OPTIMAL SIMULTANEOUS DESIGN AND OPERATIONAL PLANNING OF VEGETABLE EXTRACTION PROCESSES

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Abstract: A general multiperiod linear optimization model is proposed in this study that targets the simultaneous design and operation planning decisions of a multiproduct batch plant for the production of vegetable extracts. A multiperiod environment is considered because of the market and/or seasonal fluctuations. Thereby, the model considers changes from period to period of demands, costs, prices and raw materials supplies. The objective function maximizes the net present value of the profit considering incomes, investments and resources costs, and both product and raw material inventory costs. In the plant design problem, the sequence of operations is already defined and the pursued goal is to determine both unit sizes and its configuration in the plant. Besides the usual duplication in parallel option, a novel design alternative is included which allows adding units in series to perform a given operation. The optimal design is determined by taking into account available discrete sizes of units which corresponds to the real procurement of equipments. The model is formulated by using the linear generalized disjunctive programming (LGDP). A particular plant that produces oleoresins (solvent extracts of herbs and spices) is used to illustrate the proposed approach. Nevertheless, the developed model is general and can thus be applied to any vegetable extraction process.

Keywords: vegetable extraction; multiperiod model; multiproduct batch plants; disjunctive programming.

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INTRODUCTION

The batch mode of operation in food and biotechnological industries has received a renewed interest particularly because of the market which has become more uncertain, complex and competitive. Batch plants are considered an efficient means of production especially due their flexibility to use the available resources for manufacturing relatively small amounts of several different products within the same facilities.

In this study, efforts are focused on multiproduct batch plants where several products with similar recipes are produced. Each product is manufactured at a time, in a sequence of one or more operations. Every operation is carried out in a single equipment unit or in several units either working in parallel or in series. A product uses all units available in the plant and therefore parallel production of different products is not possible. Batch units are characterized by a processing time and no simultaneous feed and removal is performed.

Regarding multiproduct batch plants optimization models, an extensive bibliography with varying degree of detail is available (Voudouris and Grossmann, 1992; Ierapetritou and Pistikopoulos, 1996; Ravemark and Rippin, 1998; Montagna *et al.*, 2000; Athimulam *et al.*, 2006; Tsang *et al.*, 2006). In general, the proposed models have focused on one decision type, design or production planning, but not both simultaneously.

In recent years multiperiod optimization models have been also object of great deal research effort (Birewar and Grossmann, 1990; Varvarezos *et al.*, 1992; Voudouris and Grossmann, 1993; Van den Heever and Grossmann, 1999; Ryu, 2006). This major class of problems permits design and planning under variations in model parameters along the time horizon. Multiperiod model are suitable when costs, availability of raw materials, demands, and so on, typically vary from period to period due to market or seasonal changes. In particular, Moreno *et al.* (2007) presented a general multiperiod mixed-integer linear programming model that simultaneously optimizes design and production planning decisions for multiproduct batch plants. These authors include in their formulation batch and semicontinuous units, allocation of intermediate storage, and the usual structural decisions of adding units in parallel. The objective is the maximization of the net benefit of the plant accounting for parameter variations due to seasonal or market fluctuations. Examples of multiperiod plants include plants that produce pharmaceutical products where the demand pattern changes over the time horizon, and also the food industry where both raw materials and product demands have a seasonal dependence, as is the case of vegetable extraction processes.

In many cases, the above mentioned optimization problems involve continuous as well as discrete decision variables that are modelled as either mixed-integer nonlinear programming (MINLP) or mixed-integer linear programming (MILP) problems.

More recently, generalized disjunctive programming (GDP) has been proposed as an alternative modelling approach to the mixed-integer optimization program (Raman and Grossmann, 1994; Lee and Grossmann, 2000; Vecchietti *et al.*, 2003; Sawaya and Grossmann, 2005). GDP allows a combination of algebraic equations and logical expressions through disjunctions and logic propositions, which facilitates the representation of discrete decisions.

The aim of this work is to integrate production planning and design aspects of the multiproduct batch plant problem in a multiperiod scenario for the food industry. In general, previous works dealing with similar problems address only parts of the whole problem, focused on specific decisions, without considering the trade-offs among all the elements involved in the problem.

In particular, regarding the design decisions, previous works have considered only duplication in parallel of units. In this paper, the formulation includes the new option of adding units in series in the structure of the plant. Also, following the usual equipment procurement policy, in this model the unit sizes are selected from a discrete set of available units instead of the previous solutions where equipment sizes were continuous variables.

Duplication of units in parallel working out-of-phase reduces the cycle time of the operation, i.e., the time elapsed between successive batches leaving the operation. This also decreases the idle time of the other operations when the duplicated operation represents the bottleneck for the production train, thus reducing the size of these operations.

A linear generalized disjunctive problem (LGDP) model is presented and both big-M and convex hull reformulations were considered for the problem resolution. The optimal solution determines the plant design selecting the configuration in series and the number of identical parallel units from available discrete vessel sizes for each operation of the plant. Furthermore, this approach simultaneously considers planning decisions that are usually included in subsequent steps in previous works and solved through decomposition methodologies. Thus, the impact of planning decisions (purchases, inventories, and so on) can be assessed at the plant design stage. Appropriate constraints are formulated to take into account the consumption of raw materials. Different supply and inventory policies can be adopted. Demands can be satisfied using the production of that period or using stored production of previous periods. Fluctuations and seasonal variations can be assessed and the trade-offs between all the involved elements can be considered. Moreover, the problem considers the planning horizon divided into a number of time periods that may not necessarily be of the same length.

A real case of herbal extracts example is solved involving four products and several stages. The new option of duplication in series is assessed focusing on the operation of extraction.

GENERALIZED DISJUNCTIVE PROGRAMMING

The LGDP representation is going to be used in order to formulate the optimization model. This formulation can be posed as (Sawaya and Grossmann, 2005):

$$\begin{aligned}
&\text{Min } Z = \sum_{k \in K} c_k + d^T x \\
&\text{s.t. } B\,x \le b \\
&\underset{j \in J_k}{\vee} \begin{bmatrix} Y_{jk} \\ A_{jk} x \le a_{jk} \\ c_k = \gamma_{jk} \end{bmatrix}, \qquad \forall\, k \in K \qquad (1) \\
&\Omega(Y) = \text{True} \\
&0 \le x \le U,\ c_k \in \aleph^1_+,\ Y_{jk} \in \{\text{True, False}\} \\
&\quad \forall\, j \in J_k, \forall\, k \in K
\end{aligned}$$

Here, x is a vector of continuous variables bounded by a vector of upper bounds U, Y_{jk} are Boolean variables, $c_k \in \aleph^1$ are continuous variables that represent the cost associated with each disjunction and γ_{jk} are fixed charges. A disjunction $k \in K$ is composed of several disjuncts $j \in J_k$, each containing a set of linear equations and/or inequalities ($A_{jk}x \le a_{jk}$) representing the constraints of the problem, connected together by the logical OR operator ($\vee$) that enforces the contents of only one term of the disjunction. Discrete decisions are represented by Boolean variables Y_{jk} in terms of disjunctions $k \in K$ and logic propositions $\Omega(Y)$. Thus, only the constraints inside term $j \in J_k$, where Y_{jk} is true, must be satisfied; otherwise the corresponding constraints to the other terms, where Y_{jk} is false, are not enforced. Finally, $Bx \le b$ are common constraints that must hold regardless of the discrete decisions that are selected.

