UDC 539.3

BREAKING OF CHALK FALLEN ONTO THE FLOOR

V. Batyanova[1], O. Golubov[1], Y. Kalisetska[2], R. Yakubovich[1], K. Minakova[3], A. Tkachuk[4]

[1] *V. N. Karazin Kharkiv National University*

[2] *National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute», Institute of Physics and Technology*

[3] *National Technical University «Kharkiv Polytechnic Institute»*

[4] *Karlstad University*

Abstract

This problem was presented at the IPT 2022. The main goal was to find out What is the maximum height a piece of chalk might be dropped without breaking for a given surface, estimate parameters on which the height depends and suggest throwing techniques which minimize the breakage probability

Keywords: maximum hight, deformation

1. Introduction

In the article we investigate how the cylindrical piece of chalk breaks when it falls onto a flat rigid surface from a certain height. In particular, we aim to find the maximal height from which a piece of chalk survives after being dropped on the surface, i.e. no separation of the initial piece of chalk in several large pieces while small spallation, dents and crumble zones are allowed damages.

The fracture of chalk is determined by its mechanical properties. Chalk is a porous material which has a very complex microstructure on the size scale of a few microns, with individual crystals being linked together on small surface areas [1]. It explains why the strength of chalk is much less than the strength of solid crystals. Mechanical properties vary widely between over different samples. The typical compressive strength of chalk $\approx$ 4-40 MPa, tensile strength $\sim 0.4 - 4$ MPa, Young's modulus $\sim 0.4 - 20$ GPa [2].

Note the wide range of mechanical properties which are explained by different microstructures of studied samples. Also note that the compressive strength is roughly by an order of magnitude higher than the tensile strength.

In our article we investigate the maximal height from which the chalk can be dropped both theoretically and experimentally for three different cases of a falling chalk: the vertical fall, the nearly horizontal fall and fall with an arbitrary angle. Also, we perform a parametric study in which we investigate the height from which the chalk survives as a function of temperature, humidity, number of falls and other properties. In conclusions, we summarize the throwing techniques which allow to maximize the survival height.

2. Methods

2.1. Theoretical estimate for vertical fall

Let us conduct the rough estimate of the critical height for a vertical fall of chalk. The energy conservation for the chalk implies that

$$U_{\text{grav}} = SL\rho gh = E_{\text{kin}}, \tag{1}$$

if we neglect the air resistance. Here S is the surface of the cross-section of the chalk, L – its length, ρ – the density of the chalk, g – the free fall acceleration, h – the height from which the chalk is dropped. After the collision, the kinetic energy is transformed into the mechanical energy of the chalk deformation. Assuming an ideal homogeneous elastic deformation as the first estimate, we get that

$$U_{\text{elast}} \approx \frac{\sigma^2}{2E} SL, \tag{2}$$

where σ is the maximal stress achieved in the deformation and E is the Young's modulus. Equating U_{elast} to U_{grav} and assuming that compressive strength σ achieves the maximum compressive strength of chalk σ_{compr}, we get the following expression for the critical height:

$$h = \frac{\sigma^2}{2\rho g E}. \tag{3}$$

Substituting for the estimate $\sigma = 1.3\,\text{MPa}$, $\rho = 1500\,\frac{\text{kg}}{\text{m}^3}$, $E = 460\,\text{MPa}$, we get $h \approx 13$ cm. This critical height does not depend on the chalk length, which contradicts our everyday experience saying that longer chalk breaks easier.

This apparent inconsistency can be remedied if we assume that the chalk crumbles at its point of contact with the surface either due to the roughness of the chalk or the surface or due to fall of the chalk on its corner due to a tilt angle.

2.2. Amortization of fall by chalk crumbling

A part of mechanical energy can be dissipated in crumbling of material. Let a be the length at which the chalk crumbles. The work of dissipating forces during the crumbling can be estimated as

$$A_{\text{crumbl}} = S\sigma a, \tag{4}$$

where S is again the surface area of the chalk cross-section. Adding A_{crumbl} to the energy conservation law (1) we get

$$U_{\text{grav}} = U_{\text{elast}} + A_{\text{crumbl}}. \tag{5}$$

Substituting the appropriate values, we get that

$$SL\rho gh = \frac{\sigma^2}{2E}SL + S\sigma a, \tag{6}$$

and, as a result, we get a modified expression for the critical height

$$h = \frac{\sigma^2}{2\rho gE} + \frac{\sigma a}{\rho gL}. \tag{7}$$

As we can see, the first term in this expression is independent of the chalk length, while the second term in inversely proportional to the chalk length.

