DECOMPOSITION OF A CERTAIN CASH FLOW STREAM: SYSTEMIC VALUE ADDED AND NET FINAL VALUE

Carlo Alberto Magni

Dipartimento di Economia Politica
Universit di Modena e Reggio Emilia

Abstract. This paper proposes a method for evaluating a project under certainty by means of a systemic outlook, which borrows from accounting the way of representing economic facts while replacing accounting values with cash values. The investor's net worth is regarded as a system whose structure changes over time. On this basis, a profitability index is presented, here named Systemic Value Added (SVA), which lends itself to a periodic decomposition. While as an overall index the Systemic Value Added coincides with the Net Final Value (NFV) of an investment, the systemic partition of a SVA is shown to differ from the Net Present Value (NPV) decomposition model proposed by Peccati (1987, 1992), which in turn bears a strong resemblance to Stewart's (1991) EVA model. The different assumptions the three models rely on are analysed: Some inconsistencies arise in the NFV-based approach, which give rise to Peccati's and Stewart's model, but they can be healed (only in a certain sense) by re-shaping the model and taking account of the systemic approach. To this end, the introduction of a *shadow project* is needed which enables us to avoid compounding. An interesting result is that we can decompose the SVA of a project by applying Peccati's argument to its *shadow*, or which is the same, by computing the *shadow project*'s Economic Value Added.

The paper then generalizes the approach allowing for a portfolio of projects, multiple debts and multiple synchronic opportunity costs of capital, for which a tetra-dimensional decomposition is easily obtained.

Introduction

This paper deals with investment evaluations under certainty. The decision maker faces the opportunity of undertaking a project and she aims at evaluating both the overall profitability and the periodic performance, i.e. the so-called residual income. A widely accepted evaluation index is the well-known Net Present Value (NPV), or Net Final Value (NFV) if compounded, which evaluates the (overall) differential profit between the two alternatives of investing in the project or in alternative comparable investment. The NPV (NFV) approach focuses on cash flows seen as increases or decreases of wealth. An alternative index is here proposed by means of a *systemic* approach: The investor's net worth is seen as a system whose structure consists of multiple accounts (one of which is the project at hand), which are periodically activated for withdrawals and reinvestments of cash flows. This is just the environment of accounting. Yet (not accounting itself but) the systemic perspective accounting relies on can be quite useful in appraising alternatives of action. In this paper I will show that, in order to evaluate the project, the decision maker needs only use sorts of balance sheets provided that she makes use of cash values rather than accounting values. By comparing different sequences of prospective double-entry sheets (one for each alternative) she gets the same result as the evaluator who discounts or compounds cash flows at the opportunity cost of capital to obtain the Net Present Value or Net Final Value. The accounting-like index presented is named Systemic Value Added. The way the decision maker obtains it makes it amenable to a periodic decomposition in shares, giving helpful information about how that value is generated. This index coincides with the NFV but the decomposition collides with the decomposition proposed by Peccati (1987, 1992) and the two models' assumptions will be analysed. Also, Stewart's EVA model is equivalent to Peccati's model, Albeit some inconsistencies arise in the NPV-based approach (Peccati's and Stewart's model), the latter can be retrieved, to a certain extent, by the introduction of a so-called *shadow project*.

The paper is organized as follows. In Sec.1 the NPV approach is presented clarifying the evaluation process we are concerned with. Sec.2 presents the decomposition model of Peccati and briefly shows the formal equivalence with Stewart's model. Sec.3 proposes the use of double-entry sheets for evaluating the project and decomposing it, and Sec.4 shows that the Net Final Value and the Systemic Value Added are equivalent as overall indexes, while leading to dichotomic periodic partitions. Sec.5 and Sec.6 shed lights on the differences between the two models. In particular the implicit assumption of Peccati's model is unmasked, which is shown to be illicit. In Sec.7 the latter is removed by a reframing of the evaluation process which involves the introduction of the concept of *shadow project*. Peccati's model is retrieved so as to be integrated into the systemic model. Sec.8 focuses attention on the the relations between the shadow project and the evaluator's financial system. Sec.9 provides some generalizations of the previous results. Among them, we consider a portfolio of projects and a plurality of synchronic opportunity costs of capital. Some remarks conclude the paper.

1. The NPV approach

Assume that a decision maker currently invests funds at a rate of interest i and that she faces the opportunity of a nondeferrable investment, say P:[1] For the sake of simplicity we can assume that the project consists of an initial outlay $-a_0$ at time 0, and that equidistant cash flows $a_s \geq 0$ will be available at time s respectively, $s=1, \dots n$. The evaluator's initial wealth is E_0, with $0<a_0 \leq E_0$, and she aims at maximizing her terminal wealth at time $T=n$ (the term net worth will be also used as a synonym of wealth).

To enrich our analysis we can also assume that she finances her investment with a loan contract, whose cash flows are f_0 at time 0 and $-f_s \leq 0$ at time s, with $0 \leq f_0 \leq a_0$. According to the NPV approach, the decision maker will accept project P if the investment undertaking will leave her better off than investing funds at the rate i. In the latter case, denoting with E^n her net worth at time n, she will hold

$$E^n = E_0(1+i)^n; \tag{1}$$

conversely, if she decides to forego the latter opportunity in order to obtain the sequence

[1] I shall not be concerned with real options.

$$(-a_0+f_0,\ a_1-f_1,\ \ldots,\ a_n-f_n),$$

her net worth, denoted with E_n, will be

$$E_n = (E_0 - a_0 + f_0)\,(1+i)^n + \sum_{s=1}^{n} (a_s - f_s)\,(1+i)^{n-s} \tag{2}$$

where we have assumed that each net cash flow will be reinvested (or withdrawn if negative) at the constant rate of interest i, which is the so-called opportunity cost of capital.[2] If (2) is greater than (1), i.e.

$$(E_0 - a_0 + f_0)\,(1+i)^n + \sum_{s=1}^{n} (a_s - f_s)\,(1+i)^{n-s} > E_0(1+i)^n, \tag{3}$$

the project should be accepted, otherwise it should be rejected. The comparison in (3) between two final values can be disguised as a comparison between present values by dividing both sides of (3) by $(1+i)^n$:

$$(E_0 - a_0 + f_0) + \sum_{s=1}^{n} (a_s - f_s)\,(1+i)^{-s} > E_0,$$

whence

$$(-a_0 + f_0) + \sum_{s=1}^{n} (a_s - f_s)\,(1+i)^{-s} > 0. \tag{4a}$$

The left-hand side of (4a) is the well-known Net Present Value (NPV) of the investment at hand; multipying it by $(1+i)^n$ we get the Net Final Value (NFV), so that

$$(-a_0 + f_0)\,(1+i)^n + \sum_{s=1}^{n} (a_s - f_s)\,(1+i)^{n-s} > 0. \tag{4b}$$

It is worthwhile noting that the NFV of an investment is nothing but the difference between the two alternative terminal net worths (see (3)):

$$\text{NFV} = E_n - E^n \tag{4c}$$

that is the difference between (2) and (1). This allows us to see the NFV as an index measuring the residual income: The investors faces a project P which would leave her with the sum E_n. She can alternatively invest the same capital at the opportunity cost of capital i. The difference between the two opportunities gives rise to the global residual income referred to the entire lenght of the project.

[2]To be rigorous, if the investor's net worth is negative in some periods, the rate i is not an *opportunity cost*, it is a genuine rate of cost. However, this is irrelevant to our ends. The assumptions $a_0 \leq E_0$ above is made just for the sake of a better verbal explanation of the decision process at hand. Formally, nothing would change if it did not hold, but in the latter case we could not speak of "opportunity to invest at the rate i", as i would be, at least for the first period, a genuine rate of cost.

2. The financial decomposition

The NPV (NFV) is an overall measure which shows the global value of a project and is referred to the whole investment's length. But how is it generated as time passes? How much of it should be ascribed to one period or another? That is, how can we subdivide this measure in order to obtain periodic values g_1, g_2, ..., g_n, such that g_s refers to the s-th period and such that NPV$=g_1+g_2+\cdots+g_n$? A periodic decomposition is proposed by Peccati (1987, 1991, 1992). We can summarize this decomposition model by making use of the relations among the cash flows, the *project balance* and the *debt balance*. The *project balance* at time s, at the rate of interest x, is

$$\begin{aligned} &w_0 = a_0 \\ &w_s = w_{s-1}(1+x) - a_s \qquad s = 1, 2, \ldots, n. \end{aligned} \tag{5}$$

I will also call it *outstanding balance* or *outstanding capital*,[3] interpreting it as an account yielding interest at the rate of interest x, where a_0 is invested and the subsequent a_s are withdrawn. Likewise, the *debt balance* at time s at the rate of interest δ is

$$\begin{aligned} &D_0 = f_0 \\ &D_s = D_{s-1}(1+\delta) - f_s \qquad s = 1, 2, \ldots, n. \end{aligned} \tag{6}$$

I will also call it *residual debt* or *outstanding debt.* If x is P's internal rate of return and δ is the debt's contractual rate it is easy to see that:[4]

$$w_n = a_0(1+x)^n - \sum_{s=1}^{n} a_s(1+x)^{n-s} = 0.$$

and

$$D_n = f_0(1+\delta)^n - \sum_{s=1}^{n} f_s(1+\delta)^{n-s} = 0.$$

Using (5) and (6) the Net Final Value boils down to

$$\begin{aligned} \text{NFV} = -w_0(1+i)^n &+ \sum_{s=1}^{n} \left[\left(w_{s-1}(1+x) - w_s \right)(1+i)^{n-s} \right] \\ &+ D_0(1+i)^n - \sum_{s=1}^{n} \left[\left(D_{s-1}(1+\delta) - D_s \right)(1+i)^{n-s} \right], \end{aligned}$$

[3] w_s coincides in absolute value with the *project balance* introduced in Teichroew, Robichek and Montalbano (1965a, 1965b), but has opposite sign.