PROBLEM DESCRIPTION

The problem addressed in this work can be stated as follows. A multiproduct batch plant processes $i = 1, 2, \ldots, I$ products though $p = 1, 2, \ldots, P$ operations. This plant operates over a time horizon H which is divided into a finite number of time periods $t = 1, 2, \ldots, T$ that may not necessarily be of the same length H_t. Each operation p may be performed by a different number of units in series corresponding to configurations $h = 1, 2, \ldots, H_p$ (see Figure 1). Also, the configuration of units in series selected can be duplicated in parallel operating out of phase. Then each operation p may consist of $m = 1, 2, \ldots, M_p$ sets of units of the same size operating out of phase. Another design decision considered in this model is the selection of equipment sizes for batch units in each operation p, which are restricted to take values from a set $SV_p = \{\nu_{p1}, \nu_{p2}, \ldots, \nu_{pk_p}\}$ of available discrete sizes.

For each product i, lower and upper bounds on its demands in every period t, DE^L_{it} and DE^U_{it}, are known.

The plant operates in single product campaign (SPC) mode under a zero wait (ZW) policy in every time period.

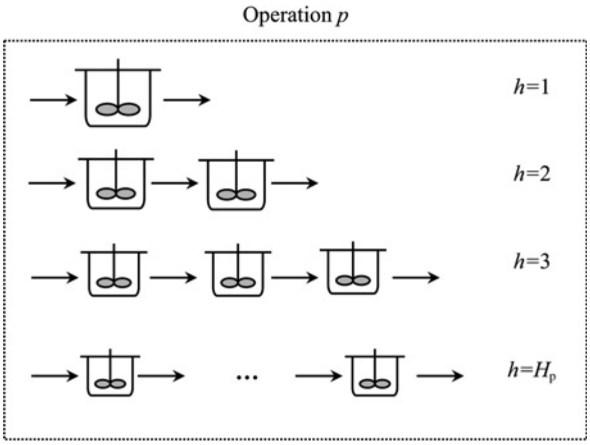

Figure 1. Configurations in series *h* for operation *p*.

With the single product campaign approach, all the batches of a product are processed without overlapping with other products. The ZW policy for scheduling consists of transferring a batch to the next unit as soon as the processing is completed in the current unit. Figure 2 shows the design options for each operation considered in this model.

The basic data for representing the operations are the size factors S_{ipt} and processing time t_{ipt} required for each product *i* at each operation *p* in every period *t*. The constant time and size factor model is the most widespread used in literature to design multiproduct batch processes. The size factor S_{ipt} is obtained from the production recipe and corresponds to the volume needed in a piece of equipment of the operation *p* to produce 1 kg of final product *i*. The subscript *t* takes into account fluctuations due to the period. When adding units in series to perform a given operation *p*, the yield for the operation is maintained at the end of the series. As a consequence of this, the size factor for product *i* at operation *p* is the same in all the configurations, and also, the size factors of downstream operations in the plant are not affected by the duplication in series. A useful application of this approach is the multistage countercurrent extraction operation. Staged extraction, i.e., series of identical units, is a common engineering practice used to overcome disadvantages of single stage extraction such us long extraction time, high solvent consumption and low extraction efficiency. In order to include the option of duplication in series in the formulation, maintaining the constant size factor model of the literature, the yield at the end of all alternative configurations has to be the same. Thus, the downstream conditions are the same without being affected by the configuration in series selected. However, each configuration *h* of units in series now has a different extraction time t_{iph}. Obviously, the cost of each alternative strongly depends on the number of included units.

The optimization of production recipes, which include the ratio of solvent to solids and the yield in the extraction, is not approached in this paper. The ratio of solvents to solids, processing times and yields are adopted from recipes that assure a single extraction stage. Then, keeping the same ratio of solvent to solids and final yield of the operation, the cycle times required by a train of 2, 3... and so forth, extractors arranged countercurrent were computed through an extraction model.

In the present formulation duplication in series implies adopting extractors with the same size factor of the original one, but with a shorter operating cycle time. This economic trade off affects not only the extraction, but also—through the modified stage cycle time—the stages up and down stream of the extraction.

In this model two cases are considered. In the first case, the elaboration of product *i* depends on a unique raw material that is identified with the same subscript *i* of product. In the second general case, production of product *i* requires a set of raw materials *c*.

On the other hand, production planning decisions allow us to determine at each period *t* and for each product *i*, the amount to be produced q_{it}, the number of batches n_{it}, and the total time T_{it} to produce product *i*. Furthermore, at the end of every period *t*, the levels of both final product IP_{it} and raw material inventories IM_{it} are obtained. Moreover, the total sales QS_{it}, the amount of raw material purchased C_{it}, and the raw material to be used for the production RM_{it} of product *i* in each period *t* are determined with this formulation.

The model then consists in selecting simultaneously the design of the plant and the operation planning involving the maximization of the net present value along the global time horizon, taking into account incomes from product sales, expenditures from raw materials purchases, inventory and investments. If time periods are equal, waste disposal costs are also added to the objective function. Bounds on products demands, costs and availability of raw materials vary from period to period.

MODEL FORMULATION USING GENERALIZED DISJUNCTIVE PROGRAMMING

In this section, the LGDP formulation for the optimal design and planning operation of multiproduct batch plants in a multiperiod scenario is presented. The selection of discrete alternatives of the design problem is modelled through the following embedded disjunctions:

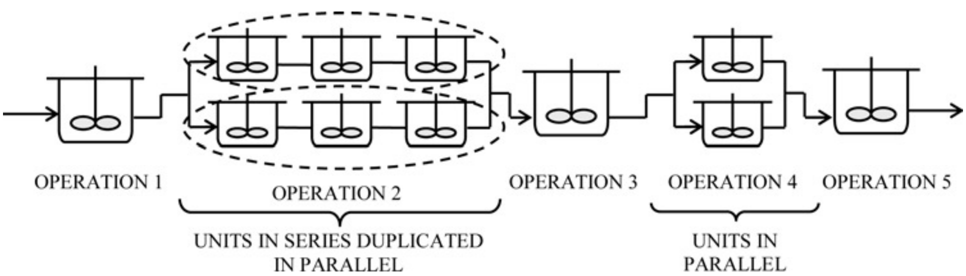

Figure 2. Design options for the batch plant.

$$\bigvee_{h \in H_p} \left[\begin{array}{c} Z_{ph} \\ \bigvee_{s \in SV_p} \left[\begin{array}{c} W_{phs} \\ n_{it} \geq \left(\dfrac{S_{ipt}}{\nu_{ps}} \right) q_{it} \quad \forall\, i, t \\ CO_p = h\,(\gamma_p + \alpha_p \nu_{ps}^{\beta_p}) \end{array} \right] \\ \bigvee_{m \in M_p} \left[\begin{array}{c} Y_{phm} \\ T_{it} \geq \dfrac{t_{iph}}{m} n_{it} \quad \forall\, i, t \\ CB_p = CO_p m \end{array} \right] \end{array} \right] \quad \forall\, p \tag{2}$$

Disjunctions have been defined for each operation p included in the process. For each operation p there are different configurations h of units in series to perform it. Let H_p be the set of configurations h that can be used to perform operation p. A Boolean variable Z_{ph} is true when configuration h is chosen for operation p and is false in the opposite case. In the optimal solution only one alternative will be selected and only the constraints included in the corresponding term must be satisfied.

