A Hybrid Approach to Operational Planning in Home Health Care*

Filipe Alves[1,2,3][0000-0002-8387-391X], António J. S. T. Duarte[4][0000-0003-3759-3850], Ana Maria A. C. Rocha[3][0000-0001-8679-2886], Ana I. Pereira[1,2][0000-0003-3803-2043], and Paulo Leitão[1,2][0000-0002-2151-7944]

[1] Research Centre in Digitalization and Intelligent Robotics (CeDRI), Instituto Politécnico de Bragança, Campus de Santa Apolónia, 5300-253 Bragança, Portugal
[2] Laboratório para a Sustentabilidade e Tecnologia em Regiões de Montanha (SusTEC), Instituto Politécnico de Bragança, Campus de Santa Apolónia, 5300-253 Bragança, Portugal
[3] ALGORITMI Center, University of Minho, 4710-057 Braga, Portugal
[4] Applied Management Research Unit (UNIAG), Instituto Politécnico de Bragança, Bragança, Portugal
{filipealves,aduarte,apereira,pleitao}@ipb.pt, arocha@dps.uminho.pt

Abstract. Home health care (HHC) management needs to plan their operations to synchronize professionals and allocate resources to perform several HHC services needed by patients. The growing demand for this type of service dictates the interest of all the stakeholders (professionals and patients) in finding high-quality daily solutions and logistics. Routing and scheduling are problems of combinatorial nature, extremely complex, and require sophisticated optimization approaches. This work aims to contribute to cost-efficient decision-making in the general improvement of the service quality. Thus, a mixed integer linear programming model, a genetic algorithm, and a hybrid approach were used to solve the operational planning through test instances of different sizes for public home care providers. Computational results are presented, followed by a discussion on the advantages and shortcomings, highlighting the strength of each approach.

Keywords: Home Health Care · Optimization · MILP · Genetic Algorithm.

1 Introduction

The world population is increasingly aging and a decrease in informal care leads to increasing demand for HHC services [18]. In this context, health institutions

* The authors are grateful to the Foundation for Science and Technology (FCT, Portugal) for financial support through national funds FCT/MCTES (PIDDAC) to CeDRI (UIDB/05757/2020 and UIDP/05757/2020), SusTEC (LA/P/0007/2021) and ALGORITMI Center (UIDB/00319/2020). Filipe Alves thanks the Foundation for Science and Technology (FCT, Portugal) for supporting its research with the PhD grant SFRH/BD/143745/2019.

need help to provide these services in order to reduce costs and increase service quality, since it is economically advantageous to keep people at home instead of in health units or hospitals [5, 17]. HHC aims to provide medical, paramedical, or social services to patients at their homes. In this sense, and taking Portugal as an example, there has been a steady increase in demand for HHC services, becoming an important topic of public, social and health concern. HHC operations need to plan human and physical resources, such as health professionals (mainly nurses) and vehicles, considering working hours, and operational constraints, among other assumptions. The HHC problem arises when a variety of health care services must be provided in patients' homes in cases of illness or injury, by health professionals from the same geographic area, aiming to optimize the scheduling and routes design, considering time or distance traveled [2]. Also, some problems are characterized by an inherent uncertainty in travel times, care duration, evolution of patients' needs, etc., and a wide variety of workers with different skills and constraints (nurse, auxiliary nurse, social technician, among others). However, these problems present some challenges to optimization, since most of these home visiting services still operate manually and without resorting to computational methods to support the scheduling and routing. Thus, these optimization problems have become increasingly complex and the search for improvements has been gaining impact in recent years [3].

This emerging sector has opened new research avenues in the field of industrial engineering, optimization and operational research (OR). In this sense, it is possible to cite some works that investigated, for example, a proper allocation of resources to each district, complying with various criteria [1], choice and dimensioning of internal resources [12], assignment of the various workers to patients [14], optimization of workers' routes to patients' homes, operational scheduling, and others. In terms of this work, the main focus is on the HHC routing and scheduling, at an operational level. These problems consist in assigning tasks to staff members of the HHC units, planning visiting hours for a set of patients, and designing the routes of visits while respecting regulatory and logistical and operational constraints [6, 22]. Other works are more focused on HHC scheduling and routing problems, for example, a comprehensive overview of existing works in the field of routing and scheduling in HHC [5]. In a later work, Cissé et al. [2], analyzed the literature on OR models applied to this topic.

In this sense, mathematical programming techniques, such as deterministic methods including (integer) linear programming, and stochastic or evolutionary approaches are two highly successful streams for optimizing combinatorial problems [20, 4]. These types of approaches are frequently applied in many highly important, practical fields, for example, classic vehicle routing problem (VRP), transportation logistics, scheduling, designing efficient communication networks, supply chain and food delivery [9, 11, 23]. Formulations of HHC problems dealing with real configurations and capturing all the complexity are a challenge for optimization researchers. In the past, optimization techniques have played a fundamental role in the research of scheduling of resources in HHC crew and emergence vehicles routing [17, 19, 21].

Therefore, the main goal of this work focuses on the solution of a real routing and scheduling problem in HHC in an interior region of Portugal. The strategy is based on two different approaches, evolutionary algorithm and exact method, with a subsequent hybrid approach of the two and respective analyses. Besides that, this work will contribute to the discussion of the two techniques producing a sensitivity analysis in post optimization to the results of different test instances that may result in better decision support on a benchmark of HHC real cases.

The rest of the paper is organized as follows: Section 2 describes the problem under study, including problem sets, parameters, and the mathematical model. In Sect. 3 the optimization approaches, which will support the test instances in the Sect. 4, will be presented. The numerical results will be presented and analyzed in Sect. 5. Finally, Sect. 6 rounds up the paper with some conclusions and future work perspectives.

