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On the Continuous Relaxation of Packing Problems – Technical Note

by

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On the Continuous Relaxation of Packing Problems – Technical Note [1]

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Abstract

We consider the classic Bin Packing Problem an we generalize it to the $m-$dimensional case. Then we consider the lower bound on the optimal solution value obtained with the continuous relaxation of the problem. We show that the *asymptotic worst-case performance* of this bound is 2^{-m}, thus generalizing previous results obtained for $m = 1$ and $m = 2$.

Key words: Bin Packing, Continuous Relaxation, Worst-Case Analysis.

1 Introduction

A well-known problem in Combinatorial Optimization is the one-dimensional *Bin Packing Problem* (BPP) in which we are given an unlimited number of bins with unit-capacity, a sequence $J = (a_1, a_2, \ldots, a_n)$ of items, each with a given *size* $s(a_i) \in (0, 1]$, and we are asked to pack all the items in the minimum number of bins (i.e. we are asked to partition the items into a minimum number b of subsets $B_1, B_2, \ldots, B_b$ such that $\sum_{a_i \in B_j} s(a_i) \leq 1$, for $j = 1, \ldots, b$). BPP is know to be unary NP-hard (see Garey and Johnson 1979) and has been intensively studied. The chapter devoted to BPP, in the book by Martello and Toth (1990), is the more complete reference for lower bounds and exact algorithms for the problem, whereas the status of the art on the approximability of BPP is summarized in the excellent survey of Coffman, Garey and Johnson (1996).

In this paper we study the simple lower bound L_0 obtained by solving the relaxed version of BPP in which any item is allowed to be splitted in pieces which can be packed into different bins. In any optimal solution to this problem all the bins, but the last one, are completely filled with the items. Allowing an item to be splitted is equivalent to remove the integrality constraint from a mathematical model of BPP, so giving the *continuous relaxation* of the problem. If we call z^* the

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value of the optimal solution to the continuous relaxation, then $L_0 = \lceil z^* \rceil$, where $\lceil \alpha \rceil$ denotes the minimum integer greater or equal to α

Given an instance I of a minimization problem P, let $L(I)$ be a lower bound value for the optimal solution value $OPT(I)$ of I. The *absolute worst-case performance ratio* of the procedure producing $L(I)$ is defined as the maximum real number ρ such that

$$\frac{L(I)}{OPT(I)} \geq \rho \qquad \text{for each instance I of P}$$

If the the bound ρ is achieved only when the number of items is sufficiently large we speak of *asymptotic worst-case performance.*

It is known that the absolute worst-case performance ratio of L_0 is $\frac{1}{2}$ (see Johnson 1973).

BPP is a basic packing problem: immediate generalizations are the *Two-Dimensional Bin Packing Problem* (2D-BPP) and the *Three-Dimensional Bin Packing Problem* (3D-BPP). In 2D-BPP we are given rectangular items, each with a given height and width, that must be packed into the minimum number of identical rectangular bins. In 3D-BPP the items are "small" boxes to be packed into the minimum number of "large" identical boxes. For 2D-BPP and 3D-BPP two different versions can be considered: in the first one the orientation of every item is fixed, whereas in the second one 90^o rotations of the items are allowed. Other variants ask to obtain the n items by cutting standardized stock pieces. In this case we have additional constraints due to technological restrictions, for example the so called *guigliottine-cut* constraint (see e.g. Dowsland and Dowsland, 1992; Dyckhoff and Finke, 1992 and Dyckoff, Scheithauer and Terno, 1997, for recent surveys).

Extending the definition of BPP to a vectorial space $\mathcal{R}^m$ (with $m \geq 1$ and integer) we obtain the $m-$dimensional BPP (mD-BPP). We are interested to study mD-BPP with possible rotation of the items and no additional restrictions. In particular we will study the asymptotic worst-case performance ratio of the lower bound L_0, applied to mD-BPP. The value of this ratio is known from more than twenty years, for $m = 1$, whereas for $m = 2$ it has been studied only recently by Martello and Vigo (1996). To our knowledge no result exists for the general problem.

In the next Section 2 we formally describe mD-BPP, and we briefly summarize the results previously obtained. In the next Section 3 we give an heuristic algorithm which is used to show that the asymptotic worst-case performance of L_0 is 2^{-m}.