2.3. Nearly horizontal fall kinematics

We do not consider the case of an ideally horizontal fall, which is highly improbable. It is virtually impossible to attain an ideally simultaneous contact of the entire length of the chalk with the surface. Rather, one corner of the chalk hits the surface, rebounds from the surface and the other corner hits the surface [3].

The velocity change of the chalk's center of mass after the first contact with the surface is

$$\Delta v = \frac{P}{m}, \tag{8}$$

where P is the impulse of the normal reaction force.

$$P = \int N\,\mathrm{d}t. \tag{9}$$

From the equation of rotational motion of the chalk, it follows that its change of the angular velocity is equal to

$$\Delta\omega = \frac{LP}{2I}, \tag{10}$$

where $\frac{L}{2}$ is the lever arm of the normal force acting on the chalk from the surface and I is the moment of inertia of the chalk, assuming that chalk length is much larger than its radius

$$I = \frac{mL^2}{12}. \tag{11}$$

It means that after the first contact of the chalk with the surface, its translation velocity v_{trans} will be

$$v_{\text{transl}} = v_0 - \Delta v = v_0 - \frac{P}{m}, \tag{12}$$

where v_0 was its initial velocity. The rotation velocity v_{rot} at the two ends of the chalk will be

$$v_{\text{rot}} = \frac{L}{2}\Delta\omega = \frac{3P}{m}. \tag{13}$$

Thus, after the left end of the chalk rebounds, its velocity v_{left} will be

$$v_{\text{left}} = v_{\text{rot}} - v_{\text{transl}} = \frac{4P}{m} - v_0. \tag{14}$$

And the velocity of the right end of the chalk v_{right} will be

$$v_{\text{right}} = v_{\text{rot}} + v_{\text{transl}}. \tag{15}$$

We assume that v_{left} equals to ev_0, where e is the restitution coefficient. From this condition we find that

$$\frac{P}{m} = \frac{(1+e)v_0}{4}. \tag{16}$$

Substituting (16) to (15) we get that

$$\frac{(3+e)v_0}{2} > v_0. \tag{17}$$

In typical cases, the restitution coefficient is between 0 and 1. Certainly, it is bigger than -1, otherwise P would be negative, implying a negative normal reaction force. It means that v_{right} is greater than v_0. Implying that the second contact of the chalk with the surface occurs at higher velocity, it is more destructive for the chalk than the first contact.

2.4. Nearly horizontal fall dynamics

Still, let us ignore this detail and conduct an order of magnitude estimate for the horizontal fall of the chalk similar to the one we conducted for its vertical fall. Again, we write the energy conservation as (4), where U_{elast} of a horizontally fallen chalk is computed as the U_{elast} of bent bar:

$$U_{\text{elast}} = \frac{JL\varphi^2}{2E}, \tag{18}$$

where φ is the bending angle and J is the geometric moment of inertia of the chalk cross-section.

$$J = \frac{\pi R^4}{4}. \tag{19}$$

The critical bending angle is determined by the tensile strength of the chalk, where $\sigma_{\text{tens}} < \sigma_{\text{compr}}$:

$$\varphi_{\text{crit}} = \frac{\sigma_{\text{tens}}}{E}. \tag{20}$$

Substituting (1), (4) and (18) to (5) we get

$$SL\rho gh = \frac{JL\varphi^2}{2E} + S\sigma_{\text{compr}}a. \quad (21)$$

Solving it for h, we find that

$$h = \frac{\sigma_{\text{tens}}^2}{8\rho gE} + \frac{\sigma_{\text{compr}}a}{\rho gL}. \quad (22)$$

As we can see, h again consists of a constant term and term which is inversely proportional to L.

2.5. Theoretical estimate for an arbitrary angle

If the chalk falls at an arbitrary angle from the vertical line α, we can estimate that its potential energy ρVgh will be distributed between the compression of chalk and the bending of the chalk roughly as

$$U_{\text{press}} = \rho Vgh\cos^2\alpha = \frac{V}{E}\frac{\sigma_{\text{press}}^2}{2}, \quad (23)$$

$$U_{\text{bend}} = \rho Vgh\sin^2\alpha = \frac{V}{E}\frac{\sigma_{\text{bend}}^2}{8}. \quad (24)$$

Such approximate expressions with coefficients $cos^2\alpha$ and $sin^2\alpha$ are chosen because they satisfy the periodicity condition: at the vertical fall all energy goes into compression, and at the horizontal fall the energy goes into bending.