2 Problem Statement

This study deals with the problem that arises from the HHC services, needing to automatically find the optimal operational planning that combines routes design, health professional's scheduling and patients allocation. The goal is to find the best schedule, considering the criteria, resources and constraints presented in the problem, in a reasonable time.

For that, it is important to define the general properties and/or assumptions of the problem, such as the HHC planning process, which includes, geographical area, resource dimension and their different instances, the care workers and patients. Therefore, the number and characterization of health professionals, the number of patients and treatments they need, and patient locations, are considered. This information and data allow the creation of a mathematical model, that represents the time spent on patient visits and travel.

Thus, considering a public Health Unit in Bragança, with a domiciliary team that provides home care to patients requiring different treatments, all the entities involved in the problem were identified.

Consider the following fixed parameters:

- $\bar{P}$ is the set of $np \in \mathbb{N}$ patients that receive home care visits, $\bar{P} = \{p_1, \ldots, p_{np}\}$ and $P = \{1, \ldots, np\}$ is the corresponding index set;
- $\bar{N}$ is the set of $nn \in \mathbb{N}$ nurses that perform home care visits, $\bar{N} = \{v_1, \ldots, v_{nn}\}$ and $N = \{1, \ldots, nn\}$ is the corresponding index set;
- $\bar{L}$ is the set of $nl \in \mathbb{N}$ locations for home care visits, $\bar{L} = \{l_1, \ldots, l_{nl}\}$ and $L = \{1, \ldots, nl\}$ is the corresponding index set;
- $\bar{T}$ is the set of $nt \in \mathbb{N}$ treatments required by patients, $\bar{T} = \{t_1, \ldots, t_{nt}\}$ and $T = \{1, \ldots, nt\}$ is the corresponding index set;
- Q is the vector with treatments duration, $Q \in \mathbb{R}^{nt}$;
- $D_{nl \times nl}$ is the time distance matrix between the nl locations. d_{ij} represents the distance (travel time) from node i to node j.

Taking into account the information regarding these sets and parameters, a mathematical programming problem can be formulated to minimize the travel time spent on visits.

The mathematical model presented is an extension of the Cumulative Routing Problem model (CumVRP) described in [10]. Considering a routing network $G = (L_0, A)$ with nodes $L_0 = \{0, 1, 2, \dots, nl\}$ (node 0 is the healthcare unit and the others are patient locations) and $A = \{(i, j) : i, j \in L_0,\ i \neq j\}$ is the set of routes. An instance of the problem is defined by the following parameters:

- u_{ik}: equal to 1 if patient $i \in P$ can be attended by nurse $k \in N$ and 0 otherwise;
- MQ: maximum duration of any nurse route (maximum shift duration).

This work proposes a mixed integer flow formulation with two sets of decision variables:

- x_{ijk} – binary variable, 1 if the nurse $k \in N$ goes from $i \in L_0$ to $j \in L_0$ and attends patient that is located in the location j and 0 otherwise;
- y_{ijk} – flow variables, they are used to accumulate the time spent on travel and patient care after each visit.

The cost or the objective function aims to minimize the longest route. A compact mathematical formulation of the proposed model follows. The index $i, j \in L_0$ are associated with the nodes (L_0 set) and $k \in N$ to nurses index.

$$\textbf{Min} \ \max_{i,k} y_{i0k} \tag{1}$$

$$\textbf{s.t.} \ \sum_i x_{0ik} = 1, \quad \forall k \in N \tag{2}$$

$$\sum_i x_{i0k} = 1, \quad \forall k \in N \tag{3}$$

$$\sum_i \sum_k x_{ijk} = 1, \quad \forall j \in L \tag{4}$$

$$\sum_j \sum_k x_{ijk} = 1, \quad \forall i \in L \tag{5}$$

$$\sum_i x_{ijk} = \sum_i x_{jik}, \quad \forall j \in L,\ \forall k \in N \tag{6}$$

$$y_{ijk} \leq MQ \cdot x_{jik}, \quad \forall i \in L_0,\ \forall j \in L_0,\ \forall k \in N \tag{7}$$

$$y_{0ik} = x_{0ik} \cdot d_{0i}, \quad \forall i \in L_0,\ \forall k \in N \tag{8}$$

$$\sum_i y_{jik} - \sum_i y_{ijk} = \sum_i x_{jik} \left(d_{ji} + q_j\right), \quad \forall j \in L,\ \forall k \in N \tag{9}$$

$$\sum_i x_{ijk} \leq u_{jk}, \quad \forall j \in L_0,\ \forall k \in N \tag{10}$$

$$x_{ijk} \geq 0 \text{ and integer}, \quad \forall i \in L_0,\ \forall j \in L_0,\ \forall k \in N \tag{11}$$

$$y_{ijk} \geq 0, \quad \forall i \in L_0,\ \forall j \in L_0,\ \forall k \in N \tag{12}$$

Equation (1) is the objective function. Equations (2) ensure that all nurses start at the depot, while equations (3) ensure that all nurses arrive at the depot. Equations (4) guarantee that exactly one nurse arrives at any patient node, while equations (5) guarantee that exactly one nurse departs from every patient node. Equations (6) state that the nurse arriving at some patient node must also depart that same node. Equations (7) put an upper bound to the flow in the arcs. Equations (8) initialize the flow in the arc from the depot to the first patient to be equal to the travel time. After that, equations (9) are flow conservation constraints and assure that the flow in each arc accumulates travel and patient care times of all the previous visits in one route. Finally, equations (10) ensure that every patient is visited by a specialized nurse, according to the compatibility coefficients defined before. Equations (11) and (12) define the lower bounds and types of the variables.