Once the configuration in series is selected, the discrete sizes of units utilized must be chosen. Thus, an embedded disjunction is included where variable W_{phs} is true when configuration h is used for operation p and discrete size s is employed to carry it out. Let SV_p be the set of available discrete sizes s for carrying out operation p. In this case, appropriate constraints are posed in each term to take into account the cost of this configuration, CO_p, and assure a production level q_{it} of product i in period t so as to satisfy product demands constraints. In the same way, only one Boolean variable W_{phs} must be true and the corresponding production and cost constraints must be fulfilled.

Finally, another set of embedded disjunctions is added, where variable Y_{phm} is true when m parallel units operating out-of-phase are used in operation p with configuration in series h. Let M_p be the maximum number of units that can be duplicated in each operation p. Each term includes constraints about the investment cost of operation p, CB_p, and the total time dedicated to production of product i in period t.

Embedded disjunctions in equation (2) shows that the selection of a discrete size s and the number of units m operating out of phase are only necessary for the configuration h selected for operation p. Now, the constraints included in each term will be presented.

Constraints in the first embedded disjunction of equation (2) are the design equation of the units and the cost of the alternative chosen in each operation. These equations are posed taking into account the original formulation for the design problem and taking advantage of the assumption about the discrete available sizes for the batch units, as it is described in the following paragraphs.

According to general batch literature (Biegler *et al.*, 1997) the sizing equation for each operation p that is applied for each product i over all time periods, is given by

$$V_p \geq S_{ipt} B_{it} \quad \forall\, i, p, t \tag{3}$$

V_p is a continuous variable corresponding to the size of the unit that perform operation p and B_{it} is the batch size of product i in period t. S_{ipt} is the size factor corresponding to product i in operation p for each period t. The amount of product i produced in time period t, q_{it} that depends on the number of batches n_{it}, is defined by

$$q_{it} = B_{it} n_{it} \quad \forall\, i, t \tag{4}$$

By combining equations (3) and (4) the following constraints are obtained:

$$n_{it} \geq \frac{S_{ipt} q_{it}}{V_p} \quad \forall\, i, t \tag{5}$$

As already mentioned, variables V_p are considered available in a set SV_p of discrete sizes ν_{ps}, which correspond to the usual commercial procurement of equipments. Taking advantage of the LGDP, each alternative can be formulated separately. Therefore, equation (5) can be posed for each available alternative. Then, the following linear expression is obtained:

$$n_{it} \geq \left(\frac{S_{ipt}}{\nu_{ps}} \right) q_{it} \tag{6}$$

where the size of the unit ν_{ps}, is a constant value. If the variable W_{phs} is true in the solution, the discrete size ν_{ps} is selected in operation p for the configuration in series h. Therefore, taking into account that in each operation only one option W_{phs} will be true, then only one (6) must be satisfied. The remaining expression where W_{phs} is false will not be considered. Next section introduces the reformulation used to transform the disjunctive expressions.

The following constraints in the first disjunction of equation (2) represent the equipment cost of this alternative, CO_p, using a power law expression on the capacity plus a fixed cost γ_p independent of the unit size in operation p. Parameters α_p and β_p are cost factors used to evaluate the cost of unit volume ν_{ps}.

Constraints in the second embedded disjunction of equation (2) consider an additional level of decision: the addition of parallel units in each operation p for the configuration in series selected previously. The cycle time for product i, TL_i, is defined as the time elapses between two consecutive batches of product i. It is given by the longest processing time among all the operations p in the plant for product i. In order to reduce the cycle time for a particular product, units duplicated out-of-phase can be introduced. If the Boolean variable Y_{phm} is true m identical set of units in parallel, as the configuration in series assigned in the above disjunction, are selected. Then, the cycle time of product i in period t is determined by

$$TL_{it} \geq \frac{t_{ipht}}{m} \tag{7}$$

Parameter t_{ipht} is the processing time for product i in each operation p using the configuration h of units in series at period t. The total time for producing product i in period t is defined as

$$T_{it} = n_{it} TL_{it} \quad \forall\, i, t \tag{8}$$

By multiplying equation (7) by the number of batches, the expression takes the form used in the second disjunction of equation (2) given by

$$T_{it} \geq \left(\frac{t_{iph}}{m}\right) n_{it} \quad (9)$$

In this expression, m is considered as a constant value. Then a term is included in the disjunction (2) for each option m. When the boolean variable Y_{phm} is true, then the corresponding discrete value for m is used.

Last constraints in second disjunction of equation (2) correspond to the total investment cost CB_p, which accounts for the cost for the configuration in series, CO_p, and the number of parallel units m selected in each operation p.

Whereas the above equations and inequalities constraints are contained in disjunctions of equation (2), there are further inequality constraints (10) that hold irrespective of discrete alternatives. Considering the SPC-ZW policy in period t, all productions must be completed within the corresponding production horizon H_t:

$$\sum_i T_{it} \leq H_t \quad \forall\, t \quad (10)$$

Planning and Inventory Constraints

The following constraints manage inventories and force total production to satisfy product demands, over all the time periods t. Figure 3 shows a scheme of the materials flow considered in the planning of this process.

