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### **Technology Shocks and the Response of Hours Worked: Time-Varying Dynamics Matter**

by

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# Technology Shocks and the Response of Hours Worked: Time-Varying Dynamics Matter

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## Abstract

Do hours worked rise or fall after a positive technology shock? According to the existing evidence it depends on whether they enter the VAR in levels (hours rise) or growth rates (hours fall). We argue that conflicting results may ultimately arise because important structural time variations are a priori ruled out by empirical models. We identify technology shocks as the only shocks driving long-run labor productivity in a Time-Varying Coefficients Bayesian Vector Autoregressions (TVC-BVAR) estimated using postwar US quarterly data. We find that (i) under *both* (levels and growth rates) specifications hours fall, and (ii) technology shocks explain about 11-25% of total aggregate fluctuations giving rise to positive but small correlations between output and hours. Differences with respect to fixed coefficients VAR are due to instabilities in the relationship between labor productivity and the levels of hours worked.

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# 1 Introduction

The short-run dynamics of hours worked following a positive technology shock have an essential role in assessing competing theories of the business cycle. Standard versions of Real Business Cycle (RBC) models (see e.g. Prescott, 1986) predict that hours must increase: an improvement in technology raises marginal productivity of labor and the labor demand which, with an upward sloping supply, implies a rise in hours worked<sup>1</sup>. On the other hand, other theories of the business cycle, like models embodying nominal rigidities (see e.g. Gali, 1999) or RBC models with habits formation and capital adjustment cost (see e.g. Francis and Ramey, 2003) predict that hours fall. The sign of the response of hours has very important implications for the role of technology shocks in explaining aggregate fluctuations. Actually a shock that fails in generating a strong positive correlation between output and hours can hardly be considered one of the main forces driving business cycles.

In recent years an interesting and intense debate on whether, in the data, hours rise or fall after a positive technology shock has emerged. Implicitly, the contention is whether the standard RBC paradigm can correctly describe the business cycle and whether technology shocks can be considered important sources of economic fluctuations. Gali (1999), using reduced form vector autoregressions augmented with the restriction that technology shock is the only shock driving long-run labor productivity, finds that hours fall. Moreover technology shocks can account just for a very small part of total fluctuations in output and hours worked at the business cycles frequencies. The author interprets all this as compelling evidence against the RBC paradigm. Similar conclusions are reached, through different approaches, by Basu Fernald and Kimball (2004), Francis and Ramey (2003) and Francis, Owyang and Theodorou (2003), Pesavento and Rossi, (2004) and Shea (1998). The reaction to this growing consensus came soon. Christiano Eichenbaum and Vigfusson (2004) (CEV henceforth), using a similar reduced form vector autoregressions representation and the same identifying restriction, replicate the exercise by Gali (1999) and they find the opposite result: as predicted by standard RBC models, hours persistently rise<sup>2</sup> Evidence in line with the CEV conclusions is provided in the works by Dedola and Neri (2004), Fisher (2005), Peersman and Straub (2003, 2005) and Uhlig (2001).

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<sup>1</sup>Under standard calibrations, such a mechanism arise no matter when the technology shock is modeled as a persistent stationary AR or a random walk.

<sup>2</sup>Chari Kehoe and McGrattan (2005) call into question the VAR approach as a useful guide to assess the relevance of theoretical models. They show, using simulated data, that VAR analysis would imply a fall of hours when the underlying theoretical model predicts a positive response after a technology shock. However Christiano Eichenbaum and Vigfusson (2005) show that the Chari Kehoe and McGrattan model is a case of little empirical relevance since it is strongly rejected by the data. On the contrary when models with higher posterior support are employed, VAR predict the right responses. Similar findings emerge in the works by Erceg, Guerrieri and Gust (2004) and Francis, Owyang and Roush (2005).

Why are the results of Gali and CEV so different? The reason is in the different specification for the time series of hours worked used in the VAR. Gali, arguing that hours are difference stationary, uses growth rates<sup>3</sup>. On the contrary CEV, justifying their choice with an encompassing argument, specify hours in levels. The whole debate is nowadays at a standstill because of such a specification controversy<sup>4</sup>: the response of hours to a positive technology shock depends on whether they are specified in levels (hours rise) or growth rates (hours fall). Consequently much effort has been spent in trying to justify from a statistical and economic point of view each of the two specifications. CEV show that the levels specification can easily explain the growth rates specification while the converse is not true. On the other hand Gali (2005) provides empirical and theoretical evidence in favor of the nonstationarity of hours worked across industrialized countries.

This paper contributes to the debate from a completely different perspective. We investigate the effects of technology shocks on hours worked in the US using Bayesian Vector Autoregressions with drifting coefficients, thus allowing for general forms of time variations and structural changes. The basic idea of the paper comes from the simple consideration that despite the different treatment of hours worked, all empirical models used in previous contributions stand on the assumption that model coefficients are constant over time. Although standard in VAR literature, such an assumption seems to be very strong when the analysis is run over a sample of fifty years and mainly when variables describing the labor market are included. Actually important changes in labor market trends, like changes in the composition of hours worked or participation rates, and in the US economy in general, like changes in the central bank anti-inflationary preferences<sup>5</sup> or changes in labor productivity trends<sup>6</sup>, have been extensively documented in literature. Moreover these changes seem to be relevant for the transmission mechanisms of technology shocks since in many works (see Gali, Lopez-Salido and Valles 2003, GLV henceforth, CEV, Fisher 2005, and Fernald 2005) it clearly emerges that results are highly sensitive to the sample or subsamples considered in the analysis. In this paper we argue that conflicting results may ultimately arise simply because some of these changes are a priori ruled out by previous empirical models. Specifically, differences in the results that are apparently due to a different treatment of hours worked may simply originate from a more fundamental misspecification arising from the too strong assumption of model coefficients constancy. Actually we show that once one allows for changes in the US economy whether hours should be specified in levels or growth rate becomes of secondary importance since competing specifications yield the same answer: at least until mid 90's hours

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<sup>3</sup>The same results emerge when hours are detrended using quadratic trends.

<sup>4</sup>See Whelan (2004) for a detailed review and a study of the robustness of the results to alternative specifications.

<sup>5</sup>See among others Boivin and Giannoni (2002a, 2002b), Clarida, Gali and Gertler, (2000), Cogley and Sargent, (2001, 2003) documenting a change in the response of monetary authorities to inflation after 1979.

<sup>6</sup>See Brainard and Perry, (2001), Kahn and Rich, (2003) and Roberts, (2000) documenting that two big changes in labor productivity took place in early 70's and again during the mid 90's.

persistently fall.

This paper addresses the following questions. What are the effects of technology shocks on hours worked and what is the importance of technology shocks in explaining aggregate fluctuations when time variations are accounted for? Can we reach robust conclusions by allowing for time variations in the US economy? To address these questions we augment the reduced form model, which is almost identical to the one originally proposed by Cogley and Sargent (2001), with the same restriction as in Gali (1999) and CEV that technology shocks are the only shocks driving labor productivity in the long-run and we use both specifications for hours worked, levels and growth rates. Given that the specification is identical to the one used in literature, our analysis can concentrate on differences directly attributable to coefficients time variations. To conduct dynamic analysis we use conditional impulse response functions, that is we condition on all out-of-sample coefficients being equal to the end-of-sample coefficients. This is motivated, on the one hand, by the fact this is the best forecast whenever coefficients evolve according to a random walk. On the other hand, under such a definition, impulse response functions display useful long-run properties. The model is estimated using Bayesian MCMC methods: we use the Gibbs sampling algorithm augmented with a rejection sampling to generate draws from the posterior distributions of the objects of interest. We check the robustness of the results to alternative end-of-sample dates and alternative identification schemes and eventually we extend the model in order to consider also investment-specific technology shocks.

Our main findings can be summarized as follows. (i) Hours fall under *both* specifications, levels and first differences. (ii) The impact effect is more pronounced and significantly different from zero only before 1990. For the level specification also the degree of persistency of the response substantially reduces over-time. (iii) Differences with respect to fixed coefficients VAR are due to instabilities in the relationship between labor productivity and the levels of hours worked. (iv) Technology shocks generate positive but small correlations between output and hours at the business cycles frequencies and the portion of output variance explained by technology shocks over the business cycles is about 11-25%. When, in addition to aggregate sector-neutral shocks, also investment-specific technology shocks are considered the percentages relative to technology shocks as a whole raise up to 39-53%. (v) Results are robust to alternative identifying restrictions. (vi) The response of monetary policy to technology shocks has changed over time but this does not seem to affect the response of hours worked.

The paper is organized as follows: section 2 revisits the evidence from fixed coefficients VARs; section 3 describes the empirical model; section 4 discusses main results; section 5 explains the differences between time-varying and fixed coefficients VARs; section 6 provides some structural interpretations of the results; section 7 assesses the robustness of the results to various alternatives; section 8 concludes.

## 2 Revisiting the Evidence from Fixed Coefficients SVARs

Let  $y_t$  be a  $n \times 1$  vector of time series with the following VAR representation

$$A(L)y_t = \varepsilon_t \tag{1}$$

where  $L$  is the lag operator,  $A(L) = I - A_1L - A_2L^2 - \dots - A_pL^p$ ,  $A_i$  are  $n \times n$  matrices of coefficients and  $\varepsilon_t$  is a  $n \times 1$  Gaussian white noise process with zero mean and covariance  $\Sigma$ . If the roots of  $A(L)$  in modulus are outside the unit circle,  $y_t$  admits the following MA representation of infinite order

$$y_t = B(L)\varepsilon_t \tag{2}$$

where  $B(L) = A(L)^{-1}$ . Let  $S$  be the unique lower triangular matrix such that  $SS' = B(1)\Sigma B(1)'$  where  $B(1) = I + B_1 + B_2 + \dots$  and let  $K = B(1)^{-1}S$ . We can rewrite (2) in terms of orthogonal shocks

$$y_t = C(L)e_t$$

where  $e_t = K^{-1}\varepsilon$  and  $C(L) = B(L)K$ . If labor productivity growth is ordered first in the vector  $y_t$ , then the first shock,  $e_{1t}$ , is the technology shock identified by the restriction that is the only shock affecting long-run productivity.

Figure 1 plots the impulse response functions of per capita hours to a technology shock from a bivariate VAR with labor productivity growth and hours worked. Top and bottom panels refer to the specification with hours in first differences and levels respectively. When specified in first differences, hours persistently and significantly decline. On the contrary, in levels, the response is positive, significant and hump-shaped, reaching its maximum after two years after the shock. Here the terms of the controversy clearly emerge: when hours are specified in growth rates they persistently decline while in levels they persistently increase.

To motivate our interest in time variations let us consider what happens when we repeat the analysis for the subsamples considered in GLV and Fisher (2005), 1954:III-1979:IV and 1982:III-2003:IV, and corresponding to the presumed breaks in the monetary policy conduct. Instabilities are evident for the levels specification (Figure 2): in the second subsample the response is positive while in the first it becomes negative. On the other hand, in growth rates, results appear to be more robust since in both subsamples hours reduce. The lack of robustness of results is not limited to the two subsamples considered above. For instance Fernald (2005) shows that if one takes into account potential shifts in trend productivity, specifically the slow-down in 1973 and pick-up in mid 90's hours worked fall for both specifications in all the subsamples. Perhaps the most striking result is that if the analysis had been done ten years before the paper by CEV, say at the very beginning of the 90's, no debate would have emerged. Actually if we exclude from the analysis the last ten years (1994-2003), hours fall under both specifications. All these findings are hard to

explain unless we admit the possibility that when we use the whole sample we are mixing periods in which the structural features characterizing the US economy are different. This, we believe, strongly suggests that the link between structural changes and the propagation mechanisms of technology shocks and the way the formers may have influenced the seconds deserves further investigations.

