

THREE FUZZY REASONING MODELS AS A DECISIONSUPPORT AID, TO FIND AN ELECTRICAL ENERGY TARIFF

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Abstract: This contribution is a laboratory-work developed as an example of approximate (fuzzy) reasoning for students, possible to be used as a decision – support to estimate an electrical energy (EE) price for consumers. The three fuzzy tariff estimation models that are developed, integrate not only the *S.C Electrica S.A.*-single-supplier rate position, but and some (social) constraints/ compulsions of National Authority of Settlements from Energy (NASE) beginning with 1999, in this transition period from Romania. Although is possible, the paper not refer to a partial-price concrete case (internal tariff used in certain year, production price, transport price, distribution price, spot price, or an external price to be sold electrical energy, etc). This "laboratory-work-paper" shows how, by changing the parameters of *S.C Electrica S.A.* and NASE, it is possible to can perform sensitivity tests on the tariff function model, until can obtain an acceptable and true price. In this aim, the three fuzzy models use different rules for pricing: conservative, aggressive, and different order of words concerning the rules respectively, finally doing a comparison among prices and models. The paper not finished all fuzzy possibilities (rules) which can influences the expected value of a some EE tariff but, with certitude, can create a discussion base, about the way of approximate/ fuzzy reasoning, as a modality to find and to refine an EE price.

Keywords: fuzzy reasoning decision-support, fuzzy different models, EE tariff estimation

1. INTRODUCTION

The estimation of an EE price and more specifically, the price to produce, to transport or to distribute the EE, was always a major problem. Historical, in this aim were used not only modified forms of the Black-Scholes formula to find a price [Carlsson and Fuller, 2001], the games theory [Maeda et al., 1992], models which use probability theory [Pereira et al 1992], Monte Carlo models [Baughman et al., 1992], fuzzy models [Wong, 1996; Cox, 1999] or, models in optimization methods [Wong, and Wong, 1996] etc. In any decision-support application is necessary to be

considered many factors, which are different, because they can be heuristic, or can appear from numerical analyses. As a rule, the heuristic factors rise from the a priori experience of the decision factor, have a non numerical structure, and can be expressed better by linguistic values. But in the field of power systems, the concrete situation is more complicated in a transition economy as in our country in these years, because: (i) not exists some EE tariffs, i.e. from more suppliers; (ii) not exists a priori knowledge of the demand and of the EE offer, as a price function; (iii) however, must equalize the

demand with the offer of EE, and, supplementary, (iv), *must keeping the market discipline*, indifferent of all professional, social and political constraints and objectives, economy in transition, etc.

From all these motifs, to find an EE tariff involve a critical mixture of many *vague* and *uncertain* factors, as the following: (1) the demand estimation, to be possible the knowledge of the EE offer (supply); (2) competitive tariffation (pricing), when exists more offers; (3) pricing strategies; (4) market sensibility (industrial & domestic markets); (5) the cost of losses; (6) the demand peaks (daily, weekly, monthly, yearly); (7) the probable life cycle of the EE generators and turbines from the plants; (8) legal national and departmental restrictions of capitalization; (9) the singleness EE product, i.e. the monopoly position (unique producer) of *S.C Electrica S.A*; (10) social/or political restrictions, specifically the transition period above mentioned in Romania; (11) the time window and the update algorithm of the EE price etc. And, much more: all these constraints and objectives have, obviously, more or less, some degree of *imprecision*.

Because all these, and also to understand easy the modality to obtain a *fuzzy price(tariff)* for EE by the students, the three models from this laboratory-work application-paper, used only four rules in the knowledge base (KB). However, although just few, these rules to establish an EE-price contain some important economic, social and political factors : (i) the *S.C Electrica S.A* must to be profitable while sustaining high sales kWh; (ii) the average price (tariff) of the competition's MWh near our market place (the markets from Russia, Ukraine, Bulgaria, Hungary, Moldavia); (iii) the cost to manufacture, transport and distribute the MWh. In the paper, "*to be profitable while sustaining high sales kWh*" (see (i) above), is, simultaneously, a NASE constraint from [*** *Metodologia, ..., ANRE 2003*], i.e. two unconditional rules, as will see below.