The stock of product i at the end of period t, IP_{it}, will depend on the stock in the previous period, $IP_{i,t-1}$, the production during this period q_{it}, the amount sold QS_{it} and the wastes due to the expired product shelf life, PW_{it}, as follows:

$$IP_{it} = IP_{i,t-1} + q_{it} - QS_{it} - PW_{it} \quad \forall\, i, t \quad (11)$$

When the elaboration of product i require only one ingredient, equation (12) poses the inventory of raw material i at the end of period t, IM_{it}, that will depend on the stock in the previous period, $IM_{i,t-1}$, the purchases during period t, C_{it}, the consumption for production, RM_{it} and the wastes due to the limited raw material lifetime, RW_{it}.

$$IM_{it} = IM_{i,t-1} + C_{it} - RM_{it} - RW_{it} \quad \forall\, i, t \quad (12)$$

When the problem takes into account time periods of equal length, lifetime considerations of both raw materials and products can be added into the formulation (Lakhdar *et al.*, 2005). Let ζ_i and χ_i be the number of time periods during which they have to be used. Thus, in order to guarantee that the stock of both raw materials and products in each period cannot be used after the next ζ_i or χ_i time periods

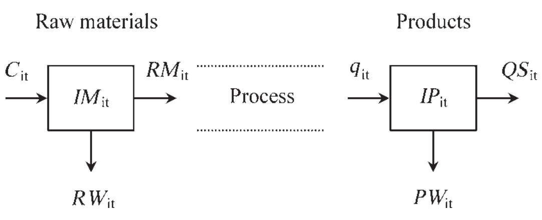

Figure 3. Planning variables.

respectively, the following constraints are imposed:

$$IP_{it} \leq \sum_{\tau=t+1}^{t+\chi_i} QS_{i\tau} \quad \forall\, i, t \quad (13)$$

$$IM_{it} \leq \sum_{\tau=t+1}^{t+\zeta_i} RM_{i\tau} \quad \forall\, i, t \quad (14)$$

Equation (13) ensures the lifetime of product by enforcing that it is sold in less than χ_i time periods from it was stored while equation (14) ensures that raw material is processed in less than ζ_i time periods from it was purchased. The above constraints have sense only when the time periods are equal in length, as well as the related term in the objective function and the last terms in equations (11) and (12).

Furthermore, stocks of both raw materials and final products stored during period t cannot exceed the respective maximum available storage capacities, IP^U_{it} and IM^U_{it}:

$$0 \leq IP_{it} \leq IP^U_{it} \quad \forall\, i, t \quad (15)$$

$$0 \leq IM_{it} \leq IM^U_{it} \quad \forall\, i, t \quad (16)$$

The initial inventories of both raw material and product IM_{i0}, IP_{i0}, at the beginning of the time horizon are assumed to be given. The use of IM_{i0} and IP_{i0} have a strong impact when this model is only used for operation planning without considering design, for example in an existing plant.

The raw material necessary for the production of the product i is obtained from a mass balance:

$$RM_{it} = F_{it} q_{it} \quad \forall\, i, t \quad (17)$$

Parameter F_{it} accounts for the process conversion, e.g., ratio of solvent to solids, time of contact, and so on.

On the other hand, when each product depend on several raw materials $c = 1, 2, \ldots, CT$, let F_{cit} be a parameter that accounts for the process conversion of raw material c to produce product i during period t. The amount consumed of raw material c in period t to elaborate product i, RM_{cit}, is obtained from a mass balance. Then,

$$RM_{cit} = F_{cit} q_{it} \quad \forall\, c, i, t \quad (18)$$

The total consumption of raw material c for production in period t, RM_{ct} is obtained from the following expression:

$$RM_{ct} = \sum_{i=1}^{I} RM_{cit} \quad \forall\, c, t \quad (19)$$

Then, equations (12), (14) and (16) must be rewritten using new variables which consider each ingredient c in every period, i.e., C_{ct}, IM_{ct}, RW_{ct} and RM_{ct}.

If a given batch of product i late meets a minimum product demand DE^L_{it}, then a late delivery ϑ_{it} takes place in that period (Lakhdar *et al.*, 2005). Late deliveries are undesirable; therefore they can be quantified by an appropriate penalty function which is minimized in the objective function. They are determined using the following expression:

$$\vartheta_{it} \geq \vartheta_{i,t-1} + DE^L_{it} - QS_{it} \quad \forall\, i, t \quad (20)$$

Objective Function

The objective function of the problem is the maximization of the present value of the benefit given by the difference between incomes due to total sales and total costs. Total costs include the raw materials purchased, units investment, inventory, operation, waste disposal, and late delivery penalties. For the case with only one ingredient for each product, the expression of the objective function can be written as

$$\begin{aligned} \text{Max } \psi = & \sum_t \sum_i np_{it} QS_{it} - \sum_t \sum_i \kappa_{it} C_{it} \\ & - \sum_t \sum_i \varepsilon_{it} \left(\frac{IM_{i,t-1} + IM_{it}}{2} \right) H_t \\ & - \sum_t \sum_i \sigma_{it} \left(\frac{IP_{i,t-1} + IP_{it}}{2} \right) H_t \\ & - \sum_t \sum_i (wp_{it} PW_{it} + wr_{it} RW) \\ & - \sum_t \sum_i (co_{it} q_{it} - cp_{it} \vartheta_{it}) - \sum_p CB_p \end{aligned} \quad (21)$$

Parameter np_{it} is the price of product i in period t and parameter κ_{it} is the cost of raw material i in period t, which takes into account market fluctuations for harvest, transportation, cooling facilities, and so on. To determine the cost of inventory for both raw materials and final products, Birewar and Grossmann (1990) proposed an average in period t, which is used here, where ε_{it} and σ_{it} are inventory cost coefficients. Waste disposal cost wp_{it} per product and wr_{it} per raw material are also considered. Operating cost includes energy consumptions in the process (steam, electricity, labour, and so on) which are proportional to the production through cost coefficients co_{it}. Late delivery penalties are included with a cost coefficient cp_{it}. Finally, the last term is the investment cost corresponding to batch units in the plant.

All the parameters in the above equations are based on given present values. Both incomes and outcomes terms of the summation that defines the objective function are discounted at specified interest rate i.

To sum up, the multiperiod model of multiproduct batch plant that is a linear generalized disjunctive problem (LGDP) is defined by maximizing the objective function (21) subject to constraints (2), (10)–(17) and (20) plus bounds constraints that may apply.

MODEL SOLUTION

For the LGDP problem solution, two methodologies are considered to transform disjunctions into a mixed integer linear program (MILP): big-M (BM) and convex hull (CH) reformulations (Vecchietti, 2000). These transformations are required taking into account that models must be formulated in a format compatible with the optimization program solvers. The performances of the two approaches are compared.

Big-M Formulation

In order to obtain the BM reformulation, the problem LGDP given in equation (1) is reformulated as an MIP problem by transforming the disjunctive constraints into big-M constraints and by replacing the Boolean variables Y_{jk} by binary variables $y_{jk} \in \{0,1\}$. The logic constraints $\Omega(Y)$ are converted into linear inequalities, which lead to the following reformulation (Sawaya and Grossmann, 2005):

$$\begin{aligned} & \text{Min } Z = \sum_{k \in K} \sum_{j \in J_k} \gamma_{jk} y_{jk} + d^T x \\ & \text{s.t. } Bx \le b \\ & A_{jk} x - a_{jk} \le BM_{jk}(1 - y_{jk}) \quad \forall j \in J_k, k \in K \\ & \sum_{j \in J_k} y_{jk} = 1 \quad \forall k \in K \\ & D\, y \le d \\ & x \in \Re^n, y_{jk} \in \{0, 1\} \quad \forall j \in J_k, k \in K \end{aligned} \quad \text{(BM)}$$

Here, BM_{jk} are the big-M scalars that render the jth system of inequalities in the kth disjunction redundant when $y_{jk} = 0$ (i.e., $Y_{jk} =$ False). The inequalities $Dy \le 0$ can be systematically derived from the logical propositions $\Omega(Y)$.