If the compatibility matrix between patients and nurses is very restrictive there will be a number of variables whose bound will be set to zero by constraints (10), effectively deleting the variables from the formulation. The number of constraints is in the order of $nl \times nn^2$, most of them being the bounds on the flow variables imposed by constraint set (7).

3 Optimization Approaches for Operational Planning

This section presents the different approaches to deal and support the operational planning for the problem presented in Sect. 2. One approach is based on the mixed integer linear programming model (MILP), where the problem is solved using a deterministic technique. Another approach is the genetic algorithm, where the problem will be solved by a meta-heuristic method. Finally, a hybrid approach will be used, which is based on the combination of the two previous approaches. The main goal is to identify the advantages and the strengths of each approach, individually and/or in combination.

3.1 Mixed Integer Linear Programming Approach

For solving the MILP, the CPLEX and optimization programming language (OPL) were used. The model presented in the Sect. 2 was implemented in OPL and optimized using the CPLEX Solver V12.10.0 with the default optimization parameters. According to the documentation [8], by default, CPLEX Solver employs preprocessing and probing of the model (coefficient reduction, elimination of redundant variables and constraints, etc.) and model strengthening by means of several types of cuts, with the idea of speeding up the solving process. The algorithm was run with the default parameters with the exception of adding stopping criteria of 1 hour (3600 seconds) in the case the optimal solution is not found within that period.

In order to strengthen the model and improve computational results, some extra sets of constraints were added to the formulation. The first one is

$$\sum_i x_{jik} \leq u_{jk}, \quad \forall j \in L_0,\ \forall k \in N \tag{13}$$

This constraint mirrors constraint (10) such that only a specialized or compatible nurse is allowed to leave a given location. Of course, this constraint was already implied by constraints (6), but preliminary testing shows their inclusion to improve computational times. We also enforced a lower limit on individual flows in all arcs by means of the following constraints:

$$y_{ijk} \geq x_{jik} \cdot (q_i + d_{ij}), \quad \forall i \in L_0,\ \forall j \in L_0,\ \forall k \in N \tag{14}$$

Similar to the previous set, these constraints are also implied by the flow conservation constraints of the compact model, but they seem to improve computational performance.

Finally, it was imposed a global lower bound (Z_{LB}) on the objective function. This bound is computed with

$$Z_{LB} = \frac{\sum_i q_i + \sum_j \min_{i \neq j} d_{ij} + nl \cdot \min_{i \neq 0} d_{i0}}{nl} \tag{15}$$

The lower bound is just the quotient of the minimum possible time required to complete all visits by the number of vehicles (nurses). The minimum possible time is computed by adding the time for treatments with the minimum time to reach a patient given an unknown location and with the minimum time for the return trip to the central hub for all nurses. This lower bound is not very tight but it seems to improve performance. Although these constraints seem to improve computational performance in preliminary testing, the tests were very limited and further examination of each one is still needed.

3.2 Genetic Algorithm Approach

A Genetic Algorithm (GA) [7] is used to solve the HHC optimization problem (1) to (12) in Sect. 2. GA is inspired by the natural biological evolution, uses a population of individuals and new individuals are generated by applying the genetic operators of crossover and mutation [13]. Genetic algorithms are particularly well suited to solve vehicle scheduling problem that is non-convex with discrete decision variable and couple with the combinatorial nature of the search space [15]. The flexibility of representation of the solutions and genetic operators allows for handling hard constraints.

The GA used in this work is summarized in Algorithm 1. Initially, a population of N_{pop} individuals is randomly generated. Each individual in the population is a vector of decision variables $\mathbf{x}$. Next, for each generation, crossover and mutation operators are applied to generate new solutions. These genetic operators were designed in order to guarantee that new solutions are feasible.

Algorithm 1 : Genetic Algorithm

1: $\mathcal{P}^0 = initialization$: randomly generate a population of N_{pop} individuals.
2: Set iteration counter $k = 0$.
3: **while** stopping criterion is not met **do**
4: $\quad \mathcal{P}' = crossover(\mathcal{P}^k)$: apply crossover procedure to individuals in population $\mathcal{P}^k$.
5: $\quad \mathcal{P}'' = mutation(\mathcal{P}^k)$: apply mutation procedure to individuals in population $\mathcal{P}^k$.
6: $\quad \mathcal{P}^{k+1} = selection(\mathcal{P}^k \cup \mathcal{P}' \cup \mathcal{P}'')$: select the N_{pop} best individuals of $\mathcal{P}^k \cup \mathcal{P}' \cup \mathcal{P}''$.
7: $\quad$ Set $k = k + 1$.
8: **end while**

The best individuals in the population have a high probability of being selected to generate new ones by crossover and mutation. Therefore, the good features of the individuals tend to be inherited by the offspring. In this manner, the population converges towards better solutions [16]. The iterative procedure terminates after a maximum number of iterations (NI) or a maximum number of function evaluations (NFE).

The model in Sect. 2 and the algorithm presented was implemented and coded in MatLab®. In complex problems belonging to non-deterministic classes, GAs are promising algorithms for searching for fast and good solutions in many applications, areas and domains planning and controlling several operations [15].

In this approach, the scheduling solution can be expressed by the vector $\mathbf{x}$ of dimension $n = 2 \times np$ with the following structure:

$$\mathbf{x} = (\mathbf{w}, \mathbf{z}) = (w_1; ...; w_{np}; z_1; ...; z_{np})$$

where the patient $w_i \in P$ will be visited by the nurse $z_i \in N$, for $i = 1, \dots, np$. In this structure, $w_i \neq w_j$ for $\forall i \neq j$ with $i, j \in P$. Therefore, for a given $\mathbf{x}$ it is possible to define the nurse's route scheduling by taking into account the order of the components of $\mathbf{x}$.