The first model is one with a conservative ('quiet') attitude concerning the strategies for tariff estimating. Contrary, in *the second model* of price the approach is with some aggressive strategies (in this aim were used hedge operators), concerning all rules (level of tariff, manufacturing costs, and competition's price per MWh). *The third model* is to explore the effects of moving the conditional rule from the end to the front of the model, to understand the difference between an expert system model and a fuzzy model in this (third) case. Finally, the three models (EE prices) are compared. In all cases are used standard functions from Matlab-Fuzzy toolbox.

2. FUZZY MODELS DESIGN

As is mentioned, the first fuzzy model has only four rules ($R_i, i = 1, \dots, 4$) as in [Solea, Ghinita, and Dugan, 2004], respectively:

(R₁): the EE tariff of the *S.C Electrica S.A* must to be *high*; **(R₂)**: the EE tariff of the *S.C Electrica S.A* and the NASE, must be *low*; **(R₃)**: the EE tariff of the *S.C Electrica S.A* must be approximately *two times*costs* of EE. **(R₄)**: IF the competition EE tariff (from the neighbouring countries in the actual Romania, - i.e. Russia, Ukraine, Bulgaria, Hungary, Moldavia etc) it is not very high, THEN the EE tariff of the *S.C Electrica S.A*, should be *approximately equal (or near)* the competition EE tariff. The rules R_1, R_2 and R_3 are non-conditional, the R_4 rule is a conditional one (IF ... THEN).

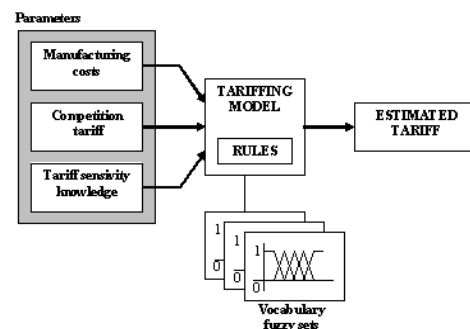


Fig. 1. Structure of the fuzzy model.

The *second model*, has the following aggressive rules (using hedge operators): the R_1 rule encodes the EE price which, now, must be to the upper value of the tariff scale, by *very* hedge; the R_2 rule by *relative/somewhat* hedge, allow the tariff to growth up the tariff sensitivity gauge; the R_3 rule, by *greater* or *above* hedge, reflects the idea that the price must be above two times costs, not approximately equal or around the costs; finally, the R_4 rule, compare with R_4 above, by using the linguistic variable *not very low*, can make as *Electrica* tariff to be close to that of the competition. Therefore the rules are:

(R₁): the EE tariff of the *S.C Electrica S.A* must be *very high*; **(R₂)**: the EE tariff of the *S.C Electrica S.A* and the NASE must be *relative/somewhat low*; **(R₃)**: the EE tariff of the *S.C Electrica S.A* must be *greater approximately two times*costs* of EE; **(R₄)**: IF the competition EE tariff (from the neighbouring countries in the actual Romania, - see R_4 above) it is *not very low*, THEN the EE tariff of the *S.C Electrica S.A*, must to be *approximately equal (or near)* the competition EE tariff. This tariff fuzzy model has the same logical modality to obtain an EE price, as the first model. The differences are only in the four rules.

The below *third model* is to see, and therefore to know, the effects of moving the unconditional rules from the front to the end of the model (or reverse). In the conventional expert systems, as in the fuzzy inference systems that contain only IF-THEN rules or only unconditional rules, is not important the order in which appear the rules in the KB. Because in this "laboratory-work-paper" the fuzzy models are mixtures of both type rules, the inference engine can't order the rules; thus, and final solution fuzzy is dependent of the order in which are executed the rules. So, the new rules are: (**R₁**): IF the competition EE tariff (i.e. for the actual Romania, the EE price from the neighbouring countries – see (**R₄**) above) is *not very high*, THEN the EE tariff of *S.C Electrica S.A.* must to be *near* the competition EE tariff; (**R₂**): the EE tariff of the *S.C Electrica S.A* must to be *high*; (**R₃**): the EE tariff of the *S.C Electrica S.A* and of the NASE, must to be *low*; (**R₄**): the EE tariff of the *S.C Electrica S.A* must be *approximately two times*costs* of EE. All the three fuzzy models used have ability to model conflicting expert rules from the knowledge base [Zadeh and Kacprzyk, 1992; Cox, 1999], which is a feature of the fuzzy system from the Figure 1. This is because each the first rule (**R₁**, **R₁**, and **R₂** respectively) ensures profitability for *S.C Electrica S.A.*, while each the second rule (**R₂**, **R₂**, and **R₃** respectively), ensures not only the social and political aspects of NASE and Government [*** *Metodologia, ..., ANRE 2003*], but and a sufficient volume of EE (MWh) sales in the market area. On the other hand, each the third rule (**R₃**, **R₃**, and **R₄** respectively) ensures that the tariff will cover the direct costs of the EE manufacturing (generating, transport, and distribution). Finally, the fourth rule (**R₄**, **R₄**, and **R₁** respectively) says that as long as the tariffs of neighbouring countries are not considered very high, the tariff of *S.C Electrica S.A* can be close (*near*) to that of competitors.

