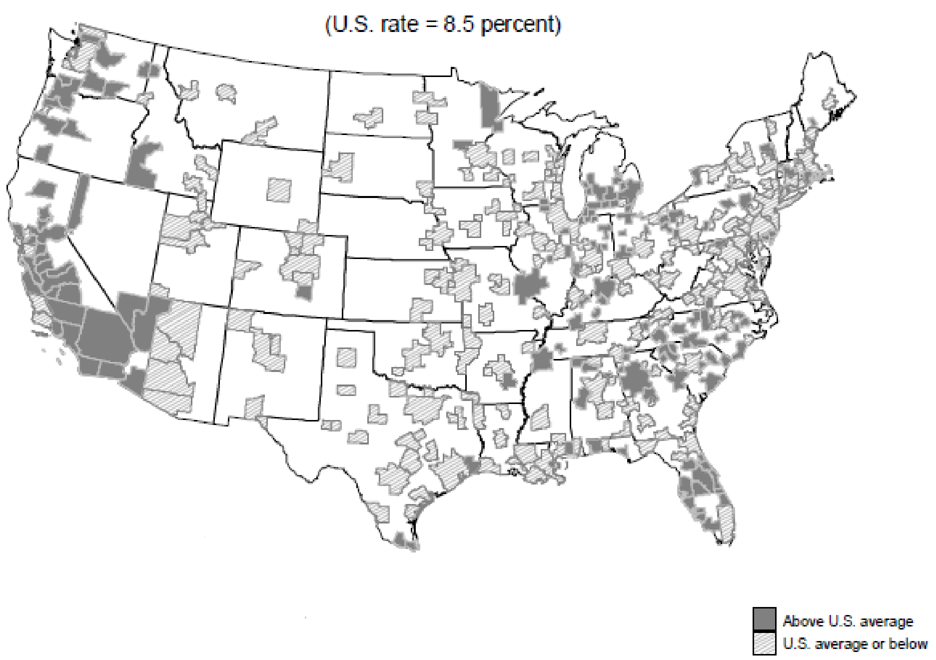


Figure 3.7: Unemployment Rates for MSA's in January 2009: differences to the US average.

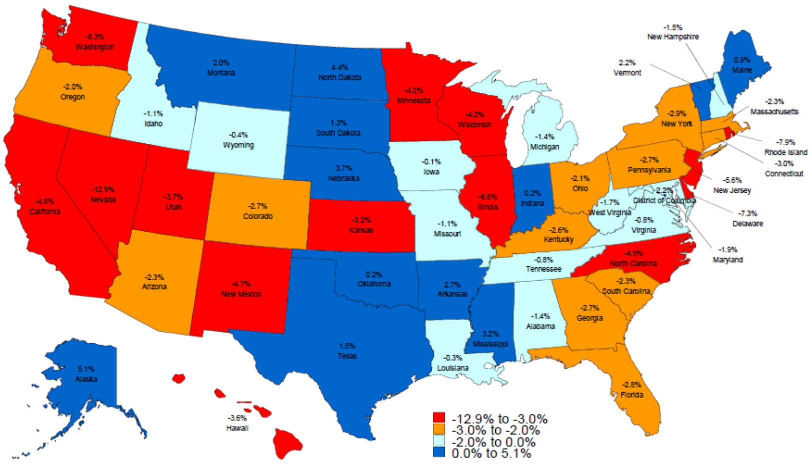


Figure 3.8: Variation in House Prices across States.

3.3.1 Where did I find the data

Annual output data at the MSA level are available at the Beaureu of Economic Analysis website for the period 2001 to 2011[3].

I use real GDP by metropolitan area in millions of chained 2005 dollars (all industry total[4]). Employment and unemployment data are taken from the Bureau of Labor Statistics (BLS). These variables are also available at monthly frequency. I construct a measure of local labor productivity for each MSA as the ratio of output to employment. Population data are taken from the "Regional Economic Accounts" of the Bureau of Economic Analysis (Table CA1-3). These are available at `http://www.bea.gov/regional/index.htm`. Population numbers reported are mid-year estimates. Quarterly data on house prices are obtained from the Federal Housing Finance Agency. I use all-transaction indexes (estimated using sales price and appraisal data). Annual estimates are computed as the average of quarterly observations.

3.3.2 Dispersion of Unemployment Rates across MSA's

If the proposed mechanism played a role during the recent Great Recession what should I observe in the data? Clearly if the housing bust in 2007 distorted the migration decisions of households and caused many unemployed households look for jobs in region with low job finding rates, this should have led to an increase in the dispersion of unemployment rates. The idea is that mobility of workers across regions should "arbitrage" differences

[3] *http://bea.gov/regional/gdpmetro/*

[4] Actually the data are available also disaggregated for industry sector.

in unemployment rates. Thus if mobility were negatively affected following the housing bust we should observe an increase in the geographical dispersion of unemployment rates.

Using monthly data on unemployment rates, Figure 3.9 plots the standard deviation of unemployment rate across MSA's for the period 2001-2010.