### 3 The Empirical Model

We use a Bayesian Vector Autoregression where the coefficients are allowed to smoothly drift over-time to describe the evolution of the US economy. Several reasons drive our choice. First, time variations and structural changes may be important. Second, there can be various features of the US economy that have changed and they should be considered simultaneously rather than separately. Third, we believe that changes in macroeconomic relationships suggest more evolution rather than sudden breaks<sup>7</sup>. Fourth, our model represents a generalization of fixed coefficients VAR and includes this as a special case.

#### 3.1 VAR Representation

Let  $y_t$  be a  $n \times 1$  vector of time series which admits the following reduced form VAR representation

$$y_t = A_{0,t} + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + \dots + A_{p,t}y_{t-p} + \varepsilon_t \quad (3)$$

where  $A_{0,t}$  is an  $n \times 1$  vector of time-varying intercepts,  $A_{i,t}$ , for  $i = 1, \dots, p$ , are  $n \times n$  matrices of time-varying coefficients<sup>8</sup> and  $\varepsilon_t$  is a  $n \times 1$  Gaussian white noise process with zero mean and covariance  $\Sigma$ . Let  $K_t$  be any, possibly time varying, nonsingular matrix such that  $K_t K_t' = \Sigma$ . Rewriting the model in terms of orthogonal shocks we have

$$y_t = A_{0,t} + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + \dots + A_{p,t}y_{t-p} + K_t e_t \quad (4)$$

where  $e_t = K_t^{-1} \varepsilon_t$  is a Gaussian white noise process with zero mean and covariance the identity matrix  $I_n$ . Equation (4) represents the class of structural representations of the vector of time series and each particular matrix  $K_t$  defines a particular representation of  $y_t$ .

#### 3.2 Dynamics

Model dynamics are summarized in the mechanisms through which shocks spread over time. Impulse response functions measure the effects of a shock on future time series relative to some

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<sup>7</sup>We do not claim that breaks from period to period are unlikely to occur but rather we argue that most of macroeconomic changes, in particular those related to the labor market, take place in a gradual way.

<sup>8</sup>The fixed coefficients VAR is a special case of the model in which  $A_{i,t} = A_i$  for all  $i$  and  $t$ .

benchmark case. Equation (3) has the following companion form

$$\mathbf{y}_t = \mu_t + \mathbf{A}_t \mathbf{y}_{t-1} + \epsilon_t$$

where  $\mathbf{y}_t = [y'_t \dots y'_{t-p+1}]'$ ,  $\epsilon_t = [\epsilon'_t 0 \dots 0]'$  and  $\mu_t = [A'_{0,t} 0 \dots 0]'$  are  $np \times 1$  vectors and

$$\mathbf{A}_t = \begin{pmatrix} & A_t \\ I_{n(p-1)} & 0_{n(p-1),n} \end{pmatrix}$$

where  $A_t = [A_{1,t} \dots A_{p,t}]$  is an  $n \times np$  matrix,  $I_{n(p-1)}$  is an  $n(p-1) \times n(p-1)$  identity matrix and  $0_{n(p-1),n}$  is a  $n(p-1) \times n$  matrix of zeros. Iterating  $k$  period forward and omitting for simplicity the constant term, we obtain

$$\mathbf{y}_{t+k} = \mathbf{A}_{t+k} \dots \mathbf{A}_t \mathbf{y}_{t-1} + \mathbf{A}_{t+k} \dots \mathbf{A}_{t+1} \epsilon_t + \mathbf{A}_{t+k} \dots \mathbf{A}_{t+2} \epsilon_{t+1} + \dots + \mathbf{A}_{t+k} \epsilon_{t+k-1} + \epsilon_{t+k}$$

Let  $\mathcal{S}_{i,j}(M)$  be a selection function, a function which selects the first  $i$  rows and  $j$  columns of the matrix  $M$ . Taking as a benchmark case the case of no-shock occurrence, and assuming that coefficients and shocks  $\epsilon_t$  are uncorrelated, the matrix of dynamic multiplier  $\mathcal{S}_{n,n}(\mathbf{A}_{t+k} \dots \mathbf{A}_{t+1})$  describes the effects of  $\epsilon_t$  on  $y_{t+k}$ , while the effects associated to structural shocks can be derived from the relation  $\epsilon_t = K_t e_t$  and are given by  $\mathcal{S}_{n,n}(\mathbf{A}_{t+k} \dots \mathbf{A}_{t+1}) K_t$ . Few important features of the impulse response functions in our setup need to be highlighted. First, the effects of the shocks depend on future coefficients: unlike in the fixed coefficients case, here propagation mechanisms are subject to future changes in the structure of the economy. Second, the effects of a shock for the same  $k$  but different  $t$  may vary over time, both because at each time period we can associate a particular reduced form dynamic multiplier, and because the identifying matrix,  $K_t$ , may change over time. Third, data provide information about model dynamics up to the end of the sample date,  $T$ , because posterior information is available only for VAR coefficients up to that date. Thus in order to study dynamics after  $T$  some forecast of future VAR coefficients is needed. To construct impulse response functions we assume  $\mathbf{A}_{T+j} = \mathbf{A}_T$  for all  $j = 1, 2, \dots$ . Three reasons motivate our choice. First, we want to use all the information contained in the data. Second,  $\mathbf{A}_T$  represents the best forecast of  $\mathbf{A}_{T+j}$  whenever coefficients evolve according to a random walk. Third, impulse response functions, under this assumption, have useful long-run properties<sup>9</sup>. Formally impulse response functions of a shock occurring at time  $t$  at horizon  $k$  are given by

$$IR(t, k) = B_{t,k} K_t$$

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<sup>9</sup>Other alternatives are available. For instance, Canova and Gambetti (2004), in a similar Bayesian approach, consider the effects of the shock under all the possible realizations of future coefficients for some finite horizon of interest. This implies drawing future coefficients from the prior density conditional to a draw for coefficients up to time  $T$  from the posterior. Actually, while useful for finite horizons, such an approach creates non-trivial complications for infinite horizons since available necessary and sufficient conditions for the convergence of  $\sum_{i=1}^k \prod_{j=1}^i \mathbf{A}_{t+j} \dots \mathbf{A}_{t+1}$  are too restrictive for our purposes.



where

$$B_{t,k} = \begin{cases} \mathcal{S}_{n,n}(\mathbf{A}_{t+k} \dots \mathbf{A}_{t+1}) & \text{if } t+k < T \\ \mathcal{S}_{n,n}(\mathbf{A}_T^{t+k-T} \mathbf{A}_T \dots \mathbf{A}_{t+1}) & \text{if } t+k \geq T \end{cases}$$

Thus for each  $t = 1, \dots, T$  we have a path of impulse response defined by the sequence  $\{B_{t,k}K_t\}_{k=1}^{\infty}$  and cumulated impulse response functions  $\{\tilde{B}_{t,k}K_t\}_{k=1}^{\infty}$  where  $\tilde{B}_{t,k} = \sum_{j=1}^k B_{t,j}$ . First note that, as in the fixed coefficients case, if all the eigenvalues of any realization of  $\mathbf{A}_T$  are smaller than one in modulus impulse response functions converge pointwise (see Appendix A). In particular the limit of cumulated impulse response functions will be varying over time, depending on  $t$ . Second, the speed of convergence and thus the long-run cumulated effects will depend on the end-of-sample date coefficients. We use the last available time period observation in order to maximize the quantity of information from the data, but in the empirical part we will investigate the sensitivity of our results to different end-of-sample dates<sup>10</sup>, that is we will end the sample at arbitrary dates different from  $T$ .

### 3.3 Identification

In order to identify the model and recover the representation of  $y_t$  in terms of structural shocks we should, in general, fix for all  $t = 1, \dots, T$  a particular matrix  $K_t$ . Since our focus is only on technology shocks we only partially identify the model, that is we only fix a column of  $K_t$  without attempting to identify all the other shocks. The restriction we use is the same as in Galí (1999) and CEV: the technology shock is the only shock affecting long-run labor productivity<sup>11</sup>. For each  $t = 1, \dots, T$ , let  $S_t$  be the unique lower triangular matrix such that  $S_t S_t' = \tilde{B}_{t,\infty} \Sigma \tilde{B}_{t,\infty}'$ . We set

$$K_t = \tilde{B}_{t,\infty}^{-1} S_t$$

Thus the path of structural impulse response functions for each  $t = 1, \dots, T$  will be given by

$$IR(t, k) = B_{t,k} \tilde{B}_{t,\infty}^{-1} S_t, \quad k = 1, 2, \dots$$

If, as in the fixed coefficients case, labor productivity is ordered first, the first shock  $e_{1t}$  is the technology shocks<sup>12</sup>.

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<sup>10</sup>Another feasible alternative would be to study local dynamics, i.e assuming that all the coefficients are constant from the period in which the shock occurs. In this case however a lot of in-sample information would not be used and for this reason we do not pursue this strategy.

<sup>11</sup>It is clear that it is the only shock affecting long-run labor productivity among the shocks in  $e_t$ . In fact in our model potentially shock to coefficients could affect variables at long-run horizons, but in this case they would have a different interpretation, since they would affect permanently the growth rates of labor productivity.

<sup>12</sup>We do not attempt to identify the others  $n - 1$  shocks and we simply fix them using an atheoretical recursive long-run ordering among the other variables. However it is important to stress that such an ordering does not affect the dynamics of the so identified technology shock, in fact it can be showed that by changing the ordering of the other variables the responses of all variables to technology shocks remain unchanged.

### 3.4 Specifications and Estimation

#### 3.4.1 A State-Space Representation

In order to understand model estimation it is useful to rewrite the model in a state space form. Let  $A_t = [A_{0,t}, A_{1,t} \dots A_{p,t}]$ ,  $x'_t = [1_n, y'_{t-1} \dots y'_{t-p}]$ , where  $1_n$  is a row vector of ones of length  $n$ , let  $vec(\cdot)$  denote the stacking column operator and let  $\theta_t = vec(A'_t)$ ,  $\theta^T = [\theta'_1 \dots \theta'_T]'$  and  $y^T = [y'_1 \dots y'_T]'$ . Then (3) can be written as

$$y_t = X'_t \theta_t + \varepsilon_t \quad (5)$$

where  $X'_t = (I_n \otimes x'_t)$  is a  $n \times (np+1)n$  matrix,  $I_n$  is a  $n \times n$  identity matrix, and  $\theta_t$  is a  $(np+1)n \times 1$  vector. Treating  $\theta_t$  as a hidden state vector, equation (3) represents the observation equation of a state space model. Let  $f(\cdot)$  be a normal density and let us assume that  $f(\theta_{t+1}|\theta_t, \phi)$  can be represented as

$$\theta_{t+1} = F\theta_t + u_{t+1}$$

where  $u_t$  is a  $(np+1)n \times 1$  Gaussian white noise process independent of  $\varepsilon_t$  with zero mean and covariance  $\Omega^{13}$ ,  $\phi = \{\Omega, \Sigma\}$  and  $F$  is a diagonal matrix of constant coefficients. We assume that  $\theta_t$  evolves according to

$$p(\theta_{t+1}|\theta_t, \phi) \propto \mathcal{I}(\theta_{t+1})f(\theta_{t+1}|\theta_t, \phi) \quad (6)$$

where  $\mathcal{I}(\theta_{t+1})$  is an indicator function assuming value zero if roots of the associated VAR polynomial are outside or on the unit circle and one otherwise. In other words the function discards path of  $\theta_t$  whenever the associated VAR polynomial roots are unstable. Such a restriction ensures convergence of impulse response functions and then makes the above discussed identification scheme implementable since it cuts the support of the distribution in correspondence of draws with unit or explosive roots<sup>14</sup>. Equation (6) represents the conditional prior for  $\theta_t$ . We assume that  $F_{jj} = 1$  if the coefficient is associated to lagged variables or equal to 0.999 for the time-varying intercept terms. The first assumption yields random walk coefficients for lagged variables provided that the roots restriction is satisfied. On the other hand, we assume that the intercept term evolve according to a very highly persistent but stationary process. This is needed since a random walk process for the intercept term would signify infinite prior variance. Except for the restriction on the unit root and the assumption of stationarity of the time-varying intercept term the above state space representation is identical to the one originally proposed by Cogley and Sargent (2001).