Using the expressions (23) and (24) we get the compressive tension in the chalk and the maximal bending stress reached at the end of the chalk:

$$\sigma_{\text{press}} = \cos\alpha\sqrt{2\rho ghE}, \quad (25)$$

$$\sigma_{\text{bend}} = 2\sin\alpha\sqrt{2\rho ghE}. \quad (26)$$

At the ends of the chalk the limiting values of the tension will be reached: $(-\sigma_{\text{press}}+\sigma_{\text{bend}})$ and $(-\sigma_{\text{press}}-\sigma_{\text{bend}})$, see (Fig. 1).

When both these values lie between σ_{tens} and $-\sigma_{\text{compr}}$, the chalk survives. If either of them exceeds the limit, the chalk brakes. This gives us such two conditions for the chalk survival:

$$\sqrt{2\rho ghE} < \frac{\sigma_{\text{compr}}}{2\sin\alpha - \cos\alpha}, \quad (27)$$

$$\sqrt{2\rho ghE} < \frac{\sigma_{\text{tens}}}{2\sin\alpha + \cos\alpha}. \quad (28)$$

In (Fig. 2) we plot the limits set by these two conditions. We see, that the vertical drop is best for the chalk survival.

3. Air friction

The Reynold's number of the chalk can be estimated as

$$\text{Re} = \frac{\rho_{\text{air}}vR}{\eta} \sim \frac{1\cdot 10^{0\ldots1}\cdot 10^{-2}}{10^{-5}} = 10^{3\ldots4}. \quad (29)$$

As the Reynold's number is sufficiently high, the air friction can be computed as the drag force

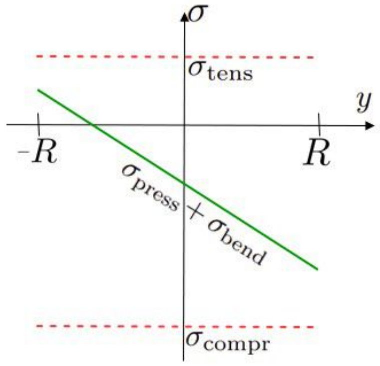

Fig. 1. Dependence of the mechanical strength of chalk σ on coordinate of chalk ends y. The red lines which are given by equations (27) and (28) show the maximal compressive and maximal tensile strenght which determine the conditions of chalk survival.

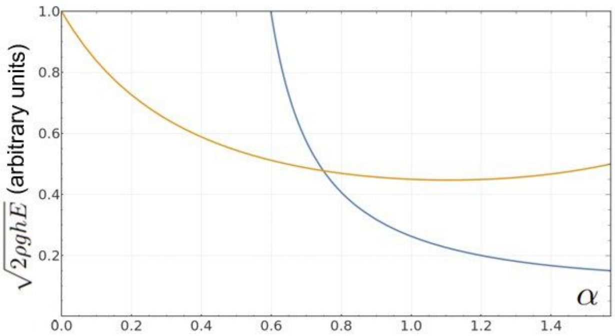

Fig. 2. Dependence of the critical mechanical strength of chalk σ on angle α at which a piece of chalk is inclined to the vertical axis.The yellow line is given by (27) and the blue line is given by (28), and these lines determine conditions for the chalk survival. As we can see, the vertical fall is best for chalk survival.

$$F_{\text{drag}} = \frac{C}{2}\rho_{\text{air}}v^2S, \quad (30)$$

where v is the speed of the chalk, S is the cross-section of the chalk, ρ_{air} is the density of air and C is the shape coefficient.

To find the limiting value of the chalk velocity, we equate drag force (29) to the gravity force

$$F_{\text{grav}} = \pi R^2 L\rho_{\text{chalk}}g. \quad (31)$$

For the vertical fall of long cylindrical chalk, we have $C_{\text{vert}} = 0.82$ and $S = \pi R^2$. Substituting it into the condition of $F_{\text{drag}} = F_{\text{grav}}$ we get

$$\frac{C_{\text{vert}}}{2}\rho_{\text{air}}v^2\cdot\pi R^2 = \pi R^2 L\rho_{\text{chalk}}g, \quad (32)$$

and expression for the chalk velocity

$$v_{\text{vert}} = \sqrt{\frac{2L\rho_{\text{chalk}}g}{C_{\text{vert}}\rho_{\text{air}}}} \approx 47\frac{\text{m}}{\text{s}}. \quad (33)$$

