Math Skills: a New Look from Functional Data Analysis

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Abstract— **Mental calculations involve various areas of the brain. The frontal, parietal and temporal lobes of the left hemisphere have a principal role in the completion of this typology of tasks. Their level of activation varies based on the mathematical competence and attentiveness of the subject under examination and the perceived difficulty of the task. Recent literature often investigates patterns of cerebral activity through fMRI, which is an expensive technique. In this scenario, EEGs represent a more straightforward and cheaper way to collect information regarding brain activity. In this work, we propose an EEG based method to detect differences in the cerebral activation level of people characterized by different abilities in carrying out the same arithmetical task. Our approach consists in the extraction of the activation level of a given region starting from the EEG acquired during resting state and during the completion of a subtraction task. We then analyze these data through Functional Data Analysis, a statistical technique that allows operating on biomedical signals as if they were functions. The application of this technique allowed for the detection of distinct cerebral patterns among the two groups and, more specifically, highlighted the presence of higher levels of activation in the parietal lobe in the population characterized by a lower performance.**

Index Terms— **EEGs, cortical activation, signal processing, functional data analysis**

I. INTRODUCTION

Difficulty and experience heavily influence cerebral activation during the completion of arithmetic assignments. Previous literature has clarified that, in right-handed individuals, the left hemisphere of the brain is the one most involved in the execution of arithmetical tasks [1], [2]. Concerning more specifically the mathematical competence, Grabner et al. [3] found a strong correlation between better skills and higher cortical activity in the middle temporal gyrus and the left angular gyrus. This fact is due to the involvement of these regions with the verbal retrieval of arithmetical facts. Moreover, when problem complexity increase, the activation area of the left angular gyrus becomes wider and extends to adjacent parietal regions. The left angular gyrus and the left temporal lobe instead reflect the level of mathematical knowledge [4], [3]. Individuals with a higher level of attention and working memory present increased activity in the prefrontal regions [1]. Lastly, the difficulty perceived by the subjects in completing the task solely influences the activation of the left parietal lobe [3].

In this paper, we focus on differences in cortical regions activation between two groups of subjects, classified according to their performance in executing an arithmetic task. In particular, we aim to investigate specific electrical potential responses evoked by these tasks in the prefrontal, parietal, and temporal areas. These regions are the most involved during attention and mathematical tasks [3]. Consequently, we expect the two groups to present different activation patterns in that lobe.

To be investigated, this context requires technologies capable of precisely assessing differences in the activation of the brain lobes. For this reason, previous literature proposed various attempts to investigate this phenomenon through functional Magnetic Resonance Imaging (fMRI) [3], [5], [6] and electroencephalograms (EEGs) [7], [8]. fMRI is a non-invasive procedure that provides a detailed report of oxygenation and deoxygenation levels in the brain that reflect, with delay, the activation of the different cortical areas. Despite these advantages, fMRI is an expensive technique and needs the patients to stay still to reduce distortions and inaccuracy to obtain the best possible data. Conversely, EEGs represent a cheaper solution and are characterized by a higher temporal resolution. Moreover, they are less invasive as they do not require the subject to be immobilized. On the other hand, they are less precise in locating the exact area of origin of the brain activities since they superficially integrate the whole underlying generated potentials. In fact, the potential of EEGs in exploring the mechanisms underlying mathematical reasoning has been previously addressed with positive results, yet leaving space to further analyses [9].

Given the respective strengths and weaknesses of fMRIs and EEGs, in this work, we propose a method to investigate EEG recordings through the use of a statistical technique able to enhance temporal and morphological changes in signals, known as Functional Data Analysis (FDA). This family of statistical techniques, comprehensively described by Ramsay [10], offers a class of methods to deal with the analysis of continuous curves and surfaces generated through the smoothing of discrete series of data. The main advantage of introducing this level of complexity lies in the possibility of treating discrete signals as continuous functions, gaining access to features other than time shifts. The quantitative assessment of morphological characteristics and the evaluation of the derivatives are two examples among the various features which can be analyzed. Moreover, curves are more natural to think through modeling problems, and they do not suffer from model misspecification [11]. In the specific context of biomedical signals, the smoothing procedure also filters out noise efficiently without altering the curve characteristics. Besides, previous studies proved the adequacy of this method for the analysis of biomedical signals and images [12] [13]. All these facts make FDA well-suited for modeling stochastic noisy signals as EEGs.