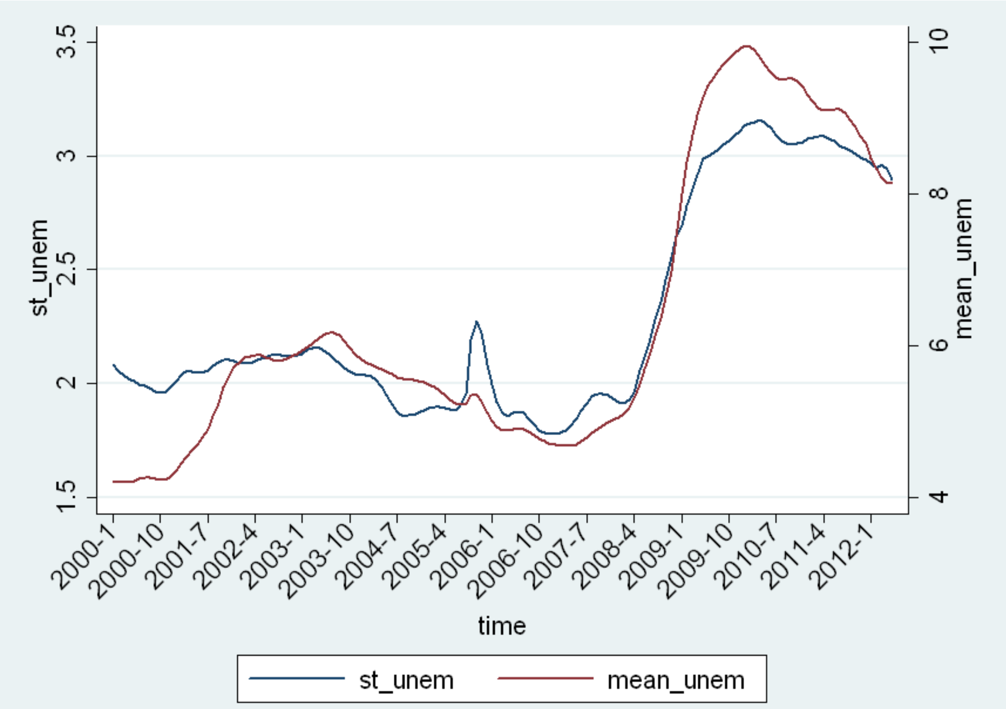
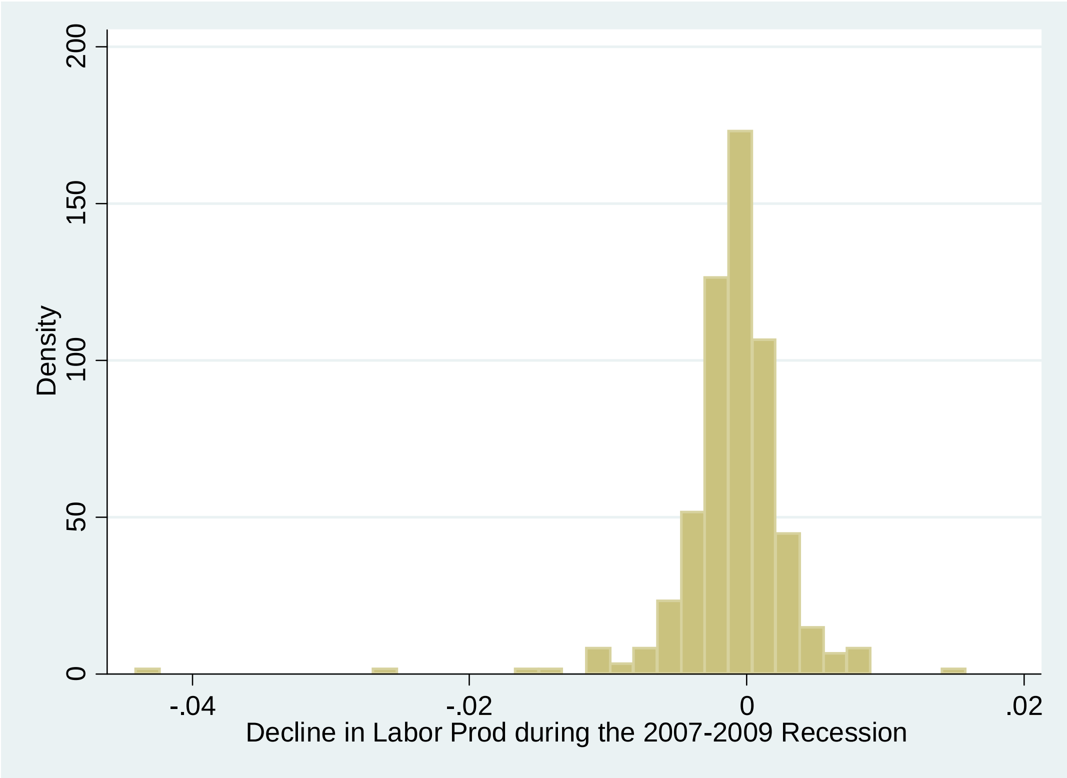


Figure 3.9: Source: Bureau of Labor Statistics (BLS) data and my calculations.

Dispersion of unemployed rate across MSA's was relatively stable around 2% until the recent recession in 2007 hit the US economy. But starting in 2008 the standard deviation increased to more than 3%.

3.3.3 Dispersion of the Decline in Labor Productivity across MSA's

At the core of my mechanism is the idea that a decline in house prices reduces geographical reallocation by distorting the migration choice of homeowners. The importance of migration for the behavior of aggregate unemployment, however, depends on the amount of regional disparities across local labor markets. Motivated by this observation, I now document the change in local and aggregate labor productivity (output per worker) during the recent recession. My data suggests that aggregate labor productivity declined by around 0,05% at the start of the recession. *But what really matters is that there is substantial variation across MSAs*: the standard deviation of the decline from 2007 to 2008 (weighted by employment) is 0,42209%. In Figure 3.3.3 I plot the histogram of the decline in labor productivity across MSA's between 2007 and 2009.



Remark. I constructed a measure of local labor productivity for each MSA as the ratio of output to unemployment.

3.3.4 Dispersion of Decline in House Prices across MSA's

I also document the large dispersion in the size of the housing bust across MSA's. I define the size of the housing bust as the percentage decline in house prices between the peak house price preceding 2007 and the lower bottom after the bust. Between 2007 and 2008, house prices declined by 28,7% on average. The standard deviation of this decline across MSAs is 14,7%. To give a more detailed picture, Figure 3.10 plots the histogram of this measure. While there are regions in which house prices declined by less than 10%, there are also a fair number of MSA's where the decline in house prices exceeded 40%. If the housing bust had an effect on the labor market during the 2007–2009 recession through its effect on local labor markets, one would expect to see this effect more pronounced in

MSAs with larger housing busts. Later in this section, I exploit this variation to provide evidence for the mechanism pursued in my work.

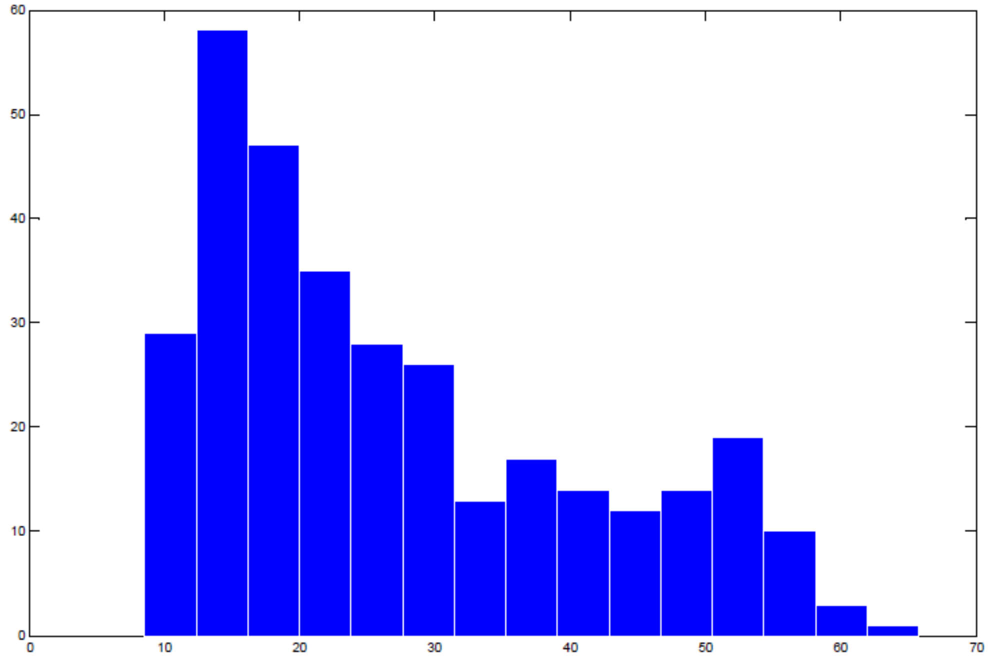


Figure 3.10: % Decline in House Prices during the Housing Bust of 2007

3.3.5 Econometric Analysis

My mechanism has two main implications that I controlled in the data:

- Unemployment should increase more in MSA's with larger house price declines, even after controlling for the decline in labor productivity.
- MSa's with larger housing busts should have smaller out-migration rates.

