MATHEMATICAL INSTRUMENTS AND THE «THEORY OF THE CONCAVE SPHERICAL MIRROR»: GALILEO'S OPTICS BEYOND ART AND SCIENCE

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RIASSUNTO

L'articolo, nel quale vengono presi in esame gli studi di ottica di Galileo precedenti all'utilizzazione del cannocchiale, cercherà di dimostrare come l'anacronistica distinzione tra arte e scienza abbia oscurato il reale significato dell'ottica galileiana. L'attenzione è concentrata in maniera particolare su di un documento poco conosciuto contenuto nel MS Gal. 83, «Theorica speculi concavi sphaerici», che è sempre risultato di difficile intepretazione, perché le sue relazioni con le reali tradizioni di ricerca in cui si è mossa l'opera galileiana non sono mai state ben comprese.

1. GETTING THE BIG PICTURE IN ART AND SCIENCE

When Lodovico Cigoli, Galileo's friend and a painter, heard about the opinion on the moon of Clavius, the mathematician of the Collegio Romano, he wrote to Galileo that «a mathematician without *disegno* is not only a mediocre mathematician, but also a man without eyes».[1] Recent scholarship has agreed with Cigoli's judgment that Galileo was such a mathematician. Edgerton has argued that Galileo perceived the light and dark spots on the moon as evidence for the moon's mountainous surface, because he was familiar with the «chiaroscuro» pictures, that are the pictures of shadows casted by illuminated regular bodies, in the books on

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[1] Cigoli to Galileo, August 11, 1611, in GALILEO GALILEI, *Le Opere di Galileo*, Firenze, G. Barberà, 11, 1968, p. 168.

linear perspective of Wenzel Jamnitzer, Daniele Barbaro, Lorenzo Sirigatti and Guidobaldo del Monte.[2] But what did Cigoli actually mean by his statement?

It's tempting to interpret Cigoli's statement in a way that makes Galileo a man who bridged the «two cultures» of art and science, and to herald Galileo's interpretation of the moonspots as an exemplary case of the formative influence of art on science. However, art and science are hardly appropriate categories to talk about the Renaissance, and thus Galileo's moon observations, because the disjunction of these «two cultures» is a modern concept that does not fit the fifteenth, sixteenth and seventeenth century. It is more historically appropriate to consider Cigoli's statement against the Renaissance background of the ubiquity of perspective. In the Renaissance, perspective was important to a whole range of practical activities, such as surveying, map-making, navigation, building and also painting.[3]

The mathematical core that provided arts like map-making, painting and surveying their disciplinary unity were projection techniques to represent space.[4] In his annotated edition of Euclid's «Optics», the Cosmographer of the Grand Duke of Tuscany in the 1560s, Egnazio Danti, also editor of Vignola's perspective manual «Le Due Regole della Prospettiva Pratica», boasted about the wide applications of perspective, that is the mathematics to represent space, to cartography, astronomy, architecture, painting and all the other arts of *disegno*.

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[2] SAMUEL Y. EDGERTON, *Galileo, Florentine «Disegno» and the «Strange spottednesse of the Moon»*, «Art Journal», 44, 1984, pp. 225-232. Reprinted in SAMUEL Y. EDGERTON, *The heritage of Giotto's geometry: art and science on the eve of the scientific revolution*, Ithaca/London, Cornell University Press, 1991, pp. 223-253. The manuals Edgerton referred to are: Guidobaldo del Monte, «Perspectivae libri sex», 1600; Daniele Barbaro, «Pratica della perspettiva», 1568; Lorenzo Sirigatti, «Pratica di prospettiva», 1596; Wenzel Jamnitzer, «Perspectiva corporum regularium», 1569.

[3] For this criticism of the application of a «two cultures» concept to the Renaissance, see JIM BENNETT, *The measurers: a Flemish image of mathematics in the sixteenth century*, Oxford, Museum of the History of Science, 1995, pp. 15-16. In a terminology more loyal to Renaissance usage, practical activities like surveying, navigation, dialling, were all «arts». Perspective has received a fair amount of attention from historians of art, but other practical activities that were involved with mathematics, have received relatively little attention from historians of science struggling with the heritage of Koyré. Now the interest for instruments in the history of science is growing, perspective is losing its privileged status. See also JIM BENNETT, *Practical geometry and operative knowledge*, «Configurations», 6, 1998, pp. 195-202, for a more detailed discussion.

[4] JIM BENNETT, *Projection and the ubiquitous virtue of geometry in the Renaissance*, in *Making space for science: territorial themes in the shaping of knowledge*, Crosbie Smith & Jon Agar ed., New York, St. Martin Press, 1998, pp. 27-38, for the substantiation of this claim, with a list of examples of practical mathematicians who wrote on perspective, but whose activity is not limited to this: Alberti, Leonardo, Piero della Francesca, Francesco di Giorgio Martini, Luca Pacioli, Egnatio Danti, Guidobaldo del Monte, Albrecht Dürer and Simon Stevin.

Also, to everyone should it be known to what extent and how perspective enriches Geography, because it alone shows the way to reduce in a plane, oval or circular, or in several other ways, the space of the complete earth, and of the particular provinces ... And not less help does it offer to Astronomy, because we know with certainty the size of the stars, and the position of the heavens, by which we know that the Moon is lower, and Saturn higher than the Sun, and lower than the fixed stars in the eight sphere. It also shows the distance from one Heaven to another, and from one star to another, and the reason why it happens that the stars appear larger in one place of the Heaven than in another. ... And leaving aside the advantages and usefulness it offers, to what extent it is necessary to infinite mechanical arts, in particular to Architecture and all the other arts of disegno, ..., I shall only say how I cannot but wonder how it is possible that this science of perspective is so low esteemed by learned men.[5]

To be a complete mathematician, means to know *disegno*, but *disegno* did not have the priviliged status it has been assigned by modern scholarship by making it «art» external to mathematics. It is just one of the many «arts». What Cigoli and Danti emphasized, was that *disegno* was «internal» to mathematics. As Bennett has argued, modern scholarhip should use this Renaissance concept of mathematics.

The ubiquity of perspective also explains why it has been argued that Galileo's acquaintance with contemporaneous cartography made him see the moon mountainous like another earth.[6]

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[5] EGNATIO DANTI, *La prospettiva di Euclide, nella quale si tratta di quelle cose, che per raggi diritti si veggono, & di quelle, che con raggi reflessi nelli specchi appariscono*, Fiorenza, Giunti, 1573, Proemio. Danti's biography is exemplary for the disciplinary unity of mathematics. Beside editing Euclid's «Optics» and Vignola's perspective manual, he is known to have been appointed cosmographer and astronomer to different patrons, to have designed and published books on instruments involving projection techniques, that is the astrolabe and the «radio latinos» of Orsini, and to have made a «Great Astronomical Quadrant». For the latter, see MARIA LUISA RIGHINI BONELLI & THOMAS SETTLE, *Eguazio's Danti's «Great Astronomical Quadrant»*, «Annali dell'Istituto e Museo di Storia della Scienza», 4, 1979, pp. 3-13.

[6] SCOTT L. MONTGOMERY, *The Scientific Voice*, New York & London, The Guilford Press, 1996, pp. 223-229. Galileo's engravings of the moon in the «Sidereus Nuncius» are commonly juxtaposed to Harriot's unpublished drawings of 1609 that lack Galileo's «chiaroscuro» rendering. Recently, Alexander has argued that Harriot's representations of the moon were influenced by his practice of mapping the coast of Virginia, together with the painter John White, on a discovery voyage with Sir Walter Raleigh. According to Alexander, Harriot never intended to make a lunar map showing topographical features like mountains, but he would have been able to if he had wanted so. See AMIR ALEXANDER, *Lunar maps and coastal outlines: Thomas Harriot's mapping of the moon*, «Studies in the History and Philosophy of Sciences», 29, 1998, pp. 345-368. At least on contemporary English topographical maps, for example, Saxton's, there is a «chiaroscuro» rendering of the mountains, but they were often done by Flemish engravers. Also the engraving by the De Bry, the Flemish publisher of Harriot's «A briefe and true report», of White's map shows a «chiaroscuro» rendering of the mountains. However, White's map does not show as much land inwards, and other drawings show that he did not master perspective. For White's

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For example, the maps of Danti in the «Guardaroba» of the Palazzo Vecchio show beautiful examples of the «chiaroscuro» rendering of mountains.[7] In fact, at the time, it was a pictorial convention for topographical maps to use little «molehills» lit from one side, mostly the left side as at sunset, and casting their shadow to the other side to represent a mountainous area. In the «Sidereus Nuncius», Galileo used the circular valley of Bohemia as a terrestrial analogy for the lunar Albategnius crater, whose dimensions he greatly exaggerated.

The area around the middle of the Moon is occupied by a certain cavity larger than all others and of a perfectly round figure. ... It offers the same aspect to shadow and illumination as a region similar to Bohemia would offer on Earth, if it were enclosed on all sides by very high mountains, placed around the periphery in a perfect circle. For on the Moon it is surrounded by such lofty ranges that its side bordering on the dark part of the Moon is observed bathed in sunlight before the dividing line between light and shadow reaches the middle of the diameter of that circle. But in the manner of the other spots, its shaded part faces the Sun while its bright part is situated towards the dark part of the Moon, which, I advise for the third time, is to be esteemed as a very strong argument for the roughness and unevennesses scattered over the entire brighter region of the Moon.[8]

The moon's crater showed the same appearances as a valley on earth at sunrise. As Montgomery has pointed out, Galileo would have seen the basin of Bohemia depicted like this on the corresponding map in Ortelius' atlas «Theatrum Orbis Terrarum». (Fig. 1) Being a mathematician, books on cartography and cosmography had their place in Galileo's library.[9]

drawings, see P. H. HULTON & D. B. QUINN, *The American drawings of John White, 1577-1590*, Chapel Hill, University of North Carolina Press, 1964; for De Bry's engravings, see THOMAS HARIOT, *A Briefe and True Report of the New Found Land of Virginia*, New York, Dover Publications, 1972.