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<sup>13</sup>We estimate the model under different assumption on  $\Omega$ : diagonal, block-diagonal with the block corresponding to the coefficients of the same equation and for the bivariate case we also specify it as full matrix. While the degree of time variation depends on the specific assumptions main results are roughly unchanged. Furthermore independently on the particular specification, structural coefficients are always allowed to evolve in a correlated manner (see Canova and Gambetti, 2004).

<sup>14</sup>The restriction on the VAR polynomial roots makes the model locally stationary at each point in time, which does not imply global stationarity.

### 3.4.2 Estimation Strategy

Estimation is done in two steps. First, we characterize the unrestricted posterior distribution  $p_u(\theta^T, \phi|y^T)$ . Second, we discard the draws that do not satisfy the restrictions on the VAR polynomial roots. Since the posterior distribution is not available in closed form, we simulate it using MCMC methods. Specifically, the first step is done using the Gibbs sampling algorithm where the time-varying parameters and the hyperparameters are treated as two different blocks, while the second is done by applying a rejection sampling to the unrestricted posterior distribution. Because of the heavy notation and the technicalities involved with the construction of posterior distributions we defer the details of the estimation to Appendix B. Once the posterior distribution is available, we draw a path for the states and the variances, we identify the technology shock and we compute the associated structural impulse response functions. After having computed a sufficiently large number of draws inference is implemented by taking the mean and 68% confidence band.

### 3.4.3 Specifications and Data

We use a bivariate VAR including labor productivity growth and per capita hours worked, and a four variables VAR in which the interest rate and inflation are added (the  $R\pi$ -specification henceforth). The bivariate VAR is important since it is the benchmark specification from which the debate originates. On the other hand, VARs that include more variables are important both because it is of interest to study the effects of technology shocks also on other macroeconomic variables, and because the results may change compared to the bivariate specification.

We use quarterly US data spanning from 1954:IV to 2003:III taken from the FRED II data base of the Federal Reserve Bank of San Louis. We initially estimate the model for the sample 1954:IV-1966:IV using fixed coefficients VAR to calibrate prior parameters and then reestimate it from 1967:I up to 2003:III. The variables used are the following: the first differences of the logs of labor productivity in the non-farm business sector (OPHNFB); the first difference of the logs of the GDP deflator (GDPDEF); the federal funds rate (FEDFUNDS); per capita hours are defined as hours worked (HOANBS) divided by the non-institutional population over 16 (CNP16OV). We use both growth rates of hours worked and the levels in logs.

## 4 Results

### 4.1 Impulse Response Functions

For each quarter we collect the posterior mean<sup>15</sup> of the impulse response functions for horizons up to 20 quarters. All 3D IRF are plotted using the following convention: on the  $x$ -axis there are quarters after the shock, on the  $y$ -axis there are the time periods, from 1967:I up to 2003:II and the  $z$ -axis there is the value of the response.

#### 4.1.1 Bivariate VARs

Figure 3 displays the response of per capita hours worked (level specification in the bottom panel and the growth rates in the top panel) to a positive technology shock in the bivariate VAR. No matter the specification used, levels or growth rates, before early 90's the response of hours worked is negative and significant on impact. It is also quite persistent, lasting on average about one year and reaching the minimum at about two or three quarters after the shock. In the levels specification the degree of persistency gradually reduces from mid 80's. Starting from mid 90's, the response becomes positive on impact, although not significant, and hump shaped. On the other hand in the growth rates specification the response is always negative and after mid 80's also permanent. While generating slightly different dynamics in the last part of the sample, until early 90's both empirical specifications point to a persistent reduction of hours.

#### 4.1.2 Larger VARs

Figure 4 displays the response of per capita hours worked (level specification in the bottom panel and the growth rates in the top panel) to a positive technology shock in the  $R\pi$  VAR. Figure 5 focuses on the posterior mean of the impact effect with 68% confidence bands. The mean response of hours at all dates and for both specifications is negative on impact although significant only until mid 90's. As in the bivariate case, in the levels specification the response is much more persistent and pronounced before mid 80's while after that date the degree of persistency tends to reduce. A similar pattern concerns the size of the response. At the end of the 90's the response is about one fifth of the response during the 70's and the reversion to the pre shock level is completed after one year, while in the first part of the sample it occurs after two or more years. The response for the growth rate specification is almost identical to the one in the bivariate case, it is negative at all horizon and permanent after mid 80's. In sum, two important results arise. First, no matter the specification for hours worked, until early 90's technology shocks are contractionary, hours worked fall. Second the response on impact displays a break dated early 90's: before that date

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<sup>15</sup>Results are very similar when the medians are considered instead of the means.

it is very pronounced and statistically different from zero while after it is much smaller and not significant.

Figure 6, 7 and 8 display the response of labor productivity and output and inflation respectively for the levels specification<sup>16</sup>. Labor productivity and output increase on impact, the former increasing more than the second because of the reduction in the labor input. At few quarters after the shock, both responses begin to climb to their new steady state level. Notice that, consistently with the response of hours, the response of output in the levels specification is smaller on impact in the first part of the sample and it takes more quarters to reach the new long-run level. Interestingly, at all dates, the impact effect of labor productivity is smaller than the long-run effect. Hence technology shocks appear to spread gradually or, at least, they affect both labor productivity and output gradually. It should be stressed that while the same finding emerges in the fixed coefficients case with hours worked in growth rates, in the level specification the response of labor productivity is substantially different (see CEV): when hours enters in levels the impact effect of labor productivity generally overshoots its new steady state. So that, labor productivity gradually declines to the new long-run equilibrium instead of increasing to it. Thus, when one takes into account time variations, not only the dynamics of hours change compared to fixed coefficients VAR, but also those of labor productivity. Inflation falls on impact and for few quarters after the shock at all dates. The response of inflation is much more persistent before 1980 than after, in particular before 1980 the response is hump shaped reaching the minimum after one year, while after it steadily reduces after a large initial effect. The result suggests that technology shocks could have contributed substantially to the changes in terms of volatility and persistence of inflation after mid 80's confirming results by Canova, Gambetti and Pappa (2005).

## 4.2 Technology Shocks and the Business Cycle

Are technology shocks important for business cycles? Are technology shocks responsible for the pattern of output and employment fluctuations associated with the business cycle? The empirical framework we use allows us to address these questions by decomposing historical fluctuations in output, labor productivity and hours into a technology and a non-technology component. From the posterior distribution we draw realizations for structural coefficients and for each realization we collect the particular realization of structural shocks. Then using only the estimated technology shocks and the structural coefficients we compute the predicted time series for output, labor productivity and hours worked. Using a bandpass filter, we extract from the resulting series the component associated with business cycle frequencies and we compute correlations and variances. We repeat the same exercise for the non-technology component. We perform the analysis using

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<sup>16</sup>We omit impulse response functions for first differences specifications, available upon request, since they are very similar.

both the levels and the growth rates specification for hours worked.

Table 1 reports the results for the technology shock. Point estimate of the correlation between output and hours attributable to technology shocks is 0.76 in the bivariate and 0.55 in the multivariate case when hours are specified in levels. Only in the bivariate case the correlation generated by technology shocks is similar to the correlation arising in actual data and it is entirely attributable to the dynamics arising in the last ten years of the sample. Correlations reduce substantially when hours are specified in first differences. In this case they are 0.46 in the bivariate and 0.28 in the larger VAR. On the other hand, non-technology shocks produce correlations between output and hours which are of the order of about 0.9. The picture is even more clear if we look at the portion of explained variance. In the levels specification technology shocks account for about 15-28% of the hours variance and 14-25% of the output variance, while in first difference they are even smaller, 9-15% and 11-14% respectively. This means that the non-technology component account for at least the 75% of cyclical output fluctuations.

By investigating the pattern of output fluctuations and the component associated with technology shocks under various specifications two robust facts emerge. First, the amplitude of total output fluctuations substantially reduces over time, particularly starting from mid 80's. Second, the size of fluctuations due to technology shocks are roughly constant over time. This has two main implications. First, technology shocks can hardly be considered the main cause of the changes observed in the US business cycles in terms of size of fluctuations. Second, because fluctuations associated to technology shocks are roughly constant while those associated to the non-technology component reduce over time, this means that contribution of technology shocks must have increased after mid 80's.

A new interesting feature emerges in our framework. The non-technology component includes two elements: the non-technology shocks ( $e_{2t}, \dots, e_{4t}$ ), and a second part resulting from shocks in the time-varying intercept term propagated by the stochastically time-varying coefficients. Adding the portion of variance explained by technology and non-technology shocks a portion of output variance of about 5-15%, depending on the particular specification, is left unexplained. This means that even if no such shocks occur, nonetheless we could observe fluctuations in output and hours accounting for about the 5-15% of the variance of actual output fluctuations and generating correlations of about 0.8-0.9. This finding is clearly ruled out in fixed coefficients. However when the linear structure is replaced by a non-linear one in which non-linearity comes from stochastically varying coefficients, multiplicative disturbances and shocks to the intercept term play a role in shaping US business cycles fluctuations. This evidence is consistent with the idea that transition dynamics arising from changes in trends or means are gradual instead of abrupt and they generate substantial movements in output and hours which are recognizable at the business cycles frequencies.

### 4.3 Testing Time-Variations

We perform two types of test: the first is an informal test on the rate of drift of the reduced form coefficients, while the second is based on posterior intervals for the differences in impulse response functions. Recall that  $\Omega$  represents the variance of the shocks in the unrestricted law of motion of the coefficients. As shown in the appendix, we calibrated the prior scale matrix,  $\Omega_0$ , so that a priori there is a high probability of small changes in the coefficients. In all the specifications the posterior distribution of  $tr(\Omega)$  shifts to the right of  $tr(\Omega_0)$ , with a 80-90% of posterior mass concentrated on values higher than  $tr(\Omega_0)$ . This means that the data are shifting the distribution toward a region of higher, compared to our prior, coefficients time variations. In other words data seem to favor a specification in which coefficients are varying over time. Figure 9 exemplifies the result for the bivariate case with hours in levels: the trace of the prior scale matrix,  $tr(\Omega_0)$  (the segment) lays on the left tail of the posterior histogram of  $tr(\Omega)$ .

The second test is a simple posterior interval test. The idea is to test whether the responses are different over the sample. Let  $\bar{t}$  be some fixed date. For all  $t = 1, \dots, \bar{t} - 1, \bar{t} + 1, \dots, T$  we draw from the posterior distribution of the impulse response functions to characterize the posterior of

$$D(t, \bar{t}, k) = IR_{2,1}(t, k) - IR_{2,1}(\bar{t}, k)$$

which is the difference between the response of hours at time  $t$  and  $\bar{t}$  at lag  $k$  to a technology shock. We take a posterior interval centered at the posterior mean of  $D(t, \bar{t}, k)$  and we check whether the zero is included. In case of no significant time-variations we should find that zero is included in the interval for all  $t$ . In the levels specification we set  $\bar{t} = 1998:III$ <sup>17</sup>. For  $k = 0$ , we find that there are 42 dates, concentrated between 1972 and 1981, for which the difference is significantly different from zero. At such dates the posterior probability of the impact effect to be smaller than the impact effect in 1998:II is on average about 0.9. For  $k = 4$ , one year after the shock, there are 4 dates, between 1978 and 1979, for which the response is different from zero. For the bivariate case numbers are very similar: we find 36 dates for  $k=0$  and 4 for  $k=4$  in which the differences are significant. For the growth rates specifications we choose 2003:III. In this case we do not find significant differences in the responses, probably because the high uncertainty surrounding the response after early 90's makes the confidence band for  $D(t, \bar{t}, k)$  extremely wide.