For a nurse schedule $\mathbf{x}$, the function $d^l(\mathbf{x})$, for $l = 1, .., nn$, gives the total distance (in time) required to perform all visits of the nurse l

$$f(\mathbf{x}) = \max_{l=1,\dots,nn} d^l(\mathbf{x}) \tag{16}$$

Then, the optimization problem can be defined as:

$$\min_{\mathbf{x} \in \Omega} f(\mathbf{x}) \tag{17}$$

where $\mathbf{x} \in \Omega$ is the decision variable space and $\Omega = \{(\mathbf{w}, \mathbf{z}) : \mathbf{w} \in P^{np}, \mathbf{z} \in V^{np}$ and $w_i \neq w_j$ for all $i \neq j\}$ is the feasible set. For that reason, GA was used to solve the model presented previously.

3.3 Hybrid Approach

Operations research (OR) is often described as a toolbox of methods, from which the most appropriate method for solving any particular problem can be selected. Since all OR methods have different strengths and weaknesses, a hybrid approach (mixing methods) offers the potential to overcome some of the drawbacks of using a single approach. For example, it is expected that working with a MILP model of the identified problem (HHC), it may take a long time to solve to optimality (even for a small case and/or instance).

In this sense, the hybrid approach will be based on the combination of the two previously mentioned approaches (GA and MILP), with the expectation of applying a computational technique that will provide more advantages to detect better solutions for the HHC problem than using the approaches individually. Thus, the idea is to initialize the MILP approach with a feasible solution (quickly obtained) from the GA (feed into the model), also known as "*warm* start". The strategy goes through supply hints to help MILP find an initial solution. The *warm* start comes from the same problem, already solved and with a feasible solution.

4 Test Instances

The HHC service at this Health Unit provides several types of care that can be classified into five different treatments. Clinical data was properly collected and treated following strict anonymity and confidentiality procedures.

The data allowed to combine different types of treatments according to their areas of application and/or performance required by patients, their average time and the different health professionals who perform them (Table 1).

The treatments are thus divided according to their specificity. Thus, Treatment 1 (T.1) refers to curative care (characterized by pressure ulcers, traumatic wounds, and burns, among others) with an average time of 30 minutes, while Treatment 2 (T.2) refers to Surveillance and Rehabilitation (characterized by evaluations and patient monitoring), with an average duration of 60 minutes. Treatment 3 (T.3) is Curative and Surveillance care (characterized by wound treatment, frequency and tension monitoring, among other pathology) averaging 75 minutes, while Treatment 4 (T.4) is only Surveillance care (assess the risk of falls, self-care, dietary, among others) and has average care of around 60 minutes. Finally, Treatment 5 (T.5) concerns more general health care (characterized by support mourning for example) such as support and monitoring and has an average of 60 minutes as well. The health unit has a total of 12 health professionals (mostly nurses) assigned to the various days of home visits.

The complete dataset has a total of 40 patients, which can be assigned to a single day or divided over several days of home visits. The same is true for health professionals, who have specific hours of primary health care at home and, therefore, may not always be on the same working day. Thus, the need arose not only to solve the combinatorial problem computationally according to the approaches in HHC planning but also to analyze the sensitivity of different cases/instances (Table 2). Each patient required specific medical assistance consisting of one or more different treatments from the 5 treatments that nurses can perform.

In total, 9 cases will be solved (from different instances) and draw the appropriate conclusions thereof, according to the optimization techniques and gener-

Table 1. Treatments performed by the nurses.

	T.1 (30 min)	T.2 (60 min)	T.3 (75 min)	T.4 (60 min)	T.5 (60 min)
Nurse 1	X			X	
Nurse 2	X	X		X	
Nurse 3	X			X	
Nurse 4	X		X	X	
Nurse 5	X			X	
Nurse 6	X	X		X	X
Nurse 7	X		X	X	
Nurse 8	X			X	
Nurse 9	X			X	
Nurse 10	X			X	
Nurse 11	X			X	
Nurse 12	X			X	

Table 2. List of test instances.

Dataset	**Parameters**
Case 1	20 patients, 8 nurses, 19 locations
Case 2	20 patients, 10 nurses, 19 locations
Case 3	20 patients, 12 nurses, 19 locations
Case 4	30 patients, 8 nurses, 25 locations
Case 5	30 patients, 10 nurses, 25 locations
Case 6	30 patients, 12 nurses, 25 locations
Case 7	40 patients, 8 nurses, 28 locations
Case 8	40 patients, 10 nurses, 28 locations
Case 9	40 patients, 12 nurses, 28 locations

ated approaches. From Table 2, it is verified that the routes of nurses allocation and scheduling take into account a total of 28 locations, representative of the patients homes in the region under study.

The travel time (in minutes) between the locations is shown in Table 3 (appropriately numbered locations for data protection and used as a time matrix resource), whose diagonal indicates the typical time required to attend to several patients in the same area (e.g., same streets).