The substantial heterogeneity in the decline of labor productivity and the decline in house prices allow me to test these implications.

Dependent variable	*Change in unemployment rate* $\Delta u_{i,t}$
Size of the housing bust $\Delta p_{i,t}$	0.022***
Standard Error	(0.0045)

Table 3.1: MSA-wide correlation between house prices and unemployment

3.3.6 Housing Bust and the Rise in Unemployment

I now look at the statistical relationship between the increase in unemployment rates during recent recession and the decline in house prices.

I regress the change in local unemployment during the recession on the change in house prices (which captures the size of the housing bust),

$$\Delta u_{i,t} = \beta_0 + \beta_1 \Delta \log p_{i,t} + e_{i,t}. \tag{3.1}$$

Of course, the correlation between the increase in unemployment and the decline in house prices can be spurious. More specifically, a larger decline in local labor productivity can both cause a larger fall in house prices and a larger increase in local unemployment. One should control for potential omitted variable bias arising from the exclusion of labor productivity. Indeed, I find that the correlation between the decline in labor productivity and the decline in house prices is $-0,21$ and significant, suggesting that part of the unemployment differences across MSAs may be driven by the differences in labor productivity.

To partially solve this omitted variable problem and go deeper into the analysis, I enrich the specification of equation (3.5) to include:

- the change in labor productivity (omitted variable), Δy_i
- a linear interaction term between change in labor productivity Δy_i and the size of the housing bust, $\Delta p_{i,t}$
- a quadratic interaction term $\Delta y_i \times (\Delta p_{i,t})^2$

This new specification leads to the estimation of the following equation:

$$\Delta u_{i,t} = \beta_0 + \beta_1 \Delta \log p_{i,t} + \beta_2 \Delta y_i + \beta_3 \Delta y_i \times \Delta p_{i,t} + \beta_4 \cdot \Delta y_i \times (\Delta p_{i,t})^2 + e_{i,t}. \tag{3.2}$$

These terms are intended to capture the variation across MSAs in the increase in unemployment as a response to the same decline in local labor productivity. More specifically, if MSAs with larger housing busts systematically differ from others in how much unemployment increased as a response to the same decline in local labor productivity, these differences will be captured by the interaction terms. The coefficient on the linear term is positive but not significant, whereas the coefficient on the second interaction term is negative and significant at 5%. This means that, holding the change in labor productivity constant, unemployed increased more in MSA's that experienced larger housing busts.

3.3.7 Housing Bust and Migration

My mechanism argues that a decline in house prices affects the labor market by decreasing migration. The first question to ask, therefore is the following: "did inter-state migration during the recession decrease?". If we look at the time series depicted in Figure 3.11 the answer seems to be positive:

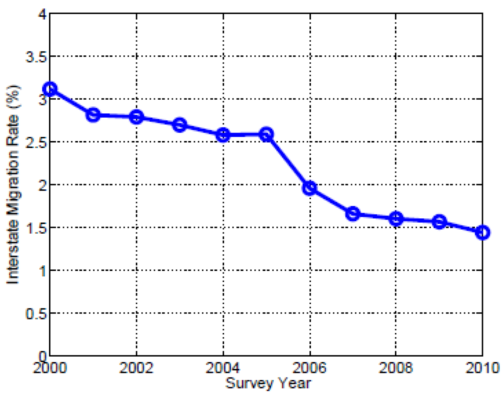


Figure 3.11: Source: Kaplan and Schulhofer-Wohl (2010). The figure plots the annual *interstate migration* rate.

However, Kaplan and Schulhofer-Wohl (2012) show that the significant drop in the annual interstate migration rate between the 2005 and 2006 Current Population Surveys is a statistical artifact, due to the change in the imputation of missing data. More precisely, the two authors document that the Census Bureau's imputation procedure for dealing with missing data before the 2006 survey year inflated the estimated interstate migration rate. They show furthermore that the change in these imputation procedures for missing data explains 90 percent of the reported decrease in interstate migration between 2005 and 2006. I report the corrected numbers in Figure 3.12.

My findings instead show that the housing bust in 2007 has had a nontrivial effect on local and aggregate unemployment and that this effect is driven by changes in net and gross migration rates that are consistent with the data. The key to resolving the apparent contradiction between my results and the paper by Kaplan and Schulhofer-Wohl (2012) is

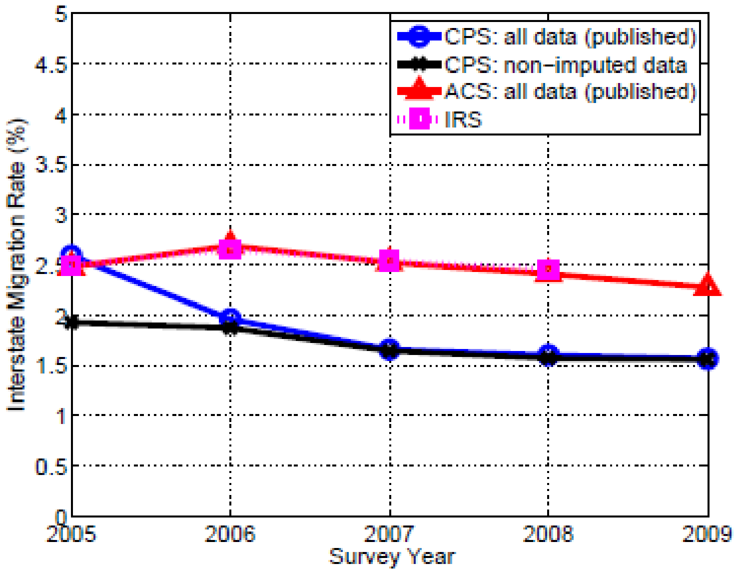


Figure 3.12: Source: Kaplan and Schulhofer-Wohl (2010).

the idea that the decline in migration observed during the recession may not represent the total decline in migration caused by the housing bust. The decline in labor productivity is asymmetric across MSAs and one might expect, absent the housing bust, net and gross migration to *increase*. In other words what really matters is not the *actual* decline in interstate migration but the difference between the actual interstate migration rate and what we would observe *absent the housing bust (counterfactual interstate migration rate)*.