[4] Under our assumptions x exists and both x and δ are unique, as the project and the loan contract have strictly monotonic Discounted Cash Flows.

whence we get

$$\begin{aligned}\text{NFV} &= \sum_{s=1}^{n} w_{s-1}(x-i)(1+i)^{n-s} + \sum_{s=1}^{n} D_{s-1}(i-\delta)(1+i)^{n-s} \\ &= \sum_{s=1}^{n} \big(w_{s-1}(x-i) + D_{s-1}(i-\delta)\big)(1+i)^{n-s}. \qquad (7a)\end{aligned}$$

We can generalize allowing the project balance and the outstanding debt to have periodic rates of interest x_s and δ_s such that

$$w_s = w_{s-1}(1+x_s) - a_s \qquad s = 1, 2, \ldots, n \qquad (5')$$

$$D_s = D_{s-1}(1+\delta_s) - f_s \qquad s = 1, 2, \ldots, n. \qquad (6')$$

If we fix the outstanding capitals and residual debts first (as Peccati suggests as plausible in some cases), then the rates x_s and δ_s rare univocally determined representing periodic internal rates of return. So doing we rule out any problem of existence and uniqueness of the internal rate of return. Therefore we can relax one of our assumptions by allowing $a_s \in \mathbb{R}$ for any $s \geq 1$ (while maintaining $-a_0 < 0$). (7a) is then replaced by

$$\text{NFV} = \sum_{s=1}^{n} \big(w_{s-1}(x_s - i_s) + D_{s-1}(i_s - \delta_s)\big)(1+i)^{n-s}. \qquad (7b)$$

Letting

$$G_s := \big(w_{s-1}(x_s - i) + D_{s-1}(i - \delta_s)\big)(1+i)^{n-s}$$

and

$$g_s := \frac{G_s}{(1+i)^n},$$

we have

$$\text{NFV} = G_1 + G_2 + \cdots + G_n \qquad (7c)$$

or, which is the same in present terms,

$$\text{NPV} = g_1 + g_2 + \cdots + g_n, \qquad (7d)$$

as we wished. The decomposition of the NPV tells us that G_s (or g_s) is the value added or subtracted in the s-th period by the project with respect to the alternative course of action (i.e. investing funds at the opportunity cost of capital i). When positive, it indicates that the investment is favorable in the s-th period, when negative it shows a decrease in net worth with respect to the alternative opportunity.

To see why g_s (G_s) should be considered the s-th period Net Present Value (Net Final Value) we can focus attention on a generic period from time $(s-1)$ to time s. At the outset of period s, our investor (fictitiously) invests the cash flow w_{s-1}, partly financing it with the residual debt D_{s-1}

received from her creditor. At the end of the period she takes back the outstanding capital w_s net of the outstanding debt D_s along with the net cash flow $a_s - f_s$. Such a situation can be considered a (fictitious) uniperiodic sub-project whose NPV is

$$\frac{-w_{s-1} + D_{s-1}}{(1+i)^{s-1}} + \frac{w_s - D_s + a_s - f_s}{(1+i)^s}. \tag{8a}$$

Using (5') and (6') the latter can be manipulated so as to obtain g_s, as we wished:

$$g_s = \frac{w_{s-1}(x_s - i) + D_{s-1}(i - \delta_s)}{(1+i)^s} \tag{8b}$$

(to get G_s we just have to compound until time n)

Peccati's decomposition is formally equivalent to Stewart's Economic Value Added (EVA) model. The Economic Value Added is a periodic residual income, i.e. it is the residual income generated in a determined. To see the formal equivalence, let us compute the EVA of P. We just have to calculate the total cost of capital, given by the product of the Weighted Cost of Capital (WACC) and the total capital invested (TC). Then the total cost of capital is subtracted from the Net Operating Profit After Taxes (NOPAT). Notationally, we have, for period s,

$$\text{EVA}_s = \text{NOPAT} - \text{WACC} * \text{TC}. \tag{8c}$$

It is easy to show that (8c) is just the numerator of g_s. In fact, (8c) can be rewritten as

$$\text{EVA}_s = \text{ROA}{*}\text{TC} - \frac{(\text{ROD}{*}\text{Debt}{+}\text{i}{*}\text{Equity})}{\text{Debt}{+}\text{Equity}} * \text{TC}$$

whence

$$\begin{aligned}\text{EVA}_s &= \text{ROA}{*}\text{TC}{-}\text{ROD}{*}\text{Debt-i}{*}(\text{TC}{-}\text{Debt}) \\ &= \text{TC}{*}(\text{ROA}{-}i){+}\text{Debt}{*}(i{-}\text{ROD})\end{aligned}$$

where ROA is the Return on Assets, ROD is the Return on Debt, and i is the opportunity cost of capital. All values refer to period s. Since TC=w_{s-1}, ROA=x, Debt=D_{s-1}, ROD=δ, the relation between EVA and g_s is established:

$$g_s = \frac{\text{EVA}_s}{(1+i)^s}$$

We have then:

$$\text{NPV} = \sum_{s=1}^{n} g_s = \sum_{s=1}^{n} \text{EVA}_s (1+i)^{-s}$$

3. An accounting-like perspective.

Let us abandon the NPV approach and forget for the moment all we know about present or future values. Let us also assume that the decision does not know what a Net Present Value is and that she is only a good accountant. She can only rely on her skillness of drawing up balance sheets. This skillness is a sufficient tool for evaluating the aforementioned project and decompose it. In fact, she compares two lines of action:

(i) undertaking project P
(ii) investing funds at the opportunity cost of capital i.

The evaluation will be based on the fact that she is willing to maximize her wealth at time n. How can she compute the value of her wealth subject to (i) or (ii) if she ignores the concept of (net) present or final value? She can only make use of the double-entry book-keeping system and draw up prospective sheets describing the evaluator's financial system for each alternative. She will therefore be able to have a glance on the two net worths at time n and choose the course of action associated with a higher net worth.

Let us begin to construct the financial system for (ii). Let C be the asset yielding interest at the rate i. As the decision maker invests her funds in C her net worth E^s at time s is given by the sum C^s, representing the value of account C:

$$\begin{array}{r|l} \underline{\text{Uses}} & \underline{\text{Sources}} \\ C^s & E^s \end{array} \tag{9a}$$

for $s = 0, 1, 2, \ldots, n$. As for case (i), our investor/accountant will record two accounts in the debit side and two accounts in the credit side, expressing the fact that she holds an asset C (whose rate of return is i), an asset P whose periodic rate of return is x_s, a loan contract D whose periodic rate (of cost) is δ_s and her own net worth E. At time s we have

$$\begin{array}{r|l} \underline{\text{Uses}} & \underline{\text{Sources}} \\ C_s & D_s \\ w_s & E_s \end{array} \tag{9b}$$

where C_s, w_s, D_s, E_s are the values of accounts C, w, D, E respectively, and where $s = 0, 1, \ldots, n$. For (9a) we state the relations

$$\begin{aligned} C^0 &= E_0 \\ C^s &= C^{s-1}(1+i) \qquad s \geq 1 \end{aligned} \tag{10a}$$

whereas for (9b) we have

$$
\begin{aligned}
&C_0 = E_0 - a_0 + f_0 \\
&w_0 = a_0 \\
&D_0 = f_0 \\
&C_s = C_{s-1}(1+i) + a_s - f_s \\
&w_s = w_{s-1}(1+x_s) - a_s \\
&D_s = D_{s-1}(1+\delta_s) - f_s \qquad \text{for } s \geq 1
\end{aligned} \tag{10b}
$$

While (10a) is obvious, it is worth clarifying the meaning of (10b). In case (i) the investor records in the balance sheet the following facts: the cash flows $f_1, f_2, \ldots, f_n$ are withdrawn from account C; the cash flows $a_1, a_2, \ldots, a_n$ are invested in account C; the capital invested in the project (outstanding capital) increases by the operating profit $x_s w_{s-1}$ and decreases by the receipt a_s; the residual debt for the loan contract increases by the periodic interest $\delta_s D_{s-1}$ and decreases by the payment f_s made to repay the debt. As for the income statement at time s, $s \geq 1$ (10a) leads to

$$
\begin{aligned}
\text{revenues} &= 0 \\
-\text{expenses (depreciation)} &= 0 \\
------ & \\
\text{net operating profit} &= 0 \\
+\text{interest on } C &= iC^{s-1} \\
-\text{interest on } D &= 0 \\
------ & \\
\text{net profit} &= iC^{s-1}
\end{aligned}
$$

whereas (10b) leads to

$$
\begin{aligned}
\text{revenues} &= a_s \\
-\text{expenses (depreciation)} &= -(w_{s-1} - w_s) \\
------ & \\
\text{net operating profit} &= x_s w_{s-1} \\
+\text{interest on } C &= iC_{s-1} \\
-\text{interest on } D &= -\delta_s D_{s-1} \\
------ & \\
\text{net profit} &= x_s w_{s-1} + iC_{s-1} - \delta_s D_{s-1}
\end{aligned}
$$

This means

$$E^s = E^{s-1} + iC^{s-1} \tag{11a}$$

$$E_s = E_{s-1} + x_s w_{s-1} + iC_{s-1} - \delta_s D_{s-1} \tag{11b}$$

for $s \geq 1$. We have, in general, $E_s \neq E^s$ for all $s \geq 1$. (11a) and (11b) are non-homogeneous first-order difference equations whose solutions are, respectively,

$$E^n = E_0 + \sum_{s=1}^{n} iC^{s-1} \tag{12a}$$

$$E_n = E_0 + \sum_{s=1}^{n} \big(x_s w_{s-1} + iC_{s-1} - \delta_s D_{s-1}\big). \tag{12b}$$

Subtracting (12a) from (12b) our accountant is able to evaluate the project: The sum

$$E_n - E^n = \sum_{s=1}^{n} \Big(x_s w_{s-1} - \delta_s D_{s-1} - i\left(C^{s-1} - C_{s-1}\right)\Big) \tag{13}$$

shows the profitability of P. If (13) is positive, the decision maker accepts the project, otherwise she rejects it.