3. THE USE AND SIMULATION OF THE FUZZY MODELS TO FIND AN EE TARIFF

This laboratory work-paper, simulate the above three fuzzy models in Matlab [***, *Stud. Ver., "Learning Mat-7 (Release 14)", Mathworks, 2005*], using standard functions as *trimf*, *trapmf*, *pimf*, *smf*, *mf*, *desigmf*, *defuzz*, *max*, *min*, etc. Although another more easy way should be possible by using a Fuzzy Inference System (FIS or three FISs with Matlab-Fuzzy Toolbox) to obtain an EE price, here is selected this way only from didactic motifs. The models shows how the base fuzzy sets are combined with fuzzy regions, these being created with the current data points from [***, *Metodologia, ANRE, 2003*], and competition costs. If the Matlab programs are running, the fuzzy models request the manufacturing costs and the competition's tariff and, after they are executed, an estimated tariff is returned. Below, the figure numbers with the indexes

a and b, are for the first and for the second model, respectively.

3.1 Fuzzy sets

The base fuzzy sets *high* and *low* of the EE tariff, (Figure 2a), indicate what points are considered for EE to be a *high* tariff and a *low* tariff.

The second fuzzy model used *hedge operators* [Zadeh, and Kacprzyk, Eds.,1992; Beale, M et al, 1994; Cox,1999 etc], which make possible to control the *restrictive* or *permissive* qualities of a fuzzy set: in this case, an aggressive attitude of *SC Electrica SA* toward market positioning, compare with the first model. Recall that the hedges are classified in *concentrators* (*very* and *extremely*, they make fuzzy sets more restrictive), and *diffusers/ diluters* (*somewhat*, *quite*, *rather*, and *sort of*, which make fuzzy sets more permissive). In this case, the fuzzy sets in Figure 2b are formed by the mixture of standard *very* and *somewhat* edges, with the base price sensitivity fuzzy sets *high* and *low* respectively (Figure 2a). The *very* hedge intensifies the fuzzy set *high* (reducing the truth membership of values normally being *high*), while *somewhat* hedge dilutes/diffuses the fuzzy set *low* (increasing the truth membership of values normally being *low*), see the Figure 2b. These are obtained by squaring the membership function (*very*) and by square root of the membership function at each point in fuzzy region.

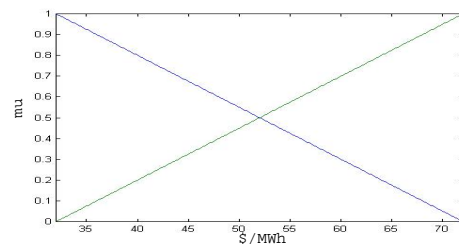


Fig. 2a: The price sensitivity fuzzy sets *high* and *low* for the tariff

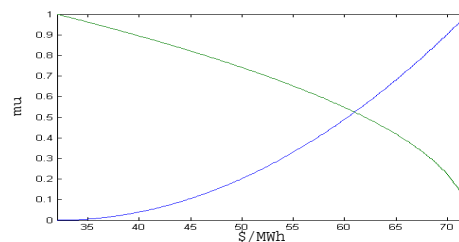


Fig. 2b: The tariff constraint fuzzy sets *very high* and *somewhat low* (with *very* and *somewhat* standard edges)

3.2 The fuzzy model sets

The *model-base fuzzy sets* from the Figures 3a and 4a depend on the actual run-time data, because each new value of manufacturing costs and competition tariffs gives new fuzzy sets. The difference in the width of

the fuzzy sets is because of the model semantics. The same fuzzy sets, but used in the second model are in Figures 3b, 4b.