Of course, decline in migration rates alone are not sufficient to conclude they were caused by housing bust (in 2007). This is because net migration is pro-cyclical, as shown by Molloy et al. (2014). To bring further evidence, I need to switch from aggregate numbers to an investigation of worker flows at the MSA level and how they are related to size of the local housing bust.

Test if among places with the same labor productivity, MSA's with *larger housing busts* had *smaller out-migration rates.* To this purpose, I run

Another implication of my mechanism is that outflows should respond less to adverse local conditions in MSAs with larger housing busts. In regression (3.3), I investigate the relationship between population flows and local labor productivity during the recent recession. In the first specification, I regress the out-migration rate on the productivity gap (i.e. the difference between aggregate (log) labor productivity and MSA level labor productivity.

$$out_i = \beta_0 + \beta_1 \left(\log \overline{z}_t - \log z_{i,t}\right) + e_{i,t} \tag{3.3}$$

Notice that of course a larger gap means that labor productivity in that MSA is

Dependent variable	*Out-migration rate* out_i
productivity gap $\log \overline{z}_t - \log z_{i,t}$	*0.0091****
Standard Error	(*0.0019*)

Table 3.2: Results, baseline regression

Dependent variable	*Out-migration rate* out_i
productivity gap $\log \overline{z}_t - \log z_{i,t}$	0.0216***
Standard Error	(0.0040)
Interaction: $(\log \overline{z}_t - \log z_{i,t}) \times \Delta p_{i,t}$	-0.0434^{***}
Standard Error	(0.0125)

Table 3.3: Results, extended regression

lower. The coefficient is positive and significant, showing that MSA's with lower labor productivity tend to have larger outflows. In a second specification, I added the interaction of the productivity gap $(\log \overline{z}_t - \log z_{i,t})$ with the size of the housing bust $\Delta p_{i,t}$:

$$\begin{aligned} out_i &= \beta_0 + \beta_1 (\log \overline{z}_t - \log z_{i,t}) + \\ &\quad + \beta_2 (\log \overline{z}_t - \log z_{i,t}) \times \Delta p_{i,t} + e_{i,t} \end{aligned} \tag{3.4}$$

The interaction term is intended to capture differences across MSAs in how outflows are related to local conditions. In particular, I am interested in documenting if, after controlling for local labor productivity, MSA's with larger housing busts have more or less outflows compared to others. The coefficient on the interaction term is negative and significant, and gives support to the hypothesis that a decline in house prices constrains the mobility of homeowners (See also Ferreira et al. (2010)).

3.4 Theoretical Model: Environment and Notation

The economy is composed of a continuum of islands inhabited by a measure one of workers and a continuum of firms. Workers and firms are both risk-neutral and in infinitely lived. Time is discrete. Workers are either employed or unemployed. Being employed means being matched with a firm. Each period an unemployed worker decides whether to stay on her current island to search for a job or to move to another island to look for a better opportunity. When moving between any two islands, an unemployed worker incurs a fixed moving cost C. For the sake of clarity I define below the notation that I follow throught the remainder of this paper.

- Unemployed workers in a certain island can either stay in that island and search for a job in the lacal labor market or move to another island (paying fixed cost C)
- Let x be worker's productivity which is equal to firm's output from a match
- Let k_x be the flow cost of posting a vacancy (depends on productivity)
- $\lambda =$ exogenous job destruction rate
- $b =$ unempl benefit.
- $v(x), u(x)$: number of vacancies and unemployed workers searching at the productivity level x on the island.
- $\Lambda(v(x), u(x)) =$ number of new matches.
- $q(x) = \frac{u(x)}{v(x)} =$ queue length (inverse of market tightness)
- $f(q(x)) =$ job finding rate, $f'(q) < 0$
- $\alpha(q(x)) =$ vacancy filling rate, $\alpha'(q) > 0$
- The vacancy filling rate can also be written as $\alpha(q(x)) = \frac{\Lambda(x)}{v(x)} \frac{u(x)}{u(x)} = f(q(x))q(x)$.
- γ is workers' bargaining power.
- Define $F(x)$ as cdf of $U[1-\omega, 1+\omega]$.

3.5 Timing of the Model

My model is a *Lucas-Prescott* island model with search-matching frictions within each island. Each time period t is divided into four stages:

1. A fraction λ of old matches are destryed.
2. Workers observe x.
3. (t.3) Some of unemployed workers on island i (those who get a lower realization of x) decide to move. If they move they must pay a fixed moving cost C in the current period; they arrive to another island in period $t+1$ and start searching for a job in this new island in $t+1$. If instead they decide to stay, they search for a job in the local labor market.
4. Production and vacancy creation take place.
5. New matches are realized.

3.6 Bellman Equations

3.6.1 Bellman equations for workers

Definitions for value functions

$H(x)$ is the value of being unemployed (before taking moving decision). $S(x)$ is the value of staying in the current island and search for a job there. M is the value of moving to another island. Hence

$$H(x) = \max\{S(x), M\} \tag{3.5}$$

where the maximization is taken over two actions: *stay* or *move* (*Should I stay or should go?*). $W(x)$ is the value of being employed at w.

I can write the value of an unemployed person who decided to stay as

$$S(x) = b + \beta f(q(x))W(x) + \beta\left[1 - f(q(x))\right]H(x) \tag{3.6}$$

i.e. with $pr = 1 - f(q(x))$ they are unlucky and don't get a job; next period they still have option to migrate, hence value is $H(x)$.

I can write the Bellman for employed workers as:

$$W(x) = w + \beta(1-\lambda)W(x) + \beta\lambda\int_{x'} H(x')dF(x') \tag{3.7}$$

Notice that productivity x does not change if remain employed. If you lose the job and become unemployed, instead, you draw a new productivity from the distribution $F(x')$. For simplicity and for the ease of analytical tractability I assume that $F(x')$ is the cdf of a Uniform with support $[1-\omega, 1+\omega]$. The parameter ω indexes the standard deviation (hence dispersion) of productivity shocks.