We can see that this accounting-based profitability index is already naturally decomposed in n shares to be ascribed to each period. To see how, just think that we aim at answering the following question: what's the difference between what the investor earns in the period s if she undertakes P and what she would earn should she invest her funds at the rate i? To answer the question we must compute the difference between net earnings *sub* (i) and net earnings *sub* (ii). We have

$$\overline{M}_s := (E_s - E_{s-1}) - (E^s - E^{s-1}). \tag{14a}$$

But, from (11a) and (11b) we realize that this difference is just the s-th addend in (13):

$$\overline{M}_s = x_s w_{s-1} - \delta_s D_{s-1} - i\left(C^{s-1} - C_{s-1}\right). \tag{14b}$$

Thus, (14b) represents the differential gain (or loss) of (i) over (ii) for period s. It is actually a periodic residual income, consistent with the income statements we have just drawn up. As the latter are derived by the investor's financial system, we call such a differential gain *Systemic Value Added for period* s. Using (13) we have then

$$E_n - E^n = \sum_{s=1}^{n} \big((E_s - E_{s-1}) - (E^s - E^{s-1})\big). \tag{15}$$

Denoting with SVA (Systemic Value Added) the total differential gain $E_n - E^n$, we have the following decomposition of SVA:

$$\text{SVA} = \overline{M}_1 + \overline{M}_2 + \cdots + \overline{M}_n. \tag{16}$$

$\overline{M}_s$ is therefore that part of the (overall) Systemic Value Added which is generated in the s-th period. $\overline{M}_s$ itself can be decomposed in three components: the change in wealth accomplished by the operating profit $x_s w_{s-1}$ (project factor), net of interest payments $\delta_s D_{s-1}$ (debt factor), and the amount $-i\big(C^{s-1}-C_{s-1}\big)$ (opportunity factor), which represents an opportunity cost or return (depending on the sign of $(C^{s-1}-C_{s-1})$), namely the interest the investor gives up (or which accrues to the investor) in the s-th period if she undertakes the project.

4. Net Final Value and Systemic Value Added

The decomposition of the SVA in n sub-indexes is accomplished by means of a systemic outlook. While the NPV (NFV) decomposition induces the evaluator to compute the s-th share as a (fictitious) sub-project's Net Present (Final) Value, (see (8a) and (8b)), our accounting-like reasoning enables us to consider the decision maker's net worth as a system, whose structure consists of assets and equities (or, in financial terms, uses and sources of funds) gradually changing in value over time. The time dimension is grasped by the evolution of the system: The net worth is periodically invested and the structure is modified so as to take account of reinvestments and withdrawals of cash flows. This changes periodically the decision maker's wealth. Different courses of action are described by different evolutions of the system's structures.

The philosophy of accounting is a natural tool for a systemic approach: What we have done is just to express the s-th net profit for (i) in terms of a surplus $\overline{M}_s$ (positive or negative) with respect to the alternative (ii):

$$E_s - E_{s-1} = E^s - E^{s-1} + \overline{M}_s$$

The sum of the net profits related to (i) can then be thought of as the sum of all net profits for (ii) and all surpluses so that

$$\sum_{s=1}^{n}\big(E_s - E_{s-1}\big) = \sum_{s=1}^{n}\big(E^s - E^{s-1}\big) + \text{SVA}.$$

The Systemic Value Added is consistent with the Net Final Value (and Net Present Value) but subsumes a different partition. To see how, we will dwell on $\overline{M}_s$ and, in particular, on the opportunity factor $i\,\big(C^{s-1} - C_{s-1}\big)$. We have

$$C_{s-1} = C_0(1+i)^{s-1} + \sum_{k=1}^{s-1}(a_k - f_k)(1+i)^{s-k-1}$$

and

$$C^{s-1} = C^0(1+i)^{s-1} = (C_0 + w_0 - D_0)(1+i)^{s-1}$$

so that

$$\big(C^{s-1} - C_{s-1}\big) = (w_0 - D_0)(1+i)^{s-1} - \sum_{k=1}^{s-1}(a_k - f_k)(1+i)^{s-k-1}.$$

Using (5') and (6') and rearranging terms, the latter reduces to

$$\big(C^{s-1} - C_{s-1}\big) = (w_{s-1} - D_{s-1}) - M_1(1+i)^{s-2} - M_2(1+i)^{s-3} - \cdots - M_{s-2}(1+i) - M_{s-1}$$

where

$$M_s := w_{s-1}(x_s - i) + D_{s-1}(i - \delta_s) = \mathrm{EVA}_s, \qquad s = 1, \dots n.$$

Substituting in (14b) we obtain the s-th share of project P's SVA

$$\overline{M}_s = M_s + \sum_{k=1}^{s-1} iM_k(1+i)^{s-k-1}. \tag{17}$$

Letting $A_s := \sum_{k=1}^{s} \overline{M}_k$, we now prove, by induction, that

$$A_s = \sum_{k=1}^{s} M_k(1+i)^{s-k} \qquad \text{for every } s \geq 1. \tag{18}$$

Setting s=1, we have, from (17), A_1=M_1. Pick m arbitrary and assume that (18) holds for every $s \leq m$, we find

$$\begin{aligned} A_{m+1} &= [\text{for additivity}] = A_m + \overline{M}_{m+1} \\ &= [\text{for (17)}] = A_m + M_{m+1} + \sum_{k=1}^{m} iM_k(1+i)^{m-k} \\ &= \sum_{k=1}^{m} M_k(1+i)^{m-k} + M_{m+1} + \sum_{k=1}^{m} iM_k(1+i)^{m-k} \\ &= M_{m+1} + \sum_{k=1}^{m} M_k(1+i)^{m+1-k} \\ &= \sum_{k=1}^{m+1} M_k(1+i)^{m+1-k} \end{aligned} \tag{Q.E.D.}$$

Hence, we obtain

$$\mathrm{SVA} = A_n = \sum_{s=1}^{n} M_s(1+i)^{n-s}. \tag{20}$$

Since

$$M_s = \frac{G_s}{(1+i)^{n-s}} \qquad \text{for all } s \geq 1$$

we finally get back to

$$\mathrm{SVA} = \sum_{s=1}^{n} \overline{M}_s = \sum_{s=1}^{n} G_s = \sum_{s=1}^{n} \mathrm{EVA}_s(1+i)^{n-s} = \mathrm{NFV}. \tag{19}$$

This result is consistent with the NFV (NPV) rule in that it states that the total relative gain SVA coincides with the financial-type index Net Final Value, and the decision maker will accept the project if and only if

$$\text{SVA} = \text{NFV} = \text{NPV}(1+i)^n > 0.$$

But while coinciding in overall terms, they give rise to different partitions. As we have seen, the s-th share of the NFV is the compound amount of $M_s(=\text{EVA}_s)$, i.e. G_s, whereas the "accounting-flavored" partition provides us with $\overline{M}_s$, with $\overline{M}_s \neq G_s$. The SVA model is grouded on a systemic/accounting way of reasoning which makes no use of NFVs nor compounding processes, whereas Peccati's model rests on financial arguments, in particular on the concept of Net Final (Present) Values and on capitalization processes. The two perspectives lead to different partitions of the two indexes. In a sense, by using a systemic perspective we are able to sum cash regardless of its maturity. This result, far from being illicit, suggests that we can create a cognitive outlook where there is no need of capitalization: time dimension is considered, as seen, by means of the system's time evolution, that is through periodic double-entry sheets.

5. Why do the two decomposition models differ?

Using (17) and (18), it is easy to see that $\overline{M}_s$ can be rewritten as

$$\overline{M}_s = M_s + i\left(\sum_{h=1}^{s-1} \overline{M}_h\right). \tag{21a}$$

This reformulation enables us to intepret the Systemic Value Added for period s $\overline{M}_s$ as the sum of a direct factor M_s (the periodic Economic Value Added generated by the capital invested w_{s-1} and by the residual debt D_{s-1}) and the periodic interest on the $(s-1)$ indirect factors $\overline{M}_h$: the latter represent the gain generated in period s by those shares referring to the previous periods, which yield returns at the rate i. These returns are borne in the s-th period: That is, $\overline{M}_1$, $\overline{M}_2$, $\ldots$,$\overline{M}_{s-1}$ can be considered assets that add up value to the global relative gain. Therefore, each share depends on all the preceding ones, which keep on bearing interest at the rate i. Such an imputation collides with the NFV-based imputation. To see why, let us assume, for the sake of simplicity, n=3, and let us decompose both SVA and NFV. We have the following decomposition table:

$$\begin{array}{lll} G_1 = M_1(1+i)^2 & G_2 = M_2(1+i) & G_3 = M_3 \\ \overline{M}_1 = M_1 & \overline{M}_2 = M_2 + i\overline{M}_1 & \overline{M}_3 = M_3 + i\overline{M}_1 + i\overline{M}_2 \end{array} \tag{21b}$$

or, which is the same,

$$\begin{array}{lll} G_1 = M_1 + (iM_1) + (iM_1 + i^2M_1) & G_2 = M_2 + (iM_2) & G_3 = M_3 \\ \overline{M}_1 = M_1 & \overline{M}_2 = M_2 + (iM_1) & \overline{M}_3 = M_3 + (iM_1 + i^2M_1) \\ & & \qquad + (iM_2) \end{array}$$

(21c)

where the first row decomposes the SVA, the second one decomposes the NFV.