2 The Problem and Previous Results

The multi-dimensional BPP can be formulated as follows. Given is an unlimited set of unitary hypercubes, and a sequence $J = \{a_1, a_2, \ldots, a_n\}$ of items each of which is characterized by an $m-$dimensional vector of sizes $s(a_j) = (s_1(a_j), s_2(a_j), \ldots, s_m(a_j))$, for $j = 1, \ldots, n$, with $s_i(a_j) \in (0, 1]$. The problem is to pack the items in the minimum number of hypercubes, in such a way that no two items assigned to the same hypercube intersect and the total size of the items assigned to the hypercube, is smaller or equal to one, for each dimension. We first consider the version of the problem in which no rotation of the items is allowed. At the end of our discussion we will generalize our results to the case with rotations.

In the following we call *m-hypercube* an hypercube in the $m-$dimensional space ($\mathcal{R}^m$), but we continue to use the word *bin* to indicate a $1-$hypercube, and we use the word *square* to denote a $2-$hypercube.

In order to have a mathematical model of the problem we can associate to an unitary hypercube the convex set: $B = \{x \in \mathcal{R}^m : 0 \leq x_i \leq 1, \text{ for } i = 1, \ldots, m\}$. Since the items cannot be rotated the position of an item a_j packed into B is completely defined by giving the coordinates of the vertex of a_j closest to the origin, i.e. the vertex $y = (y_1(a_j), y_2(a_j), \ldots, y_m(a_j))$ with minimum euclidean norm. It follows that item a_j coincides with the convex set $I_j = \{x \in \mathcal{R}^m : y_i(a_j) \leq x_i \leq y_i(a_j) + s_i(a_j), \text{ for } i = 1, \ldots, m\}$.

An item a_j is entirely contained in hypercube B if

$$y_i(a_j) + s_i(a_j) \leq 1, \qquad \text{for } i = 1, \ldots, m.$$

Two items a_j and a_k assigned to the same hypercube B do not overlap iff:

$$\exists i \in \{1, \ldots, m\} : y_i(a_j) + s_i(a_j) \leq y_i(a_k) \ \text{ or } \ y_i(a_k) + s_i(a_k) \leq y_i(a_j).$$

Let $v_j = \prod_{i=1}^{m} s_i(a_j)$ denote the *volume* of item a_j. Using the lower bound from continuous relaxation the minimum number of hypercubes necessary to pack the n items is

$$L_0 = \left\lceil \sum_{j=1}^{n} v_j \right\rceil \tag{1}$$

We have already recalled that when $m = 1$ the worst-case performance of L_0 is $\frac{1}{2}$.

Recently Martello and Vigo (1996) have determined the asymptotic worst-case performance ratio of L_0, for $m = 2$. Their proof utilizes a new and not trivial heuristic algorithm which consists of two phases. In the first phase the algorithm packs all the items into a *strip* of heigh one (and infinity length), in such a way that the total occupied area before a certain length $\overline{x}$ is no less than $\overline{x}/2$. The second phase derives a feasible bin solution dividing the strip in *slices* of length one. For each slice k two squares are created and the items entirely contained in slice k or assigned across the boundary between slices k and $k+1$ are feasibly packed in the two new squares. When all the items above have been packed, at most two additional squares are used to pack some items assigned after $\overline{x}$. It is proved that the total number of squares used is not larger than $4L_0+2$, so giving the asymptotic worst-case performance ratio $\frac{1}{4}$.

3 The Worst-Case Performance Ratio of L_0

In this section we give an heuristic algorithm for the solution of mD-BPP, and we show that the number of hypercubes used by this algorithm never exceeds $2^m L_0 + 2^{m-1} + 2^{m-2} - 1$, so proving that the asymptotic worst-case performance ratio of the continuous relaxation is 2^{-m}.

We call m_FF our algorithm, since it uses the *First-Fit* algorithm (FF) developed for BPP. First-Fit is a greedy algorithm that considers the items by increasing indices and assigns each one to the first bin in which it can be feasibly packed.

Algorithm m_FF is a recursive procedure which receives as input the number d of dimensions of the space we want to consider and a set J' of items to be packed. It gives as output a packing of the items into a number of $d-$dimensional hypercubes. To solve an instance of mD-BPP we run m_FF giving it as input $d = m$ and $J' = J$.