[7] Danti was appointed cosmographer of the Grand Duke of Tuscany, Cosimo I, in 1562. Danti had the intent to make 52 maps for the «Guardaroba», at that time housing the Medici collection of mathematical instruments, but the project was only finished by his successor Buonsignori. See THOMAS B. SETTLE, *Egnazio Danti and mathematical education in late sixteenth-century Florence*, in *New perspectives on Renaissance thought: essays in the history of science, education and philosophy*, John Henry & Sarah Hutton ed., London, Duckworth, 1990, pp. 24-37. Danti's and Buonsignori's maps have been reproduced in EGNAZIO DANTI & STEFANO BUONSIGNORI, *Le tavole geografiche della Guardaroba Medicea di Palazzo Vecchio in Firenze/The geographical panels in the Medici Guardaroba of Palazzo Vecchio in Florence*, Bernucci Editore, 1995.

[8] GALILEO, *Sidereus Nuncius*, in GALILEO, *Opere*, 3, p. 68. Translation in ALBERT VAN HELDEN, *Galileo Galilei: Sidereus Nuncius or the sidereal messenger*, Chicago/London, The University of Chicago Press, 1989, p. 47.

[9] ABRAHAM ORTELIUS, *The theatre of the whole world*, London, R. A. Skelton ed., Amsterdam, Theatrum Orbis Terrarum Ltd., 1968. In Galileo's library there were indeed a fair amount

1. The basin of Bohemia, from: Ortelius, Theatrum Orbis Terrarum, 1592. (Photo Courtesy of Ghent University Library).

There is no incompatibility between this claim for cartography as Galileo's source and Edgerton's claim for *disegno* when there is not made a modern and ahistorical distinction between «art» and «science».

This big picture of mathematics has hardly come into view, in particular in studies of Galileo's optics. Galileo's educational background in optics has been reduced to a training in linear perspective, considered to be «art». However, a large folio, preserved among the papers of the MS Galileiana 83, that is entitled «Theorica speculi concavi sphaerici», has received very little attention, neither has its content ever been analyzed. When it has received attention, it has resisted interpretation, because giving it a place in Galileo's science has never been successful. This is due to considering Galileo's training in linear perspective unconnected to a larger mathematical framework that focussed on instrument making. It will be shown that the mathematics of painting did not have the privileged status

of cosmographical books, for example several editions of Ptolemy's «Geography». Ortelius' «Theatrum Orbis Terrarum» was one of them. See ANTONIO FAVARO, *La libreria di Galileo Galilei*, «Bulletino di Bibliografia e di Storia delle Scienze Matematiche e Fisiche», 19, p. 261.

it was assigned by modern scholarschip in Galileo's education or early research. Instead, it will be argued that the disciplinary unity of mathematics was preserved by its central aim to develop measuring instruments, and that Galileo fits this tradition. Once this context is described, the «Theorica speculi concavi sphaerici» will be analysed against this background of mathematical instrument making, shown to be the core of two optical traditions, painting and natural magic, which, at the end of the sixteenth and the beginning of the seventeenth century, were part of one optical continuum. Being almost the only document relating to Galileo's sources in optics that is preserved today, the «Theorica» will offer a privileged glimpse into the instrument-oriented context of Galileo's early optics.

2. GALILEO'S EDUCATION: MATHEMATICAL INSTRUMENTS

The claims for Galileo's training in linear perspective go back to what is known about his education under Ostilio Ricci, after Galileo had left the Pisan university. As Settle has shown, Ricci introduced Galileo to the study of Euclid and Archimedes, while Galileo also studied geometry and perspective under Ricci, together with Cigoli, in the house of Bernardo Buontalenti. What this education must have been like, is known through the MS Galileiana 10, that is the «Problemi Geometrici», attributed to Ricci. Settle has convincingly argued that this manuscript was a copy of the «Ludi Matematici» of Alberti, available in several manuscript copies and in a published edition of 1568 by Cosimo Bartoli. Settle perceived a connection between Alberti's «Ludi Matematici» and the tradition of artists' manuals on linear perspective.[10] Although Settle has emphasized that the «Ludi Matematici» conveyed an «advertisement for measurement» in general, his exclusive enumeration of perspective manuals in this context has turned out unfortunate, because, herewith, a priviliged connection between Galileo and the visual arts was considered established.[11]

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[10] All references are to THOMAS B. SETTLE, *Ostilio Ricci, a bridge between Alberti and Galileo*, in *XII° Congrès international d'histoire des sciences: actes*, Congrès international d'histoire des sciences, Paris, Librairie scientifique et technique Albert Blanchard, pp. 122-126. References are to the modern edition of the «Ludi matematici» by Cecil Grayson in LEON BATTISTA ALBERTI, *Opere volgari*, translation Cecil Grayson, Bari, Laterza e Figli, 1973, pp. 131-173, almost identical with Ricci's copy in the MS Gal. 10.

[11] «But the Ludi offers more than the knowledge of simple techniques. It conveys also certain attitudes ultimately derived within and very much part of the technical-*artistic* tradition of

However, although Alberti is the founding father of the tradition of artists' manuals with his «Della Pittura», the «Ludi Matematici» is not such a manual.[12] The «Ludi Matematici» showed how to measure by sight depths, heights and distances. To that end, techniques of triangulation and instruments were introduced. Already Euclid had proposed to use a mirror to measure the height of a tower. Alberti explained how a mirror was to be placed horizontally on the ground at a known distance from the foot of the tower. The measurer withdrew from the mirror until the top of the tower was seen reflected in the mirror. The distance from the mirror to the measurer has the same proportion to the height of the measurer as the distance from the mirror to the tower has to the height of the tower.[13] Even when Kemp's proposal is accepted that this surveying technique was the ground for Brunelleschi's panels, considered to be the invention of linear perspective, there is no reason to qualify them as «art» in a modern sense.[14] Later links between surveying, topography and perspective, instanced in the work of Leonardo, Luca Pacioli and Francesco di Giorgio Martini, rather suggests a continuum of mathematical disciplines, including perspective as used by painters, with at its core the making of measuring instruments.[15] Which were these instruments and how were they used?

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the 15th and 16th centuries, a tradition which included the perfection of perspective as one of the first «sciences» in a recognizably modern sense. *From the hands of Alberti himself, Piero della Francesca, Leonardo da Vinci and Albrecht Dürer in the earlier phases and Daniele Barbaro, Iacomo Barozzi da Vignola, Guidobaldo dal Monte and Cigoli in Galileo's own day* (only to mention a few) issues treatises on perspective.» SETTLE, *Ostilio Ricci*, p. 124. However, Settle elaborated on the influence of measurement techniques on Galileo's science, when discussing his musical education. See, in particular, THOMAS B. SETTLE, *La rete degli esperimenti galileiani*, in *Galileo e la scienza sperimentale*, Milla Baldo Ceolin ed., Padova, Dipartimento di Fisica «Galileo Galilei», 1995, pp. 12-62.

[12] Alberti's «Della Pittura» was the first codification of perspective. The Latin version was written in 1435 (first printed in 1540), the Italian version in 1436. Translated in LEON BATTISTA ALBERTI, *On Painting*, translation Cecil Grayson, introduction Martin Kemp, London, Penguin Books, 1991.

[13] ALBERTI, *Opere volgari*, pp. 138-139. Euclid's prototype is in the «Optics», not in the «Catoptrics». See PAUL VER EECKE, *Euclide: l'Optique et la Catoptrique*, Paris, Librairie scientifique et technique Albert Blanchard, 1959, p. 14.

[14] MARTIN KEMP, *Science, non-science and non-sense: the interpretation of Brunelleschi's perspective*, «Art History», 1, 1978, pp. 134-161. Today, Brunelleschi's panels are lost and for the construction method he may have used, a wide variety of sources have been proposed, such as surveying skills, projective techniques of Ptolemy's geography, medieval optics and many more. For a convenient list, see MARTIN KEMP, *The science of art: optical themes in western art from Brunelleschi to Seurat*, New Haven/London, Yale University Press, 1990, p. 345. Although I'm sympathetic to it, the argument for the ubiquity of perspective or the disciplinary unity of mathematics does not depend on Kemp's proposal for surveying being the origin of linear perspective.

[15] The final section of Francesco di Giorgio Martini's «Practica di geometria» on surveying dealt with painter's perspective. Surveyor's rods and strings were used to show the principles of

By the end of the sixteenth century a range of measuring instruments was available on the market. Some of these instruments were highly specialized, for example, Francesco Pifferi's *monicometro*,[16] but often the same instrument was used for several practical mathematical activities. Instruments better known for their astronomical purposes, for example, the astrolabe and the geometrical quadrant, were also used for purposes of terrestrial measurement.[17] In a very popular sixteenth century book, Cosimo Bartoli explained that to measure heights, depths and distances, the surveyor could make use of the shadow square on the back of the astrolabe's mater and of the alidade in its centre.[18] Also, Bartoli showed how the astrolabe could be used as a drawing aid by using it to project a building onto a plane intersecting the visual cone, «a thing of much use for Architects and those who delight in painting a perspective».[19] At the same time, the use of the astrolabe was advised to military men to check whether the enemy's troops were approaching or receding.[20] It is hardly

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perspective. KIM H. VELTMAN, *Studies on Leonardo da Vinci I: Linear perspective and the visual dimensions of art and science*, München, Deutscher Kunstverlag, 1986, pp. 87-106; KIM H. VELTMAN, *Military surveying and topography: the practical dimension of Renaissance linear perspective*, «Revista da Universidade de Coimbra», 27, 1979, pp. 330-332.

[16] Pifferi was a carmelite monk and mathematician, who taught shortly at the university of Pisa before Galileo's arrival, and thereafter at the University of Siena. Pifferi's «Monicometro instromento da misurar con la vista stando fermo», was published in 1595. The monicometro allowed to measure heights, depths and distances without the measurer needing to move to take more than one point of view. Pifferi, well known to Clavius of the Collegio Romano, was present at Galileo's demonstration of the telescope in Rome in 1611. See JAMES M. LATTIS, *Between Copernicus and Galileo: Christoph Clavius and the Collapse of Ptolemaic Cosmology*, Chicago & London, The University of Chicago Press, 1994, p. 188.