As we can see, the NFV decomposition accomplishes a two-step evaluation. The idea is the following: M_1, M_2, M_3 are the three shares for period 1, 2, 3 respectively. As this is money referred to the dates 1, 2, 3, respectively, the basic principles of financial calculus force the evaluator to compound (or discount) flows to take time into consideration. After capitalization (and only after) the evaluator may sum the three shares. Conversely, in the light of our systemic perspective the decision maker can construct, in a gradual way, the three shares of the SVA. The first share is M_1 (=EVA$_1$), which exactly represents the difference between what the investor receives in the first period and what she would receive should she decide to forego the project opportunity and invest her funds at the opportunity cost of capital i. In the second period the difference between what she receives and what she would have received must take into account that, in addition to M_2, the first share does not disappear, but yields interest equal to iM_1. That is, in the second period the difference between what she receives and what she would have received must take into account that, in addition to the second EVA$_2$, the first share yields interest on the first EVA$_1$. Iterating the argument, the third share must consider the return on the two first shares M_1 and M_2, as well as the interest gained on iM_1 itself, which are produced just in the third period. Financially speaking, we can interpret every Systemic Value Added for period s as a capital invested at time s, yielding linear interest at the rate i until n, for a total interest of $(i(n-s)\overline{M}_s)$ each. In fact, we can easily check that

$$\begin{aligned}\text{NFV} = \text{SVA} &= \sum_{s=1}^{n} \overline{M}_s \\ &= \sum_{s=1}^{n} M_s + \sum_{s=1}^{n} i\left(\sum_{h=1}^{s-1} \overline{M}_h\right) \\ &= \sum_{s=1}^{n} M_s + \sum_{s=1}^{n-1} i(n-s)\overline{M}_s\end{aligned}$$

You can see that this line of argument is not obeyed by the “financial-flavored” decomposition. G_1 embodies the term iM_1 which, as we have seen, is to be ascribed to the second share, since it is generated in the second period. In addition, it comprehends the term $iM_1 + i^2M_1$ which in turn is related to the third period. At the same time G_2 includes iM_2, which is pertinent to the third period, but lacks the term iM_1 (previously embodied in G_1). Finally, the third share G_3 forgets the return on previous periods’ shares.

The financial-flavored decomposition rests on the basic principles of financial calculus, according to which cash flows cannot be summed unless they refer to the same maturity. To decompose a project’s NFV this model takes into consideration the sum

$$x_s w_{s-1} - \delta_s D_{s-1} - i(w_{s-1} - D_{s-1}) \tag{22}$$

which is conceived as the periodic differential gain; since it represents money available at time s, it must be compounded at time n (or discounted at time 0). But this brings about two anomalies:

first of all, it does not consider the return yielded by the preceding shares. Secondly, capitalizing M_s through the factor $(1+i)^{n-s}$ means anticipating money that will be earned in future periods, i.e. money that cannot be ascribed to the s-th period. Our systemic perspective provides a tool which properly imputes the differential gain and overcomes the issue of time uniformation of shares: In this sense the n shares are homogeneous, so our investor/accountant can safely sum them.

Now we investigate thouroughly the assumptions implicit in the two decomposition models. For the sake of simplicity, we will assume D_s=0 for every s, but the line of argument is not invalidated by this restriction, as we will see in Sec.8. Both the NFV and the SVA decompositions aim at answering the following question:

> "*What is the differential gain of (i) over (ii) that we are to ascribe to the s-th period?*" $(*)$

The answer to the question above is just the difference between alternative net profits, i.e.

$$(E_s - E_{s-1}) - (E^s - E^{s-1}).$$

The SVA outlook suggests us to answer to $(*)$ by directly drawing up two sequences of (cash) balance sheets for alternative (i) and (ii) respectively, whose result is

$$\overline{M}_s = \big(x_s w_{s-1} + iC_{s-1}\big) - iC^{s-1}. \tag{23a}$$

According to the NFV decomposition the evaluator argues as follows: At the beginning of the period s the investor has the opportunity to invest the sum w_{s-1}. She can select alternative (i) or alternative (ii). If (i) is selected the net profit will be $x_s w_{s-1}$; if (ii) is chosen, the net profit will be iw_{s-1}. The differential gain is, in time s value,

$$M_s = w_{s-1}(x_s - i) \tag{23b}$$

(23a) and (23b) are just (14b) and (22) under our zero debt assumption.

We call the latter argument the *financial* argument, as opposed to the former one, which we call the *systemic* argument. The next section is entirely devoted to show that the financial argument cannot provide us with the answer to $(*)$.

6. The financial and the systemic arguments

The *financial* argument assumes that we are to evaluate a (fictitious) uniperiodic project starting at time $(s-1)$ and terminating at time s:

time	0	1	2	....	$s-1$	s
cash flows	0	0	0	....	$-w_{s-1}$	$w_s + a_s$

as described in section 3. Let NFV(s) denote this sub-project's Net Final Value, calculated at time s (or, which is the same, the Net Present Value compounded until time s)

$$\text{NFV}(s) = -w_{s-1}(1+i) + w_s + a_s = w_{s-1}(x_s - i) \tag{24a}$$

(obviously, NFV(s)=EVA$_s$, as we expect). If this is to be the answer to (∗), then we must have

$$\text{NFV}(s) = (E_s - E_{s-1}) - (E^s - E^{s-1}). \tag{24b}$$

But we know that a Net Final Value is the difference between alternative final net worths. So, lookig back at (4c), we have

$$\text{NFV}(s) = E_s - E^s. \tag{24c}$$

The latter two entail

$$E_{s-1} = E^{s-1} \tag{25a}$$

(25a) tells us that if project P is undertaken the net worth at time $(s-1)$ (left-hand side) coincides with the net worth produced if the project is not undertaken (right-hand side). As this is true for *every* s, (25a) boils down tuns to n equalities

$$E_1 = E^1 \qquad E_2 = E^2 \qquad \dots \qquad E_n = E^n. \tag{25b}$$

Assume now the realistic case $w_{s-1} \neq 0$ for all s. We distinguish two exhaustive cases:

(a) $x_s \neq i$ for at least one s
(b) $x_s = i$ for all s.

If (a) holds, we have two kinds of contradictions: the *mathematical* and the *factual* contradiction. As for the *mathematical* contradiction, let s^* be an index such that $x_{s^*} \neq i$. For (25b), we must have

$$E_{s^*-1} = E^{s^*-1} \qquad \text{and} \qquad E_{s^*} = E^{s^*}.$$

As (24b) must also hold, this entails

$$\text{NFV}(s^*) = 0,$$

whence $x_{s^*} = i$, thanks to (24a), but this contradicts the assumption.

In addition, the *financial* argument leads to a *factual* contradiction, due to a vitiated interpretation of facts. In fact, the latter accomplishes the decomposition by calculating the NFV(s) for period s, which presupposes that the following assumption is made:

> *at time 0 the investor invests her net worth E_0 in asset C at the opportunity cost of capital until time $(s-1)$. At time $(s-1)$ the sum w_{s-1} is withdrawn by account C and invested in a uniperiodic project with rate of return x_s. At time s, the investor holds the final amount w_s alongside the value of account C, given by $C_s = \big(C^{s-1} - w_{s-1}\big)(1+i) + a_s$.*
> $\triangle_s$

As $\triangle$ is assumed to hold for *every* period s=1, 2,. . . , n, then it boils down to a set of n incompatible assumptions, $\triangle_1$, $\triangle_2$, . . . , $\triangle_n$: In fact for s=1 we have that

> *at time 0 the investor invests her net worth E_0 in asset C at the opportunity cost of capital until time 0. At time 0 the sum w_0 is withdrawn ... [etc.]*
>
> $\triangle_1$,

for s=2 we have that

> *at time 0 the investor invests her net worth E_0 in asset C at the opportunity cost of capital until time 1. At time 1 the sum w_1 is withdrawn ... [etc.],*
>
> $\triangle_2$

fore s=3 we have that

> *at time 0 the investor invests her net worth E_0 in asset C at the opportunity cost of capital until time 2. At time 2 the sum w_2 is withdrawn ... [etc.],*
>
> $\triangle_3$

and so on until s=n: We are clearly facing n mutually exclusive courses of action, as only one of them can be true.

As for(b), it causes the decision process to be an idle issue, as alternative (i) coincides, *from a mathematical-financial point of view*, with alternative (ii): There is no difference, financially speaking, in investing at the opportunity cost of capital the whole net worth or only a part of it, if the remainder is invested in a project whose rate of return is the opportunity cost of capital itself. This situation can be viewed as different only under a *factual* perspective, for (i) and (ii), though financially equivalent, represent distinct courses of action. In this case the two arguments lead to the same obvious (and uninteresting) result. Further, the *factual* contradiction persists, as $\triangle$ holds regardless of (a) and (b) (it is inherent in the very idea of decomposing a Net Final Value by means of n fictitious sub-projects' Net Final Value, which is the core of the *financial* argument).