If m_FF is run with $d = 1$ it simply applies the First Fit algorithm. If otherwise $d > 1$ then the algorithm starts by considering the last dimension, d, and partitions the items into classes, according to their sizes, in this dimension. For each class of items the algorithm call itself, or algorithm First Fit, to pack the items of the class, along the previous $d-1$ dimensions. More precisely, given an integer $c \geq 0$ and a dimension index d, let

$$J_d^c = \{j \in J : 2^{-c} \geq s_d(a_j) > 2^{-(c+1)}\} \tag{2}$$

denote the $c-$th class of items, for the $d-$th dimension. Moreover let $\gamma(d) = \max\{c : J_d^c \neq 0\}$ be the index of the class containing the smallest items, in the $d-$th dimension. Algorithm m_FF partitions the item set J' into classes $J_d^0, J_d^1, \ldots, J_d^{\gamma(d)}$. The items of each class are projected to the $(d-1)-$dimensional space and are packed into $(d-1)-$hypercubes. Then the items packed into each $(d-1)-$hypercube are lifted to $\mathcal{R}^d$ and assigned to a single *slice* having the first $d-1$ sizes equal to one and the $d-$th size equal to 2^{-c}. When all the classes have been considered, then we have a feasible packing of all the items into slices which differ only in the last size (the first $d-1$ sizes being equal to one). But the possible values assumed by the last sizes are the discrete values $1, 2^{-1}, 2^{-2}, \ldots, 2^{-\gamma(d)}$ (see (2)), therefore we can pack these slices into a number, say $b(d)$, of $d-$hypercubes such that all hypercubes, but possibly one, are completely filled along the $d-$th dimension (simply order the slices by non-increasing sizes in the $d-$th dimension and apply the First-Fit algorithm). Remembering (2) we can easily see that the total volume occupied by the items in the first $b(d)-1$ $d-$hypercubes is $\frac{1}{2}(b(d)-1)V(d-1)$ where $V(d-1)$ denotes the minimum volume occupied in any $(d-1)-$hypercube. It follows that if we have an algorithm which packs the $(d-1)-$dimensional items in such a way that $V(d-1) \geq 2^{-(d-1)}$, then the total volume occupied by the items in $\mathcal{R}^d$ is at least $(b(d)-1)/2^{-d}$. Since m_FF packs the items into $b(d)$ hypercubes, then the claimed asymptotic worst-case holds.

Unfortunately our algorithm for packing the items in a space $\mathcal{R}^{d-1}$ has worst-case $2^{-(d-1)}$ as asymptotic value, and not as absolute value. This means that not all the hypercubes are correctly filled, but there are some hypercubes with a total item's volume smaller than $2^{-(d-1)}$. We call *residual* these hypercubes whilst we call *normal* the hypercubes in which there are packed items with total volume not smaller than $2^{-(d-1)}$. If we lift items packed into normal $(d-1)-$hypercubes, then we obtain *normal slices*, in $\mathcal{R}^d$, if instead we lift items packed into residual $(d-1)-$hypercubes, then we obtain *residual slices*. Let us call residual also the items packed in a residual hypercube.

In the following we describe how to manage the residual hypercubes and we prove that, in the final packing, the number of such hypercubes is independent of n, thus it has no influence when n is sufficiently large.

A possible pseudo-pascal description of algorithm m_FF follows, the only difference with the previous description being the separation of normal and residual slices. The normal slices are packed as above, whereas the residual slices are packed as shown in the next two theorems. The packing of residual slices produce, in any case, residual hypercubes.

Algorithm $m\_\mathrm{FF}(d, J, P)$
input: d = dimension of the space; J=set of items;
output: a feasible packing P;
begin
  if $d = 1$ **then**
    apply algorithm First Fit giving the packing P';
    mark as "residual" a possible bin with less than $\frac{1}{2}$ units assigned;
  else
    $S := \emptyset$; $R := \emptyset$;
    for $c := 0$ **to** $\gamma(d)$ **do**
      project each item of J_d^c to $\mathcal{R}^{d-1}$ and run $m\_\mathrm{FF}(d-1, J_d^c, P')$
      lift the items to $\mathcal{R}^d$;
      for each normal $(d-1)$–hypercube define the corresponding normal slice and put this slice into set S;
      for each residual $(d-1)$–hypercube define the corresponding residual slice and put this slice into set R;
    endfor;
    determine the packing P as follows:
      pack the normal slices of S into d–hypercubes; mark as "residual" the possible hypercube with total items volume smaller than 2^{-d};
      pack the residual slices of R into additional d–hypercubes (see the proof of Theorem 3.1 and 3.2 below) and mark these hypercubes as "residual";
  endif
end.