## 5 Fixed vs. Time-Varying Coefficients VARs

We compare our findings with those arising from standard VAR. In order to make the comparison clear and simple we limit the attention to the bivariate specification.

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<sup>17</sup>We choose  $\bar{t}$  to be the date in which the 68% lower bound for the impact effect is higher. That is the date in which is more probable to find differences with the responses at other dates.

## 5.1 What Explains the Differences?

Once time variations are allowed for, hours significantly reduce on impact at least until mid 90's also when specified in levels. Why do results change with respect to the fixed coefficients case? The goal here is to investigate what are the reduced form coefficients responsible for the switch in sign of the response. We proceed as follows: first we divide all the reduced form coefficients in four blocks, each corresponding to the coefficients of the lags of the same variable in one equation; second, we set all the coefficients belonging to the same block constant and equal to the corresponding fixed coefficient estimates; third, we draw from the posterior for all the remaining time-varying coefficients and we compute the implied impulse response functions; we repeat this procedure for all the blocks. The switch from negative to positive occurs when the block corresponding to hours worked in the labor productivity equation is set to be constant over time. In this case the implied impulse response functions at all dates are positive and hump-shaped (see Figure 10), whereas when the other coefficients are replaced the resulting dynamics are roughly unchanged, in particular the sign of the response is unaffected. Moreover, time variations in the response of hours completely disappear, the impact effect being nearly constant over the whole sample. Therefore, such coefficients not only account for the switch of the sign, but they also seem to drive time variations in the response of hours.

Figure 11 focuses on both the fixed and time-varying estimates of the coefficients of the lags of hours worked in the labor productivity equation. Few features are worth noting. First, all the time-varying estimates, apart the coefficient for lag one which is roughly constant over-time, display the same pattern. They are U-shaped with a clear upward trend starting from mid 80's and crossing fixed coefficients estimates at some date around the end of the 80's (for lag 2) and the beginning of the 90's (for lag 3 and the sum of lagged coefficient). Second, the long-run coefficient, the sum of lagged coefficients, seems to be the most important, from a quantitative point of view, in tracking time variations in the response of hours since it exactly matches the pattern of variations in the impact effect. Interestingly we find that the correlation between this coefficient and the impact effect is 0.9. Third, fixed coefficients estimates resemble a sort of weighted average of the time-varying estimates in which the weight attributed to the last part of the sample is higher than that attributed to the first part. This is probably due to the sharp and synchronized increase in labor productivity growth and per capita hours worked starting from early 90s. This is consistent with the finding that by running the analysis with fixed coefficients and hours in levels excluding from the sample the last ten years, hours fall.

As an additional check we re-estimate the model constraining the intercept term to be constant over time while letting all other coefficients vary. This is an important exercise since, as shown by Fernald (2004), once one allows for trend breaks in fixed coefficients VAR, hours fall also in levels. The reason, he claims, is that short-run dynamics are dominated by a non causal low frequencies



correlation between labor productivity growth and the levels of hours worked attributable to synchronized changes in the means of the two series. Therefore, it could be that these, rather than changes in VAR coefficients, are the true responsible for the negative response of hours. If actually trend changes are responsible for the switch, by constraining them we should observe a rise of hours. We find that hours reduce and the response both in terms of persistence and size is nearly identical to the benchmark case. Thus, although probably important, changes in trend labor productivity do not seem to be the main, or at least the only, factor affecting dynamics of hours worked.

## 5.2 Encompassing Fixed Coefficients Specifications

CEV show that when the true model is the VAR with hours in levels and the analysis is performed using hours in growth rates, hours fall. The converse is not true: when the growth rates is the true model and hours are specified in levels again hours reduce. Therefore, they argue that the specification with hours in levels is more plausible since it can explain also the results of the misspecified model while the growth rates specification does not. Here, using a similar approach we investigate whether our model can encompass fixed coefficients VARs. Specifically we study whether, by running the analysis with fixed coefficients and data generated by the time varying-coefficients model, we can replicate the two basic facts: hours fall in growth rates and increase in levels. We proceed as follows. We assume that the time varying coefficients VAR is the true model and we set all the coefficients at their posterior mean values. Using the *true* model we generate 500 new time series data for labor productivity growth and hours worked. Then for each new vector of time series we estimate the response of hours to technology shocks under fixed coefficients using both specifications, levels and growth rates. Finally we average over the 500 responses.

Figure 12 displays the results when the time varying coefficients VAR with hours in growth rates is assumed to be the true model. Solid and dotted lines represent the responses of hours, specified in levels and growth rates respectively, estimated with fixed coefficients VARs and actual data. Starred lines, solid and dotted, represent the same responses but arising with simulated data and averaged over the 500 realizations. When the true model is the time varying coefficients VAR with hours in growth rates hours decline under both specifications. This means that, while easily explaining the Gali's results, the model fails in explaining the CEV's results, since the response of hours is negative instead of being positive. Figure 13 displays the same responses but when the time varying coefficients VAR with hours in levels is assumed to be the true model. In this case hours reduce when specified in first differences and increase when specified in levels, exactly as with actual data. The model encompasses both fixed coefficients specifications, since the misspecified VAR exactly reproduces the results of Gali and CEV. This means that using fixed

coefficients VAR and hours in levels we would conclude that hours increase while the true model implies a significant decline in hours for most of the sample period. Under this encompassing criterion the time-varying levels specification seems to perform better than the growth rates one since it is able to explain all the results previously found in literature.

## 6 Structural Explanations for the Dynamics of Hours Worked

### 6.1 Explaining the Decline of Hours

There exist basically two classes of structural explanations of why hours worked may fall after a positive technology shock. The first relies on the presence of some frictions in the economy, while the second relies on frictionless models in which technology generates large wealth effects. Here we investigate whether theoretical predictions match our empirical findings.

#### 6.1.1 Nominal vs. Real Frictions

A first explanation of the decline of hours relies on the presence of sticky prices and a not completely accommodative monetary policy. The intuition originally provided by Gali (1999) is the following. Consider an economy where in equilibrium output equals real balances, prices are set in advance and the monetary policy follows a simple money rule<sup>18</sup>. If, in response of a positive technology shock, monetary policy is not sufficiently accommodative and aggregate demand expands less than the increase due to the technological improvement, then employment must fall in order to keep supply and demand in the goods market in equilibrium<sup>19</sup>. A second explanation relies on the presence of some real rigidities. Francis and Ramey (2001) propose a modification of the standard RBC model which can potentially account for the reductions of hours after a technological improvement. The basic ingredients are habit formation in consumption and capital adjustment costs. The authors show that the response of consumption and investment is much more sluggish than in the standard case because consumers prefer not to change consumption by too much and investment is made expensive by the capital adjustment costs. Thus if the resulting increase in output is smaller than the increase in productivity hours must fall.

The two explanations have, as stressed by Francis and Ramey (2004), very different implications in terms of the response of real wages. In the sticky price model real wages either fall or at most increase by little on impact and then they gradually converge to a new higher steady state

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<sup>18</sup>Similar mechanisms generate from more complete dynamic models in which price predeterminacy is substituted with a Calvo-type random price adjustment, see e.g. King and Wolman (1996).

<sup>19</sup>While monetary policy is a crucial ingredients for such an explanation, it should be stressed that a money target rule is not a necessary condition to generate the fall in hours worked; actually some authors (Basu, 1998, Gali and Rabanal, 2004) show that sticky prices model with more realistic policy rules, like a Taylor rule, are still able to generate the decline in hours.

level. On the other hand the habit formation adjustment costs model predicts that real wages immediately rise overshooting the long-run level slightly. We reestimate the model adding the real wage. The resulting response of wages closely track from a qualitative point of view that of productivity. Specifically real wages slightly increase on impact and then gradually rise until reaching the new long-run level. While in sharp contrast with the predictions of the model embedding real rigidities, the behavior of wages appears to be roughly consistent with sticky prices models.

### 6.1.2 Wealth Effect and Slow Technological Change

In an important paper, Campbell (1994) showed that technology improvement may generate "perverse effects" on labor inputs. Contrary to common wisdom, in a RBC model a persistent and permanent negative technology shock (what the author calls a "productivity slowdown") may actually increase hours worked for some quarters. The reason is that, due to its slow diffusion, the shock triggers a large wealth effect that dominates the substitution effect in the short-run and makes consumers to substitute leisure for work. The increase in hours can be so sustained that output can rise in the very short-run. By reversing the sign of the shock, the above mechanism could explain why a technological improvement may actually reduce, instead of raising, hours worked. A similar mechanism emerges in the recent works by Linde (2004) and Rotemberg (2000).

One of the main implications of the dynamics arising from those models is that both consumption and consumption-to-output ratio must increase on impact. The former increases because of the wealth effect, while the second increases because investment reduces since agents anticipate that marginal productivity of capital will be higher in the future. In order to assess the relevance of this explanation we estimate the model adding consumption and investigating the response of hours and both consumption and consumption-to-output ratio. As in previous specification hours fall on impact and the dynamics are almost unchanged. Consumption rises on impact over all the sample although the response is not significantly different from zero except for few year at the end of the 90s. On the contrary consumption-to-output declines on impact for all the dates and until mid 80s the response is also significantly different from zero. The sign of the response of consumption-to-output ratio is at odds with the predictions of RBC models with slow technological changes. Therefore while it cannot be excluded that large wealth effects stand behind the reduction of hours worked, such effects do not seem to be generated by technological progress diffusing slowly throughout the economy.

## 6.2 Explaining Time Variations: the Fed’s Time-Varying Response

From an econometric point of view the reduction in magnitude of the response of hours on impact seems to depend to a large extent on changes in the reduced form coefficients in the labor productivity equation. Nevertheless there could be several possible structural explanations for this pattern. As mentioned earlier, in a sticky prices model the response of hours worked crucially depends on the monetary policy conduct. The more expansionary is the monetary policy after the technological improvement, the smaller is the decline of hour worked because the higher is the expansion in the aggregate demand. Therefore, changes in monetary policy could explain why the response of hours has changed over time. Some authors (see e.g. Orphanides and GLV) argue that before 1979 monetary authorities had a less aggressive stance against inflation and were giving more importance to output stabilization. Due to mismeasurements of potential output, movements in interest rate overshooted those prescribed by the optimal rule and policies adopted before 1979 turned out to be overstabilizing. Such a conjecture could explain why the fall of hours worked is more pronounced before mid 80’s than after<sup>20</sup>.