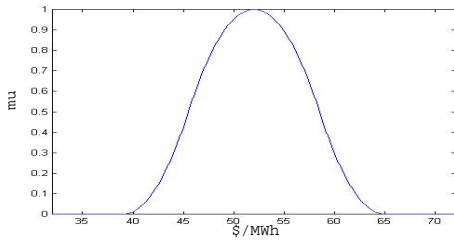


Fig. 3a: Fuzzy set of the *manufacturing costs*, the first model

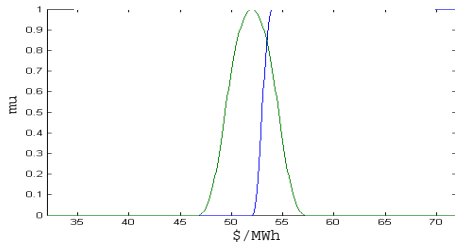


Fig.3b: Fuzzy set of *around* and *above around* two times *manufacturing costs*, the second model

For example, the fuzzy set of the *manufacturing costs* (Figure 3a, with a value of \$52.00) has a relative great percentual diffusion to account for a basic uncertainty at this point (obviously for full manufacturing costs of EE), but, and for the degree to which we want this factor to contribute to the default tariff value. By contrast, the fuzzy set *near competition's tariff* has a thinner diffusion (much small percentual) to account for the model's assumption (the rule R_4 with the *S.C Electrica's* tariff *near/close* to the competition's tariff). Recall that by changing the width of these dynamically created fuzzy sets (see Figures 3a,b and 4a,b), can be obtained a modality to refine the precision of the fuzzy model [Zadeh, and Kacprzyk, Eds., 1992; Cox, 1999; Yan et al., 1994].

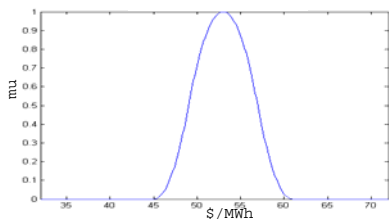


Fig. 4a and 4b: The fuzzy sets of the *near to competition's tariff* are identical (4a, 4b) for the two models

3.3 Execution Sets

The executions of the tariff estimation rules with the Matlab programs *cost_1,2,3* are with linear fuzzy sets (Figure 2a - *high* and *low*) and with nonlinear fuzzy sets (Figure 2b - *very high* and *somewhat low*)

respectively. In both cases the domains are between \$/MWh (32.00 – 72.00).

After evaluating and applying of the unconditional rules (R_1), (R_2), and (R_3), the solution fuzzy sets are shown in Figure 5a, and Figure 5b, respectively, for the two models. Being executed non-conditional propositions, the solutions are generated by the intersection of the two by two sets (AND operation).

In Figure 5a, after the rules R_1 , R_2 , and R_3 execution, the model has a middle up triangular fuzzy region with a $\mu[0.5]$ height, obtained by the intersection of the (linear) *high* and (linear) *low* fuzzy regions. In Figure 5b, for the second model are used the hedges *very* (high) and *somewhat* (low), and as a result of these nonlinear linguistic variables and the execution of the R_1 , R_2 and R_3 rules, the price fuzzy region of the tariff it is not triangular.

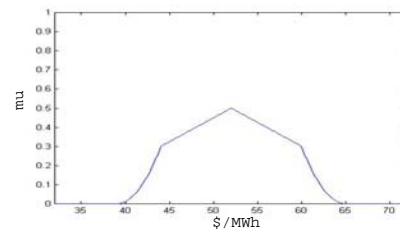


Fig. 5a: Fuzzy set of the *tariff* solution after executing rules (R_1), (R_2) and (R_3) for the first model

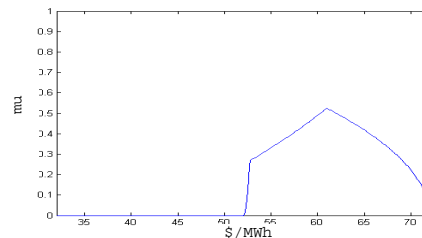


Fig. 5b: Fuzzy set of the *tariff* solution after executing rules (R_1), (R_2), and (R_3) for the second model

Figures 5a and 5b are obtained because the rule R_3 overlays the current working fuzzy regions with the bell-shaped fuzzy regions from Figures 3a and 3b respectively (*manufacturing costs*). Because R_3 is an unconditional rule, is used the minimum operator (AND), to have the minimum of the solution fuzzy set. The 'mechanism' to obtain the tariff solution from Figures 5a and 5b can be see in Figures 6a,b.