No such contradictions, mathematical or factual, arise in the *systemic* argument, which presupposes the following hypothesis:

> *at time 0 the investor invests the sum (C^0-w_0) in asset C at the opportunity cost of capital and the sum w_0 in project P.* $\square$

According to $\square$ no fictitious sub-projects are introduced. The starting point is that the investor undertakes P at time 0. Consequently, the double-entry sheets are gradually drawn up step by step from the first one to the last one, by considering that the cash flow a_s are reinvested, as they are generated, in account C, and generate in turn interest at the rate of interest i:

$$\begin{array}{r|l} \underline{\text{Uses}} & \underline{\text{Sources}} \\ C_s = C_{s-1}(1+i) + a_s & E_s = E_{s-1} + iC_{s-1} + x_s w_{s-1} \\ w_s & \end{array} \tag{26a}$$

It could seem that the contradiction is healed with the assumption of w_{s-1}=0 for some s. On the contrary, regardless of whatsoever assumption on the outstanding capitals the *financial* argument is fallacious: in fact it introduces n sub-projects and therefore n isolated situations of the following kind:

$$\begin{array}{r|l} \underline{\text{Uses}} & \underline{\text{Sources}} \\ C_s = \big(C^{s-1} - w_{s-1}\big)(1+i) + a_s & E_s = C^{s-1} + iC^{s-1} + w_{s-1}(x_s - i) \\ w_s & \end{array} \tag{26b}$$

(26b) holds for *every* s, implying C_{s-1}=C^{s-1} and therefore E_{s-1}=E^{s-1}. To focus on the NFV(s) means to erase all the *differential* past prior to time $(s-1)$, as if the evaluator had not undertaken project P at time 0, investing instead her wealth at the opportunity cost of capital. That is, the two different courses of actions are made to coincide until time $(s-1)$. To accept $\triangle_s$ for *every* s means accepting the *financial* argument's relation

$$C_s = \big(C^{s-1} - w_{s-1}\big)\,(1+i) + a_s$$

for *every* s. But, will it or not, $\triangle_s$ cannot either escape the *systemic*'s recurrence equation

$$C_s = C_{s-1}(1+i) + a_s$$

for *every* s. The latter two are mathematically incompatible since they can be rewritten, respectively, as

$$\begin{aligned} C_s &= \big[E_0(1+i)^{s-1} - a_0\prod_{k=1}^{s-1}(1+x_k) + a_1\prod_{k=2}^{s-1}(1+x_k) + \cdots + a_{s-1}\big]\,(1+i) + a_s \\ C_s &= \big[E_0(1+i)^{s-1} - a_0(1+i)^{s-1} + a_1(1+i)^{s-2} + \cdots + a_{s-1}\big]\,(1+i) + a_s, \end{aligned}$$

which differ as long as there exists at least one k such that $x_k \neq i$.[5]

7. Reshaping the financial argument with a systemic perspective

We might ask whether we are able to heal the mathematical contradiction inherent in the *financial* argument with the help of the *systemic* approach. In other terms, can the *systemic* perspective be incorporated in the *financial* argument so as to provide a correct partition of the NFV on the basis of an NFV(s) analysis? The answer is the object of the present section.

Let $\overline{w}_s$ be the value of w_s obtained by replacing each x_s with i:

$$\overline{w}_s := a_0(1+i)^s - a_1(1+i)^{s-1} - \cdots - a_s \qquad s = 1, 2, \ldots, n$$

[5] The two C_s can be equal *for some* s, but not *for every* s, as assumed by the *financial* argument.

Note that the following then hold:

$$\overline{w}_s = C^s - C_s$$

$$\overline{w}_s = \overline{w}_{s-1}(1+i) - a_s$$

The *systemic* C_s can be then rewritten as

$$\begin{aligned} &C_0 = C^0 - \overline{w}_0 \\ &C_s = C_{s-1}(1+i) + a_s = \big(C^{s-1} - \overline{w}_{s-1}\big)(1+i) + a_s \qquad s = 1, 2, \ldots, n \end{aligned} \tag{27a}$$

where $\overline{w}_0{:=}a_0$. Now let

$$\begin{aligned} &\overline{a}_0 := a_0 \\ &\overline{a}_s := x_s w_{s-1} - i\overline{w}_{s-1} + a_s \qquad s = 1, 2, \ldots, n. \end{aligned} \tag{27b}$$

Suppose that the investor undertakes a project $\overline{P}$, which we name here *shadow project* of P, consisting of the cash flow stream

$$(-\overline{a}_0, \overline{a}_1, \ldots, \overline{a}_n).$$

The Net Final Value of $\overline{P}$ is

$$\overline{\text{NFV}} = -\overline{a}_0(1+i)^n + \sum_{s=1}^{n} \overline{a}_s(1+i)^{n-s}$$

It easy to demonstrate that we can correctly decompose P by applying Peccati's decomposition to the *shadow* project $\overline{P}$ provided that we avoid to compound the Net Final Value so obtained.

From (27b) we obtain

$$\begin{aligned} &\overline{w}_0 = \overline{a}_0 \\ &\overline{w}_s = \overline{w}_{s-1}(1+\overline{x}_s) - \overline{a}_s \qquad s = 1, 2, \ldots, n \end{aligned} \tag{28}$$

where

$$\overline{x}_s := x_s \, \frac{w_{s-1}}{\overline{w}_{s-1}}.$$

We can then interpret $\overline{w}_s$ as the *project balance* of $\overline{P}$ at the rate $\overline{x}_s$, and the $\overline{a}_s$ as withdrawals from (if positive) or investments in (if negative) an account yielding interest at the periodic rate $\overline{x}_s$, s=1,2, $\ldots n$.

Let us decompose $\overline{P}$ by using the *financial* argument: the investor invests $\overline{w}_{s-1}$ at the beginning of the s-th period and receives the sum $\overline{w}_s + \overline{a}_s$ at the end of the period:

time	$s-1$	s
cash flows	$-\overline{w}_{s-1}$	$\overline{w}_s + \overline{a}_s$

At time s the periodic net final value of the project is

$$\begin{aligned}\overline{\mathrm{NFV}}(s) &= -\overline{w}_{s-1}(1+i) + \overline{w}_s + \overline{a}_s \\ &= -\overline{w}_{s-1}(1+i) + (\overline{w}_{s-1}(1+\overline{x}_s) - \overline{a}_s) + \overline{a}_s \\ &= \overline{w}_{s-1}(\overline{x}_s - i). \end{aligned} \tag{29}$$

Note that (29) is just the Economic Value Added of the shadow project (henceforth $\overline{\mathrm{EVA}}_s$), as we expected, due to equivalence of Peccati's model and Stewart's. We state that such an $\overline{\mathrm{EVA}}_s$ measures the periodic residual income for P answering question $(*)$. If this is true, we shall find that

$$\overline{\mathrm{EVA}}_s = \overline{\mathrm{NFV}}(s) = (E_s - E_{s-1}) - \left(E^s - E^{s-1}\right).$$

In fact,

$$\begin{aligned}\overline{\mathrm{EVA}}_s &= \overline{\mathrm{NFV}}(s) \\ &= \overline{w}_{s-1}(\overline{x}_s - i) \\ &= x_s w_{s-1} - i\overline{w}_{s-1} \\ &= x_s w_{s-1} - i\left(C^{s-1} - C_{s-1}\right) \\ &= \overline{M}_s \\ &= (E_s - E_{s-1}) - \left(E^s - E^{s-1}\right). \end{aligned} \tag{30}$$

Hence,

$$\mathrm{SVA} = \sum_{s=1}^{n} \overline{w}_{s-1}(\overline{x}_s - i) = \sum_{s=1}^{n} \overline{\mathrm{NFV}}(s) = \sum_{s=1}^{n} \overline{\mathrm{EVA}}_s = \mathrm{NFV} \tag{31}$$

and the two models are, to a certain extent, reconciled with no need of compounding.[6]
It is worthwhile noting that

$$\overline{a}_s = \overline{M}_s + a_s \qquad \text{for } s \geq 1$$

[6]We can decompose P by directly applying the *financial* argument to P provided that we take account of the proper net worth's structure by means of a correction factor, which takes into consideration that the *financial* argument uses w_{s-1}, whereas it should use $\overline{w}_{s-1}$. This discrepancy arises at the beginning of period s, so what the *financial* argument disregards is the interest generated during the period. The correction factor must then be $i(w_{s-1} - \overline{w}_{s-1})$ and it allows us to avoid compounding once we have found the periodic NFV of P. So, we first compute G_s. Since the latter does not fit the correct system's structure, we need take account of the correction factor, compounded until time n. We

so that

$$-a_0(1+i)^{n-s} + \sum_{s=1}^{n} a_s(1+i)^{n-s} = -\overline{a}_0 + \sum_{s=1}^{n} \overline{a}_s(i) - (-a_0 + \sum_{s=1}^{n} a_s) \tag{32}$$

where the dependence of $\overline{a}_s$ on i is pointed out. Changing i, $\overline{a}_s(i)$ adjusts itself so as to avoid the need of compounding. According to (32) the Net Final Value of project P, calculated at the rate i, equals the difference between the Net Final Value of the cash flow stream produced by project $\overline{P}$ and the Net Final Value of the cash flow stream generated by P, both calculated at a zero rate (which means, in other terms, that we do not compound). Therefore, the *shadow* project is that (fictitious) project wich enables us to overlook capitalization processes. As its cash flows are just differerential net profits calculated from (cash) balance sheets, this approach links accounting and financial calculus, often thought of as two incompatible frameworks. And more, it seems that persisting on a NFV-based outlook leads to antinomies, whereas the use of an accounting-like perspective is so natural and safe that is capable of embracing the mathematical structure of the NFV healing the latter's inconsistencies.

It is also worthwhile noting that (30) provides us with the interesting result that the Economic Value Added of $\overline{P}$ coincides with the Systemic Value Added of P. We are dealing then with three models, two of them are NFV-flavored (Peccati's and Stewart's), the other is accounting-flavored. The relations we have found among the three models can be summarized as follows:

$$\begin{aligned}
M_s &= \text{EVA}_s = g_s(1+i)^s = \frac{G_s}{(1+i)^{n-s}} \\
\overline{M}_s &= \overline{\text{EVA}}_s = \overline{\text{NFV}}(s) \\
\overline{\text{EVA}}_s &= \text{EVA}_s + \sum_{k=1}^{s-1} i\text{EVA}_k(1+i)^{s-k-1} \\
\overline{\text{EVA}}_s &= \text{EVA}_s + i\left(\sum_{h=1}^{s-1} \overline{\text{EVA}}_h\right).
\end{aligned}$$

Therefore, we can answer to $(*)$ by means of our SVA model, whereas (*) is not answered by the other two models. We can nonetheless retrieve the concept of Economic Value Added (and therefore

---

sum the two and then offset the previous compounding process by discounting back to time s: We obtain, as a result,

$$\begin{aligned}
\text{Periodic NFV of } P &= \frac{G_s + i(w_{s-1} - \overline{w}_{s-1})(1+i)^{n-s}}{(1+i)^{n-s}} \\
&= M_s + i(w_{s-1} - \overline{w}_{s-1}) \\
&= \overline{\text{NFV}}(s) \\
&= \overline{M}_s
\end{aligned}$$

We have then

$$\text{SVA} = \sum_{s=1}^{n} \frac{G_s + i(w_{s-1} - \overline{w}_{s-1})(1+i)^{n-s}}{(1+i)^{n-s}}.$$

the NFV-based models) by introducing our shadow project $\overline{P}$. In addition, we get to a complete partition of the NFV, as we do not need compounding. So we have the overall residual income SVA (=NFV) which is subdivided in n periodic residual incomes, each of which is an Economic Value Added.