In this paper, we propose a FDA-based approach to explore cortical activation patterns at the basis of differences in the perceived difficulty in the completion of an arithmetical task. In Section II, we describe the dataset analyzed in this work, the tools employed, and the proposed method. In Section III, we present the results obtained, and in Section IV we discuss them alongside possible future works.

II. Methods

In this Section, we describe the dataset employed and the analysis carried out in this study. Our approach consists of three main steps: the representation of the signals as curves, the outliers removal, and a statistical test to verify if the curves characterizing the two populations differ significantly.

A. Dataset

The EEG signals treated in the present analyses come from the "EEG During Mental Arithmetic Tasks" dataset, made available by the National Taras Shevchenko University of Kyiv (Ukraine) [7]. The signals contained in this dataset are recorded using the international 10/20 scheme [14] sampled at 500 Hz and are provided in `edf` format [15]. For this study, we only take into consideration the channels Fp1, P3 and T3, as they are situated respectively in the left prefrontal, left parietal, and left temporal region. The experimental population includes 36 right-handed university students (9 males and 27 females) attending the Biology or Medicine faculties. The recordings were acquired through two experimental conditions: resting state and the execution of a subtraction task. In particular, during the recording of the resting state, the participants sat in a dark, soundproof chamber for 6 minutes with their eyes closed. During the subtraction task, the participants had to count mentally, without speaking or moving fingers for 4 minutes. The arithmetic task consisted of the serial subtraction of a 2 digits number from a 4 digits one (e.g., 17 and 5143). The authors considered a participant to succeed in the task if their result did not differ by more than 20% from the correct value. Based on their performance, participants were divided into two groups: "Bad Counters" (Group B, 10 patients) and "Good Counters" (Group G, 26 patients), based on the number of correct operations per minute [7]. Age did not significantly differ between the two subpopulations (B: 18.40 ± 2.01 and G: 18.19 ± 2.26 years). All the participants had normal or corrected-to-normal visual acuity and color vision and did not present any clinical manifestations of mental, cognitive, or psychiatric impairment nor learning disabilities. None of them were addicted to medications, alcohol, or drugs. Each subject signed an informed consent. The study adheres to the principles of the Declaration of Helsinki for medical research involving human subjects.

B. Preprocessing

All the steps hereby described are performed through the Python libraries: `MNE`, `Numpy` and `scikit-fda` [16], [17], [18]. Firstly, we divide the recordings into segments of 40 seconds each. Then, to remove the majority of artifacts and noise, we perform an averaging technique on them [19].

C. Curve Representation

In order to highlight the characteristics of the electrical activity elicited by the task, we subtract the resting state EEG from the one acquired during the task. The result is the object of our analysis and from now on in this paper, we will refer to it as the Extracted Functional Potential (EFP). As mentioned before, one of the advantages correlated to the use of FDA methods is that it allows the analysis of discrete biomedical signals as continuous functions; hence, the recordings are smoothed, fitting a basis function representation through the minimization of the sum of squared errors. This procedure not only filters out remaining noise but prepares the data for the application of FDA, by allowing their representation as functions. Basis representation consists of the use of a set of basis functions evaluated on sub-intervals defined by knots [10]. Basis function procedure consists in representing a function $x(t)$ by a linear expansion in terms of K known basis functions ϕ_k, weighted by the coefficients c_k, as below:

$$x(t) = \sum_{k=1}^{K} c_k \phi_k(t)$$

Among all possible basis functions, we choose to use the B-splines. In general, spline-basis are easier to compute and allow direct computation of the derivatives [20]. In particular, we use B-Spline basis elements [10], [21] recursively defined as follows:

$$B_{i,1}(x) = 1 \quad \text{if} \quad t_i \leq x < t_{i+1}, \quad \text{otherwise}$$

$$B_{i,k}(x) = \frac{x - t_i}{t_{i+k} - t_i} B_{i,k-1} + \frac{t_{i+k+1} - x}{t_{i+k+1} - t_{i+1}} B_{i+1,k-1}(x)$$

where: i represents the number of knots, defined in the interval $[1, n]$ with n set by the user ($n = 17$ in the present work); k indicates the order of the spline, which is set to 4 for B-Spline, meaning that each spline is a third order polynomial. The performance of the smoothing procedure was evaluated through visual inspection over traces, the evaluation criteria were the reduction of noise and a good approximation of the curves. The smoothed EFP curves are then employed to investigate differences in the activation patterns of the two groups.

D. Outliers removal and ANOVA test

We aim at detecting and removing outliers that would harm the validity of the work. For this purpose we exploit a functional boxplot of the smoothed EFPs for each of the three channels considered in the analyses. This informative exploratory tool is based on the center outward ordering induced by depth measures for functional data, which allows us to order curves from the most central to the most outlying in the distribution [22]. Depth measures are functions that assign, to each possible observation, a value measuring how central that observation is in a given distribution of curves. Thanks to this tool, we can efficaciously detect the outliers, which are classified through the application of the 1.5 times the 50% central region empirical rule, analogously to classical boxplots [23]. Finally, we remove the outlying signals from the two groups.

In order to evaluate if the smoothed EFP of the groups differ significantly, and in which channels, we use a functional one-way ANOVA test [24], a statistical test specifically developed to deal with functional data. This method assesses the null hypothesis of equality between the means of groups B and G. For this test we set the level of significance to $\alpha = 0.05$. Thus, we refuse the null hypothesis in channels with a p-value inferior to α.

III. Results

In this Section, we present the curves obtained through the preprocessing pipeline previously illustrated, the functional boxplots relative to the smoothed EFP and the outputs of the functional ANOVA test.

Fig. 1 illustrates the subtraction procedure through which we obtain the EFPs curves for the P3 electrode for two subjects exemplifying the B and G groups.

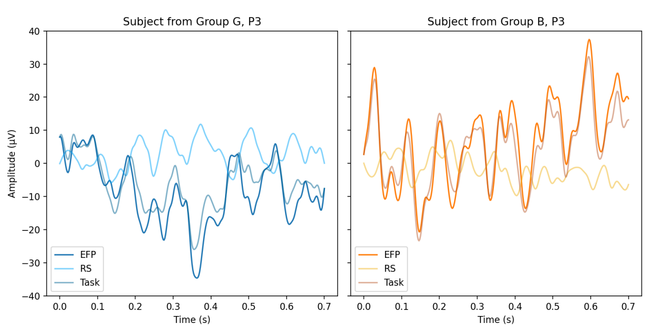

Fig. 1. Resting state (RS), Task and Extracted Functional Potential (EFP) recordings of one subject from group G (Female, 18 years old) and of one from group B (Female, 21 years old). EFP results from the subtraction of RS from Task.

The functional boxplots which allowed for the outlier inspection and removal are shown in Fig. 2.

Fig. 3 highlights the difference between the mean of the two sub-populations for the P3 channel, obtained after the removal of the outliers. It is evident that the arithmetical task elicits higher levels of activity in the parietal lobe of the B group.

Table 1 summarizes the results of the functional ANOVA test. Only the differences in the P3 recordings, the electrode posed in the parietal region of the scalp, reach a statistical significance. On the contrary, the activity recorded by the electrodes Fp1 and T3 (respectively in the prefrontal and temporal regions) result to be similar.