Our framework allows us to study whether changes in the response of hours depend on shifts in monetary policy preferences. We estimate a simple Taylor rule in which the interest rate responds only to contemporaneous inflation and output growth<sup>21</sup>

$$i_t = a_t \pi_t^{tech} + b_t \Delta y_t^{tech} + \varepsilon_t \quad (7)$$

where  $i_t$  is the federal funds rate and  $\pi_t^{tech}$ ,  $\Delta y_t^{tech}$  are respectively the component of inflation and output growth associated to technology shocks<sup>22</sup>. The previous explanation would hold if the coefficient on output is high before mid 1980 than after. Figure 14 displays the two coefficients,  $a_t$   $b_t$ , along with the 68% confidence bands. First, consistently with a large amount of evidence in empirical literature, we find that monetary policy stance becomes more aggressive against inflation from early 80’s, the coefficients raising from about 1 during the 70’s up to 2.5-3 during the 80’s. However, differently from what is argued by the majority of works, and consistently with a growing stream of literature (see e.g., Bernanke and Mihov, 1998, Canova, 2004, Canova and Gambetti, 2004, Primiceri, 2005, Sims, 2001, and Sims and Zha, 2004) the change does not represent a permanent break. Interestingly around 1992 the coefficient reduces again, being about 1.4, and in 2001 is not significantly different from the 70’s level, around 1. Second, the coefficient on output is almost constant over all the sample, in particular we do not find a significant change after mid 80’s. Hence the result is hardly consistent with a primary role of monetary policy in

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<sup>20</sup>Similar results can arise in a framework where monetary authorities are learning, see Lansing (2000).

<sup>21</sup>An alternative strategy would be to compute the ratio between the response of the interest rate and inflation and output growth. We do not follow this strategy because in that case we would not control for the other variables.

<sup>22</sup>Note that by construction the regressors exogenous and orthogonal to the residuals justifying the Kalman Filter estimation.

shaping changes in the transmission of technology shocks<sup>23</sup>.

## 7 Robustness and Extensions

We perform a number of robustness checks. We first check whether our results are robust to the choice of the end-of-sample date and definitions of IRF and second whether alternative identification schemes give qualitatively similar results. In addition, we extend the model to consider also investment-specific technology shocks.

### 7.1 Alternative End-of-Sample Dates and IRF Definitions

The definition of impulse response functions used in the paper has a potential drawback when identification is achieved with long run restrictions. Structural short-run dynamics for each date in the sample depend on the end-of-sample coefficients matrix  $\mathbf{A}_T$ . We choose as end-of sample date the last available observation for the data in order to maximize the available information. However, it could be that different choices of  $T$  yield different results, in particular for the last part of the sample. Therefore, we cut the sample at arbitrary dates, we choose two and four years before the last available observation and we run the analysis using  $\mathbf{A}_{T-8}$  and  $\mathbf{A}_{T-16}$ . As expected small quantitative differences emerge mainly for the quarters in the last part of the sample. However our main conclusions are very robust. First, the response of hours worked is still negative at all horizons with shapes almost identical to those resulting from the benchmark case. Second, the size of the impact effect is reducing over time in absolute value, in particular the response of hours is not significantly different from zero after early 90's.

We also check if results are robust using a different definition of impulse response functions. Specifically, we sample future coefficients from the prior density conditional to a draw from the posterior for the in-sample-coefficients. In so doing we take into account future coefficients variation. Clearly, we have to discard the draws which yield impulse response functions which do not satisfy some convergence criterion. Also in this case, mean impulse response are almost identical to the benchmark case.

### 7.2 Sign Restrictions

Recently, a number of papers questioned the validity of the conclusions drawn using long-run restrictions (see e.g. Uhlig, 2003). Some authors (Francis, Owyang and Theodorou, 2003, Dedola and Neri, 2004, and Peersman and Straub, 2004, 2005), taking a radically different approach,

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<sup>23</sup>The same conclusion is reached by looking at the response of the real interest rate. Actually we do not find evidence that the real interest rate reacts more before 1979 than after. Nevertheless we do find a clearly declining trend in the response of the real rate but only starting from early 90's and lasting until 2000.

suggest to use inequalities restrictions directly derived from DSGE models, in the spirit of the restrictions originally proposed by Canova and De Nicolò (200) and Uhlig (2005). Here we check the robustness of results when long-run identifying restrictions are replaced with sign restrictions. We take as identifying restrictions a set of sign inequalities which are robust under different specifications of the technology process. Specifically we assume that a positive technology shock (i) does not raise inflation and the interest rate for three quarters and (ii) does not decrease labor productivity for 40 quarters after the shock. We leave all the other shocks unidentified. We use the same draws for reduced form coefficients used under long-run restrictions and the implementation of the restrictions is identical as Canova Gambetti and Pappa (2005). Again per capita hours worked fall after a positive technology shock. The impact effect is negative, the responses reach their minimal level between the first and second quarter after the shock and then they begin to climb back toward the pre-shock level and after between one and two years the responses become positive. Responses are qualitatively similar to those found under long-run restrictions while time variation seem to be relatively limited: in fact responses are roughly similar at all dates.

### 7.3 Investment-Specific Technology Shocks

Greenwood, Hercowitz and Krusell (2000) (GHK henceforth) put forward a version of the RBC model in which the main source of technological progress is not of the aggregate sector neutral kind as we identified but rather is specific to the investment sector. Using a calibrated version of the model, the authors find, that investment-specific technology shocks explain about 30% of output fluctuations. Similarly, Fisher (2005) through VAR analysis finds that unlike neutral shocks investment specific technological change contribute for about 40-60% to aggregate fluctuations. We investigate how results change when also investment-specific technology shocks are considered in the analysis. We estimate the TVC-BVAR using, in the following order, real price of investment, labor productivity and per capita hours worked. Following the identification scheme proposed by Fisher (2005), we assume that (i) neutral and investment-specific technology shocks are the only shocks affecting long run labor productivity and that (ii) investment-specific technology shocks the only shock affecting long run real price of investment. Using the previous recursive long run scheme, the first shock will be the investment-specific and the second the sector-neutral shock. Differently from the benchmark case here both shock may affect long run labor productivity.

Unlike the case of neutral technological progress, hours increase at all dates after an investment-specific technology shocks and except for some quarters around early 80's the response is particularly persistent and hump-shaped. Furthermore, in response to neutral technology shocks hours fall in both specifications. Table 2 documents the contribution of the two types of technology shocks to aggregate fluctuations and the implied correlations among variables. Panel A refers to the levels specification, panel B to the first difference specification. The two technology

shocks together explain about 39-53% of the total volatility of output and hours worked at business cycles frequencies, depending on the particular specification. In particular neutral technology shocks, as in the benchmark case, account for about 10-20% while investment-specific for about 20-30% of the total variability at the business cycles fluctuations for both variables. Interestingly investment-specific shocks generate a high correlation between output and hours, about 0.8-0.9, which is similar to the one found in actual data, while correlation generated by neutral shocks are similar to the previous case, about 0.5-0.6. When also investment-specific shocks are included in the analysis the importance of technology shocks on the whole in explaining aggregate fluctuations is remarkably increased. On the other hand, results for neutral technology found previously are confirmed here.

#### 7.4 Sensitivity to the Choice of Variables

Finally we check whether results are sensitive to the choice of variables. CEV argue that it is important, at least in fixed coefficients VARs, to include consumption-to-output and investment-to-output ratio. Taking their suggestion we estimate the model using a different specification including labor productivity growth rates, hours consumption-to-output and investment-to-output ratio. Results using the new specification are qualitatively very similar to previous results. In the growth rates specification hours reduce persistently at all dates. In the levels specification the pattern of the response of hours worked is almost identical to the bivariate case. The response is negative, particularly persistent and significant on impact until mid 90's. From mid 90's the mean response turns positive and humped shaped but not significantly different from zero on impact. Table 3 displays the implied correlations and percentages of variances explained by technology shocks. When hours are specified in levels technology shocks generate a correlation between output and hours of 0.79 and explain about the 38% of the total output variance. Numbers are slightly higher than in the benchmark specifications. In the growth rates specification technology shocks generate a correlation between output and hours of 0.52 and the percentage of explained output variance is about 17%. Also under the new specification main conclusions are confirmed.

## 8 Conclusions

The response of hours worked to technological improvements is a key issue in assessing the relevance of different theoretical characterizations of the business cycle. From the point of view of the empirical research, evidence in favor of both a decline and a rise of hours worked emerges. Results crucially depend on how the time series for hours worked is specified in the VAR. In this paper we argue that conflicting results may arise because important time variations and structural changes the US economy underwent during the postwar period are a priori ruled out by standard models.

In other words, we argue that differences in the results depending on the particular specification for hours worked may simply originate from a more fundamental misspecification arising from the too strong assumption of model coefficients constancy. We investigate the effects of technology shocks on hours worked using a Bayesian Vector Autoregression with drifting coefficients augmented with the same standard restriction used in the literature, that is the technology shock is the only shock affecting long-run labor productivity.

Time-varying dynamics matter. Once time variations are allowed for, competing empirical specifications (levels and growth rates) yield similar results: hours fall at least until mid 90s. The decline is particularly pronounced and statistically different from zero until early 90's, while after that date hours are less responsive to technology shocks. We argue that the differences between fixed and time-varying coefficients are due to instabilities in the coefficients of hours worked in the labor productivity equation. Other findings complement our main result. Aggregate sector neutral technology shocks of the kind emphasized by RBC proponents can hardly be considered the only force driving business cycles since they can only explain about 11-25% of the total output variance. Nevertheless when also investment-specific technology shocks are considered, the percentage of output variance accounted for by technology shocks as a whole is remarkably increased.

The decline of hours worked is in line with models of nominal rigidities or with RBC models in which technology generates large wealth effects. However while the negative sign of the response has reliable structural explanations, time variations in the size of the response are left unexplained. Actually changes in the monetary policy conduct are not able to account for the reduction in absolute value of the impact effect on hours. So why are technology shocks less and less contractionary beginning from early 90's? We leave the answer to this question to future investigations.



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## Appendix

### A

Consider any  $\tau < T$ . Long-run impulse response and cumulated impulse response functions are given respectively by the limits

$$\lim_{k \rightarrow \infty} \mathbf{A}_T^k \mathbf{A}_T \dots \mathbf{A}_{\tau+1}$$

$$\lim_{k \rightarrow \infty} \mathcal{A}_\tau + (I + \sum_{j=1}^k \mathbf{A}_T^j) \mathcal{B}_\tau$$

where  $\mathcal{A}_\tau = I + \mathbf{A}_{\tau+1} + \mathbf{A}_{\tau+2} \mathbf{A}_{\tau+1} + \dots + \mathbf{A}_{T-1} \mathbf{A}_{T-2} \dots \mathbf{A}_{\tau+2} \mathbf{A}_{\tau+1}$  and  $\mathcal{B}_\tau = \mathbf{A}_T \mathbf{A}_{T-1} \dots \mathbf{A}_{\tau+2} \mathbf{A}_{\tau+1}$ . If for any realization of  $\mathbf{A}_T$  the largest eigenvalue is smaller than one in absolute value then impulse response converge pointwise to zero while long-run cumulated impulse response converge pointwise to  $\mathcal{A}_\tau + (I - \mathbf{A}_T)^{-1} \mathcal{B}_\tau$ . This comes from

$$\lim_{k \rightarrow \infty} \mathbf{A}_T^k = 0$$

$$\lim_{k \rightarrow \infty} (I + \sum_{j=1}^k \mathbf{A}_T^j) = (I - \mathbf{A}_T)^{-1}$$

### B

#### Priors

We assume  $\theta_0$ ,  $\Sigma$  and  $\Omega$  to be independent. We specify the following prior distributions

$$p(\theta_0) = N(\bar{\theta}, \bar{P})$$

$$p(\Sigma) = IW(\Sigma_0^{-1}, \nu_0)$$

$$p(\Omega) = IW(\Omega_0^{-1}, \nu_0)$$

For the block diagonal and diagonal specification we use respectively  $p(\Omega_i) = IW(\Omega_{i0}^{-1}, \nu_0)$ , where  $i$  refers to the  $i$ -th equation and  $p(\Omega_{ii}) = IG(\frac{1}{2}, \frac{\Omega_{ii0}}{2})$ . We "calibrate" the prior by estimating a fixed coefficients VAR using data from 1954:IV up to 1966:IV. We set  $\bar{\theta}$  equal to the point estimates of the coefficients and  $\bar{P}$  to the estimated covariance matrix.  $\Sigma_0$  is equal to the estimated covariance matrix of VAR innovations and  $\Omega_0 = \varrho \bar{P}$  and  $\nu_0$  equal to the number of observations of the initial sample. The parameter  $\varrho$  measures how much the time variation is allowed in coefficients. We set  $\varrho = 0.01$ .