We end this section by pointing out that the decomposition based on the shadow project is only a particular case of a more general scheme. This means that we have not one but infinite *shadow* projects. Let $\underline{P}$ be any of these, consisting of the sequence of cash flows

$$(-\underline{a}_0, \underline{a}_1, \ldots, \underline{a}_n).$$

Denote with $\underline{w}_s$ its outstanding balance at rate $\underline{x}_s$. Fix all $\underline{w}_s$ arbitrarily; then pick $a_0 = \underline{w}_0$. As for the other flows, they must satisfy

$$-\underline{w}_{s\text{–}1}(1+i) + \underline{w}_s + \underline{a}_s = \overline{M}_s \tag{33}$$

whence

$$\begin{aligned} \underline{a}_0 &= a_0, \\ \underline{a}_s &= \underline{w}_{s\text{–}1}(1+i) - \underline{w}_s + \overline{M}_s \end{aligned} \tag{34a}$$

The rate $\underline{x}_s$ is thus univocally determined from the outstanding balance equation

$$\underline{w}_s = \underline{w}_{s\text{–}1}(1+\underline{x}_s) - \underline{a}_s,$$

so that we can rewrite (34a) as

$$\begin{aligned} \underline{a}_0 &= a_0, \\ \underline{a}_s &= \underline{w}_{s\text{–}1}(\underline{x}_s - i) + \overline{M}_s \end{aligned} \tag{34b}$$

We have found infinite sequences of cash flows $\underline{a}_s(\underline{w}_{s\text{–}1}, \underline{w}_s)$, depending on the outstanding capitals selected, and infinite *shadow* projects $\underline{P}$. $\overline{P}$ is just one of these, obtained by choosing $\underline{w}_s$=$\overline{w}_s$, which implies $\underline{w}_s$=$\underline{w}_{s\text{–}1}(1+i)$−$a_s$ and therefore $\underline{a}_s$=$a_s + \overline{M}_s$=$\overline{a}_s$ for all s≥1.

8. The shadow project and the financial system

In reframing the decision/evaluation process we have applied the *financial argument* to project $\overline{P}$. In this way, the contradiction seen arises for project $\overline{P}$, but not for project P. To clear the issue, the reader can turn to (26) and apply (26b) to project $\overline{P}$. That is, the evaluator applies $\triangle_s$ by pretending that alternative (i) is referred to $\overline{P}$, not to P. This means that we have n sub-projects and n situations of the following kind:

$$
\begin{array}{rl}
\underline{\text{Uses}} & | \quad \underline{\text{Sources}} \\
\overline{C}_s = \big(C^{s-1} - \overline{w}_{s-1}\big)(1+i) + \overline{a}_s & | \quad \overline{E}_s = C^{s-1} + iC^{s-1} + \overline{w}_{s-1}(\overline{x}_s - i) \\
\overline{w}_s & |
\end{array}
\tag{26c}
$$

where the new notations $\overline{C}_s$, $\overline{E}_s$ remind us that account C and the net worth E are measured by pretending that alternative (i) refers to P's shadow project. (26c) implies, as we know, $\overline{C}_{s-1}{=}C^{s-1}$ and $\overline{E}_{s-1}{=}E^{s-1}$ *for every* s, as well as (26b) implies $C_{s-1}{=}C^{s-1}$ and $E_{s-1}{=}E^{s-1}$. We know that this means that (26b) leads to the contradiction we have studied for project P, as it holds for *every* s. Likewise, (26c) leads to the same contradiction for project $\overline{P}$. But the net profit in (26c) coincides with the net profit in (26a):

$$iC^{s-1} + \overline{w}_{s-1}(\overline{x}_s - i) = iC_{s-1} + x_s w_{s-1}.$$

This entails that while the net profit in (26c) is incorrect for $\overline{P}$, it is correct for P.

So doing, we *shift* the contradiction, moving it from P to $\overline{P}$. Peccati's *financial argument* can be now safely applied (without capitalization) because its contradictory assumptions invalidate the decomposition of $\overline{P}$, while recovering at the same time the decomposition of P, which now coincides with the SVA model here introduced. To say it in Stewart's terms: to decompose a project P take $\overline{\text{EVA}}_s$ not EVA_s (and forget capitalization)!.

9. Generalizations

In this section we take some generalizations of the aforementioned results. First, we relax our zero debt assumption assuming $D_{s-1}{\neq}0$. Secondly, we assume that the opportunity cost of capital changes over time and denote it with i_s for period s. With such assumptions, if we refer alternative (i) to P and apply the *systemic argument*, the financial system's structure at time s is depicted as follows:

$$
\begin{array}{rl}
\underline{\text{Uses}} & | \quad \underline{\text{Sources}} \\
C_s = C_{s-1}(1+i_s) + a_s - f_s & | \quad D_s \\
w_s & | \quad E_s = E_{s-1} + i_s C_{s-1} + x_s w_{s-1} - \delta_s D_{s-1}
\end{array}
$$

which is nothing but (9b). Conversely, if we refer alternative (i) to $\overline{P}$ and apply the *financial argument*, the financial system's structure at time s is described as follows:

$$
\begin{array}{rl}
\underline{\text{Uses}} & | \quad \underline{\text{Sources}} \\
\overline{C}_s = \big(\overline{C}^{s-1} - \overline{w}_{s-1} + \overline{D}_{s-1}\big)(1+i_s) + \overline{a}_s - \overline{f}_s & | \quad \overline{D}_s \\
\overline{w}_s & | \quad \overline{E}_s = C^{s-1} + i_s C^{s-1} + \overline{w}_{s-1}(\overline{x}_s - i_s) + \overline{D}_{s-1}(i_s - \overline{\delta}_s)
\end{array}
$$

where

$$\overline{D}_0 := D_0 = f_0$$
$$D_s := D_{s-1}(1+\delta_s) - f_s$$
$$\overline{D}_s := \overline{D}_{s-1}(1+i_s) - f_s$$
$$\overline{\delta}_s := \delta_s \, \frac{D_{s-1}}{\overline{D}_{s-1}}$$

and where $\overline{w}_s$ is now such that

$$\overline{w}_s := \overline{w}_{s-1}(1+i_s) - a_s.$$

Financially speaking, we introduce the shadow project $\overline{P}$ whose cash flows $\overline{a}_s$ are diminished by the debt cash flows $\overline{f}_s$, so that the net sequence is

$$(-\overline{a}_0 + \overline{f}_0, \overline{a}_1 - \overline{f}_1, \ldots, \overline{a}_n - \overline{f}_n)$$

with

$$\overline{a}_s = x_s w_{s-1} - i_s \overline{w}_{s-1} + a_s$$

and

$$\overline{f}_s = \delta_s D_{s-1} - i_s \overline{D}_{s-1} + f_s,$$

whence

$$\overline{a}_s - \overline{f}_s = a_s - f_s + \overline{M}_s$$

where the Systemic Value Added for period s is now redefined by replacing i with i_s:

$$\overline{M}_s := x_s w_{s-1} - \delta_s D_{s-1} - i_s \left(C^{s-1} - C_{s-1}\right) = x_s w_{s-1} - \delta_s D_{s-1} - i_s(\overline{w}_{s-1} - \overline{D}_{s-1})$$

At every time $(s-1)$ the investor invests the sum $\overline{w}_{s-1}$ in a uniperiodic project whose rate of return is $\overline{x}_s$, which is partly financed with debt ($\overline{D}_{s-1}$) and partly with her own net worth ($\overline{w}_{s-1} - \overline{D}_{s-1}$), i.e. by a withdrawal from account C.[7] Following Peccati's *financial argument*, the situation is

time	$s-1$	s
cash flows	$-\overline{w}_{s-1} + \overline{D}_{s-1}$	$\overline{w}_s + \overline{a}_s - \overline{D}_s - \overline{f}_s$

Using the fact that

$$\overline{w}_s = \overline{w}_{s-1}(1+\overline{x}_s) - \overline{a}_s$$

[7] If the value of account C is negative, account C can be seen as external financing as well as the debt. But the former looks like a current account, whereas the latter is a loan contract.

$$\overline{D}_s = \overline{D}_{s-1}(1+\overline{\delta}_s) - \overline{f}_s$$

we get to

$$\begin{aligned}\overline{\text{NFV}}(s) &= \overline{\text{EVA}}_s \\ &= -(\overline{w}_{s-1} - \overline{D}_{s-1})(1+i_s) + \overline{w}_s + \overline{a}_s - \overline{D}_s - \overline{f}_s \\ &= \overline{w}_{s-1}(\overline{x}_s - i_s) - \overline{D}_{s-1}(\overline{\delta}_s - i_s)\end{aligned}$$

which coincides with P's Systemic Value Added for period s.

Let us further generalize and assume that the investor finances the project turning to m creditors. Now, the financial system at time s for project P is

Uses	Sources
C_s	D_s^1
	D_s^2
	...
	D_s^m
	E_s

whereas for $\overline{P}$ we have the same structure, but the symbols $\overline{C}_s$, $\overline{D}_s^l$, $\overline{E}_s$ replace, respectively C_s, D_s^l, E_s Letting f_s^l be the cash flow for debt l at time s, $l = 1, \ldots, m$, and δ_s^l its contractual rate, we have now

$$\overline{\text{NFV}}(s) = \overline{w}_{s-1}(\overline{x}_s - i_s) - \sum_{l=1}^{m} \overline{D}_{s-1}^l(\overline{\delta}_s^l - i_s)$$

where

$$\begin{aligned}\overline{D}_0^l &:= D_0^l := f_0^l \\ D_s^l &:= D_{s-1}^l(1+\delta_s^l) - f_s^l \\ \overline{D}_s^l &:= \overline{D}_{s-1}^l(1+i_s) - f_s^l \qquad\qquad l = 1, 2, \ldots, m \\ \overline{\delta}_s^l &:= \delta_s^l \, \frac{D_{s-1}^l}{\overline{D}_{s-1}^l}\end{aligned}$$

with $f_s^l \geq 0$.