The value of a mover is given by:

$$M = b - C + \beta\int_{x'} H(x')dF(x') \tag{3.8}$$

3.6.2 Bellman equations for Firms

Definitions

$J(x)$ = value of being matched with a worker
$V(x)$ = value of having an open vacancy

The Bellman equations for firms are:

$$J(x) = x - w + \beta(1-\lambda)J(x) \tag{3.9}$$

and

$$V(x) = -k_x + \beta\alpha(q(x))J(x) + \beta\left[1 - \alpha(q(x))\right]V(x) \tag{3.10}$$

I assume as standard in Diamond-Mortensen-Pissarides model that the wage is given by Nash bargaining over workers' and firms' surpluses:

$$w = \arg\max_{\widetilde{w}} \left\{ (W(x;\widetilde{w}) - H(x))^{\gamma} (J(x;\widetilde{w}) - V(x))^{1-\gamma} \right\} \tag{3.11}$$

3.7 Solution of the Model

I start imposing the free-entry condition on vacancies:

$$V(x) = 0, \quad \forall x \tag{FE}$$

Hence from (3.10) I get:

$$J(x) = \frac{k_x}{\beta\alpha(q(x))} \tag{3.12}$$

Then define $\overline{H}(x) = \int_{x'} H(x')dQ_u(x'|x)$, hence from (3.7) I get

$$W(x) = \frac{w + \beta\lambda\overline{H}(x)}{1 - \beta(1-\lambda)}$$

Let $\widetilde{\lambda} = 1 - \beta(1-\lambda)$. From (3.9) I get

$$J(x) = \frac{x - w}{1 - \beta(1-\lambda)} \tag{3.13}$$

Combining (3.12) and (3.13) I get:

$$\begin{aligned} \frac{x-w}{1-\beta(1-\lambda)} &= \frac{k_x}{\beta\alpha(q(x))} \\ \Longrightarrow \quad J(x) &= \frac{x-w}{\widetilde{\lambda}} = \frac{k_x}{\beta\alpha(q(x))} \end{aligned} \tag{3.14}$$

Let's turn to wage bargaining. The FOC for Nash bargaining is:

$$\frac{\gamma}{W(x) - H(x)}\frac{\partial W(x;w)}{\partial w} + \frac{1-\gamma}{J(x) - V(x)}\frac{\partial J(x;w)}{\partial w} = 0;$$

Since $\frac{\partial W(x;w)}{\partial w} = \frac{1}{\widetilde{\lambda}}$ and $\frac{\partial J(x;w)}{\partial w} = -\frac{1}{\widetilde{\lambda}}$, we have

$$\begin{aligned} \frac{\gamma}{W(x) - H(x)} &= \frac{1-\gamma}{J(x)} \\ \Longrightarrow \quad & \frac{\gamma}{1-\gamma}J(x) = W(x) - H(x) \end{aligned} \tag{NASH RULE}$$

Combining this with eq(3.14) I get

$$W(x) - H(x) = \frac{\gamma}{1-\gamma}\frac{k_x}{\beta\alpha(q(x))}. \tag{3.15}$$

3.7.1 Stayers and firms

For stayers of course we have $H(x) = \max\{S(x), M\} = S(x)$, hence (11) becomes:

$$W(x) - S(x) = \frac{\gamma}{1-\gamma}\frac{k_x}{\beta\alpha(q(x))} \tag{3.16}$$

Also, (3.6) becomes

$$S(x) - b = \beta f(q(x))\left[W(x) - S(x)\right] + \beta S(x) \tag{3.17}$$

Plugging equation (3.17) into (3.16) delivers:

$$(1-\beta)\,S(x) - b = \frac{\gamma}{1-\gamma}\frac{k_x}{q(x)}, \tag{3.18}$$

where I also used the relation $\alpha(q(x)) = f(q(x))q(x)$. Equation (3.18) tells how queue length $q(x)$ and the value of staying $S(x)$ are related at a given x. In words, it says that the longer is the queue, the lower is the value of staying, the stronger the incentives to move.

Now insert equations (3.7) and (3.18) into eq (3.6) for $W(x)$ and $S(x)$ yields:

$$\left[1-\beta\left(1-f(q)\right)\right]\left[\frac{b}{1-\beta} + \frac{\gamma}{(1-\gamma)(1-\beta)}\frac{k_x}{q(x)}\right] = b + \beta f(q)\frac{w + \beta\lambda\overline{H}(x)}{1-\beta(1-\lambda)}. \tag{3.19}$$

Furthermore from (3.14) we get the productivity specific wage:

$$w(x) = x - \frac{k_x\widetilde{\lambda}}{\beta\alpha(q(x))}; \tag{wage}$$

Equation (wage) can be interpreted as a kind of labor demand equation: if the bargained wage w is higher then $\alpha(q(x))$ must be higher, but a higher pr that a vacancy is filled can be obtained if the queu length is higher, which means less vacancy posting. Substituting (wage) into (3.19) yields an implicit equation in

$$\frac{\widetilde{\lambda} - \beta\lambda\psi}{1-\beta}\left(b + \frac{\gamma}{1-\gamma}\frac{k_x}{q(x)}\right) + \frac{\widetilde{\lambda}k_x}{\beta(1-\gamma)\alpha\left(q(x)\right)} = x + \beta\lambda(1-\psi)H_0 \tag{queue}$$

Since LHS of (queue) is strictly decreasing in $q(x)$ and the RHS is a constant, the above equation uniquely pins down $q(x)$ for every x. Given H_0 the labor market variables are determined by equations (**??**) and (**??**).

Hp: $q(x) = q_1$, $\forall x$ (I assume that the queue length is the same across productivity levels; or is it a guess?)

$\Longrightarrow$ $P_f = f(q_1(x))$, $\forall x$ assume a constant job finding rate (guess and verify method).

$\Longrightarrow$ since $\alpha(q(x)) = f(q(x))q(x)$, it follows from (queue) that k_x is linear in x.

$\implies$ wage $w(x)$ is linear in x from (wage)

Hence also $S(x)$ is linear in x. In fact, from (3.6) we have

$$\begin{aligned} S(x) &= b + \beta f(q(x))W(x) + \beta\left[1 - f(q(x))\right]H(x) \\ &= b + \beta f(q(x))W(x) + \beta\left[1 - f(q(x))\right]S(x) \end{aligned}$$

where the second follows since we are considering stayers, hence $H(x) = \max\{S(x), M\} = S(x)$.

$$\begin{aligned} S(x) &= b + \beta f(q(x))W(x) + \beta\left[1 - f(q(x))\right]S(x) \\ S(x)\left[1 - \beta\left(1 - f(q(x))\right)\right] &= b + \beta f(q(x))W(x) \\ S(x) &= \frac{b + \beta f(q(x))W(x)}{1 - \beta\left(1 - f(q(x))\right)} \\ S(x) &= \frac{b + \beta P_f \frac{w + \beta\lambda\overline{H}(x)}{\widetilde{\lambda}}}{1 - \beta\left(1 - P_f\right)} \end{aligned}$$

and $\overline{H}(x) \equiv \int H(x')dF(x') = \int S(x')dF(x')$.

After some boring algebra (get ξ_0, ξ_1 and ξ_2 with the undetermined coefficient method) we get the following linear equation for $S(x)$ the value of staying:

$$S(x) = \xi_0 + \xi_1 H_0 + \xi_2 x \tag{S}$$

where $H_0 = \int H(x)dF(x)$, $F(x) \sim U[1-\omega, 1+\omega]$. The value of moving instead is given by:

$$M = b - C + \beta H_0. \tag{M}$$

Then remember the value of being unemployed at the beginning of the period:

$$\begin{aligned} H(x) &= \max_{stay,move}\{S(x), M\} \\ &= \max_{stay,move}\{\xi_0 + \xi_1 H_0 + \xi_2 x, b - c + \beta H_0\} \end{aligned}$$

- **Case 1**: If C is high enough so that $M \leq S(1-\omega)$ then we have NO MOBILITY $\forall x \in [1-\omega, 1+\omega]$. In fact from (S) we see that $S(x)$ is strictly increasing in x, hence $S(1-\omega) \leq S(x)$, $\forall x \in [1-\omega, 1+\omega]$, $\implies M \leq S(x)$, $\forall x$

- **Case 2**: Instead if $S(1-\omega) < M$, then $\exists$ threshold x_c such that

$$\begin{aligned} S(x) &< M \iff x < x_c \\ S(x) &\geq M \iff x \geq x_c \end{aligned}$$

as represented in the figure below.