[17] The astrolabe was used for astronomical calculation and observation, that is to compute the movements of the stars and the sun and to measure altitude. In the sixteenth century, different projection techniques were used on an astrolabe. See J. A. BENNETT, *The divided circle: a history of instruments for astronomy, navigation and surveying*, Oxford, Phaidon & Christie's, 1987, pp. 14-16; BRUCE CHANDLER & SARA SCHECHNER GENUTH, *Western Astrolabes*, Chicago, Adler Planetarium & Astronomy Museum, 1998; ANTHONY TURNER, *Early scientific instruments: Europe 1400-1800*, London, Sotheby's Publications, 1987, pp. 11-16. The astrolabe used for terrestrial measurement was widely discussed, for example in Johann Stöffler, «Elucidatio fabricae ususque astrolabii», 1524; Juan de Rojas, «Commentariorum in astrolabium quod planispharium vocant», 1551; Egnazio Danti, «Trattato dell'uso et della fabbrica dell'astrolabio», 1569; Gemma Frisius, «De astrolabo catholico liber», 1583.

[18] COSIMO BARTOLI, *Del modo di misurare le distantie, le superficie, i corpi, le piante, le provincie, le prospettive, & tutte le altre cose terrene, che possono occorrere a gli huomini: secondo le vere regole d'Euclide, & de gli altri più lodati scrittori*, Portland (Oregon), Collegium Graphicum, 1972, pp. 16-49. See also FILIPPO CAMEROTA, *Giorgio Vasari Il Giovane: Raccolto fatto dal Cav: Giorgio Vasari di varii instrumenti per misurare con la vista*, «Biblioteca della Scienza Italiana», 14, Firenze, Giunti Gruppo Editoriale, 1996, pp. 44-48.

[19] BARTOLI, *Del modo di misurare*, pp. 47-48.

[20] BARTOLI, *Del modo di misurare*, p. 49.

surprising to find military concerns side by side with «artistic» concerns. It has often been observed that the perspective manuals of the end of the sixteenth century had an audience mainly interested in military fortifications, that the men, often known today as «artists», also were assigned the function of supervising the military fortifications of their patron, and that instruments, often considered helpful in a painter's workshop, like Alberti's veil or Dürer's *sportello*, were used for military topography and surveying.[21]

In general, in the fifteenth and sixteenth centuries, the military concerns of the courtly patrons of the mathematicians provided the framework and the motivation for the boom in measuring instruments.[22] There was a natural link between surveying and military concerns, for example range-finding in gunnery.[23] The elevation of the cannon depended on the distance of the target, which was, of course, inaccessible at the risk of one's life. Surveying instruments, incorporating triangulation techniques, allowed to measure the distance to the target without approaching the enemy. To that end, Tartaglia invented the *squadra dei bombardieri*, presented in two versions. To measure the height and distance of a target, the *squadra dei bombardieri* was equipped with two equal legs and a shadow square, that was operated in the same way as the shadow square of a quadrant, while to point a cannon to its target, one of the legs of the *squadra dei bombardieri* was made longer so it could be inserted into the cannon's mouth. A plumb line suspended from the vertex showed the elevation of the cannon on the quadrant arc, divided into twelve equal parts, called «points».[24] (Fig. 2-3)

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[21] In general, any attempt to make a distinction between «painters» and «military engineers» would be a fruitless enterprise. For painters with military concerns, see J. R. HALE, *Renaissance fortification: art or engineering?*, London, Thames and Hudson, 1977, for a list of examples, among others, Bernardo Buontalenti, who housed Ricci when teaching perspective to Cigoli and Galileo. See also VELTMAN, *Military surveying*, pp. 357-367, and pp. 339-344, for the military application of Alberti's veil, used to draw the perspective image of a fortification. For the military audience of the perspective manuals of Commandino, Benedetti and Guidobaldo at the end of the sixteenth century, see J.V. FIELD, *Perspective and the mathematicians: Alberti to Desargues*, in *Mathematics from manuscript to print: 1300-1600*, C. Hay ed., Oxford, Clarendon Press, 1988, pp. 237-263. Also, Galileo taught military fortification to his students. For the textbook he used, see GALILEO, *Opere*, 2, pp. 2-146.

[22] PAMELA O. LONG, *Power, Patronage and the Authorship of Ars: From Mechanical Know-how to Mechanical Knowledge in the Last Scribal Age*, «Isis», 88, 1997, pp. 1-41.

[23] JIM BENNETT & STEPHEN JOHNSTON, *The geometry of war 1500-1700*, Oxford, Museum of the History of Science, 1996, p. 11.
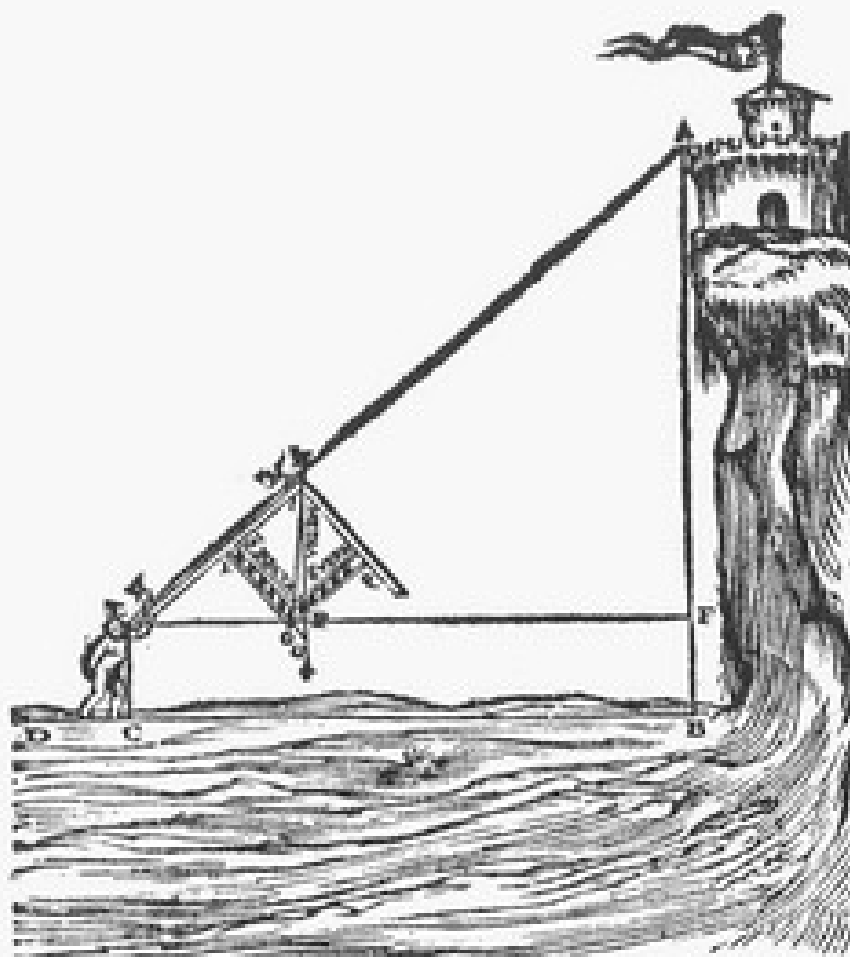
[24] NICCOLO TARTAGLIA, *Quesiti et inventione diverse*, Arnaldo Masotti ed., Brescia, Ateneo di Brescia, 1959, pp. 5-35; and the third book of NICCOLO TARTAGLIA, *Nova scientia inventa da Nicolo Tartalea*, In Vinegia, Per Stephano da Sabio ad instantia di Nicolo Tartalea, 1537.
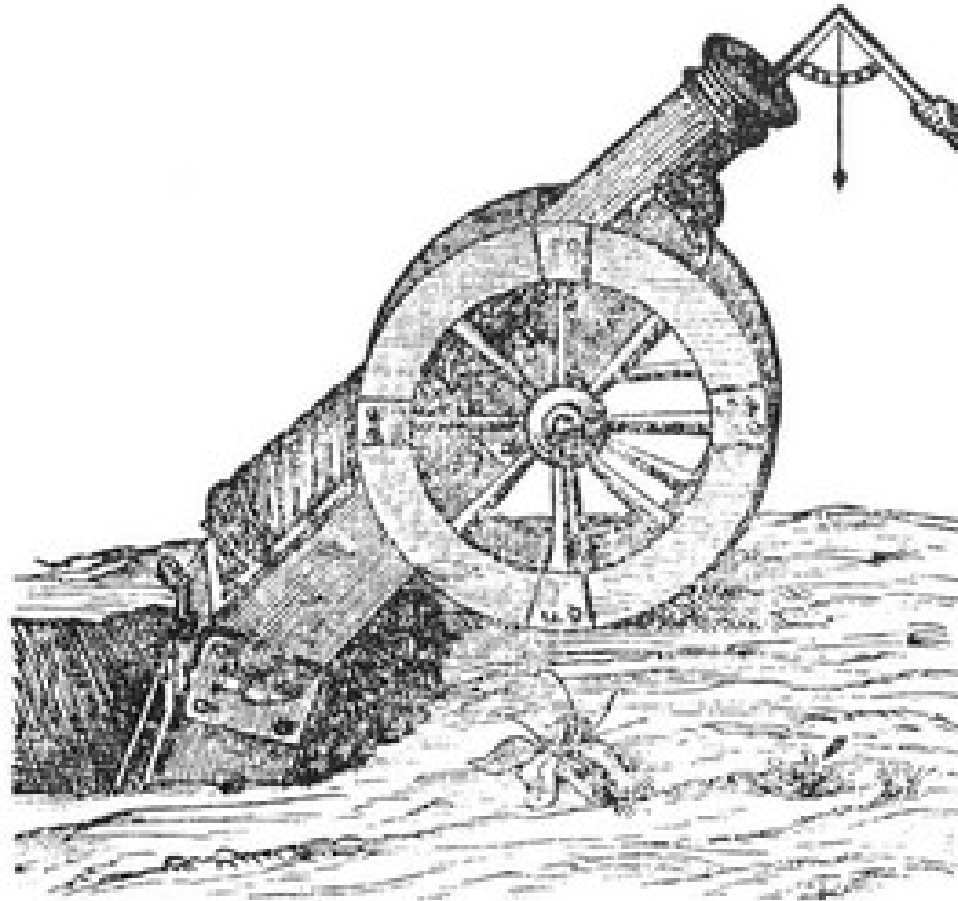
2. The *squadra dei bombardieri*, from: Tartaglia, La Nova Scientia, 1583. (Photo Courtesy of Ghent University Library).