Table 3. Time matrix with sources and targets to list locations.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
1	7	32	12	11	10	29	9	30	24	9	18	8	27	13	35	14	27	7	10	24	11	14	26	10	12	12	6	28
2	32	7	33	35	31	31	33	43	41	35	29	29	29	35	27	31	31	33	35	44	35	28	30	33	32	32	32	40
3	12	33	7	12	13	30	10	33	24	6	21	14	29	15	37	9	24	11	7	18	9	17	28	6	7	6	10	25
4	11	35	12	7	11	32	10	35	27	12	24	14	31	19	39	10	25	13	10	24	10	15	30	11	12	11	12	26
5	10	31	13	11	7	27	8	32	28	13	20	11	26	17	34	15	26	11	11	26	11	10	25	13	15	14	10	27
6	29	31	30	32	27	7	30	37	38	32	26	26	16	32	24	28	36	30	32	41	32	25	18	30	29	29	29	37
7	9	33	10	10	8	30	7	33	25	10	21	11	8	15	36	12	26	9	8	23	8	12	26	9	11	11	9	27
8	30	43	33	35	32	37	33	7	28	32	27	28	38	26	46	31	42	28	33	43	7	29	37	32	31	32	29	43
9	24	41	24	27	28	38	25	28	7	26	28	27	36	23	44	22	37	24	25	22	27	28	35	23	22	23	23	38
10	9	35	6	12	13	32	10	32	26	7	23	12	30	14	38	11	25	10	8	20	9	17	29	8	8	8	9	26
11	18	29	21	24	20	26	21	27	28	23	7	16	24	21	32	19	30	19	22	31	23	17	23	20	19	19	18	31
12	8	29	14	14	11	26	11	28	27	12	16	7	25	12	33	17	25	10	12	26	13	10	24	13	15	14	9	26
13	27	29	29	31	26	16	8	38	36	30	24	25	7	30	12	25	34	27	29	38	30	22	13	28	26	27	26	35
14	13	35	15	19	17	32	15	26	23	14	21	12	30	7	38	16	30	11	15	28	16	18	29	17	16	17	11	31
15	35	27	37	39	34	24	36	46	44	38	32	33	12	38	7	33	42	35	37	46	38	30	21	36	34	35	35	43
16	14	31	9	10	15	28	12	31	22	11	19	17	25	16	33	7	21	14	9	20	9	14	25	9	8	9	13	22
17	27	31	24	25	26	36	26	42	37	25	30	25	34	30	42	21	7	28	26	33	25	22	34	25	24	25	27	18
18	7	33	11	13	11	30	9	28	24	10	19	10	27	11	35	14	28	7	10	24	11	15	27	11	13	12	5	29
19	10	35	7	10	11	32	8	33	25	8	22	12	29	15	37	9	26	10	7	19	6	15	29	6	8	8	9	26
20	24	44	18	24	26	41	23	43	22	20	31	26	38	28	46	20	33	24	19	7	22	26	38	18	17	18	23	35
21	11	35	9	10	11	32	8	7	27	9	23	13	30	16	38	9	25	11	6	22	7	13	28	6	8	7	9	25
22	14	28	17	15	10	25	12	29	28	17	17	10	22	18	30	14	22	14	15	26	13	7	23	18	20	19	14	25
23	26	30	28	30	25	18	26	37	35	29	23	24	13	29	21	25	34	27	29	38	28	23	7	28	27	28	27	35
24	10	33	6	11	13	30	9	32	23	8	20	13	28	17	36	9	25	11	6	18	6	18	28	7	6	5	11	24
25	12	32	7	12	15	29	11	31	22	8	19	15	26	16	34	8	24	13	8	17	8	20	27	6	7	7	12	23
26	12	32	6	11	14	29	11	32	23	8	19	14	27	17	35	9	25	12	8	18	7	19	28	5	7	7	12	24
27	6	32	10	12	10	29	9	29	23	9	18	9	26	11	35	13	27	5	9	23	9	14	27	11	12	12	7	28
28	28	40	25	26	27	37	27	43	38	26	31	26	35	31	43	22	18	29	26	35	25	25	35	24	23	24	28	7

The health unit prefers to have average values between locations in the same area (which are not always reached) and assigned these values on the diagonal to make sure they are not exceeded. In this way, each node (e.g., location 1 (first row) and location 1 (first column) are the same location, and so on) identifies a set of origins and destinations between each address or residence. The response is computed using optimization techniques for $M : N$ routes computation.

In this sense, a sensitivity analysis was generated to the uncertainty in the output of a set of different instance sizes. The strategy involves an evaluation of the robustness of the deterministic, stochastic and hybrid approaches, in terms of their behaviors and impacts when subject to different parameters.

5 Numerical Results

In this section it is intended to qualitatively evaluate the flexibility and effectiveness of each of the approaches, taking into account their variability and solutions. The range of values of the numerical results is examined by analyzing how the output value (from all approaches) behaves.

5.1 MILP Results

In order to test the proposed MILP formulation, all the datasets in the Table 2 have been solved. The developed OPL model was instantiated with data from the different problems and optimization was run until the optimal solution was found or when a time limit of 1 hour (3600 seconds) was reached. The experiment was carried out using a laptop equipped with an Intel® Core™ i7-10510U CPU with 16 GB of memory. Table 4 summarizes the results.

Table 4. Summary of optimization results for the MILP model.

Case	P	N	R	C	B	Best	LB	Gap	Time
1	20	8	4119	3942	2032	186	186	0.0%	45
2	20	10	4963	4754	2452	186	186	0.0%	58
3	20	12	5807	5566	2872	186	186	0.0%	39
4	30	8	10819	10542	5372	264	210	20.5%	3601
5	30	10	13223	12894	6572	220	169	23.2%	3601
6	30	12	15627	15246	7772	187	148	20.9%	3601
7	40	8	17354	17003	8630	349	285	18.3%	3601
8	40	10	21078	20663	10490	296	229	22.6%	3604
9	40	12	24802	24323	12350	284	284	0.0%	2938

For each case the following information is presented: the number of patients to be treated (P) and the number of vehicles or nurses (N); the number of rows (R), columns (C) and binaries (B) after the resolve and probing as reported by CPLEX, which reflect the effective size of the problem; the objective value for the best solution found (Best), the lower bound (LB), the percentage gap (Gap) between the best solution and the lower bound and the time (in seconds) CPLEX spent to solve the problem. In a case where CPLEX found the optimal solution, the gap is zero (cases 1, 2, 3, and 9).

5.2 GA Results

Following the previous approach, the proposed meta-heuristic (GA) was also applied to the dataset in Table 2.