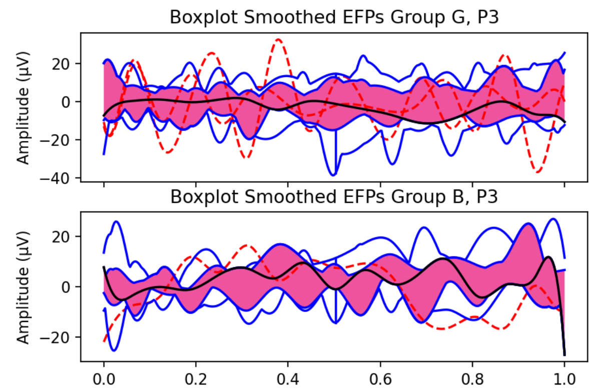

Fig. 2. Functional boxplots of EFP from channel P3 of groups G and B. It is possible to distinguish the central region (in magenta), the median (in black), the maximum non-outlying envelope (in blue) and the full outliers (the red dotted lines).

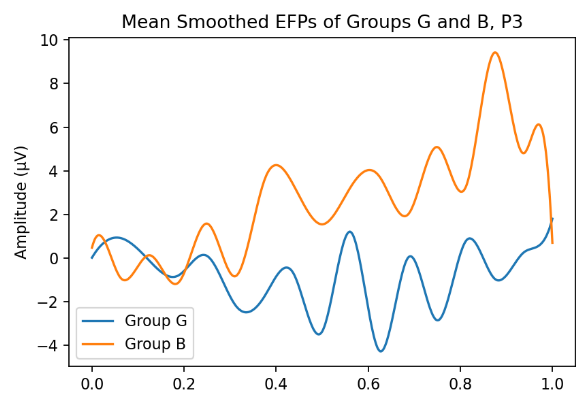

Fig. 3. Functional means of smoothed EFPs of groups G (in blue) and B (in orange) from channel P3 after the preprocessing and after the outliers removal.

TABLE I
Output of the one-way functional ANOVA test.
$*$: statistically significant.

Channel	P-value	Statistical Significance
Fp1	0.58	
P3	0.04	$*$
T3	0.87	

IV. Discussion & Conclusions

In this work, we propose a FDA-based method to explore cortical activity through EEGs recordings. To evaluate the potentiality of the presented approach, we tackle the detection of the discrepancies in the task complexity experienced by two groups of university students, classified as Good and Bad counters, during a subtraction task. In our analyses, we took into special account three brain regions by considering a channel each: the parietal (through the P3 electrode), the prefrontal (Fp1), and the temporal (T3), as they are the most involved in mathematical tasks. For the same reason, and since all the subjects are right-handed, we refer to the brain's left hemisphere.

A difference in the parietal region's activity of the two groups emerges from our result. More specifically, the P3 electrode registers significantly higher ($p = 0.04$) levels of activation in the subjects considered to be "Bad counters". This fact is compatible with previous findings in literature and with data made available by the creators of the dataset themselves. As stated in Section I, this lobe is known to increase its activity in correspondence of complicated tasks. Analogously, the temporal lobe is known to increase its activity according to the arithmetical competence of the patient. Moreover, the subjects enrolled in the study attend similar university faculties and, therefore, it is realistic to expect them to have similar capabilities and competencies. Hence, it is reasonable for T3 recordings not to show major discrepancies in the activity of the temporal lobe ($p = 0.87$). Likewise, the Fp1 electrode does not register significant differences between the levels of prefrontal activation of the two groups ($p = 0.58$). Consequently, we did not find any significant differences in attentiveness and working memory between the groups.

From these results, we can conclude that the proposed method can successfully detect the divergence in the task complexity experienced by the Good and Bad counters groups. In conclusion, the analysis of cerebral activation through EEG recordings and FDA can represent a useful addition to the more common practice of fMRI.

In the future, we aim to extend the work hereby presented in two ways. On the one hand, we plan to expand the method and the analysis proposed by implementing additional FDA procedures, such as clustering and functional PCA. This would allow us to better detail the differences between the groups. On the other hand, we plan to employ a more extensive dataset, characterized by more balanced groups B and G and by higher resolution EEGs, also considering more than one channel for cortical region.

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