Tartaglia's *squadra dei bombardieri* had many rivals on the sixteenth century market, for example, the *radio latino*. Danti's annotations to the «Trattato del Radio Latino», an instrument invented by Latino Orsini, explained all the uses such a measuring device could have for military men, like measuring heights, depths and distances, pointing a cannon at its target, making a ground plan of a fortification, mapping the environment or determining the position of the sun or the stars.[25] It was this tradition of making measuring instruments applicable to several practical activities, and presented in the context of military concerns, which was Galileo's background and into which he continued to work after he finished his studies and became a professor of mathematics at the Paduan university.

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[25] EGNATIO DANTI, *Trattato del radio latino: Istrumento giustissimo & facile più d'ogni altro per prendere qual si voglia misura, & positione di luogo, tanto in Cielo come in Terra*, In Roma, Apresso Marc'Antonio Moretti & Iacomo Brianzi, 1586.

3. Measuring the height of a tower using the squadra, from: Tartaglia, Quesiti et inventioni diverse, 1583. (Photo Courtesy of Ghent University Library).

Galileo inherited this background from his teacher Ostilio Ricci.[26] Alberti's «Ludi Matematici» served for Ricci as a textbook in his classes for pages at the Tuscan court who were interested in military applications. Also, Ricci's own research continued in this tradition. Ricci invented the *archimetro*, an instrument intended to measure by sight heights, depths and distances.[27] Galileo's interest continued after he had finished his

[26] Even before starting his classes with Ostilio Ricci, that is, already as a medical student at the university of Pisa, Galileo most likely used mathematical instruments, in particular an astrolabe. On the Hartman astrolabe, modified by Giusti, that may have been owned by Galileo, see GERARD L. E. TURNER, *An astrolabe belonging to Galileo?*, «Nuncius», 12, 1997, pp. 87-92, while the large astrolabe, now in the Museum of the History of Science in Florence (inv. no. 3361), often referred to as "Galileo's Astrolabe", is not related to Galileo. See GERARD L. TURNER, *The Florentine workshop of Giovan Battista Giusti*, «Nuncius», 10, 1995, pp. 134, 157-159.

[27] Ricci's «L'uso dell'archimetro», reprinted in FEDERICO VINCI, *Ostilio Ricci da Fermo, maestro di Galileo Galilei*, Fermo, 1929, pp. 23-29.

studies with Ricci. Galileo's library contained several books on mathematical instruments, for example, Gemma Frisius' and Stöffler's book on the astrolabe, Danti's «Trattato del Radio Latino», and Tartaglia's «Nova Scientia» and his «Quesiti et inventioni diverse».[28] Tartaglia's *squadra dei bombardieri* was known to Galileo and informed his own invention of the geometric and military compass.[29] Although the manual that went with the instrument was only published in 1606, a first version of the instrument already existed in 1597. At that time, Galileo's compass resembled Tartaglia's *squadra dei bombardieri*, but Galileo made it foldable for ease of use like Ricci's *archimetro*, and he arranged that the gunner would no longer have to expose himself to danger by applying the *squadra dei bombardieri* to the cannon's mouth.[30]

Galileo's compass also had the functions of Guidobaldo's proportional compass to divide lines and circumferences into equal parts and to construct regular polygons.[31] After 1599, Guidobaldo's scales had been removed for other scales which made Galileo's compass into a universal mechanical calculator.[32] For surveying purposes, detailed arithmetical calculations of a triangulation were no longer necessary, because Galileo's compass allowed to do this mechanically by using the arithmetic scale.[33]

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[28] The books referred to are: Gemma Frisius, «De astrolabio catholico», 1556; Johannes Stöffler, «Elucidatio fabricae, ususque Astrolabii», 1585; Tartaglia's «Quesiti et inventione diverse» and «Nova Scientia». Beside these, Galileo's library also contained several books on sundials and on cosmography, including its instruments. See FAVARO, *La libreria*, especially pp. 257-238, pp. 260-261. For the application of Galileo's compass to surveying, see also FILIPPO CAMEROTA, *Giorgio Vasari*, pp. 126-130.

[29] On Galileo's geometric and military compass and Tartaglia's squadra dei bombardieri, see STILLMAN DRAKE, *Tartaglia's squadra and Galileo's compasso*, «Annali dell'Istituto e Museo di Storia della Scienza», 2, 1977, pp. 40-43.

[30] GALILEO, *Le Operazioni del Compasso Geometrico e Militare*, in GALILEO, *Opere*, 2, p. 412. Translation in STILLMAN DRAKE, *Galileo Galilei: Operations of the geometric and military compass*, Washington, Smithsonian Institution Press, 1978, p. 79.

[31] The proportional compass, invented by Guidobaldo del Monte, is a further development of the reduction compass invented by Fabrizio Mordente. The reduction compass allowed to divide lines in equal parts or according to a given proportion and to divide circumferences in grades and minutes. Beside the operations of the reduction compass, the proportional compass also allowed to find the length of the sides of regular polygons inscribed in a circle. See PAUL L. ROSE, *The origins of the proportional compass from Mordente to Galileo*, «Physis», 10, 1968, pp. 53-69; and also E. ROSEN, *The invention of the reduction compass*, «Physis», 10, 1968, pp. 306-308.

[32] The final version of the geometric and military compass was more than a military instrument. The legs of the compass were equipped with various scales that allowed to solve every mathematical operation useful for practical men, like the calculation of interests, the duplication of area and volume, the calculation of the proportion between volume and weight, the construction of polygons, the rule of three, the extraction of square and cubic roots, and so on.

[33] GALILEO, *Operazioni*, in GALILEO, *Opere*, 2, p. 415. Translation in DRAKE, *Operations*, p. 81. For the use of the arithmetic scale and the conversion of umbra retta into umbra versa, see also CAMEROTA, *Giorgio Vasari*, pp. 128-130.

Galileo's compass could also be used as an astronomical quadrant, while Marolois showed its application to topography, and Desargues argued for its relevance for linear perspective. Galileo's compass had the ambition to be a universal instrument.[34] Drake and Camerota have argued that it was the culmination point of a Renaissance tradition of making instruments applicable to all areas of measuring.[35] Galileo's background in this tradition will prove essential to understand his involvement with the «Theorica speculi concavi sphaerici». Before analyzing the «Theorica» in relation to Galileo's geometric and military compass, the history of this document will be described by comparing it to other copies of the same document.

3. A HISTORICAL ANALYSIS OF THE «THEORICA SPECULI CONCAVI SPHAERICI»

A large folio, today preserved among the papers of the MS Galileiana 83 in the National Library of Florence, has hardly been mentioned in the vast English language scholarship on Galileo's science. Lindberg once reserved a footnote for it, limiting his comment to an evaluation as «brief but competent», but this is the exception that confirms the rule.[36] The «Theorica speculi concavi sphaerici», as the folio is entitled, has been readily available, because a facsimile was published by Favaro in the third volume of Galileo's collected works.[37] (Fig. 4) Favaro dated it to 1610, because he noted some similarities in subject with Magini's «Breve instruttione sopra l'apparenze et mirabili effetti dello specchio concavo sferico», published a year later.[38] Magini presumably informed Galileo

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[34] None of these applications was explained by Galileo. The compass had a scale in order to allow it to be used as an astronomical quadrant, but Galileo did not allow himself to go into something that he considered widely known: «Next following the division of the Astronomical Quadrant, whose use (having been dealt with by many others) will not be explained here». GALILEO, *Operazioni*, in GALILEO, *Opere*, 2, p. 412. Translation in DRAKE, *Operations*, p. 79. For the topographical and painterly use of Galileo's compass, see F. CAMEROTA, *Giorgio Vasari*, pp. 131-135.

[35] DRAKE, *Tartaglia's squadra*; F. CAMEROTA, *Giorgio Vasari*, p. 126.

[36] DAVID C. LINDBERG, *Optics in sixteenth century Italy*, in *Novità celesti e crisi del sapere: atti del convegno internazionale di studi Galileiani*, Paolo Galluzzi ed., Firenze, Giunti Barbèra, 1984, p. 131.

[37] GALILEO, *Opere*, 3, pp. 865-870. The «Theorica» is in the MS Gal. 83, together with fragments of uncertain date, now published in GALILEO, *Opere*, 8, pp. 559ff. Antinori, who in the nineteenth century reorganized the Galilean collection, claimed that «this autograph work of Galileo was found in a book, containing various recipes for lenses and telescopes. It was not known that Galileo ever dealt with this subject». If Antinori is reliable, this MS, presumably, has not survived. On Antinori, see MICHELE CAMEROTA, *Gli scritti De motu antiquiora di Galileo Galilei: il Ms. Gal. 71: Un'analisi storico-critica*, Cagliari, CUEC Editrice, 1992, pp. 28-29.