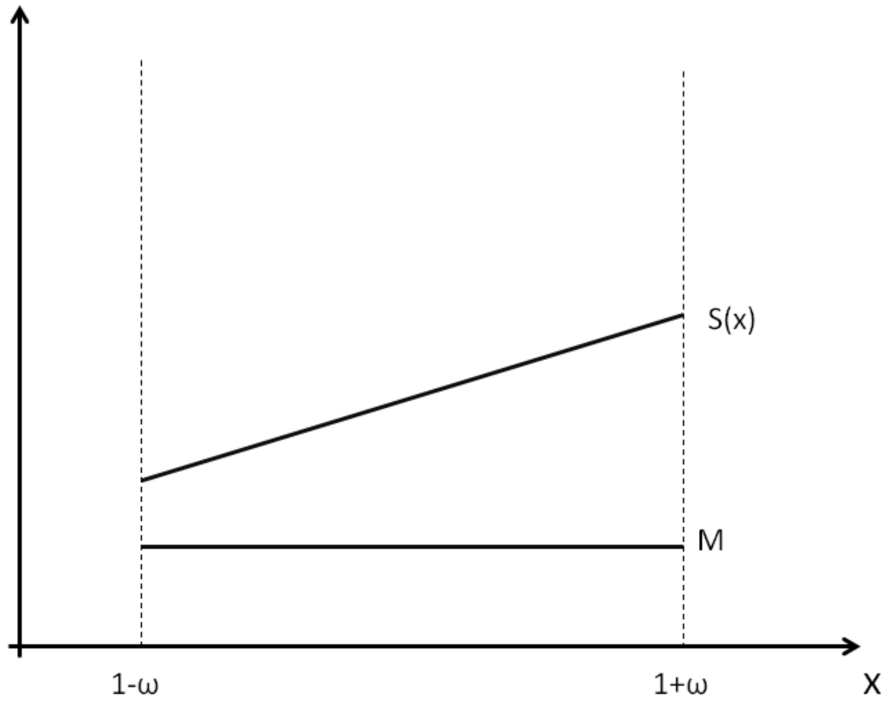


Figure 3.13: Case $M \leq S(1-\omega)$: No Mobility.

3.7.2 Case 1: No Mobility

Our three equations:

$$S(x) = \xi_0 + \xi_1 H_0 + \xi_2 x \tag{S}$$

where $H_0 = \int H(x)dF(x),\ F(x) \sim U[1-\omega, 1+\omega]$.

$$M = b - C + \beta H_0. \tag{M}$$

$$H(x) = \max_{stay,move} \{S(x), M\}$$

Claim: $H_0 = S(1)$

Proof

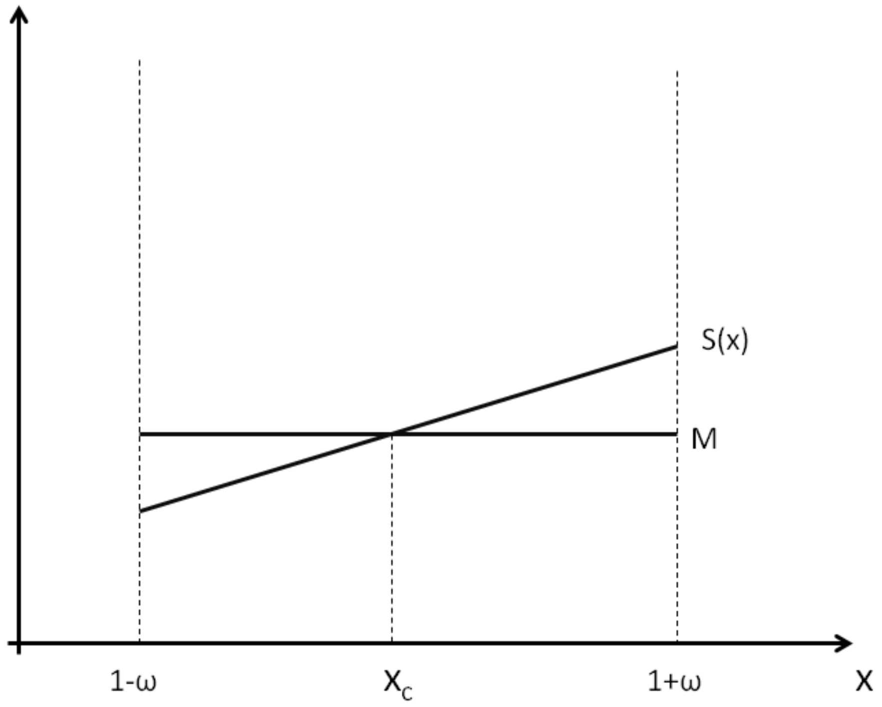


Figure 3.14: Case $S(1-\omega) < M$: if $x < x_c$ unemployed leave, if $x \geq x_c$ unemployed stay in their current location.

$$\begin{aligned}
H_0 &= \int H(x)dF(x) = \\
&= \int \max\{S(x), M\}\, dF(x) = \\
&= \int_{1-\omega}^{1+\omega} S(x)dF(x) = \\
&= \int_{1-\omega}^{1+\omega} \{\xi_0 + \xi_1 H_0 + \xi_2 x\}\, dF(x) = \\
&= \int_{1-\omega}^{1+\omega} \{\xi_0 + \xi_1 H_0\}\, dF(x) + \xi_2 \int_{1-\omega}^{1+\omega} x dF(x) = \\
&= \xi_0 + \xi_1 H_0 + \xi_2 \cdot 1 = S(1). \qquad \text{(H0_no_mob)}
\end{aligned}$$

where the last line follows from

$$\begin{aligned}
\int_{1-\omega}^{1+\omega} x dF(x) &= \int_{1-\omega}^{1+\omega} x \frac{1}{2\omega} dx = \\
&= \frac{1}{2\omega} \int_{1-\omega}^{1+\omega} x dx = \\
&= \frac{1}{2\omega} \frac{x^2}{2} |_{1-\omega}^{1+\omega} = \\
&= \frac{1}{4\omega} \left[(1+\omega)^2 - (1-\omega)^2\right] = \\
&= \frac{1}{4\omega} 4\omega = 1
\end{aligned}$$

The last derivation is really trivial, since the expected value of x is just the midpoint of the interval $[1-\omega, 1+\omega]$.