This result was obtained using 8 cm long chalk. For the horizontal fall of a long cylindrical chalk, we have $C_{\text{hor}} = 1.17$ and $S = 2RL$. And again we have

$$\frac{C_{\text{hor}}}{2}\rho_{\text{air}}v^2 \cdot 2RL = \pi R^2 L \rho_{\text{chalk}} g. \tag{34}$$

Solving (34) for v and substituting appropriate values, we get:

$$v_{\text{hor}} = \sqrt{\frac{\pi R \rho_{\text{chalk}} g}{C_{\text{hor}}\rho_{\text{air}}}} = 12\frac{\text{m}}{\text{s}}. \tag{35}$$

In most of our experiments, the velocities are several times less than the limiting velocities, which allows us to neglect the air friction in our theory. Still, we must acknowledge that if the chalk is sufficiently short or the surface on which it falls is sufficiently soft, the breaking velocity can be smaller than the terminal velocity of the chalk falling in the air. Thus, the chalk can survive a fall from any height. We checked it experimentally by dropping short chalks from large heights.

4. Results

We can check obtained dependencies experimentally. We take chalks of different lengths, raise them to a height h and release them, controlling the initial angle of the chalk orientation. We repeat the experiment many times for the same height and if the certain percentage of chalks does not break, we increase the height by 10 cm and repeat the experiment.

As the result of such experiment, we get the following plot (Fig. 3). The green dots mark the heights and lengths at which the majority of chalks did not break, and the red dots mark heights and lengths at which the majority of chalks broke. The theoretical line is in a good agreement with the boundary between the red and green dots. The line was given by equation (7), using crumbling length a as a fitting parameter.

We conduct the same experiments for the horizontal fall of the chalk, and find out that the data are in a decent agreement with the hyperbola predicted by (22) if a is used as a fitting parameter (Fig. 4). For very long chalks, the experimental critical heights seem to be lower than the theoretical prediction. Overall, the critical height for the horizontal fall of the chalk is found to be lower than the one for vertical fall.

5. Parametric study

The proposed theory is valid only if the surface has much higher critical stress and Young's modulus than the chalk. If the surface is softer than the chalk, it deforms and softens the fall. If the surface is more fragile than the chalk, it breaks and deformation softens the fall. The resulting elastic or plastic deformation of the surface absorbs a part of the chalk energy and allows the chalk to brake less compared to a strong surface.

It is shown experimentally, that both tensile and compressive strength of chalk grow at temperatures less than zero, see (Fig. 6). This is explained by the ice cementing monocrystals in the chalk. It can have an

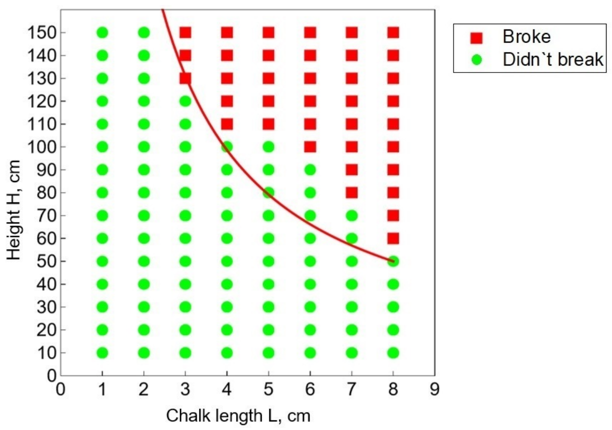

Fig. 3. Dependence of the chance of the survival of a piece of chalk on the height from which it was dropped H and chalk length L for the vertical fall case. The theoretical line is given by (7).

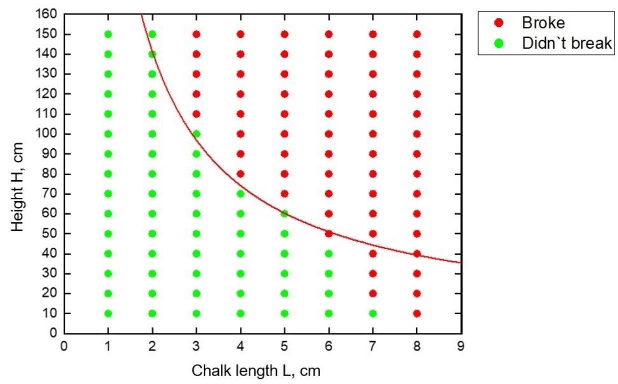

Fig. 4. Dependence of the chance of the survival of a piece of chalk on the height from which it was dropped H and chalk length L for the horizontal fall case. The theoretical line is given by (22).