The experiment was carried out using a laptop equipped with an Intel® Core™ M-5Y71 CPU 1.4GHz with 8 GB of memory. The numerical results were obtained using the MatLab®. The values of the control parameters used in GA for this problem were tuned after preliminary experiments. A population size of 30 individuals ($N_{pop} = 30$) and a probability rate of 50% for crossover and mutation procedures were used. The stopping criterion was based on the maximum number of iterations defined as 100 ($NI = 100$) or the maximum number of function evaluations of 5000 ($NFE = 5000$). Since GA is a stochastic algorithm, 30 runs were carried out with random initial populations. GA found feasible solutions for all runs in a reasonable time. This study analyses the effect of defined parameters on the output directly and on model performance, thus the analysis is evaluated by comparing the corresponding observations.

Table 5 gives the sensitivity rank of all the parameters for all criteria, starting with criteria on the performance in all runs for each case, such as best or minimal solution (f_{min}), the solution average (f_{avg}), the standard deviation (SD), the number of different best solutions found (N_{min}) and, finally the average time to solve the optimization problem ($Time$), in seconds.

Table 5. Summary of optimization results for the GA meta-heuristic.

Case	f_{min}	f_{avg}	SD	N_{min}	$Time$
1	233	262	18	2	29.9
2	212	240	17.2	1	31.4
3	188	228	14.4	1	37
4	333	361	16	1	38.4
5	272	324	19.4	1	43.8
6	246	287	21.3	1	48.6
7	425	483	22.8	1	48.7
8	372	430	25.6	2	54.9
9	331	397	22.9	1	57.5

Table 5 presents the results for the different defined criteria that allow to statistically evaluate the balance of the solution outputs. The goal of applying GA is to characterize how the meta-heuristic programming responds to changes in input, with an emphasis on finding the input parameters to which outputs are more or less sensitive to different cases from different instances.

5.3 Discussion

In this section, the results of the two previous approaches are analyzed and compared. In this sense, the illustration of a scheduling example on a Gantt chart, according to the two approaches under study, for the instance of case 2 is shown in Figure 1. It is possible to see in Figure 1, the different allocations of patients to their respective nurses and to view the complete routes of home visits, which include travel times, patient treatment times, and consequent return to the point of origin (Health Unit). Both approaches present optimized management of the routes with a good workload balancing between the different entities.

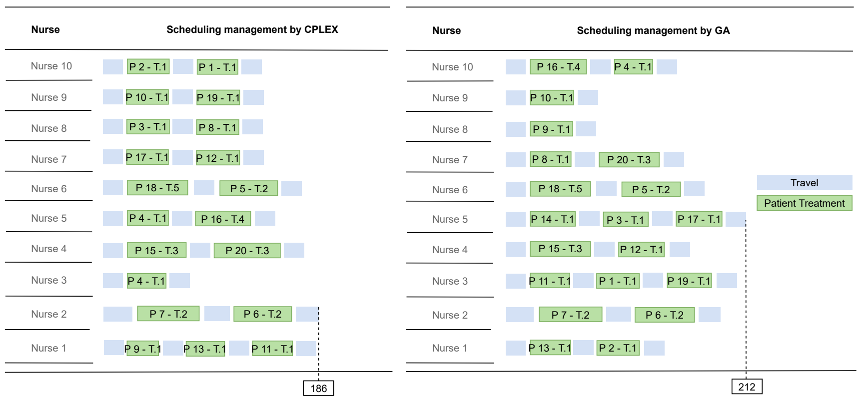

Fig. 1. Example of a scheduling management by the two approaches for case 2.

Regarding the maximum value reached by the two approaches, concerning the total time of home visits, it has a value of 212 minutes for the GA meta-heuristic and the optimal solution of 186 minutes for the MILP model by CPLEX. On the other hand, Figure 2 summarizes Tables 4 and 5.

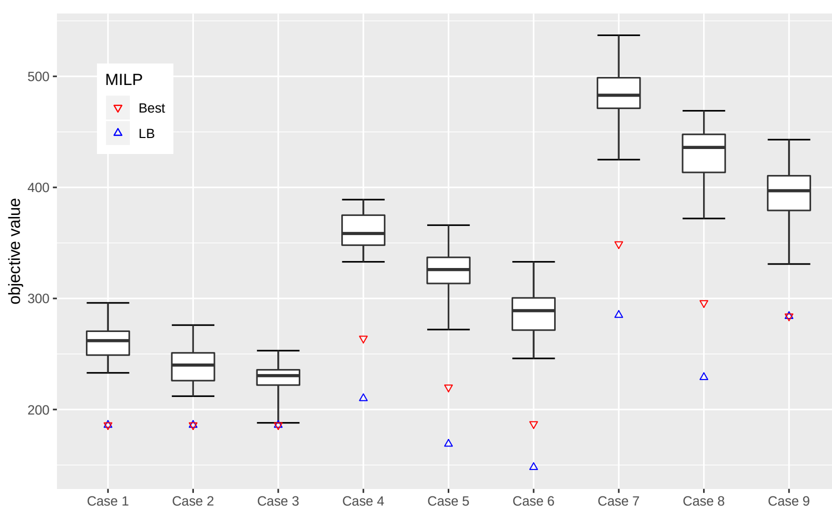

Fig. 2. Objective values for GA and MILP.

Figure 2 compares the results for the GA meta-heuristic (boxplots for the 30 runs of the algorithm) and for the MILP model (best solution and lower bound). It can be seen that the MILP model finds either optimal or better solutions than the GA. However, computational times presented by GA reach solutions more quickly (never exceeding 60 seconds even in the largest instances), and in some cases with solutions close to the optimum, which makes it possible to support the decision-making in emergency scheduling needs.