Rearranging (H0_no_mob), we get

$$H_0 = \frac{\xi_0 + \xi_2}{1 - \xi_1}.$$

Also notice that:

$$\begin{aligned}
S(1-\omega) &= \xi_0 + \xi_1 H_0 + \xi_2 (1-\omega) \\
&= \xi_0 + \xi_1 \left(\frac{\xi_0 + \xi_2}{1 - \xi_1}\right) + \xi_2 (1-\omega)
\end{aligned}$$

Furthermore, notice that

$$\begin{aligned}
M &= b - C + \beta H_0 = \\
&= b - C + \beta \frac{\xi_0 + \xi_2}{1 - \xi_1}
\end{aligned}$$

Find the value C_1 such that if $C \geq C_1$ then $M \leq S(1-\omega)$ (so that NO MOBILITY).

$$\begin{aligned} M &\leq S(1-\omega) \\ b - C + \beta\frac{\xi_0+\xi_2}{1-\xi_1} &\leq \xi_0 + \xi_1\left(\frac{\xi_0+\xi_2}{1-\xi_1}\right) + \xi_2(1-\omega) \\ C &\geq C_1 \equiv b + (\beta - \xi)\left(\frac{\xi_0+\xi_2}{1-\xi_1}\right) - \xi_0 - \xi_2(1-\omega) \end{aligned}$$

3.7.3 Case 2: Mover's problem

Let $C < C_1$. Then there exists $1-\omega < x_c < 1+\omega$ such that $S(x_c) = M$. Then to pin down the unique value of x_c we use precisely this condition $S(x_c) = M$.

Start from the equations:

$$S(x) = \xi_0 + \xi_1 H_0 + \xi_2 x \tag{3.20}$$

and

$$M = b - C + \beta H_0$$

Nota Bene: these equations are valid in both cases (No Mobility and Mobility). Then

$$\begin{aligned} S(x_c) &= M \\ \xi_0 + \xi_1 H_0 + \xi_2 x_c &= b - C + \beta H_0 \end{aligned}$$

Solving for H_0 we get:

$$H_0 = \frac{\xi_0 + \xi_2 x_c + C - b}{\beta - \xi_1} \tag{H$_0$\_1}$$

On the other hand, remember that we have another equation for H_0 as a function of x_c (then I equate the two and solve for x_c!!). The equation is just the definition of H_0:

$$\begin{aligned} H_0 &= \int H(x)dF(x) \\ H_0 &= \int \max_{stay,move}\{S(x), M\}\, dF(x) \end{aligned} \tag{3.21}$$

From figure 2 we notice that

$$\max\{S(x), M\} = \begin{cases} M & \text{if } 1-\omega < x \leq x_c \\ S(x) & \text{if } x_c < x \leq 1+\omega \end{cases}$$

Hence (3.21) can be rewritten as

$$\begin{aligned} H_0 &= \int_{1-\omega}^{1+\omega} \max\{S(x), M\}\, dF(x) = \\ &= \int_{1-\omega}^{x_c} \max\{S(x), M\}\, dF(x) + \int_{x_c}^{1+\omega} \max\{S(x), M\}\, dF(x) = \\ &= M\int_{1-\omega}^{x_c} dF(x) + \int_{x_c}^{1+\omega} S(x)dF(x) \end{aligned}$$

Hence we get

$$H_0 = M\left[\frac{x_c-(1-\omega)}{2\omega}\right] + \int_{x_c}^{1+\omega} S(x)dF(x)$$

and using (3.20):

$$\begin{aligned} H_0 &= M\left[\frac{x_c-(1-\omega)}{2\omega}\right] + \int_{x_c}^{1+\omega} (\xi_0+\xi_1 H_0+\xi_2 x)\, dF(x) = \\ &= M\left[\frac{x_c-(1-\omega)}{2\omega}\right] + \int_{x_c}^{1+\omega} (\xi_0+\xi_1 H_0)\, dF(x) + \xi_2 \int_{x_c}^{1+\omega} xdF(x) \\ &= M\left[\frac{x_c-(1-\omega)}{2\omega}\right] + (\xi_0+\xi_1 H_0)\left[\frac{(1+\omega)-x_c}{2\omega}\right] + \xi_2 \int_{x_c}^{1+\omega} xdF(x) \end{aligned}$$

Plugging $M = \xi_0 + \xi_1 H_0 + \xi_2 x_c$ into the last equation we get

$$\begin{aligned} H_0 &= (\xi_0+\xi_1 H_0+\xi_2 x_c)\left[\frac{x_c-(1-\omega)}{2\omega}\right] + (\xi_0+\xi_1 H_0)\left[\frac{(1+\omega)-x_c}{2\omega}\right] + \xi_2 \int_{x_c}^{1+\omega} xdF(x) \\ &= (\xi_0+\xi_1 H_0)\frac{x_c}{2\omega} - (\xi_0+\xi_1 H_0)\frac{1-\omega}{2\omega} + \xi_2 x_c\left[\frac{x_c-(1-\omega)}{2\omega}\right] + \\ &\quad + (\xi_0+\xi_1 H_0)\frac{1+\omega}{2\omega} - (\xi_0+\xi_1 H_0)\frac{x_c}{2\omega} + \xi_2 \int_{x_c}^{1+\omega} xdF(x) \\ &= -(\xi_0+\xi_1 H_0)\frac{1-\omega}{2\omega} + \xi_2 x_c\left[\frac{x_c-(1-\omega)}{2\omega}\right] + (\xi_0+\xi_1 H_0)\frac{1+\omega}{2\omega} + \xi_2 \int_{x_c}^{1+\omega} xdF(x) \\ &= \frac{(\xi_0+\xi_1 H_0)}{2\omega}\left[1+\omega-(1-\omega)\right] + \xi_2\left[\frac{x_c^2-(1-\omega)x_c}{2\omega}\right] + \xi_2 \int_{x_c}^{1+\omega} xdF(x) \\ &= \frac{(\xi_0+\xi_1 H_0)}{2\omega}2\omega + \xi_2\left[\frac{x_c^2-(1-\omega)x_c}{2\omega}\right] + \xi_2 \int_{x_c}^{1+\omega} xdF(x) \\ &= \xi_0+\xi_1 H_0 + \xi_2\left[\frac{x_c^2-(1-\omega)x_c}{2\omega} + \int_{x_c}^{1+\omega} xdF(x)\right] \end{aligned}$$

Hence

$$H_0 = \xi_0+\xi_1 H_0 + \xi_2\left[\frac{x_c^2-(1-\omega)x_c}{2\omega} + \int_{x_c}^{1+\omega} xdF(x)\right]$$