The joint prior is

$$p(\theta^T, \phi) = p(\theta^T | \phi) p(\phi)$$

$$\propto \mathcal{I}(\theta^T) f(\theta^T | \phi) p(\theta_0) p(\Sigma) p(\Omega) \quad (8)$$

where  $\mathcal{I}(\theta^T) = \prod_{t=0}^T \mathcal{I}(\theta_t)$  and  $f(\theta^T | \phi) = f(\theta_0 | \phi) \prod_{t=0}^{T-1} f(\theta_{t+1} | \theta_t, \phi)$ .

## Posterior Density

In a Bayesian approach the goal is to summarize the posterior density of the objects of interests. The posterior density is  $p(\theta^T, \phi|y^T)$ . This density is very complicated but it can be decomposed into more tractable pieces. First we can express it as

$$p(\theta^T, \phi|y^T) \propto p(y^T|\theta^T, \phi)p(\theta^T, \phi)$$

where the first term of the right hand side is the likelihood and the second the joint posterior density. Conditional to the states up to time  $T$  and the hyperparameters the measurement equation is linear with Gaussian innovation, thus the conditional likelihood is Gaussian. The second term can be splitted into a conditional and a marginal density thus we have

$$\begin{aligned} p(\theta^T, \phi|y^T) &\propto f(y^T|\theta^T, \phi)p(\theta^T|\phi)p(\phi) \\ &\propto \mathcal{I}(\theta^T) \left[ f(y^T|\theta^T, \phi)f(\theta^T|\phi)p(\phi) \right] \end{aligned}$$

where from the first to the second line we used the restricted prior distribution for the states. Note that the term in brackets is the posterior without the restriction on impulse response functions. Thus our posterior distribution is proportional to the unrestricted posterior density,  $p_u$ ,

$$p(\theta^T, \phi|y^T) \propto \mathcal{I}(\theta^T)p_u(\theta^T, \phi|y^T)$$

This is particularly convenient since we can first characterize the unrestricted posterior and then perform the rejection sampling (see below) to collect the draws satisfying the restriction.

## Drawing from the posterior of reduced form parameters

The Gibbs Sampler we use to compute the posterior for the reduced form parameters iterate on two steps. The implementation is identical to Cogley and Sargent (2001).

- Step 1: States given hyperparameters

Conditional on  $y^T, \phi$ , the unrestricted posterior of the states is normal and  $p_u(\theta^T|y^T, \phi) = f(\theta_T|y^T, \phi) \prod_{t=1}^{T-1} f(\theta_t|\theta_{t+1}, y^t, \phi)$ . All densities on the right end side are Gaussian they their conditional means and variances can be computed using the Kalman backward filter. Let  $\theta_{t|t} \equiv E(\theta_t|y^t, \phi)$ ;  $P_{t|t-1} \equiv Var(\theta_t|y^{t-1}, \phi)$ ;  $P_{t|t} \equiv Var(\theta_t|y^t, \phi)$ . Given  $P_{0|0}$ ,  $\theta_{0|0}$ ,  $\Omega$  and  $\Sigma$ , we compute Kalman filter recursions

$$\begin{aligned} \theta_{t|t-1} &= F\theta_{t-1|t-1} \\ P_{t|t-1} &= FP_{t-1|t-1}F' + \Omega \\ K &= (P_{t|t-1}X_t)(X_t'P_{t|t-1}X_t + \Sigma)^{-1} \\ \theta_{t|t} &= \theta_{t|t-1} + K_t(y_t - X_t'\theta_{t-1|t-1}) \\ P_{t|t} &= P_{t|t-1} - K_t(X_t'P_{t|t-1}) \end{aligned} \tag{9}$$

The last iteration gives  $\theta_{T|T}$  and  $P_{T|T}$  which are the conditional means and variance of  $f(\theta_t|y^T, \phi)$ . Hence  $f(\theta_T|y^T, \phi) = N(\theta_{T|T}, P_{T|T})$ . The other  $T-1$  densities can be computed using the backward recursions

$$\begin{aligned}\theta_{t|t+1} &= \theta_{t|t} + P_{t|t}F'P_{t+1}^{-1}(\theta_{t+1} - \theta_{t|t}) \\ P_{t|t+1} &= P_{t|t} - P_{t|t}F'P_{t+1}^{-1}FP_{t|t}\end{aligned}\tag{10}$$

where  $\theta_{t|t+1} \equiv E(\theta_t|\theta_{t+1}, y^t, \phi)$  and  $P_{t|t+1} \equiv Var(\theta_t|\theta_{t+1}, y^t, \phi)$  are the conditional means and variances of the remaining terms in  $p_u(\theta^T|y^T, \phi)$ . Thus  $f(\theta_t|\theta_{t+1}, y^t, \phi) = N(\theta_{t|t+1}, P_{t|t+1})$ . Therefore, to sample  $\theta^T$  from the conditional posterior we proceed backward, sampling  $\theta_T$  from  $N(\theta_{T|T}, P_{T|T})$  and  $\theta^t$  from  $N(\theta_{t|t+1}, P_{t|t+1})$  for all  $t < T$ .

- Step 2: Hyperparameters given states

Since  $(\Sigma, \Omega)$  are independent, we can sample them separately. Conditional on the states and the data  $\varepsilon_t$  and  $u_t$  are observable and Gaussian. Combining a Gaussian likelihood with an inverse-Wishart prior results in an inverse-Wishart posterior, so that

$$\begin{aligned}p(\Sigma|\theta^T, y^T) &= IW(\Sigma_1^{-1}, \nu_1) \\ p(\Omega|\theta^T, y^T) &= IW(\Omega_1^{-1}, \nu_1)\end{aligned}$$

where  $\Sigma_1 = \Sigma_0 + \sum_{t=1}^T \varepsilon_t \varepsilon_t'$ ,  $\Omega_1 = \Omega_0 + \sum_{t=1}^T u_t u_t'$ ,  $\nu_1 = \nu_0 + T$ , . For the block diagonal and diagonal specification we have  $p(\Omega_{ii}|\theta^T, y^T) = IG(\frac{T+1}{2}, \frac{(T+1)\Omega_{ii1}}{2})$  where  $\Omega_{ii1} = \Omega_{ii0} + \sum_{t=1}^T u_{it}^2$  and when block-diagonal  $p(\Omega_i|\theta^T, y^T) = IW(\Omega_{i1}^{-1}, \nu_1)$ ,  $\Omega_{i1} = \Omega_{i0} + \Omega_{iT}$  and  $\Omega_{iT} = \sum_{t=1}^T u_t^i u_t^{i'}$  where  $u_t^i$  is the vector of shocks in the coefficients of equation  $i$ .

Under regularity conditions and after a burn-in period, iterations on these two steps produce draw from  $p_u(\theta^T, \Sigma, \Omega|y^T)$ . We have constructed CUMSUM graphs to check for convergence and found that the chain had converged roughly after 2000 draws for each date in the sample. The densities for the parameters obtained with the remaining draws are well behaved and none is multimodal. We keeping one every four of the remaining 8000 draws and discard all the draws generating non convergent impulse response functions. The autocorrelation function of the 2000 draws which are left is somewhat persistent. We could reduce it by taking draws more largely spaced but this comes at the price of reducing the number of draws which satisfy the VAR polynomial roots restrictions and therefore substantially reduce the precision of the estimates. In the end, we have about 250-300 draws for each date to conduct structural inference.

## The Rejection Sampling

This second step ensures that posterior density puts zero probability to draws which do not satisfy the restriction on impulse response functions convergence<sup>24</sup>. The implementation of the rejection sampling is very similar to those in Cogley and Sargent (2001). First we need a candidate density  $g(\theta^T, \phi)$ , satisfying three properties: i) must be non negative and well defined for all  $(\theta^T, \phi)$  for which  $p(\theta^T, \phi|Y^T) > 0$ ; ii) it must have finite integral; iii) the importance ratio  $R(\theta^T, \phi)$  must have an upperbound  $Z$

$$R(\theta^T, \phi) = \frac{p(\theta^T, \phi|Y^T)}{g(\theta^T, \phi)} \leq Z < \infty$$

where

$$p(\theta^T, \phi|Y^T) = \frac{\mathcal{I}(\theta^T)p_u(\theta^T, \phi|y^T)}{\int \int \mathcal{I}(\theta^T)p_u(\theta^T, \phi|y^T)d\theta^T d\phi}$$

A natural candidate density is the unrestricted posterior  $p_u(\theta^T, \phi|y^T)$  because is a probability density, integrates to one and it is non-negative and it is defined for all  $(\theta^T, \phi)$ . Moreover we have

$$R(\theta^T, \phi) \leq \frac{1}{\int \int \mathcal{I}(\theta^T)p_u(\theta^T, \phi|y^T)d\theta^T d\phi} = Z$$

and  $Z$  is finite if the probability of a draw with associated convergent impulse response functions from the unrestricted posterior, the denominator, is non-zero. First we draw a trial  $(\theta_i^T, \phi_i)$  from the unrestricted posterior, second we accept it with probability  $\frac{R(\theta_i^T, \phi_i)}{Z} = \mathcal{I}(\theta_i^T)$  that is with probability one if it satisfies restrictions or zeros if it does not.

## Drawing Impulse Response Functions

It is very easy to draw impulse response functions from the posterior distribution. Note that impulse response functions are continuous functions of the autoregressive coefficients. For a given draw of  $\theta^T$  and  $\Sigma$  from the joint posterior we simply form the product  $B_{t,k}$ , we compute the limit  $\tilde{B}_{t,\infty}$  and the matrix  $K_t$ . We compute impulse response functions associated to that draw. We repeat this step a sufficiently large number of time and collecting the draw at each step. Eventually we compute median or mean and confidence bands.

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<sup>24</sup>See Gellman, Carlin, Stern and Rubin (1995).

## Tables

Table 1: Technology Shocks<sup>a</sup>

Specifications	Corr(lp,h)	Corr(lp,y)	Corr(h,y)	% Var(lp)	% Var(h)	% Var(y)
<i>Biv:</i>						
<i>levels</i>	-0.1022	0.5917	0.7639	0.4648	0.2852	0.2571
$\Delta logs$	-0.4909	0.6557	0.4616	0.6229	0.1576	0.1493
<i>R<math>\pi</math>:</i>						
<i>levels</i>	-0.3396	0.6075	0.5495	0.3814	0.1546	0.1489
$\Delta logs$	-0.4975	0.7627	0.2780	0.4541	0.0905	0.1142

<sup>a</sup> Correlations and variance at business cycles frequencies. lp = labor productivity, h = per capita hours, y = output.