[38] GIO. ANTONIO MAGINI, *Breve instruttione sopra l'apparenze et mirabili effetti dello specchio concavo sferico*, Bologna, Gio: Batt. Bellagamba, 1611. For Favaro's opinion, see GALILEO, *Opere*, 3, p. 867.
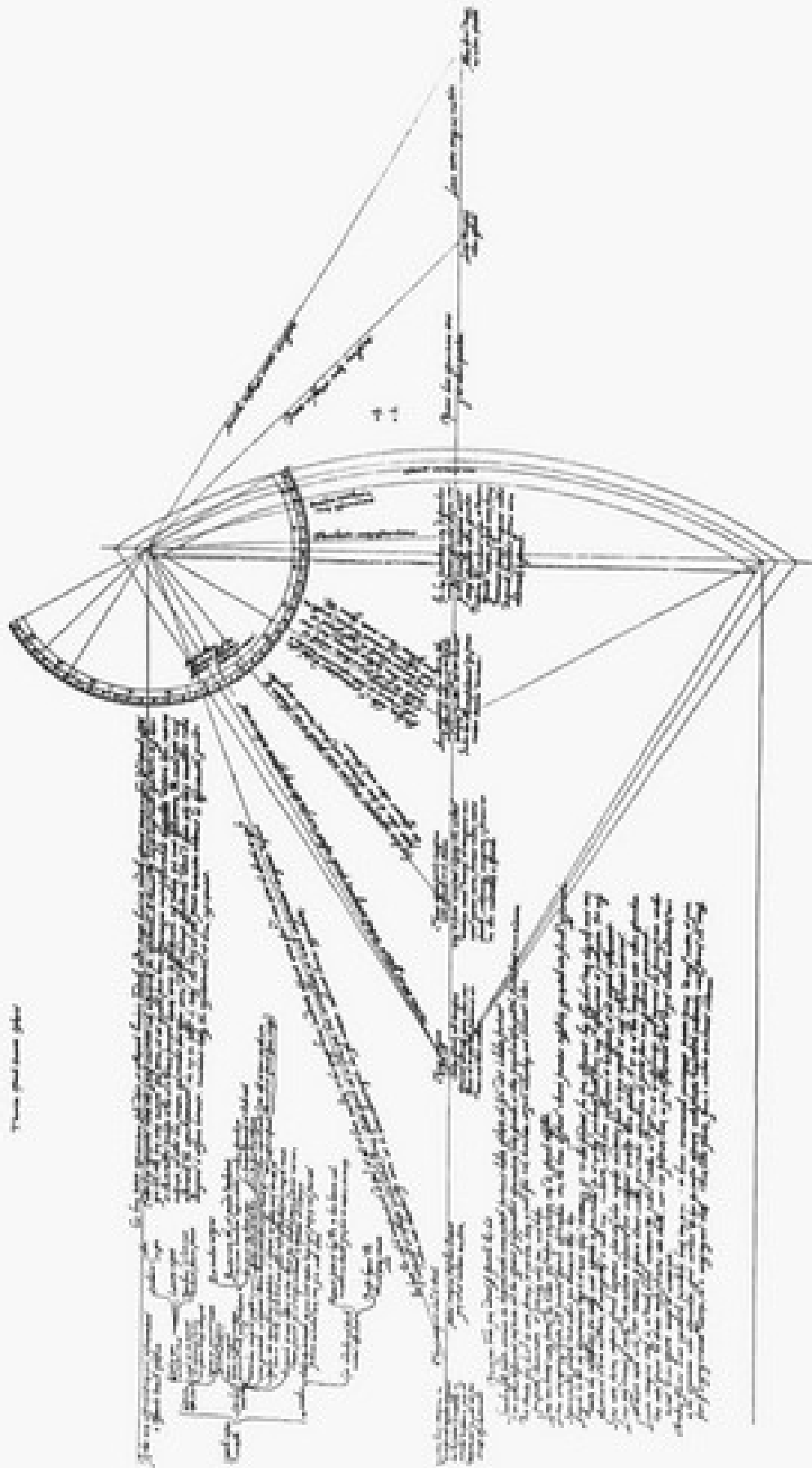
4. Galileo, Theorica speculi concavi sphaerici, 1592-1601.

about this work as early as the latter's visit to Bologna in the summer of 1610 to show Magini his telescopic discoveries. Magini's extant letters to Galileo between 28 September 1610 and 11 January 1611 show that Magini was anxious to engage Galileo as his broker in order to sell one of the large concave mirrors Magini had made for Rudolf II to the Grand Duke of Tuscany, Galileo's new patron, instead.[39] Also, Magini promised to send Galileo two copies of his newly published work, presumably to facilitate the deal. It seems Galileo indeed did receive these copies, because in his last letter Magini acknowledged Galileo's praise for his «Breve instruttione», and Galileo's library contained a copy of the book.[40] However, nothing in the correspondence between Magini and Galileo substantiates Favaro's dating of the «Theorica» to 1610.

Drake has disagreed with Favaro's dating, because «at that time Galileo was so busy day and night with the improvement and use of the refracting telescope that the elaborate calculations required for the «Theorica» are unlikely to have been carried out, nor would they have assisted him in any work he is known to have done at Padua».[41] Drake thought a dating to mid-1626 more likely, because between 1624 and 1626 Galileo was engaged in discussions about the possibility of a concave mirror to function as a telescope with Marsili and about the focal point of a concave spherical mirror with Guazzaroni.[42] However, nowhere in this

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[39] Magini's mirrors were very large, with circumferences up to 1.5 m and a radius of curvature up to 50 cm. He sent one of these mirrors to Rudolf II, using Tengnagel, Tycho's son-in-law, as a broker, as early as 1604, but his attempts to be paid for it remained unsuccessful. As late as 1610, he still had not been paid. Therefore, he decided to change his strategy and to sell one to Cosimo II, Grand Duke of Tuscany. On Magini's mirrors, see ANTONIO FAVARO, *Carteggio inedito di Ticone Brahe, Giovanni Keplero e di altri celebri astronomi e matematici dei secoli XVI e XVII con Giovanni Antonio Magini, tratto dall'Archivio Malvezzi de' Medici in Bologna*, Bologna, N. Zanichelli, 1886, pp. 161-174. On the mirror for the Grand Duke, see in particular, GALILEO, *Opere*, 10, pp. 437-439, pp. 442-443. Magini and Galileo do not seem to have been on the bad terms suggested by the unfortunate story of the attack of Horky, a student of Magini, on Galileo's telescopic discoveries. See GIOVANNI BAFFETTI, *Il «Sidereus Nuncius» a Bologna*, «Intersezioni», 11, pp. 477-500. On Magini's astronomy in general, see ENRICO PERUZZI, *Critica e rielaborazione del sistema copernicano in Giovanni Antonio Magini*, in *La diffusione del Copernicanismo in Italia 1543-1610*, a cura di Massimo Bucciantini & Maurizio Torrini, Firenze, Leo S. Olschki, 1997, pp. 83-98. However, Magini was not only known for his astronomical work, but also for his geographical and cartographical studies, see FILIPPO CAMEROTA, *Giorgio Vasari*, pp. 262 ff.

[40] On the 28 December 1610, Magini promised Galileo to send two copies of the «Breve instruttione». See GALILEO, *Opere*, 10, p. 496. The following letter opens with «de lodi che V.S. ha date al mio trattatello dello specchio concavo ...», Magini to Galileo, January 11, 1611, in GALILEO, *Opere*, 11, p. 19. For the copy of the «Breve instruttione» in Galileo's library, see FAVARO, *La libreria*, p. 263.

[41] STILLMAN DRAKE, *Galileo at Work: his scientific biography*, Chicago/London, The University of Chicago Press, 1978, p. 304.

[42] Guazzaroni to Galileo, April 20, 1624, in GALILEO, *Opere*, 13, pp. 172-174; Marsili to

correspondence is there any mention of the «Theorica». Also, Drake's dating to mid-1626 is based on his assumption, shared by Favaro and Lindberg, that the «Theorica» is an original work of Galileo. However, the Italian scholars Roberto Savelli, in a completely ignored paper, and Libero Sosio, more recently, uncovered the existence of two related documents, respectively a 1602 publication of a «Theorica speculi concavi sphaerici» by Giovanni Antonio Magini and a «Manoscritto dell'iride e del calore» by Paolo Sarpi, that never went to print until it was recently and only in part edited by Libero Sosio and Luisa Cozzi.[43]

Sarpi's «Manoscritto» is a collection of notes and diagrams of personal observations of the colors of the rainbow, reflection in mirrors and refraction through prisms and lenses. The «Theorica» is part of this collection, but Sarpi only reproduced part of it, that is the diagram dividing the phenomena observed in a concave spherical mirror in three distinct categories, presented at the top left corner in Galileo's copy. With primary light, that is direct light of the sun, the concave mirror acts as a burning mirror, for instance «it makes the lead that is divided into plates fluent». Also, it «reflects the heat, so that the difference between winter and summer is known through the reflection». Under the heading of secondary light, that is the indirect light of the sun in a room entering through a window, image formation in a concave mirror, that is the size, the orientation and the location of the images, is discussed. Moreover, it is argued that «ice is felt through its distant image», because «the species are perceptible by the sense of touch». Also, a concave spherical mirror «reflects conversations and voices, as an echo, so that those who stand very far away hear it». Third, it is noted that a camera obscura in combination with a concave mirror «depicts a marvelous picture on a piece of paper or a screen of things outside». Finally, it is argued that with a burning candle and a concave mirror «letters can be read at night in a dark place or that things which happen in the camps of the enemies can be seen».

Missing in Sarpi's copy is the mid-section of the «Theorica», that is the large drawing of a concave spherical mirror, with on the top right corner a

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Galileo, July 7, 1626, in GALILEO, *Opere*, 13, pp. 330-331. Guazzaroni does refer to Magini's «Breve instruttione».

[43] ROBERTO SAVELLI, *Intorno al rinvenimento della Theorica di Ettore Antonio pubblicata da Giovanni Antonio Magini*, «Atti della Fondazione G. Ronchi», 10, 1955, pp. 3-11; PAOLO SARPI, *Pensieri naturali, metafisici e matematici*, Luisa Cozzi & Libero Sosio ed., Milano/Napoli, Riccardo Ricciardi Editore, 1996, pp. 519-546. Neither Cozzi and Sosio nor Savelli have presented a completely satisfactory analysis of the «Theorica». Savelli's paper was not known to Cozzi and Sosio. Sarpi's original is now in the Biblioteca Marciana, while Magini's is in the Biblioteca Comunale dell'Archiginnasio in Bologna.

«semicircle divided into grades» to measure the varying angles of incidence and of reflection. Each line of the drawing represents a «line of incidence» as well as a «line of reflection», thus the principle of reversibility is acknowledged, as, for instance, when talking about the line through the focal point of the mirror: «This line has two relations to this mirror. The first is that it is the line of refraction [reflection] of the rays of the Sun, and then it burns. The second is that it is the first incidence of the candle, and then its reflected line makes the image of the light of the candle appear on the surface of the mirror. Then it is able to heat over an extended distance, and to illuminate so clearly that letters can be read at night». Adjacent to the picture, the text discusses in a competent way the various locations and orientations of the mirror images with respect to the place of the object and the focal point. Indeed, of the utmost importance, as will be seen, is the determination of the focal point of the mirror and its «point of inversion».