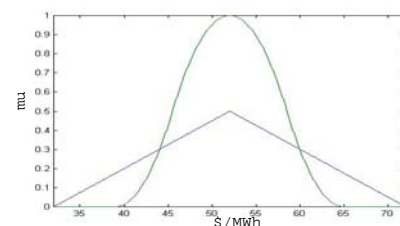


Fig. 6a: Fuzzy set of the *tariff* solution before mixed with the fuzzy set of the (R_3), *manufacturing*

costs (from Fig 3a); i.e. before the rule (R₃) is executed, 1st model.

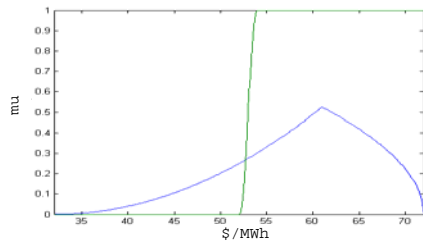


Fig. 6b: Fuzzy set of the *tariff* solution before mixed with the fuzzy set of the *more than/above around manufacturing costs* (from Fig 3b); i.e. before the rule (R₃) is executed, 2nd model.

In the both models the rules (R₄) are conditional rules (IF – THEN) and complex sentences, because these use a fuzzy linguistic variable in the predicate. Thus: to evaluating and applying of the conditional rule R₄, initially must create and evaluate the predicate linguistic variable, to be possible to determine the truth value from the following sentence(s): *the competition price of EE is not very high (low, in the second model) . . .*

To have the fuzzy region *very high* (from Figure 7a), is incorporated the *very* hedge with the original fuzzy set *high*, and to obtain the fuzzy region *not very high* (Figure 7b), was applied the Zadeh standard complement *not* ($1 - \mu(x)$), to the fuzzy set *very high* from Figure 7a [Zadeh, and Kacprzyk, Eds., 1992; Cox, 1999].

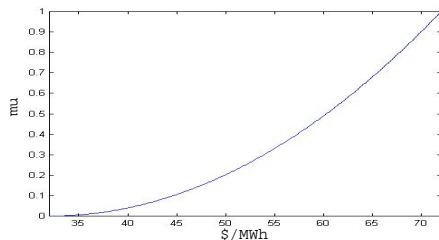


Fig. 7a: The fuzzy region *very high*

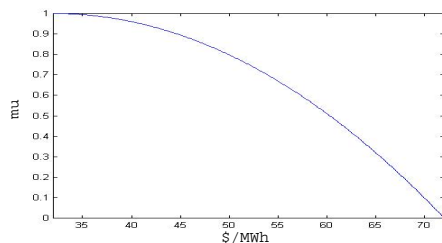


Fig. 7b: The fuzzy region of linguistic variable *not very high*

The same procedure was used and with the *very low* fuzzy region (hedge *very* to the *low* fuzzy set), and after, with the standard Zadeh *not* ($1 - \mu(x)$) at this

fuzzy set, is obtained the linguistic variable *not very low*.

From this moment we can determine the truth of predicate by finding the membership for the competition tariff. Because can be many simulations with many values, can determined many truth values with the first model. Thus, the consequent proposition:

. . . the EE tariff proposed by the *S.C Electrica S.A*, must be approximately equal (or near) the competition EE tariff, accordingly can be evaluated, and the solution fuzzy set *tariff* can be updated many times. The solution fuzzy set *tariff* after the evaluating and applying of the rule R₄ can be see in Figure 8a for the first model, and in Figure 8b for the second model. With a great truth value for the predicate, the minimum correlation process is applied to the fuzzy set *near the competition EE tariff* (see the Figures 4a and 4b) and, as a result the consequent's height is diminished, see Figure 8a.