5.4 Hybrid Results

In order to further enhance our computational findings, it was also performed a *warm* start procedure with the MILP model combined with GA solutions. The procedure consisted in starting the optimization in CPLEX from an initial solution. The solution provided to CPLEX was the best solution from the GA. Table 6 shows the results from the experiment, where for each case and/or instance the objective values of the starting solution, the best solution, the lower bound, and computation time are presented.

Table 6. *Warm* start computational results in MILP

Case	Start	Best	LB	Time
1	233	186 (0.0%)	186 (0.0%)	47 (4.4%)
2	212	186 (0.0%)	186 (0.0%)	51 (-12.1%)
3	188	186 (0.0%)	186 (0.0%)	67 (71.8%)
4	333	259 (-1.9%)	210 (0.0%)	3600 (0.0%)
5	272	222 (0.9%)	171 (1.2%)	3601 (0.0%)
6	246	186 (-0.5%)	186 (25.7%)	3565 (-1.0%)
7	425	350 (0.3%)	285 (0.0%)	3600 (0.0%)
8	372	293 (-1.0%)	232 (1.3%)	3600 (-0.1%)
9	331	284 (0.0%)	284 (0.0%)	1294 (-56.0%)

The percentage change relative to the *default* CPLEX start is shown inside the brackets. Figure 3 shows a visualization of the results in terms of boxplots.

The results have high variability and drawing definitive conclusions is difficult with such a small number of instances. The results for the best solution seemed to be about the same. The same applies to the lower bound with the exception of one case (case 6) for which it was possible to close the gap and reach the optimal solution. The computational times show a global decrease of 7.9%, mainly explained by the decrease in one case (case 9).

The time taken by the GA meta-heuristic to reach the initial solutions was 390 seconds (about 6.5 minutes) and thus the decrease in total time would be only 6%. Taking everything into account, the *warm* start seems to be beneficial for optimizing the computational time and reaching the optimal solutions.

In conclusion, it is important to mention that all solutions were validated (but not yet applied in real context) by the health institution (nurses), which positively reinforces the different strategies used. Furthermore, the different solutions achieved significant savings rates in route optimization when compared to

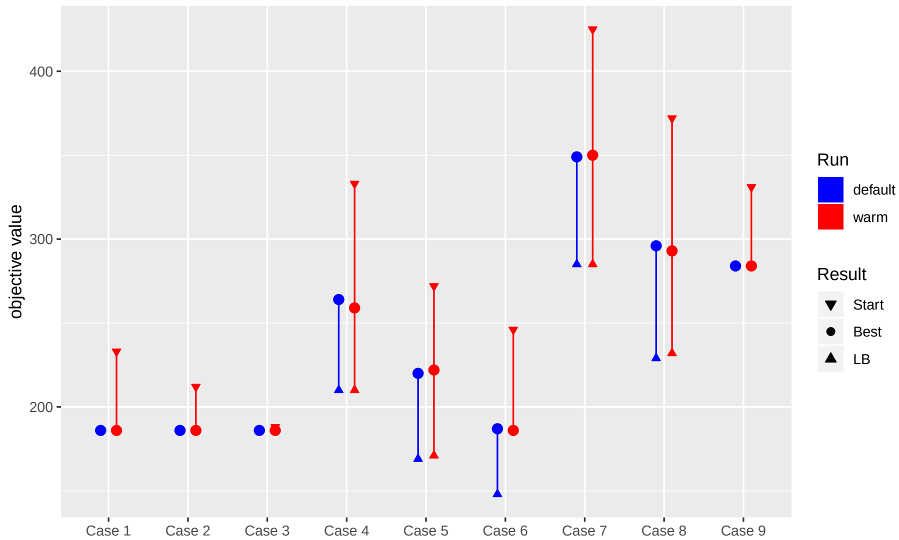

Fig. 3. *Default* and *warm* start comparison for MILP model.

existing planning in the institution (manually scheduling/calendar), in addition to simplifying the planning process with faster response rates.

6 Conclusions and Future Work

In this paper, three optimization approaches, GA, MILP, and a hybrid approach involving the first two have been proposed, formulated, and applied for the coordinated decision-making in HHC services under different test instances based on real cases. This study analyzes and quantifies the impact of one or more variables in the final results to evaluate the robustness of all approaches, reduction of uncertainty, and calibration model for the operational planning to support HHC services. The optimal solution, or the quickest solution, may depend on the needs of the moment, knowing in advance that, with health, great robustness in responses and solutions is always necessary.

According to these principles, and taking into account the numerical results obtained, it was found that the approaches are successful in route allocating and scheduling, reflecting some differences in the solutions for home care visits. It is visible that the CPLEX solution presents a better workload balancing and consequently reaches the optimal solution. In case 2 in particular, the MILP solution has a minimization of the maximum time of almost half an hour when compared to GA. In general terms, the MILP model finds either optimal or better solutions than the GA method, however, in terms of computational times, CPLEX only solved the initial 3 cases in a reasonable time. Therefore, in computational expenses, GA reaches the solutions extremely fast and in some of the test instances it is close to the optimal solution. In conclusion, in terms of screening, it is possible to state that in small instances the optimal solution is easily obtained using the deterministic technique, but when the cases become more complex, GA becomes more attractive.

Encouraged by these results, a hybrid approach was designed, which consisted of a *warm* start procedure with the MILP model. GA solutions will be used as initial solutions in CPLEX, allowing to refine and eliminate the search space, that is, to combine the advantages of both methods. Basically, in CPLEX it means installing an advanced base and using it as a starting point, allowing to prune the branching tree. The *warm* start proved to be beneficial for the performance of MILP solvers, accelerating the resolution of the instances of the HHC problem, thus enhancing the MILP solution.