Table 2: Technology shocks under the *CI* specification

Specifications	Corr(lp,h)	Corr(lp,y)	Corr(h,y)	% Var(lp)	% Var(h)	% Var(y)
<i>levels</i>	0.0062	0.6193	0.7986	0.4713	0.3632	0.3384
$\Delta logs$	-0.5691	0.4935	0.5191	0.4748	0.1185	0.1692

Table 3: Neutral and Investment-Specific shocks

Specifications	Corr(lp,h)	Corr(lp,y)	Corr(h,y)	% Var(lp)	% Var(h)	% Var(y)
Neutral						
<i>levels</i>	-0.2114	0.5580	0.7236	0.4038	0.2225	0.2066
$\Delta logs$	-0.4096	0.6441	0.4852	0.3833	0.1052	0.0981
Investment-Specific						
<i>levels</i>	0.3079	0.6447	0.9300	0.2111	0.3388	0.3271
$\Delta logs$	0.2315	0.6209	0.9116	0.1656	0.2140	0.2170
Both						
<i>levels</i>	0.0504	0.5869	0.8509	0.6239	0.5514	0.5356
$\Delta logs$	0.0716	0.6816	0.7912	0.6309	0.3418	0.3939



# Figures

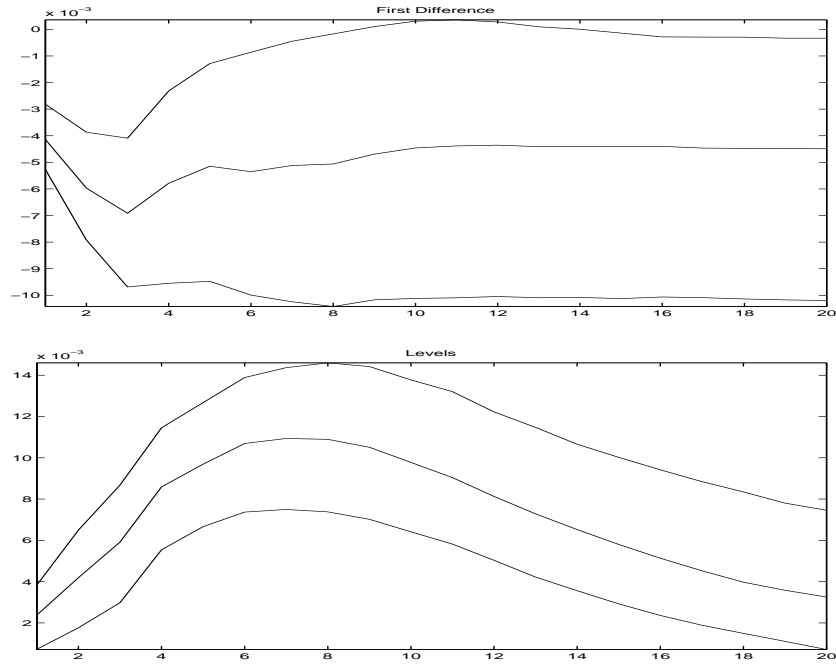


Figure 1: Effects of technology shocks on hours worked (first differences and levels specification in top and bottom panel respectively) in the bivariate VAR, full-sample.

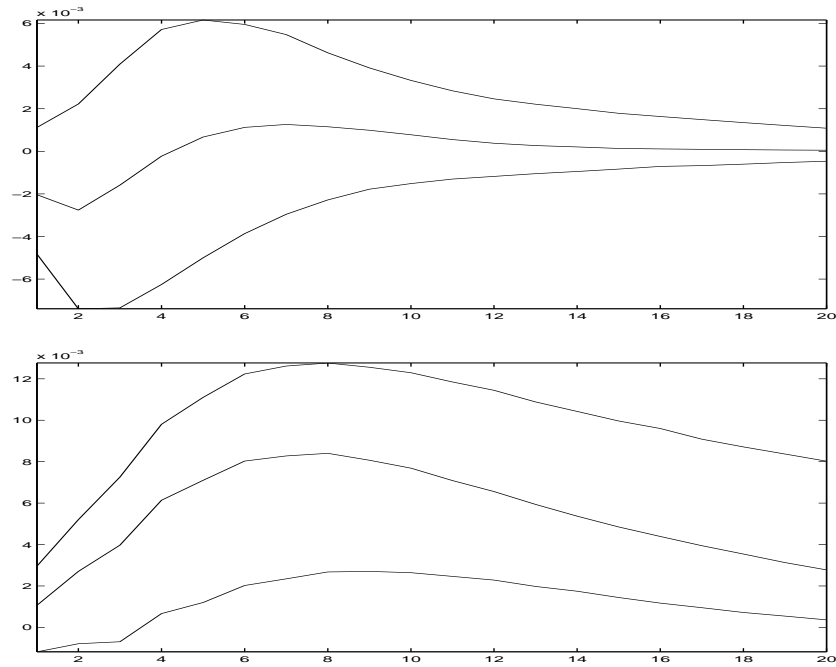


Figure 2: Effects of technology shocks on hours worked (levels) in two subsamples: 1954:III-1979:IV in the top panel, 1982:III-2003:IV in the bottom panel.

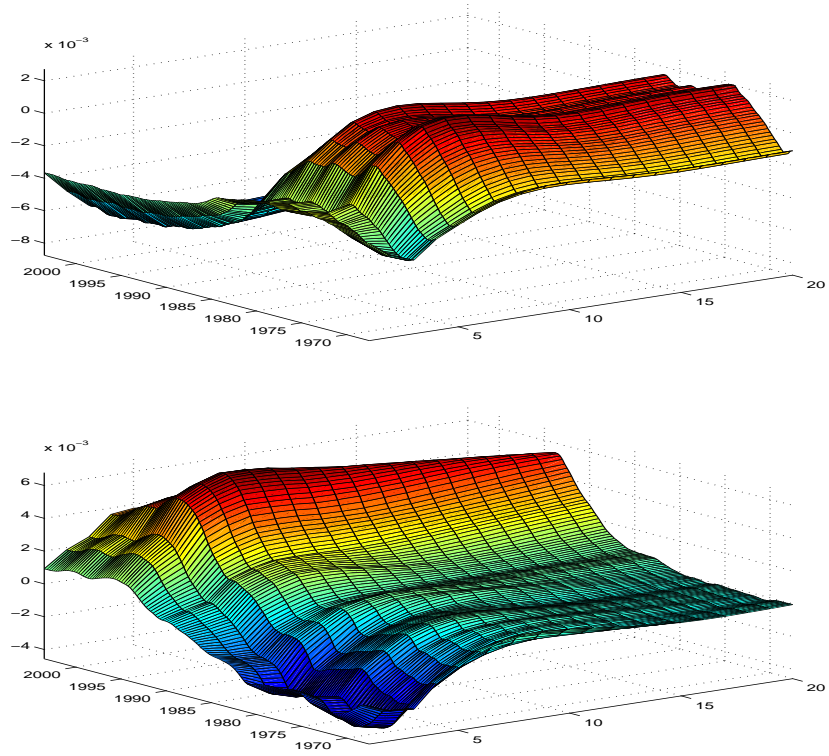


Figure 3: *Response of hours worked to a technology shock in the bivariate VAR: top panel first difference specification, bottom panel levels specification.*

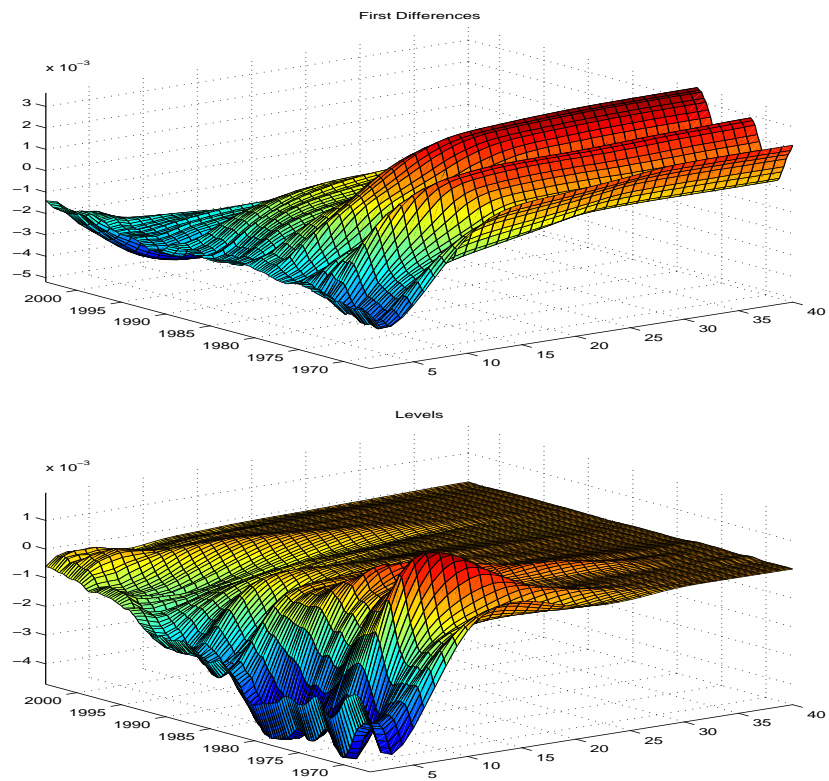


Figure 4: *Response of hours worked to a technology shock in the  $R\pi$  VAR: top panel first difference specification, bottom panel levels specification.*

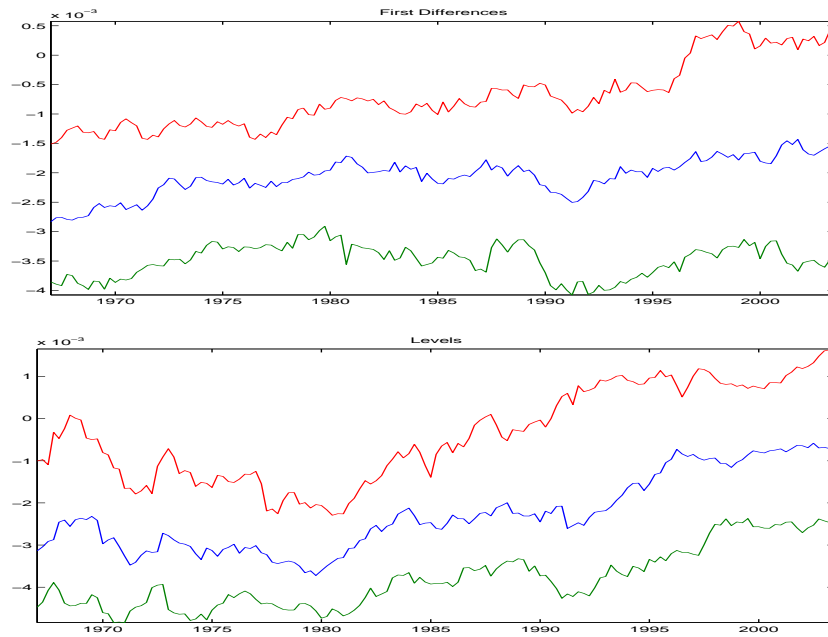


Figure 5: *Impact effects of technology shock on hours worked in the  $R\pi$  VAR, posterior median and 68% confidence bands. Top panel first difference specification, bottom panel levels specification.*

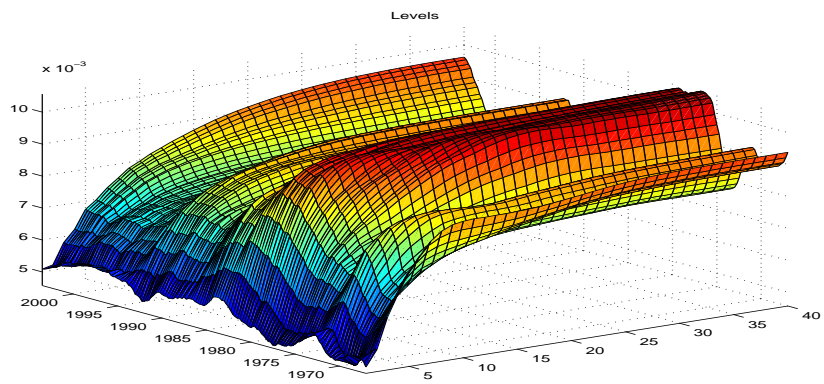


Figure 6: *Response of labor productivity to a technology shock in the  $R\pi$  VAR, levels specification.*