implication that cold chalks can survive the fall better. To test this, we conducted an experiment in which we froze chalks to sub-zero temperatures in the freezer. Indeed, the critical height increased (Fig. 9)

Survival of the chalk also depends on the prehistory of its fall. When the chalk falls multiple times, micro-cracks accumulate lessening the chalk's strength. To investigate this phenomenon, we dropped chalks from subcritical heights multiple times and counted the number of falls it takes for a chalk to brake (Fig. 7). We see that at small heights it takes a very high number of falls for a chalk to brake. But, eventually, the chalk breaks even if it is dropped from a very small height.

There is no sharp boundary between the heights at which chalks brake and at which they do not. There is a slow probability change as a function of the chalk length, the dropping height and the other parameters. As an example, in (Fig. 8) we show the statistics of the probability of chalk breakage for different values of parameters. Each point in this plot was obtained by dropping 100 chalks from the same height and counting the percentage of the chalks which broke. We see that the probability of braking increases as a function of a chalk length and is the highest for horizontal fall and lowest for vertical fall. Still, it does not abruptly go from almost zero to almost one, so increase is very smooth.

One more idea to maximize the chalk survival is sharpening of the lower end of the chalk. In (Fig. 5) we show the results of the experiment in which we dropped the chalk on the end which was sharpened. The sharp end of the chalk easily compresses, absorbs the kinetic energy of the chalk, and saves the rest of the chalk from breaking. We see that the critical height of the chalk survival in (Fig. 5) is higher than in previous experiments.

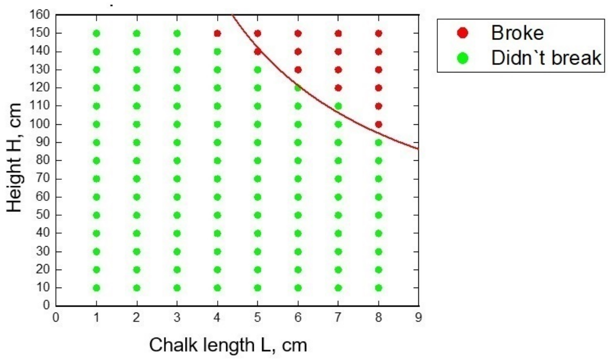

Fig. 5. Dependence of the chance of the survival of a piece of chalk on the height from which it was dropped H and chalk length L for the sharpened chalk fall case.

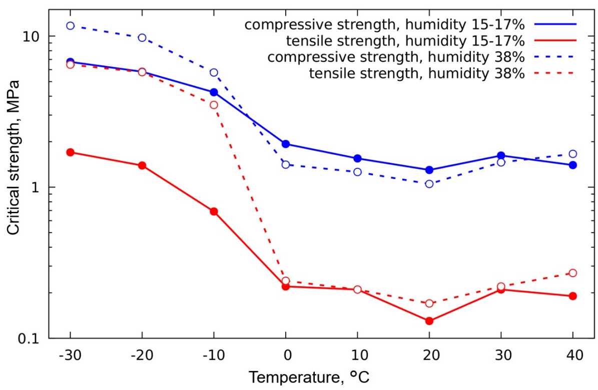

Fig. 6. Dependence of the chalk compressive strength and tensile strength on temperature. Different types of lines correspond to different humidities (dotted line corresponds to higher humidity and continuous line corresponds to lower humidity. The data is taken from [4].

6. Conclusions

Another possible explanation of the better survival of shorter chalk is the size effect. As the larger sample of chalk has a higher probability to include a critical crack, its compressive strength is smaller. This effect has been investigated by [5] and [6], and it was found that for a set of chalk samples with the same shape, but different sizes d the compressive strength is proportional to $\sigma \propto d^{-0.69}$.