The tightest values for the BM_{jk} scalar can be determined by the following expression (Vecchietti *et al.*, 2003):

$$BM_{jk} = \max\,(A_{jk} x - a_{jk},\ x^L \le x \le x^U)$$

Note that, in order to determine the best value for BM_{jk}, bounds for the continuous variables must be provided.

Big-M Reformulation of the Model for the Vegetable Extraction Process

Hence, problem in equation (2) can be transformed into the following MILP problem with positive big-M constants, which are used to represent sufficient large bounds:

$$\sum_h z_{ph} = 1 \quad \forall p \quad (22)$$

$$n_{it} \ge \left(\frac{S_{ipt}}{\nu_{ps}} \right) q_{it} - BM1_{it}(1 - w_{phs}) \quad \forall i, p, h \in H_p, s \in SV_p, t \quad (23)$$

$$\sum_s w_{phs} = z_{ph} \quad \forall p, h \in H_p \quad (24)$$

$$\sum_m y_{phm} = z_{ph} \quad \forall p, h \in H_p \quad (25)$$

$$T_{it} \ge \frac{t_{ipht}}{m} n_{it} - BM2_{it}(1 - y_{phm}) \quad \forall i, p, h \in H_p, m \in M_p, t \quad (26)$$

$$CO_p \ge h\left(\gamma_p + \alpha_p \nu_{ps}^{\beta_p}\right) - BM3_p(1 - w_{phs}) \quad \forall p, h \in H_p, s \in SV_p \quad (27)$$

$$CO_p \le h\left(\gamma_p + \alpha_p \cdot \nu_{ps}^{\beta_p}\right) + BM3_p(1 - w_{phs}) \quad \forall p, h \in H_p, s \in SV_p \quad (28)$$

$$CB_p \ge m\, CO_p - BM4_p(1 - y_{phm}) \quad \forall p, h \in H_p, m \in M_p \quad (29)$$

$$CB_p \le m\, CO_p + BM4_p(1 - y_{phm}) \quad \forall p, h \in H_p, m \in M_p \quad (30)$$

A big-M constraint as equation (23) is satisfied if $w_{phs} = 1$. Otherwise, if $w_{phs} = 0$, the corresponding constraint becomes redundant, taking into account that $BM1_{it}$ is a scalar large enough. Similar interpretations can be made for big-M constraints (26)—(30). The tightest values for big-M constants for the above equations are calculated by means of the following expressions:

$$BM1_{it} = n_{it}^U \quad \forall\, i, t \tag{31}$$

$$BM2_{it} = T_{it}^U \quad \forall\, i, t \tag{32}$$

$$BM3_p = H_p\{\gamma_p + \alpha_p(\max \nu_{ps}^{\beta_p})\} \quad \forall\, p \tag{33}$$

$$BM4_p = M_p H_p\{\gamma_p + \alpha_p(\max \nu_{ps}^{\beta_p})\} \quad \forall\, p \tag{34}$$

Due to this being a maximization problem, equations (28) and (30) can be eliminated of the BM relaxation without lacking in rigour.

The big-M reformulation to the original problem consists of objective function (21) subject to constraints (22)–(27), (29), constraint (10) about the time horizon, plus planning constraints (11)–(17) and (20). Bounds on the involved variables must be also added.

Convex Hull Formulation

In order to obtain the CH reformulation, the LGDP problem is transformed into an MILP by replacing the Boolean variables Y_{jk} by binary variables y_{jk} and disaggregating the continuous variables $x \in \Re^n$ into new variables $\omega_{jk} \in \Re^n$. Using the convex hull constraints for each disjunction, the following reformulation is obtained:

$$\begin{aligned}
&\text{Min } Z = \sum_{k\in K}\sum_{j\in J_k} \gamma_{jk} y_{jk} + d^T x\\
&\text{s.t. } Bx \le b\\
&A_{jk}\,\omega_{jk} \le a_{jk}\, y_{jk} \quad \forall\, j \in J_k, k \in K\\
&x = \sum_{j\in J_k} \omega_{jk} \quad \forall\, k \in K\\
&\omega_{jk} \le y_{jk} U_{jk} \quad \forall\, j \in J_k, k \in K\\
&\sum_{j\in J_k} y_{jk} = 1 \quad \forall\, k \in K\\
&Dy \le d\\
&x, \omega \in \Re^n, y_{jk} \in \{0, 1\} \quad \forall\, j \in J_k, k \in K
\end{aligned} \tag{CH}$$

The new variables ω_{jk} are the disaggregated variables, while the parameters U_{jk} serve as their upper bounds.

It is important to note that, whereas the big-M relaxation adds one constraint to the original formulation, for the convex hull relaxation, the continuous variables x are disaggregated into new variables.

Convex Hull Reformulation of the Model for the Vegetable Extraction Process

Using the convex hull constraints for each disjunction in equation (2), the following reformulation is obtained:

$$\left(\frac{S_{ipt}}{\nu_{ps}}\right) qb_{iphst} \le nb_{iphst} \quad \forall\, i, p, h \in H_p, s \in SV_p, t \tag{35}$$

$$n_{it} = \sum_h \sum_s nb_{iphst} \quad \forall\, i, p, t \tag{36}$$

$$nb_{iphst} \le n_{it}^U w_{phs} \quad \forall\, i, p, h \in H_p, s \in SV_p, t \tag{37}$$

$$q_{it} = \sum_h \sum_s qb_{iphst} \quad \forall\, i, p, t \tag{38}$$

$$qb_{iphst} \le q_{it}^U w_{phs} \quad \forall\, i, p, h \in H_p, s \in SV_p, t \tag{39}$$

$$\frac{t_{iph}}{m} nc_{iphmt} \le Tc_{iphmt} \quad \forall\, i, p, h \in H_p, m \in M_p, t \tag{40}$$

$$T_{it} \ge \sum_h \sum_m Tc_{iphmt} \quad \forall\, i, p, t \tag{41}$$

$$Tc_{iphmt} \le T_{it}^U y_{phm} \quad \forall\, i, p, h \in H_p, m \in M_p, t \tag{42}$$

$$n_{it} = \sum_h \sum_m nc_{iphmt} \quad \forall\, i, p, t \tag{43}$$

$$nc_{iphmt} \le n_{it}^U y_{phm} \quad \forall\, i, p, h \in H_p, m \in M_p, t \tag{44}$$

$$h(\gamma_p + \alpha_p \nu_{ps}^{\beta_p}) w_{phs} \le COb_{phs} \quad \forall\, p, h \in H_p, s \in SV_p \tag{45}$$

$$CO_p = \sum_h \sum_s COb_{phs} \quad \forall\, p \tag{46}$$

$$COb_{phs} \le CB_p^U w_{phs} \quad \forall\, p, h \in H_p, s \in SV_p \tag{47}$$

$$m\, COc_{phm} \le CBc_{phm} \quad \forall\, p, h \in H_p, m \in M_p \tag{48}$$

$$CB_p = \sum_h \sum_m CBc_{phm} \quad \forall\, p \tag{49}$$

$$CBc_{phm} \le CB_p^U y_{phm} \quad \forall\, p, h \in H_p, m \in M_p \tag{50}$$

$$CO_p = \sum_h \sum_m COc_{phm} \quad \forall\, p \tag{51}$$

$$COc_{phm} \le CB_p^U y_{phm} \quad \forall\, p, h \in H_p, m \in M_p \tag{52}$$

New variables, for example nb_{iphst}, nc_{iphmt}, are generated disaggregating previous variables, while parameters n_{it}^U serve as their upper bounds.