At the bottom of the folio, ten «principles of all things which are seen through the mirror» are discussed. These «principles» are the definitions of the lines and angles of incidence and of reflection, the law of equal angles and the cathetus rule to determine the place of an image. Of interest to the «artificer» of concave mirrors would have been the advice to make the concave spherical mirrors large, because the smaller the mirror, «the smaller the things said appear».

As Sosio has argued, Sarpi's «Manoscritto», including his copy of the top part of the «Theorica», was written during the period around 1587 and 1588. First, his personal observations of the rainbow are dated to this time, although it cannot be excluded that the «Theorica» was added at a later date to the «Manoscritto».[44] Second, Sarpi's «Pensieri» from the same period show his interest in reflection in spherical mirrors, that is image formation in convex and concave mirrors and the heat and burning caused by reflected rays.[45]

There is no exact word to word match between Sarpi's and Galileo's «Theorica». Sarpi often shortened or even paraphrased the wording as

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[44] SARPI, *Pensieri*, pp. 336-339. The drawings of the rainbow are dated 12 December 1587, 9 January 1588 and 28 March 1588.

[45] Pensiero 61 dealt with the law of equal angles; pensiero 63 with the "left-right reversal" of an image in a mirror; pensiero 65 with the cathetus rule; pensieri 62, 67, 68, 70, 73 with image formation in plane, convex and concave mirrors. See SARPI, *Pensieri*, pp. 74-84. Pensieri 53 and 124 dealt with the burning effect of a mirror, and compared the burning effect when using the secondary light of the moon with the burning effect when using reflected light on earth. See SARPI, *Pensieri*, pp. 65-66, 143. The former pensieri are dated as early as 1578, but Sarpi returned to image formation in a concave mirror and to burning mirrors in 1588, respectively in pensieri 388 and 479. See SARPI, *Pensieri*, pp. 295, 358.

found in Galileo's copy, but the similarity is close enough not to doubt that it concerns the same source. Galileo did not copy his «Theorica» from Sarpi's more limited version, neither is there any debt the other way around, if the 1587-1588 dating of Sarpi's «Manoscritto» is correct. Sarpi and Galileo first met in the house of Gian Vincenzo Pinelli shortly after Galileo's arrival in Padua in 1592.[46] During Galileo's eighteen year stay at Padua both men would have had ample opportunity to discuss the «Theorica» and the problems it dealt with. In fact, Reeves recently suggested that Sarpi and Galileo collaborated on the secondary light of the moon around 1606-1607, and to that end, experimented with plane and non-plane mirrors.[47] However, the third copy of the «Theorica» excludes a 1607 date for Galileo's and Sarpi's copy of the «Theorica», and, also, makes it more likely that Galileo and Sarpi copied independently from a same source.

Magini's copy of the «Theorica» is the only one that ever went to print. Magini made large concave mirrors for Rudolf II, and other patrons, although he was not very successful in being paid for it. His publication in 1602 of the «Theorica» presumably served as a textbook on how to use these mirrors to see the «miraculous effects» Magini claimed could be obtained with them. Magini did not claim he was the original author of the «Theorica». In the short introduction added by Magini, not present in Galileo's copy, he only claimed credit for editing it. The author of the «Theorica» was Ettore Ausonio, who, Magini claimed, was a physician and mathematician in Venice.[48] Ausonio graduated from the University of Padua in 1543. Four years later, after the death of his teacher Federico Delfino, he was scheduled to lecture at the same university, but, eventually, it was Pietro Catena, and not Ausonio, who became the new professor of mathematics.[49]

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[46] LUISA MURARO, *Giambattista Della Porta mago e scienziato*, Milano, Feltrinelli, 1978, p. 17. On the relationship between Galileo and Sarpi, see GAETANO COZZI, *Paolo Sarpi tra Venezia e l'Europa*, Torino, Piccola Biblioteca Einaudi, 1979, pp. 135-234; LIBERO SOSIO, *Galileo Galilei e Paolo Sarpi*, in *Galileo Galilei e la cultura Veneziana*, Venezia, Istituto Veneto di Scienze, Lettere ed Arti, 1995; VINCENZO FERRONE, *Galileo tra Paolo Sarpi e Federico Cesi: premesse per una ricerca*, in *Novità celesti e crisi del sapere: atti del convegno internazionale di studi Galileiani*, P. Galluzzi ed., Firenze, Giunti Barbèra, 1984, pp. 239-254.

[47] EILEEN REEVES, *Painting the heavens: art and science in the age of Galileo*, Princeton, Princeton University Press, 1997, pp. 104-112.

[48] In Magini's dedication to Fachinetti: «I was forced to publish the 'Theory of the Concave Mirror', whose author is Hector Ausonius, a physician in Venice and a mathematician of some stature.» G. A. MAGINI, *Theorica speculi concavi sphaerici*, Bononiae, Apud Ioannem Baptistam Bellagambam, 1602.

[49] For biographical information on Ausonio, see PASQUALE VENTRICE, *La discussione sulle maree tra astronomia, meccanica e filosofia nella cultura veneto-padovana del cinquecento*, Venezia,

Ausonio's work was collected in the famous library of Pinelli. Today at the Ambrosiana, two short treatises of Ausonio on catoptrics are preserved, while his «Theorica speculi concavi sphaerici» does not survive. However, the content of Ausonio's «D'una nuova invenzione d'uno specchio» and his «Secreti d'alcune apparenze in uno specchio» dealt with the same problems of image formation in a concave mirror, the use of a concave spherical mirror as a burning mirror and the use of the reflection of candle light to light up a room at night.[50] The similarity with the «Theorica» is close enough to establish Ausonio's authorship of the «Theorica» with certainty. Although Ausonio was considered to be an expert in optical instruments by his contemporaries, his interest in catoptrics, in particular burning mirrors, in the "Invenzione" and the "Secreti" was connected with his search for an explanation of the tides, an aspect absent in Magini, Sarpi and Galileo. The latter two preferred a mechanical explanation of the tides, while Ausonio argued that the tides were caused by the heat of the sun. Ausonio claimed to have performed an experiment with a burning mirror and a vessel of water to prove that the heat of the sun caused the waves of the water in the vessel.[51]

Who was Galileo's source? Did Galileo copy directly from Ausonio's original or did he consult Magini's printed edition? The key to solve this puzzle is to find out why Magini published Ausonio's «Theorica». Magini claimed to have published it, because he was afraid it would get lost.[52] Moreover, according to Magini, the «Theorica» deserved publication, because Ausonio was the first who calculated the focal point of a concave spherical mirror to be at the «fourth part of the diameter of the mirror».[53]

Istituto Veneto di Scienze, Lettere ed Arti, 1989, p. 36. On Delfino and Catena, see PAUL LAWRENCE ROSE, *Professors of mathematics at Padua University 1521-1588*, «Physis», 17, 1975, pp. 300-304.

[50] *D'una nuova invenzione d'uno specchio*, MS A71 Inf.; *Secreti d'alcune apparenze in uno specchio*, today in the MS D246 Inf. of the Ambrosiana Library in Milan. The references given by Sosio, who did not have the opportunity to consult these manuscripts, are wrong.

[51] VENTRICE, *La discussione sulle maree*, p. 40. On Sarpi's and Galileo's theory of the tides, see also SOSIO, *Galileo Galilei e Paolo Sarpi*, pp. 305-308. Other notes on optics and catoptrics of Ausonio show him highly involved with the subject, also when their seem to be no connections with his theory of the tides. See *Optica et Catoptrica*, MS G120 Inf., Ambrosiana, and his lecture notes on Witelo, that is *Lectiones Libri Vitellionis de Prospectiva*, MS R105 Sup., ff. 258r-261v, Ambrosiana.

[52] «I've said that it was important to publish this 'Theory', moreover, because it has ever been in danger, and it would certainly have perished, if a friend, for whom I once described it, would not have made a copy for me.» MAGINI, *Theorica*. A likely candidate for this "friend" of Magini is Ercole Bottrigaro, who was into contact with Pinelli around 1600, see his two letters to Pinelli of July 1600 on ecclesiastical music in MS S107 Sup., ff.106r-109v, Ambrosiana.

[53] The «fourth part of the diameter of the mirror», here and everywhere else in this paper,

Indeed, the pseudo-Euclidean «Catoptrics» confused the center of curvature of a concave spherical mirror with its focal point.[54] However, the pseudo-Euclidean «Catoptrics» was corrected by several authors, for example, in Diocles' «On Burning Mirrors» and Alhazen's «Discourse on a concave spherical mirror». They noted, long before Ausonio's «Theorica», that the focal point of a concave spherical mirror was at the fourth part of its diameter. Also, they were aware that the solar rays reflected in a concave mirror do not converge to this point, that is they were aware of the «spherical aberration» of a concave spherical mirror, claiming that the ideal burning mirror was parabolic and not spherical.[55] Most likely, Magini was unaware of these developments. Alhazen's «Discourse on a concave spherical mirror» was never translated into Latin, while Witelo only dealt with the parabolic burning mirror.[56] At the turn of the sixteenth century, it was not evident that the focal point of a spherical mirror was at the fourth part of its diameter, as it was for a parabolic mirror. Kepler criticized Magini for making this claim.[57]

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refers, in a modern terminology, to 1/2 of the radius of curvature of the mirror. «In Antiquity, everyone agreed that the focal point was the center of the mirror, or of the sphere, or the globe, of which it was part, like Euclid, Witelo, Alhazen and the others. However, this is certainly false, because nothing burns in the center of the mirror, nor is there any heat. It's amazing that Antiquity has made this mistake. Thus, the true focal point is in the fourth part of the diameter, as has been noted by Hettore Ausonio physician and eccellent mathematician in Venice in his «Theory of the concave spherical mirror', at another time published by us.» MAGINI, *Breve instruttione*, p. 13.