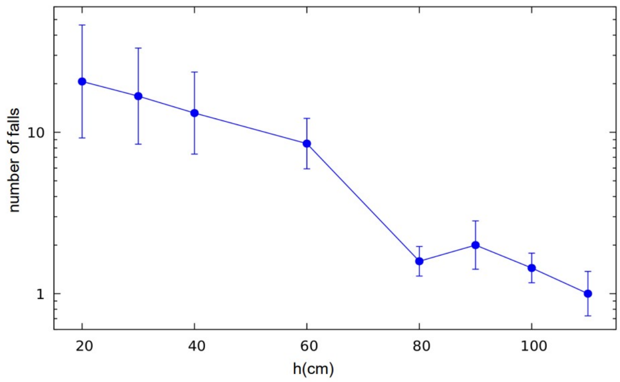

Fig. 7. Dependence of the number of falls without breaking on the height from which a piece of chalk was dropped. We see that the pieces which were dropped from smaller heights can survive more falls than ones which were dropped from the greater heights.

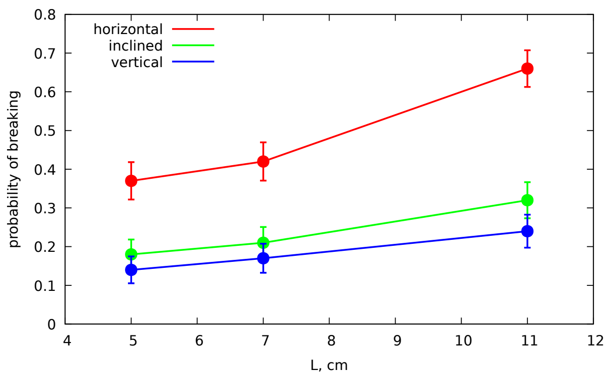

Fig. 8. Dependence of probability of chalk breaking on chalk length L. Experiments were repeated for 3 different chalk lengths and 3 different orientations.

Assuming that the strength is determined by the total volume of the sample, one can estimate that for a chalk with the same cross-section area S but varying length, L the strength varies as $\sigma \propto L^{-0.69/3} \propto L^{-0.23}$. This is insufficient to explain (Fig. 3) and (Fig. 4) without invoking the amortization of fall by chalk crumbling. Still, this is an important effect, which requires a further theoretical and experimental investigation.

7. Throwing techunique

We finally can present our results with the following recommendations of how to throw the chalk to maximize its survival rate:

- It is better to drop a piece of chalk vertically than horizontally;
- We should not give any initial velocity to the chalk, because it will only increase its collision velocity with the surface;
- If the chalk has sharper edge, is better to drop it on this sharper edge. If there's no sharper edge, one can be created by writing with the chalk;
- It is also favorable to freeze the chalk and not to drop the same chalk multiple times to avoid accumulation of microcracks.

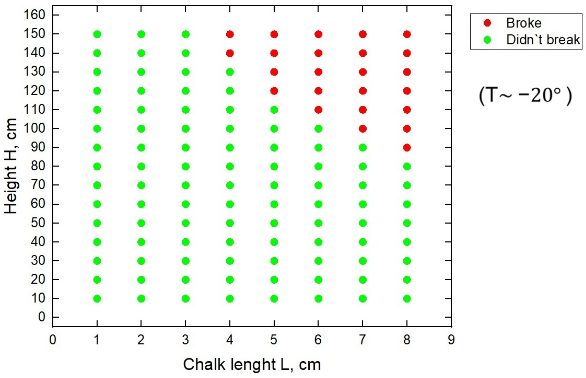

Fig. 9. Dependence of the chance of the survival of a piece of chalk on the height from which it was dropped H and chalk length L for the cooled chalk fall case.

Due to air friction, if the chalk is sufficiently small or the surface is sufficiently soft, the maximal height can be virtually infinite.

(Fig. 10) illustrates how the critical height h increases for a given chalk length L if different recommendations for maximization of the chalk survival heights are applied.

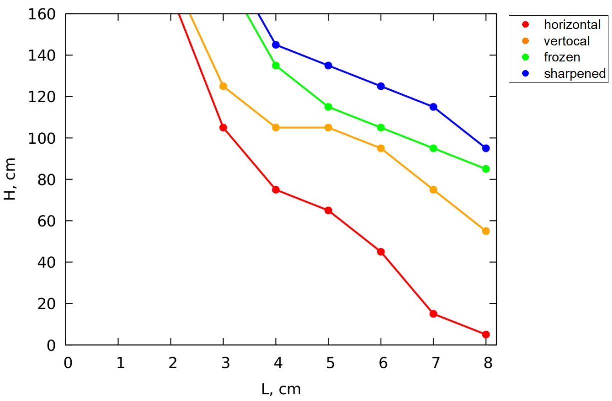

Fig. 10. Dependence of the chance of the survival of a piece of chalk on height from which it was dropped H and chalk length L for different chalk fall cases.

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