The MILP problem obtained by applying the convex hull formulation to the original model is to maximize the objective function (21) subject to constraints (10)–(17), (20), (22), (24), (25) and (35)–(52).

EXAMPLES

Example 1

To illustrate the use of the model proposed and the MILP reformulations presented in previous sections, a multiproduct batch plant for the production of four oleoresins, sweet bay (A), pepper (B), rosemary (C) and thyme (D) oleoresins, is considered. This plant consists of the following operations: (1) extraction (2) expression, (3) evaporation and (4) blending. All of these operations can be duplicated up to three units operating out of phase. In order to perform the operation 1 (extraction), several configurations of units in series are available with a countercurrent arrangement. In this example the extraction can be performed up to seven units in series.

A global horizon time of two years (12 000 h working) has been considered, that has been divided into a set of equal time periods, namely from 1 to 12 (1000 h) corresponding to two months each because of the seasonal variations of the availability and prices of raw materials.

Demands, costs, and prices differ from period to period. Tables 1, 2 and 3 contain the data for this example. The processing time of operation 1 in Table 1, corresponds to using only one unit option. In order to obtain the conversion factor F_{it} necessary for equation (17), the mass balances for batch extraction have to be solved (Moreno *et al.*, 2007).

Minimum demands of oleoresins in each period are 50% of the upper demands in Table 3. The inventory coefficient costs per ton of both final products and raw materials for all the products are \$1.5 (ton h) and \$0.2 (ton h) respectively. Cost coefficients for late delivery it is assumed as a 50% of product prices. Products and raw materials lifetimes in time periods are 3 and 2, respectively.

Table 1. Process data for the example.

	Size factors, S_{ipt} (L kg^{-1})				Processing time, t_{ipt} (h)				Conversion factor
i	1	2	3	4	1	2	3	4	F_{it}
A	20	15	12	1.5	25.95	1	2.5	0.5	11.11
B	23	15	12	1.5	39.46	2.0	1.5	2.0	11.11
C	40	25	24	1.5	34.09	1.0	3.0	1.0	22.22
D	30	20	17	1.5	27.93	1.0	2.0	1.0	15.87

Table 2. Available standard sizes.

		Discrete volumes, ν_{ps} (L) operations			
Option		1	2	3	4
1		500	500	250	50
2		1000	700	500	100
3		1500	1000	750	150
4		2500	1500	1000	200
5		3000	2000	1500	250
Cost coef	α_p	875	1150	800	700
Fixed cost	γ_p		2050		
Cost exp	β_p		0.6		

Table 3. Prices and demand bounds of example 1.

	Costs of raw materials, κ_{it} (\$/kg^{-1})				Prices of products, np_{it} (\$ kg^{-1})				Bounds on demands, DE_{it}^{U} ($\times 10^2$ kg)			
t	A	B	C	D	A	B	C	D	A	B	C	D
1	1.5	2.5	1.2	1.8	36	40	37	37	28.0	27.0	37.0	35.0
2	1.5	2.5	1.2	0.6	36	40	35	37	30.0	28.0	39.0	36.0
3	2.2	1.2	1.2	0.6	38	38	35	37	32.0	30.0	41.0	38.0
4	2.2	1.2	1.2	0.6	38	38	35	37	34.0	32.0	45.0	40.0
5	1.7	2.7	2.5	2.0	36	40	37	37	39.0	37.0	48.0	45.0
6	1.7	2.7	2.5	0.8	36	40	35	37	42.0	39.0	49.0	48.0
7	2.4	1.4	2.5	0.8	38	38	35	37	44.0	41.0	53.0	50.0
8	2.4	1.4	2.5	0.8	38	38	35	37	47.0	43.0	55.0	52.0
9	1.9	2.9	2.5	2.2	36	40	37	37	50.0	44.0	56.0	53.0
10	1.9	2.9	2.5	1.0	36	40	35	37	51.0	45.0	57.0	54.0
11	2.6	1.6	1.2	1.0	38	38	35	37	52.0	46.0	58.0	55.0
12	2.6	1.6	1.2	1.0	38	38	35	37	52.0	46.0	58.0	55.0

As mentioned, processing times for each product *i* at the extraction operation take smaller values as the number of units in series grows. In Table 4 extraction processing times for each configuration in series *h*, t_{ipht}, are summarized.

The optimal solution yields a total profit of \$2 312 906.6 with a 0% gap of termination tolerance. CPLEX was used as a solver for this problem in the GAMS package on a Pentium(R) 4 CPU 3.00 GHz. The same results were obtained applying both BM and CH reformulations. Table 5 reports the optimal results of each unit's dimensions for the operations. It also indicates the number of out-of-phase duplicated units and the number of units operating in series. Operation 1 has six units in series selected and operation 3 has two units in parallel out-of-phase, which allow reducing idle times. The results show that both duplication options, out-of-phase and in series, have been included. Both alternatives were feasible in operation 1, but duplication in series was selected due to a best solution obtained. For example, with six parallel units working out-of-phase, the processing time in operation 1 is reduced to 5.68 h for product C whereas with six units in series is only 1.67 h. Thus, the value of this new alternative in the model is validated.

A detailed analysis of the economic results for the example is presented in Table 6.

Finally, Figures 4–7 illustrates the production, sales, purchases and inventory profiles of all products at the optimal solution. These figures are divided in two diagrams corresponding to raw materials and final products.

Table 4. Extraction times (h) for different configurations.

	Number of unit in series						
i	1	2	3	4	5	6	7
A	25.95	9.28	5.35	3.47	2.37	1.63	1.11
B	39.46	9.76	5.55	3.59	2.44	1.68	1.15
C	34.09	9.63	5.50	3.56	2.42	1.67	1.14
D	27.93	9.41	5.41	3.51	2.39	1.65	1.12

Table 5. Results of the example 1.

Operation	1	2	3	4
V_p	1000	700	750	100
Units in series	6	1	1	1
Units in parallel	1	1	2	1

Table 6. Economic evaluation results of example.