Some limitations of the work have already been identified, namely the non-inclusion of the priority factor among patients, possible time windows in the attendance and periodic visits, which nowadays are parameters to improve the model of HHC. The natural continuation of this work would be the digitalization of the route definition process, which would involve the creation of an interface that would allow the user to change the problem data (e.g., number and characteristics of patients), as well as the visualization of the routes obtained by the optimization. The accuracy of the model parameters (distances, travel times and treatments) could also be improved through google micro-services. Furthermore, future work will be devoted to extending the approach to different algorithms as well as different methodologies (e.g., multi-agent systems).

References

1. Benzarti, E., Sahin, E., Dallery, Y.: Operations management applied to home care services: Analysis of the districting problem. Decision Support Systems **55**(2), 587–598 (2013). https://doi.org/https://doi.org/10.1016/j.dss.2012.10.015, 1. Analytics and Modeling for Better HealthCare 2. Decision Making in Healthcare
2. Cissé, M., Yalçındağ, S., Kergosien, Y., Şahin, E., Lenté, C., Matta, A.: Or problems related to home health care: A review of relevant routing and scheduling problems. Operations Research for Health Care **13-14**, 1 – 22 (2017). https://doi.org/https://doi.org/10.1016/j.orhc.2017.06.001
3. Davidson], P.P., Blum, C., Lozano, J.A.: The weighted independent domination problem: Integer linear programming models and metaheuristic approaches. European Journal of Operational Research **265**(3), 860 – 871 (2018). https://doi.org/https://doi.org/10.1016/j.ejor.2017.08.044
4. Fathollahi-Fard, A.M., Hajiaghaei-Keshteli, M., Tavakkoli-Moghaddam, R.: The social engineering optimizer (seo). Engineering Applications of Artificial Intelligence **72**, 267 – 293 (2018). https://doi.org/https://doi.org/10.1016/j.engappai.2018.04.009
5. Fikar, C., Hirsch, P.: Home health care routing and scheduling: A review. Computers & Operations Research **77**, 86 – 95 (2017). https://doi.org/https://doi.org/10.1016/j.cor.2016.07.019
6. Gutiérrez, E.V., Vidal, C.J.: Home health care logistics management problems: A critical review of models and methods. Revista Facultad de Ingeniería Universidad de Antioquia (68), 160–175 (2013)

7. Holland, J.H., et al.: Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence. MIT press (1992)
8. IBM Corporation: IBM ILOG CPLEX Optimization Studio CPLEX User's Manual. IBM Corporation (2019)
9. Kar, A.K.: Bio inspired computing - a review of algorithms and scope of applications. Expert Systems with Applications **59**, 20 – 32 (2016). https://doi.org/https://doi.org/10.1016/j.eswa.2016.04.018
10. İmdat Kara, Kara, B.Y., Yetiş, M.K.: Cumulative vehicle routing problems. In: Caric, T., Gold, H. (eds.) Vehicle Routing Problem, chap. 6. IntechOpen, Rijeka (2008). https://doi.org/10.5772/5812
11. Knop, D., Koutecký, M., Mnich, M.: Combinatorial n-fold integer programming and applications. Mathematical Programming pp. 1–34 (2019)
12. Koeleman, P., Bhulai, S., van Meersbergen, M.: Optimal patient and personnel scheduling policies for care-at-home service facilities. European Journal of Operational Research **219**(3), 557–563 (2012). https://doi.org/https://doi.org/10.1016/j.ejor.2011.10.046, feature Clusters
13. Kramer, O.: Genetic algorithm essentials, vol. 679. Springer (2017)
14. Lanzarone, E., Matta, A.: A cost assignment policy for home care patients. Flexible Services and Manufacturing Journal **24**(4), 465–495 (2012)
15. Lee, C.: A review of applications of genetic algorithms in operations management. Engineering Applications of Artificial Intelligence **76**, 1 – 12 (2018). https://doi.org/https://doi.org/10.1016/j.engappai.2018.08.011
16. Metawa, N., Hassan, M.K., Elhoseny, M.: Genetic algorithm based model for optimizing bank lending decisions. Expert Systems with Applications **80**, 75–82 (2017)
17. Nickel, S., Schröder, M., Steeg, J.: Mid-term and short-term planning support for home health care services. European Journal of Operational Research **219**(3), 574–587 (2012)
18. OECD. Publishing and Organization for Economic Cooperation and Development (OECD) Staff: Health at a glance 2019: OECD Indicators. OECD Publishing (2019), https://doi.org/10.1787/4dd50c09-en
19. Paraskevopoulos, D.C., Laporte, G., Repoussis, P.P., Tarantilis, C.D.: Resource constrained routing and scheduling: Review and research prospects. European Journal of Operational Research **263**(3), 737–754 (2017)
20. Raidl, G.R., Puchinger, J.: Combining (Integer) Linear Programming Techniques and Metaheuristics for Combinatorial Optimization, pp. 31–62. Springer Berlin Heidelberg, Berlin, Heidelberg (2008). https://doi.org/10.1007/978-3-540-78295-7_2
21. Rasmussen, M.S., Justesen, T., Dohn, A., Larsen, J.: The home care crew scheduling problem: Preference-based visit clustering and temporal dependencies. European Journal of Operational Research **219**(3), 598 – 610 (2012). https://doi.org/https://doi.org/10.1016/j.ejor.2011.10.048, feature Clusters
22. Sahin, E., Matta, A.: A contribution to operations management-related issues and models for home care structures. International Journal of Logistics Research and Applications **18**(4), 355–385 (2015)
23. Speranza, M.G.: Trends in transportation and logistics. European Journal of Operational Research **264**(3), 830 – 836 (2018). https://doi.org/https://doi.org/10.1016/j.ejor.2016.08.032