Suppose next that the investor holds a portfolio of q projects: The evaluation of the portfolio implies an enrichment of the system's structure so that

$$\begin{array}{r|l} \underline{\text{Uses}} & \underline{\text{Sources}} \\ C_s & D_s^1 \\ w_s^1 & D_s^2 \\ w_s^2 & ... \\ ... & D_s^m \\ w_s^q & E_s \end{array} \tag{35}$$

for project P, whereas for $\overline{P}$ we have the same structure but the symbols $\overline{C}_s$, $\overline{D}_s^l$, $\overline{E}_s$, $\overline{w}_s^r$ replace, respectively, C_s, D_s^l, E_s, w_s^r, with

$$\begin{aligned} \overline{w}_0^r &:= a_0^r \\ \overline{w}_s^r &:= \overline{w}_{s-1}^r(1+i_s) - a_s^r \qquad\qquad r = 1, 2, \ldots, q \end{aligned}$$

where a_s^r is obviously the cash flow generated by project r at time s.

The value of C for P is

$$\begin{aligned} C_s &= C_{s-1}(1+i_s) + \sum_{r=1}^{q} a_s^r - \sum_{l=1}^{m} f_s^l \\ &= \left(C^{s-1} - \sum_{r=1}^{q} \overline{w}_{s-1}^r + \sum_{l=1}^{m} \overline{D}_{s-1}^l\right)(1+i_s) + \sum_{r=1}^{q} a_s^r - \sum_{l=1}^{m} f_s^l \end{aligned}$$

whereas for $\overline{P}$ it is

$$\overline{C}_s = \left(C^{s-1} - \sum_{r=1}^{q} \overline{w}_{s-1}^r + \sum_{l=1}^{m} \overline{D}_{s-1}^l\right)(1+i_s) + \sum_{r=1}^{q} \overline{a}_s^r - \sum_{l=1}^{m} \overline{f}_s^l.$$

We have

$$\begin{aligned} w_s^r &= w_{s-1}^r(1+x_s^r) - a_s^r \\ D_s^l &= D_{s-1}^l(1+\delta_s^l) - f_s^l \end{aligned}$$

for P and

$$\begin{aligned} \overline{w}_s^r &= \overline{w}_{s-1}^r(1+\overline{x}_s^r) - \overline{a}_s^r \\ \overline{D}_s^l &= \overline{D}_{s-1}^l(1+\overline{\delta}_s^l) - \overline{f}_s^l \end{aligned}$$

for $\overline{P}$, with

$$\overline{x}_s^r = x_s^r \frac{w_{s-1}^r}{\overline{w}_{s-1}^r}.$$

The net profit relative to project P coincides with that of project $\overline{P}$, and we can find it again by applying Peccati's argument to project $\overline{P}$. We have then

$$\overline{\text{NFV}}(s) = \overline{\text{EVA}}_s = \sum_{r=1}^{q} \overline{w}_{s-1}^r (\overline{x}_s^r - i_s) - \sum_{l=1}^{m} \overline{D}_{s-1}^l (\overline{\delta}_s^l - i_s)$$

We end the section with a further extension: We replace account C with a plurality of "opportunity" accounts K^j, whose periodic rate of interest (opportunity costs of capital) are i_s^j, j=1,2,... p. The system's structure for P is now articulated as

$$\begin{array}{r|l} \underline{\text{Uses}} & \underline{\text{Sources}} \\ K_s^1 & D_s^1 \\ K_s^2 & D_s^2 \\ \ldots & \ldots \\ K_s^p & \ldots \\ w_s^1 & \ldots \\ w_s^2 & \ldots \\ \ldots & D_s^m \\ w_s^q & E_s \end{array} \qquad (36)$$

whereas for $\overline{P}$ we have the same structure but $\overline{K}_s^j$, $\overline{w}_s^r$, $\overline{D}_s^l$, $\overline{E}_s$ replace, respectively, K_s^j, w_s^r, D_s^l, E_s.

The latter generalization forces the evaluator to select one or more "opportunity" accounts K_s^j to be activated for withdrawals and reinvestment of the cash flows released by the projects and the debts. Referring to time s, denote with a_s^{rj} the quota of project r's cash flow invested in (if positive) or withdrawn from (if negative) account K^j. Likewise, denote with f_s^{lj} the quota of debt l's cash flow withdrawn from account K^j, j=1,2,... p, such that

$$\sum_{j=1}^{p} a_s^{rj} = a_s^r$$

$$\sum_{j=1}^{p} f_s^{lj} = f_s^l.$$

Let us give the following notations:

$$\begin{aligned}
\overline{w}_0^{rj} &:= w_0^{rj} := a_0^{rj}\\
\overline{D}_0^{lj} &:= D_0^{lj} := f_0^{lj}\\
w_s^{rj} &:= w_{s-1}^{rj}(1+x_s^r) - a_s^{rj}\\
\overline{w}_s^{rj} &:= \overline{w}_{s-1}^{rj}(1+i_s^j) - a_s^{rj}\\
D_s^{lj} &:= D_{s-1}^{lj}(1+\delta_s^l) - f_s^{lj}\\
\overline{D}_s^{lj} &:= \overline{D}_{s-1}^{lj}(1+i_s^j) - f_s^{lj}.
\end{aligned}$$

The value of K^j for P is

$$\begin{aligned}
K_s^j &= K_{s-1}^j(1+i_s^j) + \sum_{r=1}^{q} a_s^{rj} - \sum_{l=1}^{m} f_s^{lj}\\
&= \left(K_0^j \prod_{k=1}^{s-1}(1+i_k^j) - \sum_{r=1}^{q}\overline{w}_{s-1}^{rj} + \sum_{l=1}^{m}\overline{D}_{s-1}^{lj} \right)(1+i_s^j) + \sum_{r=1}^{q} a_s^{rj} - \sum_{l=1}^{m} f_s^{lj} \qquad l=1,2,\dots,p,
\end{aligned} \tag{37a}$$

the value of K^j for $\overline{P}$ is

$$\overline{K}_s^j = \left(K_0^j \prod_{k=1}^{s-1}(1+i_k^j) - \sum_{r=1}^{q}\overline{w}_{s-1}^{rj} + \sum_{l=1}^{m}\overline{D}_{s-1}^{lj} \right)(1+i_s^j) + \sum_{r=1}^{q} \overline{a}_s^{rj} - \sum_{l=1}^{m} \overline{f}_s^{lj} \qquad l=1,2,\dots,p, \tag{37b}$$

The q projects are split in p accounts, so that we can imagine a set of pq implicit projects P^{rj}. Each account j deals then with q projects consisting of the sequences

$$(a_0^{1j}, a_1^{1j}, \dots, a_n^{1j})$$
$$(a_0^{2j}, a_1^{2j}, \dots, a_n^{2j})$$
$$\dots\dots\dots\dots$$
$$(a_0^{qj}, a_1^{qj}, \dots, a_n^{qj})$$

implicit in projects P^1, P^2, ..., P^q respectively. We introduce q shadow projects $\overline{P}^r$ as well as pq implicit shadow projects for each account K^j. Let $\overline{P}^{rj}$ be the shadow project of P^{rj}. Focusing on a single K^j we are in the same situation described in the previous generalization. The outstanding capital for project $\overline{P}^{rj}$ is $\overline{w}_{s-1}^{rj}$. This is the capital invested in period s. The total sum withdrawn from account K^j at the beginning of the period is therefore

$$\sum_{r=1}^{q}\overline{w}_{s-1}^{rj}.$$

At the same time, the investor borrows from m creditors, whose outstanding debts are $\overline{D}^l_{s-1}$. We can impute a quota of each debt to account K^j through the ratio

$$\gamma^j_s = \frac{\sum_{r=1}^q \overline{w}^{rj}_{s-1}}{\sum_{j=1}^p \sum_{r=1}^q \overline{w}^{rj}_{s-1}} = \frac{\sum_{r=1}^q \overline{w}^{rj}_{s-1}}{\sum_{r=1}^q \overline{w}^{r}_{s-1}}.$$

Letting $\overline{D}^{lj}_{s-1}$ be the quota of debt $\overline{D}^l_s$ imputed to account K^j we have

$$\overline{D}^{lj}_{s-1} = \gamma^j_s \overline{D}^l_{s-1}$$

so that

$$\sum_{j=1}^p \overline{D}^{lj}_{s-1} = \overline{D}^l_{s-1}.$$

The total external financing for account K^j is

$$\sum_{l=1}^m \overline{D}^{lj}_{s-1}.$$

We have then p situations of the following kind, one for each account:

time	$s-1$	s
cash flows	$-W'_{s-1} + D'_{s-1}$	$+W'_s + A'_s - D'_s - F'_s$

with

$$W'_{s-1} := \sum_{r=1}^q \overline{w}^{rj}_{s-1}$$
$$D'_{s-1} := \sum_{l=1}^m \overline{D}^{lj}_{s-1}$$
$$A'_s := \sum_{r=1}^q a^{rj}_s$$
$$F'_s := \sum_{l=1}^m f^{lj}_s$$

The quota of the shadow's periodic NFV (EVA) to be ascribed to account K^j is then

$$\overline{\text{NFV}}^j(s) = \overline{\text{EVA}}^j(s) = \sum_{r=1}^{q} \overline{w}_{s-1}^{rj}(\overline{x}_s^r - i_s^j) - \sum_{l=1}^{m} \overline{D}_{s-1}^{lj}(\overline{\delta}_s^l - i_s^j). \tag{38}$$