Description	Optimal value
Incomes for sales	5 904 975.5
Purchased raw materials	2 864 187.3
Investment cost for batch units	506 376.5
Raw material inventory costs	68 225.9
Product inventory costs	132 304.0
Operating costs	20 975.0
Waste disposal costs	0.0
Late delivery penalties	0.0
Total (\$)	2 312 906.6

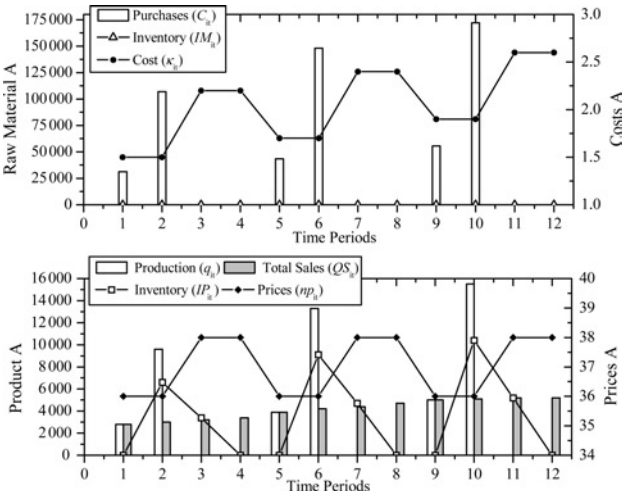

Figure 4. Profile for product A.

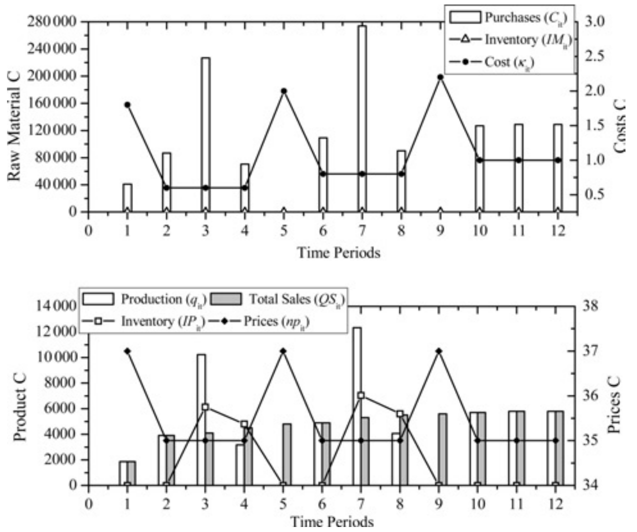

Figure 6. Profile for product C.

Results for product A are shown in Figure 4. It can be seen in the first diagram that raw material for A is purchased in periods 1, 2, 5, 6 and 9, 10 because its costs are cheaper then. Note in second diagram that product A is manufactured only in the periods above mentioned. The excess of production made in periods 2, 6 and 10 is carried forward as inventory for satisfying demands in subsequent periods.

Figure 5 illustrates product B profiles. The first diagram shows that raw material purchases are stopped during periods 5, 6, 9 and 10 where the costs increase. The extra amount purchased in periods 4 and 8 is held as inventory to allow the production in the following periods. In the second diagram, it clearly can be seen that production in time periods 5 and 9 is higher than demands. The extra amount is held as inventory to meet maximum demands in the following intervals.

Results for product C are shown in Figure 6. Note that in the first diagram, purchases are made in almost all periods except in periods 5 and 9 because of the higher prices. No inventory of raw material is kept for this product. Second diagram shows that production in periods 3 and 7 are higher than sales, so this allows building inventories of product C. The stocks are consumed to satisfy maximum demand in the following periods 4, 5, 8 and 9. Note that in periods 4 and 8, production is lower than demands while in periods 5 and 9 product C is not manufactured.

The first diagram of Figure 7 shows that purchase profile of product D reaches the maximum values in periods 4 and 8 where the costs of raw material have the lowest value. Then, when the prices suddenly raise, the purchases are stopped. The extra material is held as inventory to be used in the following period. In the first three periods, where costs have high values, purchases are performed in order to satisfy the demands. In second diagram can be seen that the amount in excess produced in periods 4, 5, 8 and 9 are kept as inventory to satisfy demands in following periods.

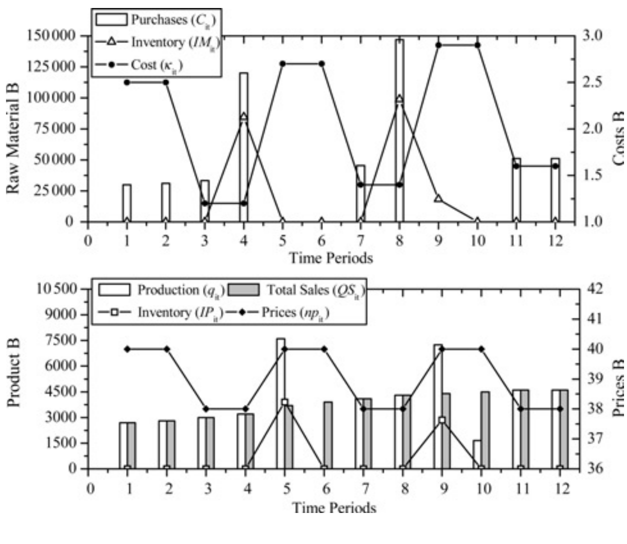

Figure 5. Profile for product B.

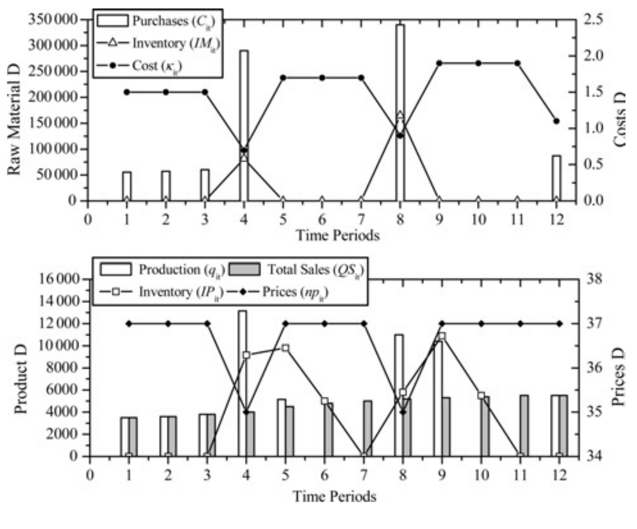

Figure 7. Profile for product D.

It should be noted that there are no late deliveries and wastage of raw material or final product in any of the time periods in this case.

Comparing reformulations

It is important to note that, whereas the big-M relaxation adds one constraint to the original formulation, for the convex hull relaxation the number of variables is increased significantly when compared to the disjunctive problem formulation equation (2), and new constraints are also added. Compared to the big-M relaxation, this can lead to a large number of variables and constraints, especially if there are many disjunctions, which becomes important for problems of large size. However, the relaxed feasible region of the CH reformulation is tighter than the BM reformulation (Vecchietti *et al.*, 2003).

Table 7 compares the two approaches. The CH reformulation requires a higher number of variables and constraints than the BM reformulation which leads to an increase in the solution time required for CH. Hence, the BM reformulation is the most effective in solving the problem presented in this work.