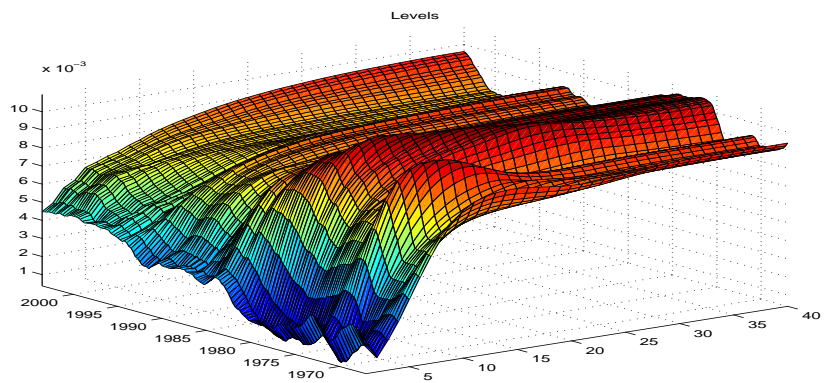


Figure 7: *Impulse response functions of output to a technology shock in the  $R\pi$  VAR, levels specification.*

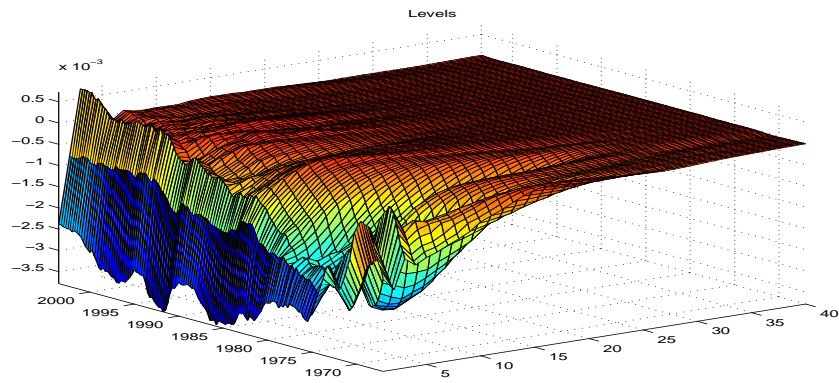


Figure 8: *Impulse response functions of inflation to a technology shock in the  $R\pi$  VAR, levels specification.*

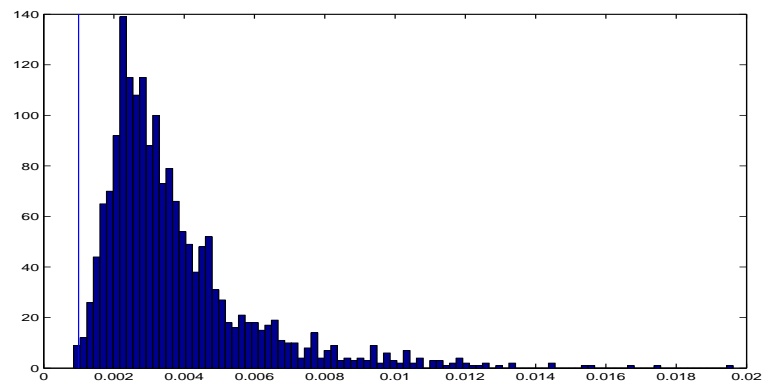


Figure 9: *Trace of the posterior (histogram) and prior (segment) variance matrix of the coefficients..*

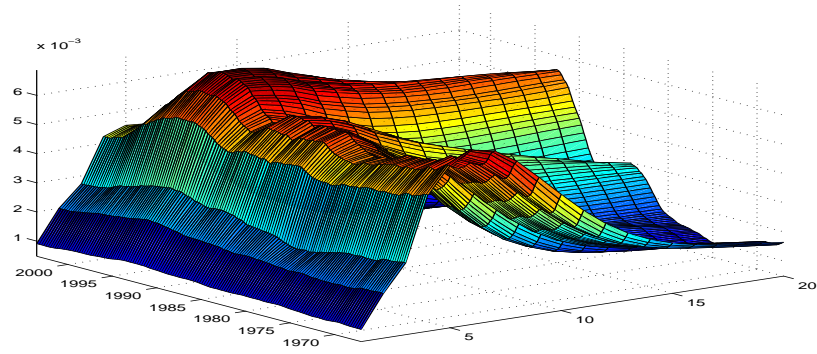


Figure 10: Top panel impulse response of levels of hours with coefficients of hours in the labor productivity equation replaced, bivariate VAR with hours in levels.

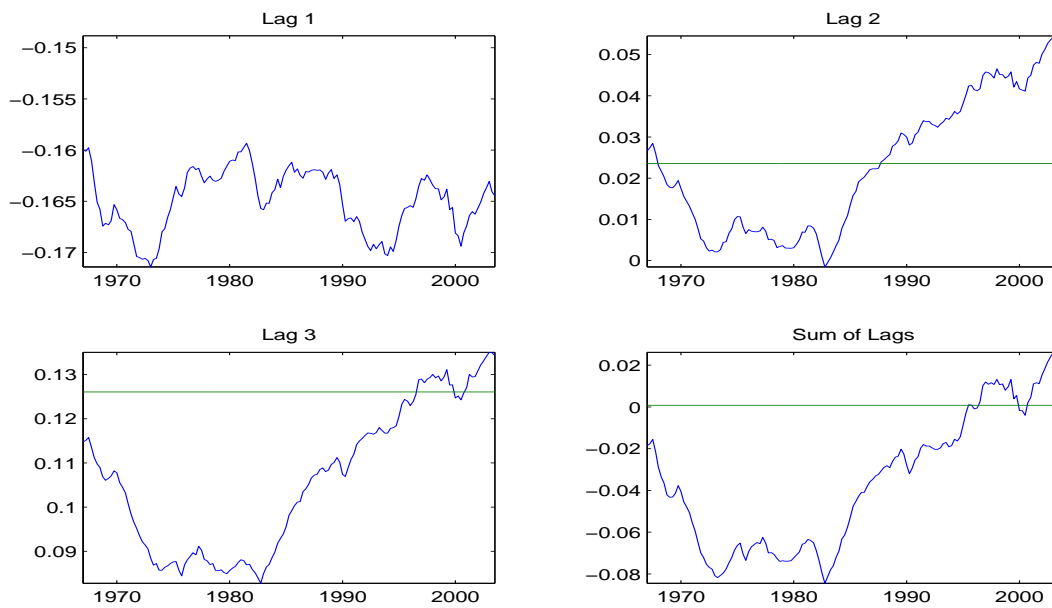


Figure 11: Estimates of lagged coefficients of hours worked in the labor productivity equation in the fixed and time-varying coefficients VAR.

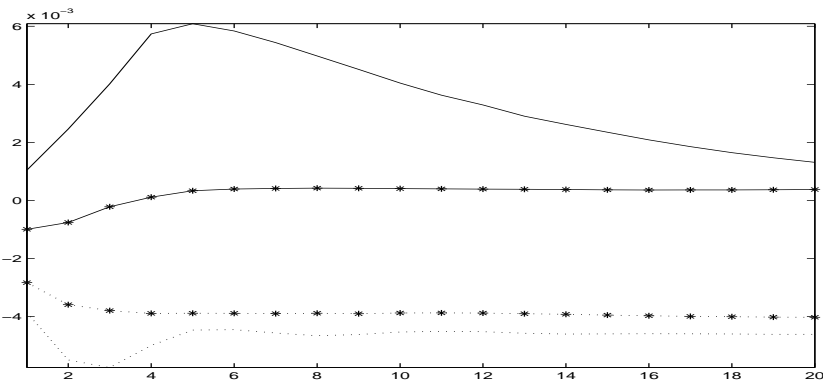


Figure 12: *Encompassing test for the growth rates specification. Dotted line: response of hours in the growth rates specification using real data. Solid line: response of hours in the levels specification using real data. Dotted starred line: response of hours in the growth rates specification using data generated by the time varying model with hours in growth rates. Solid starred line: response of hours in the levels specification using data generated by the time varying model with hours in growth rates.*

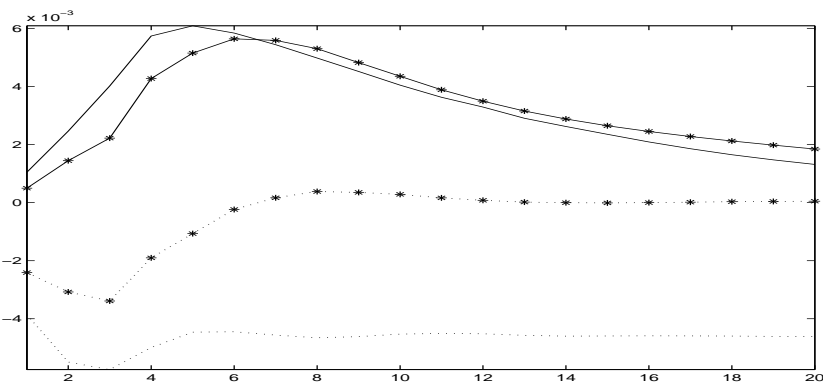


Figure 13: *Encompassing test for the levels specification. Dotted line: response of hours in the growth rates specification using real data. Solid line: response of hours in the levels specification using real data. Dotted starred line: response of hours in the growth rates specification using data generated by the time varying model with hours in levels. Solid starred line: response of hours in the levels specification using data generated by the time varying model with hours in levels.*

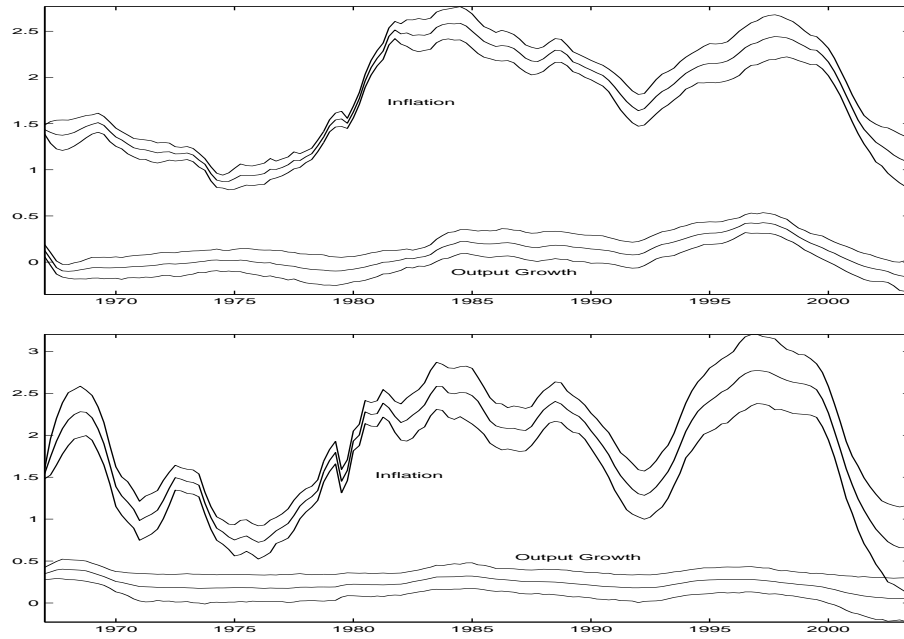


Figure 14: *Central banks preferences: coefficients on inflation and output growth in levels (top panel) and growth rate (bottom panel) specifications.*