Computing the integral in the RHS yields:

$$\begin{aligned} \int_{x_c}^{1+\omega} xdF(x) &= \int_{x_c}^{1+\omega} \frac{x}{2\omega}dx = \frac{1}{2\omega}\frac{x^2}{2}\Big|_{x_c}^{1+\omega} = \\ &= \frac{1}{4\omega}\left[(1+\omega)^2 - x_c^2\right] \\ &= \frac{1}{4\omega}\left(\omega^2+2\omega-x_c^2+1\right) \\ &= \frac{1}{2\omega}+\frac{1}{2}-\frac{x_c^2}{4\omega} \end{aligned}$$

Hence we have

$$H_0 = \frac{\xi_0+\xi_2 x_c + C - b}{\beta - \xi_1} \qquad \text{(A)}$$

and

$$H_0 = \xi_0 + \xi_1 H_0 + \xi_2 \left[\frac{x_c^2 - (1-\omega)x_c}{2\omega} + \frac{1}{2\omega} + \frac{1}{2} - \frac{x_c^2}{4\omega} \right]$$

or

$$H_0 = \frac{\xi_0}{1-\xi_1} + \frac{\xi_2}{1-\xi_1} \left[\frac{x_c^2 - (1-\omega)x_c}{2\omega} + \frac{1}{2\omega} + \frac{1}{2} - \frac{x_c^2}{4\omega} \right] \tag{B}$$

Equating (A) and (B) we get a quadratic equation for x_c:

$$\frac{\xi_0 + \xi_2 x_c + C - b}{\beta - \xi_1} = \frac{\xi_0}{1-\xi_1} + \frac{\xi_2}{1-\xi_1} \left[\frac{x_c^2 - (1-\omega)x_c}{2\omega} + \frac{1}{2\omega} + \frac{1}{2} - \frac{x_c^2}{4\omega} \right]$$

Rearranging (CHECK!!) yields:

$$\left(\frac{1}{4\omega} \frac{\xi_2}{1-\xi_1} \right) x_c^2 - \left(\frac{\xi_2}{\beta - \xi_1} + \frac{\xi_2}{1-\xi_1} \frac{1-\omega}{2\omega} \right) x_c + \frac{\xi_0}{1-\xi_1} + \frac{\xi_2}{1-\xi_1} \frac{1+\omega}{2\omega} - \frac{\xi_0}{\beta - \xi_1} - \frac{C-b}{\beta - \xi_1} = 0$$

Solving this equation wrt x_c and using $P_m = \frac{x_c - (1-\omega)}{2\omega}$ we find

$$P_m = v - \sqrt{(v-1)\left(v + \frac{\xi_0 + \xi_2}{\xi_2 \omega} \right) + \frac{v}{\omega \xi_2}(C-b)},$$

where $v = \frac{1-\xi_1}{\beta - \xi_1} > 1$.

Given P_f (the job finding rate) and P_m (the probability of moving to another island), the economy-wide mobility rate is

$$\overline{m} = \frac{1}{1 + \frac{1}{1-\psi}\left(\frac{1}{\lambda} + \frac{1}{P_f} \right)\left(\frac{1}{P_f} - 1 \right)},$$

and the aggregate unemployment rate is

$$\overline{u} = \overline{m}\left(1 + \frac{1}{(1-\psi) P_f} \left(\frac{1}{P_m} - 1 \right) \right).$$

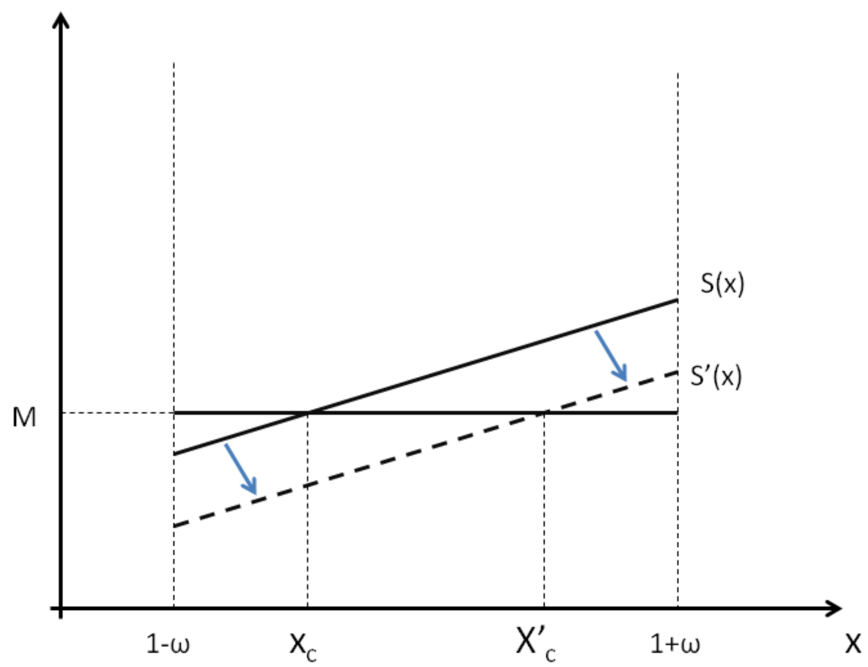
3.8 Putting the model into work: A Comparative Statics exercise

In the above economy, there are no unemployment differences across islands. However one can hit the economy with a negative idiosyncratic shock on, say, island 1 and see how mobility rates and unemployment rates adjust. For this purpose, I consider an unanticipated, permanent shock to island 1. Suppose that, due to the shock, per-period output of a firm-worker match on the island is now xz (as opposed to x in the absence of the shock), where z is a positive number close to 1. For the remainder, z is referred to as a local technology shock.

Fact. *An adverse local technology shock* ($z < 1$) *raises the queue length* $q(x)$ *and therefore lowers the job-finding rate* $f(q(x))$ *in the local market for all* x.

Proof. Inserting xz into equation (queue) for x and using the fact that the left-hand side of the equation is strictly decreasing in $q(x)$, it can be seen that $q(x)$ goes up as z declines. Consequently, the probability of finding a job on the island, $f(q(x))$, declines for all x.

Impact on mobility. Since the adverse shock reduces the match surplus at each productivity level, the productivity-specific wages of the island also decline. As both the wage and the job-finding rate go down, the value of searching for a job on this island, $S(x)$, declines for all x. However, since there are many islands, the value of leaving the island, M, remains the same (see Figure 3.8).



The Impact of an Adverse Technological Shock. The figure shows the impact of an unanticipated adverse technology shock to the island. $S(x)$ and $S'(x)$ denote the values before and after the realization of the shock.

As a result, the number of people leaving the island will sharply increase upon the realization of the shock (in fact the mobility threshold x_c, below which the value of moving is larger, shifts to the right). New workers will still come to the island from the rest of the economy, but at a lower rate. These fewer new settlers will have, on average, higher location-specific productivity for island 1 than those who were arriving before the permanent shock. So, for island 1, out-migration will be higher than in-migration until the island's labor force reaches a lower permanent level.

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