Example 2

Example 2 illustrates the advantages of the proposed multiperiod model to take into account demand fluctuations. In order to compare with the previous solution, two problems with fixed demand profiles are solved considering a constant demand for each product *i* for all the periods *t*. Two different cases are posed using data presented in Example 1. First, in problem (a) a low constant demand profile for each product is considered that correspond to the upper bounds of demands of the first time period in Example 1. Second, in problem (b) a high constant demand profile for each product corresponding to the upper demands of the twelfth time period is taken into account. In both cases, these values are fixed, no fluctuations on parameters are allowed, and the demands must be fulfilled (there is no range of acceptable demand values). In this way, two scenarios are posed: in the first case low demands have been forecasted and high demands in the second one. However, taking into account that these values are constant for all the periods poor results should be obtained when fluctuation are considered. Thus, the value of multiproduct approach is reinforced as is shown.

Due to the different demands considered in each problem the resulting configuration of the plant is different. Table 8 shows the optimal plant structure obtained in problems (a) and (b), where the first number in brackets corresponds to the number of units in series and the last one corresponds to the number of parallel units.

Table 7. Comparison of the results.

Results	Big-M	Convex hull
Optimal solution ($)	2 312 906.6	2 312 906.6
Resolution time (s)	16.87	846.43
Equations	4245	12 815
Variables	627	8417
Discrete variables	90	90

Table 8. Optimal sizes for both problems in example 2.

	Operation			
Problem	1	2	3	4
a	1000 (5)-(*1*)	700 (1)-(*1*)	750 (1)-(*1*)	100 (1)-(*1*)
b	1500 (5)-(*1*)	1000 (1)-(*1*)	1000 (1)-(*2*)	100 (1)-(*1*)

Table 9. Economic evaluation results for both problems in example 2.

	Optimal values	
Description	Problem (a)	Problem (b)
Incomes for sales	4 897 070.3	5 904 975.5
Purchased raw materials	2 359 629.1	2 863 450.6
Investment cost for batch units	404 593.4	555 130.6
Raw material inventory costs	95 619.3	65 271.3
Product inventory costs	89 114.9	132 412.4
Operating costs	17 175.1	20 975.0
Waste disposal costs	0.0	0.0
Late delivery penalties	0.0	0.0
Total ($)	1 930 938.3	2 267 735.5

These results will be used to show the mistakes made when these values are used in a multiperiod model. If a multiperiod formulation was not available, previous works used to design the plant using the results attained with the one-period model, and thus a suboptimal solution was determined. In order to show the differences between both approaches, the original problem with 12 time periods is solved again with the unit sizes fixed. Two cases are considered, where units are given taking into account both solutions in Table 8. Then the corresponding optimal production planning of the plant is obtained.

Using volumes given in problem (a), the objective function is $1 930 938.3, whereas the objective function with values of problem (b) is $2 267 735.5. Both solutions are lower than the optimal solution originally attained: $2 312 906.6. Table 9 presents the economic results of both problems in detail.

Analysing the results in detail, the poorest performance corresponds to problem (a). The plant has been designed without forecasts about following periods, and only the information of period 1 has been taken into account. Then a small plant has been obtained that cannot fulfill the growing demands of the next periods. Results of problem (b) show a reduced profit when compared to Example 1. If results of Table 9 are compared with Table 6, the same incomes for sales are obtained. However, the optimal benefit is not obtained, because investment costs by equipment units are increased.

In conclusion, this example emphasizes the value of simultaneously considering design and planning decisions over a multiperiod context.

CONCLUSIONS

This paper has addressed a multiperiod model integrating the optimization of design and production planning problems of multiproduct batch plants. A new major option of adding units in series to perform an operation was considered in the design problem. The problem was formulated as a LGDP model, where Boolean variables related to the selection of configurations of units in series, discrete sizes of equipments, and duplication in parallel out-of-phase for each operation were defined.

This model explicitly accounts for the effect of seasonal or market variations of products demands and raw materials availability. Both raw materials and products inventory costs are readily accounted for. Furthermore, the model shows the interaction between design decisions and commercial, production, sales and inventory policies simultaneously. Once the model was in disjunctive form, it was posed as an equivalent MILP problem allowing its solution with the available MILP solvers. Thus, the LGDP was reformulated into MILP problem by means of the CH or BM relaxation.

The solved examples have shown the trade-off among design and planning decisions taking into account the multi-period context. Solving problems considering only one approach or not assessing market or seasonal fluctuations achieve suboptimal and incomplete solutions.

The model proposed was applied to a case of concurrent production planning and design for vegetable extraction and is intended as a guide for the construction of similar models in other industries and for other operations like fermentation and homogenization through the application of the new option of adding units in series.

NOMENCLATURE

Subscripts

h	units in series
i	product
m	units in parallel
p	operation
s	discrete sizes for the units
t	time period
τ	time period

Superscripts

L	lower bound
U	upper bound

Parameters

co_{it}	operating cost coefficient of product i at period t
DE_{it}	demand for product i in period t
F_{cit}	parameter that accounts conversion of raw material c to produce i at period t
H	time horizon
H_t	net available production time for all products in period t
k_p	number of discrete sizes available for operation p
np_{it}	price of product i in period t
S_{ip}	size factor of product i in operation p for each period t
t_{iph}	processing time of product i in operation p with h units in series in period t
wp_{pt}	waste disposal cost coefficient per product i in period t
wr_{pt}	waste disposal cost coefficient per raw material i in period t
α_p	cost coefficient for a batch unit in operation p
β_p	cost exponent for a batch unit in operation p
γ_p	fixed cost associated with each unit in operation p
ε_{it}	Inventory cost coefficient for raw material i in period t
κ_{it}	price for the raw material of product i in period t
ν_{ps}	standard volume of size s for batch unit in operation p
σ_{it}	inventory cost coefficient for product i in period t
ζ_i	time periods during which raw materials have to be used
χ_i	time periods during which products have to be used

Binary variables

z_{ph}	it is 1 if configuration h is selected in operation p
w_{phs}	it is 1 if the unit in operation p with configuration h has size s
y_{phm}	it is 1 in operation p with configuration h has m units in parallel out-of-phase

Continuous variables

B_{it}	batch size of product i in period t
C_{it}	amount of raw material for producing i purchased in period t
IM_{it}	inventory of raw material i at the end of a period t
IP_{it}	inventory of final product i at the end of a period t
n_{it}	number of batches of product i in period t
PW_{it}	product i wasted at period t due to the limited product lifetime
q_{it}	amount of product i to be produced in period t
QS_{it}	amount of product i sold at the end of period t
RM_{it}	raw material inventory for product i in period t
RW_{it}	raw material i wasted at period t due to the limited product lifetime
T_{it}	total time for producing product i in period t
TL_{it}	limiting cycle time of product i in period t
V_p	size of a batch unit in operation p
ϑ_{it}	late delivery for product i in period t

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