[54] Pseudo-Euclid, Catoptrics, proposition 30. See VER EECKE, *L'Optique*, pp. 122-123.

[55] On Diocles, see G. J. TOOMER, *Diocles on Burning Mirrors*, Berlin/Heidelberg/New York, Springer-Verlag, 1976; WILBUR KNORR, *The geometry of burning-mirrors in Antiquity*, «Isis», 74, 1983, pp. 53-60; and WILBUR R. KNORR, *Archimedes and the pseudo-Euclidean catoptrics: early stages in the ancient geometric theory of mirrors*, «Archives Internationales d'Histoire des Sciences», 35, 1985, pp. 78-83. For Alhazen's «Discourse on a concave spherical mirror», see E. WIEDEMANN, *Ibn al Haithams Schrift über die sphärischen Hohlspiegel*, «Bibliotheca Mathematica», 10, 1910, pp. 293-307, and H. J. J. WINTER & W. ARAFAT, *A Discourse on the Concave Spherical Mirror by Ibn Al-Haitham*, «Journal of the Royal Asiatic Society of Bengal», 16, 1950, pp. 1-16; for Alhazen on the parabolic burning mirror, see J. L. HEIBERG & EILHARD WIEDEMANN, *Ibn al Haithams Schrift über parabolische Hohlspiegel*, «Bibliotheca Mathematica», 10, 1910, pp. 201-237. On the history of burning mirrors, see also, JURGIS BALTRUSAITIS, *Le miroir: révélations, science-fiction et fallacies*, Paris, Elmayan/Le Seuil, 1978, pp. 95-121.

[56] While Alhazen's «Discourse on the parabolic burning mirror» was translated into Latin during the Middle Ages, his «Discourse on the spherical burning mirror» was not. See A. I. SABRA, *The optics of Ibn al-Haytham*, 2, London, The Warburg Institute, 1989, pp. XLII-XLV. It explains why Witelo only dealt with the parabolic burning mirror. WITELO, *Opticae Thesaurus: Alhazeni arabis libri septem, nuncprimum editi, eiusdem liber de crepusculis et nubium ascensionibus, item Vitellionis Thuringopoloni libri X*, David C. Lindberg ed., «The Sources of Science», 94, New York/London, Johnson Reprint Corporation, 1972, pp. 401-402. Translation in EDWARD GRANT, *A source book in medieval science*, Cambridge (Massachusetts), Harvard University Press, 1974, pp. 417-418.

[57] Kepler was convinced that the focal point of a concave spherical mirror was in its center of curvature. Kepler to Brengger, January 17, 1605, in JOHANNES KEPLER, *Gesammelte Werke*,

Magini was a critical editor. He criticized Ausonio for identifying the focal point of the concave spherical mirror with its «point of inversion». According to Magini, this point was the center of curvature, and he carefully corrected all references to the place of the «point of inversion» in Ausonio's notes adjunct to the large drawing of the concave mirror.[58] Galileo's copy of the «Theorica» does not contain Magini's «correction». In Galileo's copy all references to the place of the «point of inversion» are to the «fourth part of the diameter» of the concave mirror.[59] Thus, Galileo did not copy from Magini, but from Ausonio's original «Theorica», as did Magini himself. When did Galileo make his copy?

Most likely, Ausonio's «Theorica» was collected by Pinelli for his library, as the other manuscripts of Ausonio were. Gian Vincenzo Pinelli arrived in Padua in 1558. He was famous in his own time for his library, which, among many other things, contained the manuscript works of the professors of mathematics at the University of Padua, but he also possessed a collection of mathematical and astronomical instruments. He had a network of correspondents all over Europe, who also purchased books for him, while his house in Padua functioned as an informal academy.[60] Galileo was

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15, Walther Von Dyck & Max Caspar ed., München, C. H. Beck'sche Verlagsbuchhandlung, 1934, p. 106. Kepler's dissatisfaction with Magini's optics might explain the financial reluctance of Rudolf II.

[58] The image of an object placed between the focal point and the center of curvature of a concave spherical mirror is inverted and magnified, while the image of an object placed at the center of curvature is inverted and of the same size as the object. However, the image of an object placed between the focal point and the mirror vertex is virtual, that is «behind the mirror», magnified and not inverted. The «point of inversion» refers to the point closest to the mirror vertex that produces an inverted image of an object, when placed at that point. Since this point is the focal point of the mirror, Magini's «correction» appears to be wrong. Even in his «Breve instruttione», Magini was still convinced that the image of an object placed between the mirror vertex and the center of curvature was always right oriented (and virtual). See MAGINI, Breve instruttione, p. 23.

[59] In the dedication: «E quidem hanc Theoricam, ut multis partibus meliorem efficere me potuisse scio, sic unum tantum in ea praestitisse fateor, videlicet erratum quoddam non leve tamen sustulisse, quod fortasse per incuriam Ausonius admiserat, nempé in loco concursus radiorum Solarium, in quarta scilicet diametri parte inversionem fieri imaginu, ibiq, omnia confundi, quod sané non hoc in loco contingit, sed in ipsius Speculi centro, idest in semidiametri termino.» In the notes additional to the drawing of the concave mirror, Magini added «ad centrum Speculi omnia videntur inversa» and made Galileo's, assumed to be Ausonio's original, «in hoc loco omnia confunduntur, quia commutantur sursum deorsum» refer to the center of curvature instead of the focal point of the mirror. See MAGINI, Theorica, and Galileo, Theorica, in GALILEO, Opere, 3, p. 863. This difference has not been noted by Sosio, but it has been by Savelli.

[60] On Pinelli's circle, see A. STELLA, Galileo, il circolo culturale di Gian Vincenzo Pinelli e la «Patavina Libertas», in Galileo e la cultura padovana, Giovanni Santinello ed., Padova, CEDAM, 1992, pp. 307-326. Pinelli's European network of book purchasers and correspondents is corroborated by Yates who identified Corbinelli as Pinelli's book purchaser in Paris. FRANCES A. YATES, Giordano Bruno and the hermetic tradition, Chicago, The University of Chicago Press, 1964, pp. 293, 346. On Galileo's introduction in Pinelli's circle and the meetings with Sarpi, see

introduced to Pinelli's circle from the moment of his arrival in Padua in 1592 by his patron Guidobaldo del Monte. In fact, Pinelli had been instrumental in Galileo's obtaining of the chair in Padua and Galileo prepared for his inaugural speech in Pinelli's house. Insofar as necessary to show for a man that well known in the contemporaneous scholarly world on both sides of the Alps, Pinelli was known by Sarpi, Galileo and Magini. Sarpi and Galileo were visitors of the academy at Pinelli's house. In fact, it is reported that Sarpi, Galileo and Porta met in his house in 1593. Also, Magini knew Pinelli, if Aquilecchia's identification of Magini's informant in a letter to Porta of 27 July 1594 as Pinelli is correct.[61] When Pinelli died in 1601, his library was dispersed and part of it was later lost at sea during shipment.[62] The fate of the Pinelli collection explains why Magini was eager to publish Ausonio's «Theorica» in 1602. After 1601, Pinelli's library would no longer be accessible to the public. Thus, Galileo presumably copied Ausonio's manuscript, maybe for his private class on optics in 1601, between 1592, that is his introduction in Pinelli's circle, and 1601, when the manuscript became inaccessible.

4. ANAMORPHIC ART, NATURAL MAGIC AND MATHEMATICAL INSTRUMENTS

Ausonio's «Theorica» has resisted interpretation. No genuine analysis of its content has ever been attempted, nor has anyone succeeded in giving it a place in Galileo's science. If mentioned at all, Ausonio's «Theorica» was marginalized by reducing its content to a subject with which Galileo would have been only familiar, because he was assumed to respond to questions raised by friends and patrons.[63] Unlike Magini, who built mirrors even prior

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ANTONIO FAVARO, *Galileo e lo Studio di Padova*, Padova, Editrice Antenore, 1964, pp. 1:38-50, 2:52-59.

[61] The letter is reproduced in GIOVANNI AQUILECCHIA, *La sconosciuto Metoposcopia di G. B. Della Porta, di una differenziata del Cardano e di quella del Magini attribuita allo Spontoni*, «Filologia e critica», 10, 1985, pp. 311-312. See also GIOVANNI AQUILECCHIA, «*In facie prudentis relucit sapientia*»: *Appunti sulla letteratura metoposcopia tra Cinque e Seicento*, in *Giovan Battista Della Porta nell'Europa del suo tempo*, Maurizio Torrini ed., Napoli, Guida Editore, 1990, pp. 218; and GIOVANNI AQUILECCHIA, *Nuovi appunti sulla Metoposcopia di Giovanni Antonio Magini*, «Quaderni Veneti», 8, 1988, p. 110.

[62] Late in 1604, en route to Naples, the ship with the Pinelli collection on board was attacked by pirates, and several chests of books and manuscripts perished. What was left, was bought on an auction by Federico Borromeo. However, only in 1609 did the Pinelli collection join the newly found Biblioteca Ambrosiana in Milan. On the fate of the Pinelli collection, see MARCELLA GRENDLER, *A Greek Collection in Padua: The library of Gian Vincenzo Pinelli (1535-1601)*, «Renaissance Quarterly», 33, 1980, pp. 389-390.

[63] Galileo's occasional taking up and dropping of a subject is never to be used as an argu-

to his publication of Ausonio's «Theorica» in 1602, Galileo's copy of the «Theorica» does not seem to fit the overall picture of his scientific interests. Recently, Sosio even has assumed a case of bad memory, when Galileo did not note the similarity between Magini's «Breve instruttione» and the copy he had made of Ausonio's «Theorica» a few years before, when Magini sent him two copies of his new book in 1611.[64] Is more evidence needed than the omittence of Ausonio's «Theorica» in Galileo's correspondence to claim that it did not make much of an impression on Galileo? A thorough analysis of the «Theorica» will show that it defined itself at the cross-road of what are often considered different optical traditions, but is, in fact, one optical tradition for which the designing of instruments was central.