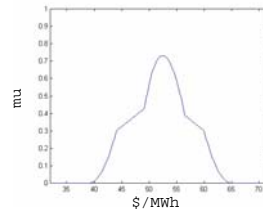


Fig. 8a The final fuzzy set of the *tariff* solution after executing rule (R₄), first model

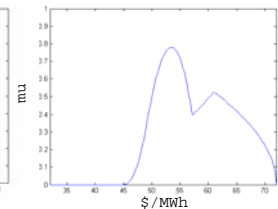


Fig. 8b: The final fuzzy set of the *tariff* solution after executing (R₄), second model

Recall [Zadeh, and Kacprzyk, Eds., 1992; Terano, Asai, and Sugeno, 1993; Cox, 1999; Yan et al., 1994; Beale and Demuth, 1994], that the fuzzy conditional propositions update the solution fuzzy set by the union of the consequent set with the solution set (when is run the OR/ MAX operation). The below eqs. are formal relations to be applied the conditional fuzzy rules (where: \otimes Pr is the Cartesian product).

$$\begin{aligned} \mu_{\text{consequent}} \otimes \text{Pr} [X_i] &= \mu_{\text{consequent}} [X_i] \times \mu_{\text{premise}} \\ \mu_{\text{solution}} [X_i] &= \max (\mu_{\text{solution}} [X_i], \mu_{\text{consequent}} \otimes \text{Pr} [X_i]) \end{aligned}$$

The first eq. is for the correlation process, and the second eq. shown the update mode of a working solution fuzzy set with a conditional proposition. Both models are now complete.

3.4 Defuzzification

The *tariff expected value* is found by defuzzification of the solution fuzzy set, i.e. the fuzzy space representing the combined knowledge of the four rules R₁ . . . R₄, from each model. This space contains the tariff fuzzy set. Defuzzification is the final phase of fuzzy reasoning. By it, is selected the

expected value of the solution variable from the consequent fuzzy region. Because the value must be the best value that represents the information contained in the solution fuzzy set, the defuzzification method is a critical design factor. From some fuzzy *defuzzification methods*, was selected the 'centroid of area method' (or *center of gravity*, or *composite moments technique*) and *largest of maximum method*, using Matlab `defuzz` function. Recall that this function can returns a defuzzified value by five defuzzification strategies: *centroid* (centroid) and *bisector* (bisector) of area methods; *mean* (mom), *smallest* (som), and *largest* (lom) of maximum methods [***, *Mathworks*, *Learning Student Matlab 7 (Release 14)*", 2005]. In our case, the centroid method mix up the four linguistic rules in an output fuzzy space which, after the first above property, (i), move smoothly if the model parameters modifying (see Figures 8a and 8b). The second, i.e. the *largest of maximum method*, bases its recommended tariff on the highest truth value of a point from the fuzzy output space. Because this, if the conditional rule (R_4) has a truth greater than (R_1), (R_2), or (R_3), the *tariff* value is mainly influenced by this rule. A contrary, when the truth of conditional rule is less than the truth of unconditionals, the tariff value is mainly influenced by the mixture of the (R_1), (R_2), and (R_3) rules. Anyhow, the selection of defuzzification method in a fuzzy model depends how we want to be the value of the result.

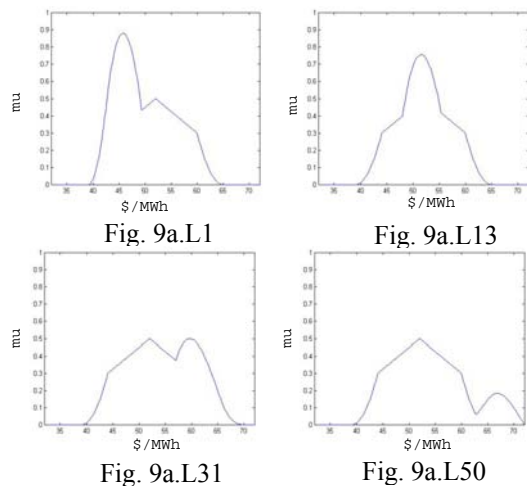
Table 1a

No. Line	Competition Tariff	Trutf fc.	Tariff (Centroid)	Trutf fc.	Tariff (Maxim)	Trutf fc.
1	46.0000	0.8775	49.9625	0.4491	45.8000	0.8795
2	46.5000	0.8686	50.2300	0.4558	46.3000	0.8708
...						
13	52.0000	0.7500	51.9299	0.7516	51.5000	0.7561
14	52.5000	0.7373	52.0213	0.7439	52.0000	0.7440
...						
31	61.0000	0.4744	54.2619	0.4435	52.0000	0.5000
32	61.5000	0.4561	54.3913	0.4402	52.0000	0.5000
...						
49	70.0000	0.0975	54.0756	0.4481	52.0000	0.5000
50	70.5000	0.0736	53.9269	0.4518	52.0000	0.5000