Theorem 3.1 *The number of residual squares in a two-dimensional packing is at most two.*

Proof. Using the First Fit algorithm each bin is filled with at least $\frac{1}{2}$ units, with the possible exception of the last bin (a residual bin). When we run m_FF to pack items in $\mathcal{R}^2$, then for each class c there could be at most one residual bin, so at most one residual slice can be generated, when the items of the residual bin are lifted to $\mathcal{R}^2$. In each of these slices the first dimension is used less than $\frac{1}{2}$ units, so it is possible to pack all the items of the residual slices into a single unitary square as follows (see Figure 1). We pack all the items of the residual bin associated with class J_2^0 in the first half of the square, cutting the square at $\frac{1}{2}$, along the first dimension. All the other items can be packed in the remaining half square with an iterative technique: at each iteration k we cut the remaining of the square in two identical parts, along the second dimension, then we pack in the first part the items of J_2^k assigned to a residual bin. So a single residual square is necessary to pack all the items of the residual bins generated by the First-Fit algorithm.

A second possible residual square could be defined when m_FF packs the normal slices. □

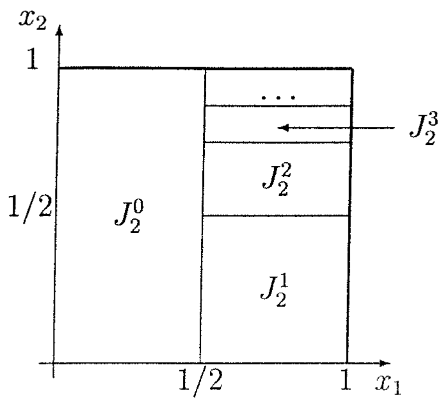

Figure 1: Packing of the residual items in $\mathcal{R}^2$

Theorem 3.2 *The maximum number of residual hypercubes generated by m_FF when it packs items in $\mathcal{R}^d$, with $d \geq 2$, it is $r(d) = 2^{d-1} + 2^{d-2} - 1$.*

Proof. If $d = 2$ then from Theorem 3.1 we have $r(2) = 2$ and the thesis holds. If instead $d > 2$ then we know that the first residual hypercube could be generated by packing the normal slices. Let $r(d-1)$ denote the maximum number of residual $(d-1)-$hypercubes which can be generated by packing the items of any class J_d^c, projected on $\mathcal{R}^{d-1}$. The items of J_d^0 associated with a single residual $(d-1)-$hypercube, require a $d-$hypercube to be packed. Instead, remembering (2) one can see that we can pack into a single $d-$hypercube the items associated with a residual $(d-1)-$hypercube for each class $J_d^1, J_D^2, \ldots, J_d^{\gamma(d)}$. So we need $r(d-1)$ $d-$hypercubes for the residual items from class J_d^0 and other $r(d-1)$ $d-$hypercubes for the residual items from the other classes, and the total number of residual $d-$hypercubes is $r(d) = 1 + 2r(d-1)$. Using this recursion and the fact that $r(2) = 2$ it is easy to obtain the thesis. □

From the above Theorem 3.2 it immediately descend that the number of normal $m-$hypercubes generated by m_FF, when it packs items in $\mathcal{R}^m$ is $b(m) - 2^{m-1} - 2^{m-2} + 1$. Reminding that in each normal hypercube we have packed items with total volume not less than 2^{-m}, we obtain

$$\frac{L_0(I)}{OPT(I)} \geq \frac{\lceil 2^{-m}(b(m) - 2^{m-1} - 2^{m-2} + 1) \rceil}{b(m)} = \frac{\lceil 2^{-m}(b(m)+1) - \frac{3}{4} \rceil}{b(m)} \qquad (3)$$

The above ratio is arbitrarily close to 2^{-m} for n sufficiently large. We have thus proved our main result.

Theorem 3.3 *The asymptotic worst-case performance ratio of the continuous relaxation of an $m-$dimensional bin packing problem is 2^{-m}.*

We can also prove that the above worst-case performance is tight. It is sufficient to consider an instance with $s_i(a_j) = \frac{1}{2} + \varepsilon$, for $i = 1, \ldots, m$, $j = 1, \ldots, n$, where ε is a small positive constant. It is easy to see that m_FF packs the items into n normal $m-$hypercubes, but the total volume is $V = n(\frac{1}{2} + \varepsilon)^m$, so the performance ratio can be arbitrarily close to 2^{-m}.

Finally note that the above example also shows that the same performance ratio holds when the rotation of the items is allowed.

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