(38) is the share generated in period s by account j. We can rearrange (38) so as to decompose the share according to the source of funds used. Let

$$\alpha_s^{rj} := \frac{\overline{w}_{s-1}^{rj}}{\sum_{r=1}^{q} \overline{w}_{s-1}^{rj}};$$

$\alpha_s^{rj}\overline{D}_{s-1}^{lj}$ is that part borrowed from creditor l financing the initial outlay $\overline{w}_{s-1}^{rj}$.[8]

It is easy to rearrange (38) and manipulate it so as to obtain

$$\overline{\text{NFV}}^j(s) = \sum_{r=1}^{q} \left[\left(\sum_{l=1}^{m} \alpha_s^{rj}\overline{D}_{s-1}^{lj}(\overline{x}_s^r - \overline{\delta}_s^l) \right) + \left((\overline{w}_{s-1}^{rj} - \sum_{l=1}^{m} \alpha_s^{rj}\overline{D}_{s-1}^{lj})(\overline{x}_s^r - i_s^j) \right) \right] \tag{39}$$

Letting $Q_s^{lrj}{:=}\alpha_s^{rj}\overline{D}_{s-1}^{lj}(\overline{x}_s^r - \overline{\delta}_s^l)$, $l{=}1,\dots m$ and $Q_s^{m+1,rj}{:=}\left(\overline{w}_{s-1}^{rj} - \sum_{l=1}^{m} \alpha_s^{rj}\overline{D}_{s-1}^{lj}\right)(\overline{x}_s^r - i_s^j)$ and summing for j and s we obtain the portfolio's Net Final Value

$$\text{NFV} = \sum_{s=1}^{n}\sum_{j=1}^{p} \overline{\text{NFV}}^j(s) = \sum_{s=1}^{n}\sum_{j=1}^{p} \overline{\text{EVA}}_s^j = \sum_{s=1}^{n}\sum_{j=1}^{p}\sum_{r=1}^{q}\sum_{l=1}^{m+1} Q_s^{lrj} \tag{40}$$

As we see, (40) leads to a natural tetra-dimensional decomposition of the portfolio's NFV:

(I) periodic decomposition (according to the n periods involved),
(II) opportunity account decomposition (according to the p opportunity accounts activated)
(III) project decomposition (according to the q projects undertaken)
(IV) financing decomposition (according to the $(m+1)$ sources of funds).

Q_s^{lrj} is the quota of the portfolio's NFV to be imputed to source l, to project r, to account j, to period s. If we sum it for l we obtain the quota of the s-th period NFV of project r generated by the opportunity account K^j. If we sum it for r we obtain the quota of the s-th period NFV of the whole portfolio generated by source l of account K^j. If we sum it for j we obtain the share of the

[8]Note that

$$\alpha_s^{rj}\overline{D}_{s-1}^{lj} = \frac{\overline{w}_{s-1}^{rj}}{\sum_{r=1}^{q}\overline{w}_{s-1}^{rj}} \frac{\sum_{r=1}^{q}\overline{w}_{s-1}^{rj}}{\sum_{j=1}^{p}\sum_{r=1}^{q}\overline{w}_{s-1}^{rj}}\overline{D}_{s-1}^{l} = \frac{\overline{w}_{s-1}^{rj}}{\sum_{j=1}^{p}\sum_{r=1}^{q}\overline{w}_{s-1}^{rj}}\overline{D}_{s-1}^{l} = \frac{\overline{w}_{s-1}^{rj}}{\sum_{r=1}^{q}\overline{w}_{s-1}^{r}}\overline{D}_{s-1}^{l}.$$

The ratio in the last member measures the amount invested in shadow project $\overline{P}^{rj}$ (outstanding capital) relative to the whole capital invested using account K^j, that is the entire amount withdrawn (if positive) from account K^j.

s-th period NFV of project r generated by source l. If we sum it for s, we obtain the quota of the portfolio's NFV generated by project r through use of source l of account K^j.

Taking the sum for more than one variable we obtain various types of information. For instance, if we wish to know project r's NFV we just have to sum A_s^{lrj} for l, j and s. If we instead wish to compute what is the total contribution of account K^j to the portfolio's NFV, we must sum it for l, r, and s; if we wish to calculate the total contribution of external financing to the s-th period NFV we must sum it for all $l \leq m$ and then for r and j; and so on. By different use of the variables we get different relevant pieces of information.

We end this section pointing out that the portfolio's NFV in (40) coincides, as we expect, with the portfolio's SVA obtained by directly using the *systemic* argument, as in Sec. 3:

$$\text{NFV} = \text{SVA} = \sum_{s=1}^{n} \overline{M}_s = \sum_{s=1}^{n} \text{project factor} + \text{debt factor} + \text{opportunity factor}$$

that is

$$\sum_{s=1}^{n} \left(\sum_{r=1}^{q} x_s^r w_{s-1}^r - \sum_{l=1}^{m} \delta^l D_{s-1}^l - \sum_{j=1}^{p} i_s^j \big(K_0^j \prod_{k=1}^{s-1} (1 + i_k^j) - K_{s-1}^j \big) \right) \tag{41}$$

whose term in brackets mirrors (14b).

Conclusions

This paper has proposed a model of periodic decomposition of a certain cash flow stream. A *systemic* outlook has been used, strictly related with an accounting-flavored approach. It has been set against the *financial* argument, but the term "systemic" is not meant to counter the term "financial": The latter has been used as a mere synonym of the locution "based on the concept of Net Present (or Final) Value". Actually, the *systemic* model does aim to a financial decomposition of a cash flow stream and heals some inconsistencies of the alternative model. A formal recovery of the *financial* argument is possible by changing interpretation and introducing the convenient concept of *shadow* project. Any project holds its own *shadow*, which we can use to correctly decompose the (overall) residual income. Referring to time n, we can alternatively apply Peccati's argument to (or compute the Economic Value Added of) the *shadow* of P (the two procedures lead to the same result), which coincides with P's Systemic Value Added for period s. We discover that the shares so obtained need not (and have not to) be compounded: The sum is taken as such. When adopting the systemic perspective, the process of compounding does not seem so natural from a decision-making point of view, as it seems, in my opinion, an unescapable device to salvage an improper framing, which anticipates some (differential) profits, while discarding others (see again (21)). Further, the recovery of Peccati's and Stewart's model enables us to obtain relevant information: The last section has generalized the integration of the *financial* model into the *systemic* framework, ending with the case of a portfolio of projects financed by multiple debts and multiple synchronic opportuniy accounts. The evaluation we have arrived to provides us with four types of decomposition:

(I) periodic decomposition (the share of portfolio's NFV generated in period s), obtained by summing Q_s^{lrj} for all variables except s

(II) opportunity account decomposition (the share of portfolio's NFV generated by the use of account K^j), obtained by summing Q_s^{lrj} for all variables except j

(III) project decomposition (the share of portfolio's NFV generated by project r), obtained by summing Q_s^{lrj} for all variables except r

(IV) financing decomposition (the share of portfolio's NFV generated by the use source l), obtained by summing Q_s^{lrj} for all variables except l.

We have therefore reached a *systemic-financial* decomposition, lending itself to the solution of problems of maximization, which further researches can dwell on. For example, we can consider each debt rate as a function of the amount borrowed by the investor, and each synchronic opportunity costs as a function of the value of the opportunity accounts. As the latter depend on the cash flows invested in and withdrawn from, the evaluator copes with the following non-trivial maximization problem

$$\max_{(D_0^l, a_s^{rj}, f_s^{lj})} \quad \sum_{s=1}^{n}\sum_{j=1}^{p}\sum_{r=1}^{q}\sum_{l=1}^{m+1} A_s^{lrj}$$

with a total of $(m+p+q+n)$ variables to be selected.

An internal rate of return for the entire portfolio can be easily calculated by looking back at Solomon (1956), who asserts that we must consider alternative courses of actions and that an internal rate of return is meaningful if it is the periodic yield produced by an entire course of action and not by a project.[9] This idea fits the *systemic* framework and we can then define the internal rate of return of the system as that rate y such that

$$E_0(1+y)^n = E_n = E^n + \text{SVA}. \tag{42}$$

This proposal is based on the *systemic* concept that "the idea of a rate of growth involves a ratio and cannot be uniquely defined unless one can uniquely value initial and terminal positions" (Hirshleifer (1958, p.347)). Hirshleifer is thinking of initial and terminal positions for the *project*, but these cannot always be univocally determined (e.g., when the initial cash flow is an inflow). In addition, to compare such generalized rates of return is meaningless unless the projects'initial outlays are the same. The systemic internal rate of return defined in (42) generalizes the idea by letting the initial *net worth* and the final *net worth* be the initial and final positions, so that we can compare two or more different courses of action, involving different projects with different initial outlays (or even projects with an initial receipt).

Future researches can dwell on multiple objectives for the evaluator. In this case the *systemic* framework could fit in the following way: The economic agent has various kinds of objectives, financial, economic, strategic etc. The preferences lead to a selection of different structures of the system, by activating a particular account preferred or borrowing from a preferred creditor or investing a certain sum in a preferred project. We might think to introduce a preference index which

[9] *"The valid comparison is [...] between two alternative courses of action. The ultimate criterion is the total wealth that the investor can expect from each alternative by the terminal date [...] If the rate of return is to be used as an index of relative profitability, then the relevant rate is the per annum yield promised by each alternative course of action from its inception to a common terminal date in the future".* (Solomon (*op.cit.*, p.127))

summarizes the preference for a particular structure of the investor's wealth. The selection of the amount of debts and the strategy of activations of the opportunity accounts K^j (that is, the policy of reinvestments and withdrawals) will determine a particular sequence of balance sheets, leading to a preference index. Changing the structure the preference index will change as well as the Systemic Value Added. It could be interesting to study how a decrease in wealth is compensated, in terms of preference, by a particular net worth's structure. The model could be then enriched to deal with uncertain cash flows, in order to render it effectively applicable to real-life situations. In this case the opportunity cost of capital i should reflect the rate of return of equivalent-risk assets.

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