With this didactic aim, these differences between the estimation tariffs by the two defuzzification methods for different competition tariffs (50 lines) can see in the Table 1a. This Table 1a, is only one and only with some lines, from a total of three Tables correspondingly to the three methods, because space economy. E.g., in the Table 1a, the manufacturing costs, from [***, *Metodologia, ..., ANRE, 2003*], are always, as value, \$26.00/MWh. The Table shows an atypical defuzzification behaviour: as the competition tariff increases (between \$(46.00 . . . 70.50)/MWh), the truth of the rule (R_4)'s predicate decreases. It can be see that at the competition tariff of \$46.00/MWh (line 1, i.e. L1 in Table 1a), the tariff of \$49.96/MWh is considered *not very high*, but at the competition tariff of \$70.50/MWh (line 50, i.e. L50 in Table 1a), the tariff of \$53.92/MWh is, as value, *very high*.

As a conclusion, the centroid method "consider" the effects of the unconditional rules and "retain the tariff" in a region towards the center of the unconditional rules regions. The *largest of maximum method* follow very closely the competition tariff between the lines 1– 30, in Table 1a. But, as soon as the importance (surface) of output fuzzy region defined by the rule (R_4) is less that the importance (surface) of the output fuzzy region formed by the unconditional rules (R_1), (R_2), (R_3), the conditional rule (R_4) no controls the fuzzification result. As we see above, the results in our case, are the maximum regions defined by the intersection of the unconditional rules that have an approximately constant height in the output fuzzy set.

This thing is at the line 30, where, with this method, the tariff has a passing from \$59.20/MWh back to \$52.00/MWh, and remains at this value although the competition tariff rises. In the third model, the conditional rule is executed first and after of the three unconditionals, the model parameters being the same as those of the first model. From space economy, the fuzzy sets are not shown here, but are the same as the first model. After defuzzification of the final *tariff* fuzzy set from model execution with centroid defuzzification method, is find a value no more different from first model (although, in literature, the two approaches produce significantly different results, see [Zadeh, and Kacprzyk,Eds., 1992; Cox,1999]. An explanation should be the fact that by running the unconditional rules last, will be restrict any sets constructed by the conditional rules, and by running the unconditional rules first is obtained a minimum solution set. The Figures 10a, Li ($i = 1, 13, 31, 50$) are the executions of the tariff models associated with the chart shown in the Tables 1a; more exactly, the graphs associated only with the lines i (Li, $i = 1, 13, 31, 50$) from the Tables 1a. From space motifs the graphs associated with lines from Tables 1b, 1c are not shown.



4. CONCLUSIONS

The paper considers three fuzzy models to obtain a tariff (price) for EE in a transition country (here Romania). In the fuzzy models were considered some objectives of SC Electrica SA (unique producer of EE), a restriction of NASE [***, Metodologia..., ANRE, 2003], and a conditional rule (IF-THEN) concerning the competition, to avoid a competition war with the countries near Romania: Russia, Ukraine, Hungary, Bulgaria, Moldavia.

All fuzzy rules used (conditional and unconditionals) are a mixture of many vague and uncertain factors, with a more or less of imprecision degree. Because the paper is a didactical one, was used three fuzzy models: *the first model* with conservative rules, *the second* with a more aggressive strategy/ rules concerning the EE price (by using hedges), and *the third*, with the unconditional rules from the front to the end of the model or reverse, to see the effects of this moving. All fuzzy models used only four rules, but they can use more additional rules if another policy for the EE tariff is necessary.

As defuzzification method was used two techniques (*centroid* and *largest of maximum method*). To see the difference in estimated tariff between centroid and largest maximum methods for different values of competition price, the *defuzzification behaviour* can be seen in the Tables 1a. After defuzzification of the final *tariff* fuzzy set for all the three models execution by centroid method (using the same manufacturing cost and competition price values), the estimated EE prices for first and third models are approximately the same, while for the second model, the tariff is more greater. Because fuzzy logic provides a sensitive approach to obtain a tariff for EE, we believe that in the future this approach can be refined by (i) use of more conditional and unconditional rules, and (ii), use of rules with more sophisticated linguistic variables.

5. REFERENCES

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