Ausonio's analysis of image formation in a concave spherical mirror is indeed very competent, as Lindberg has claimed, but Ausonio also described optical effects which seem more to be meant to entertain. When describing the optical effects of a burning mirror, Ausonio proposed to project letters on a distant wall by means of the mirror and solar light. In his «Natural Magick», published in 1589, Porta described how, using a plane mirror, «letters may be cast out and read, on a wall that is far distant».

On the superficies of a plain Glass, make Letters with black ink, or with wax, that they may be solid to hinder the light of the Glass, and shadow it; then hold the Glass against the Sun-beams, so that the beams reflecting on the Glass, may be cast upon the opposite wall of a Chamber, it is no doubt but the light and letters will be seen in the Chamber.[65]

Also, Ausonio's effects that could be obtained «in tenebris» have their parallel in Porta's «Natural Magick».[66] The camera obscura was already

ment for claiming a lack of interest in a subject, because this fragmented nature of science is a typical characteristic of the courtly patronage structure Galileo had to adapt to. Taking up a subject was often the consequence of a question asked by a patron. See MARIO BIAGIOLI, *Galileo Courtier: The Practice of Science in the Culture of Absolutism*, Chicago/London, The University of Chicago Press, 1993, for court patronage and the fragmented nature of science.

[64] SARPI, *Pensieri*, p. 527.

[65] JOHN BAPTISTA PORTA, *Natural Magick*, London, Printed for Thomas Young and Samuel Speed, 1658, p. 356. Ausonio/Galileo: «cum prima luce solis litteras in pariete remoto legendas proponit». In GALILEO, *Opere*, 3, p. 865. On Porta's optics, see THOMAS FRANGENBERG, *Perspective Aristotelianism: three case-studies of Cinquecento visual theory*, «Journal of the Warburg and Courtauld Institutes», 54, 1991, pp. 150-158; DAVID C. LINDBERG, *Theories of vision from Alkindi to Kepler*, Chicago, The University of Chicago Press, 1976, pp. 183-185; MURARO, *Giambattista Della Porta*; GIOVAN BATTISTA DELLA PORTA, *De Telescopio*, Vasco Ronchi & Maria Amalia Naldoni ed., Firenze, Leo S. Olschki, 1962, pp. 1-19; VASCO RONCHI, *The nature of light: an historical survey*, Cambridge (Massachusetts), Harvard University Press, 1970, pp. 78-87.

[66] This similarity of the effects «in tenebris» has also been noted by Sosio in SARPI, *Pensieri*, p. 530.

discussed in the context of catoptrics by Danti, whose annotations to his edition of Euclid's «Catoptrics» discussed a camera obscura and the use of a concave mirror to re-invert the image.[67] Less familiar was presumably Ausonio's use of a mirror to «reflect heat, cold, and the voice too», as Porta described it.[68] Also, Ausonio's proposal to use candle light to read letters in an otherwise dark room was clearified by Porta who claimed it could be done by placing a candle in the focal point of the mirror.

Take the Glass in your hand, and set a candle to the point of Inversion, for the parallel beams will be reflected to the place desired, and the place will be enlightened above sixty paces, and whatsoever falls between the parallels, will be clearly seen: the reason is, because the beams from the Centre to the circumference, are reflected parallel, when the parallels come to a point; and in the place thus illuminated, letters may be read, and all things done conveniently, that require great light.[69]

Although in some instances Porta confused the focal point of a concave spherical mirror with its center of curvature, as in the sentence cited above, on other occasions he showed himself aware of where the focal point of a concave spherical mirror should be, when avoiding spherical aberration by using an aperture to obstruct the incoming light.

In a Concave spherical Glass the beams meeting together, kindle fire in a fourth part of the diameter under the Centre, which are directed within the side of a Hexagon from the superficies of the circle. But a Parabolic Section, is, wherein all the beams meet in one point from all the parts of its superficies.[70]

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[67] DANTI, *La prospettiva di Euclide*, pp. 81-84. Ausonio/Galileo: «in tenebris sole illuminante ea, quae sunt extra, depingit papram, vel parietem pictura mirabili, earum rerum, quae sunt extra». In GALILEO, *Opere*, 3, p. 865. PORTA, *Natural Magick*, pp. 363-365. Porta discussed four types of camera obscura: (1) with a simple aperture, (2) with a convex lens, (3) with a concave mirror to obtain a larger image, (4) with a concave mirror to re-invert the image. See also MURARO, *Giambattista Della Porta*, p. 121.

[68] PORTA, *Natural Magick*, p. 361. Ausonio/Galileo: «sermones & voces reddit, ut echo, ii aut qui maximè distant, audiant, nisi surdastri fuerint, qui vero magis accemt ad loquentem, non audiant»; «calorem etiam ita remittit, ut cognoscatur hyemis & aestatis diversitas per reflexionem». In GALILEO, *Opere*, 3, p. 865. The Jesuit Giuseppe Biancani, who was in contact with Galileo when both men were in Padua, cited Ausonio and Porta in this respect in his work on the echo, claiming priority for Ausonio. See GIUSEPPE BIANCANI, *Sphaera mundi*, Modena, Ex Typographia Iuliani Cassiani, 1635, p. 231 (Eileen Reeves, personal communication). On the relationship between Galileo and Biancani, see WILLIAM A. WALLACE, *Galileo and his Sources: The Heritage of the Collegio Romano in Galileo's Science*, Princeton, Princeton University Press, 1984, p. 269.

[69] PORTA, *Natural Magick*, p. 362. Ausonio/Galileo: «In tenebris remotè possint legi litterae in loco obscuro vel in noctu videantur quae fiant in castris inimicorum». In GALILEO, *Opere*, 3, p. 865.

[70] PORTA, *Natural Magick*, p. 371. When writing «De Refractione», Porta is no longer con-

Also, in disagreement with Magini, Porta claimed that the focal point was also the «point of inversion», when he proposed his reader to do this: "Hold your Glass against the Sun, and where you see the beams unite, know that to be the point of Inversion".[71]

Did Porta see Ausonio's «Theorica»? At least concerning Ausonio's effects «in tenebris», Magini was convinced that Porta did, when he suggested in his «Breve instruttione» that Ausonio's effects «were extensively discussed by Mr. Batista dalla Porta in his Natural Magick».[72] Indeed, it is very likely Porta did consult Ausonio's «Theorica». Maybe he had an opportunity to see it when he visited Venice in 1580 to attend the construction of a parabolic mirror and a pair of eye-glasses. At that time he also met Sarpi, who already may have been aware of Ausonio's «Theorica».[73] On the other hand, Porta never cited Ausonio's «Theorica» and Magini's claim may just have been a way to enhance the importance of his publication of Ausonio's «Theorica» by claiming priority for it concerning the effects «in tenebris». However, the emphasis on the determination of the focal point at the "fourth part of the diameter of the mirror" and the principle of reversibility, completely alien to medieval optics, in Ausonio's "Theorica" as well as Porta's "Natural Magick" strongly suggests that Magini may have been right about Porta's borrowing from Ausonio's "Theorica".[74]

Anyway, it is hard to deny that Ausonio's «Theorica» and Porta's «Natural Magick» shared the same focus on playful effects «in tenebris» and obtained by mirrors. Even Magini's «Breve instruttione» was a clear statement about the entertainment the ever-changing appearances in a concave mirror could offer to a patron like Rudolf II willing to buy such a mirror from Magini. When writing the «Natural Magick», Porta's resources partly must have

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fused about the place of the focal point of a concave spherical mirror. There, he identifies, without hesitation, the focal point as the point at the «fourth part of the diameter» of the mirror. «In reflexionibus speculorum concavorum reflexionis linea non ascendet ultra quartam diametri». Ion. Baptista Porta, *De refractione optices parte libri novem*, Neapoli, Apud Io. Iacobum Carlinum & Antonium Pacem, 1593, p. 39.

[71] Porta, *Natural Magick*, p. 360.

[72] Magini, *Breve instruttione*, p. 31.

[73] Muraro, *Giambattista Della Porta*, p. 16. On the relationship between Porta and Sarpi and Porta's borrowing from Sarpi, see M. Gliozzi, *Relazioni scientifiche tra Paolo Sarpi e G. B. Porta*, «Archives Internationales d'Histoire des Sciences», 1, 1947, pp. 395-433.

[74] Ausonio/Galileo: «Haec linea habet duas comparationes ad ipsum speculum. Prima est, ut sit linea refractionis radiorum Solis, & tunc comburit. Secunda ut sit prima incidentia candelae, et tunc sua linea reflexa apparere facit imaginem luminis candelae in superficie Speculi, & potest calefacere per remotam distantiam, et illuminare quàm clarissimè, ita ut noctu legantur litterae. In Galileo, *Opere*, 3, p. 865. On the principle of reversibility in Porta's «Natural Magick», see Muraro, *Giambattista Della Porta*, pp. 118-119.

been derived from the optical instruments used in the painter's workshop or from manuals that discussed these instruments. Beside the camera obscura, used to draw a life-like picture without the loss of colour, Porta included among the effects to be obtained with a concave lens that a painter could use it to make a smaller image without loss of proportion.

A Painter may do it with great commodity, and proportion: for by opposition to a Concave Lenticular, those things that are in a great Plain are contracted into a small compass by it; so that a Painter that beholds it, may with little labour and skill, draw them all proportionably and exactly.[75]

Beside lenses, also mirrors were used in painter's workshops.[76] Porta drew attention to the ludic effects and deformations obtained by concave, cylindrical and conic mirrors.[77] At the same time, in manuals intended for painters, for example, Egnazio Danti's «Le Due Regole della Prospettiva Pratica», there is a boom of interest in these perceptual tricks with mirror images.[78] At the beginning of the seventeenth century, mirror anamorphoses, that are deformed pictures that are seen correctly when viewed in a cylindrical or conic mirror, appeared in painters' manuals, like Niceron's, who considered optical laws a kind of natural magic.[79] However, perspectival anamorphoses, that are deformed pictures only seen correctly from a lateral viewpoint, were already known by Leonardo

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[75] PORTA, *Natural Magick*, p. 369.

[76] Convex mirrors are shown in paintings as early as Van Eyck's «Arnolfini», but they were also used as drawing aids, that is, in the same way as a concave lens, to reduce without loss of proportion. On the use of mirrors in painters' workshops, see HEINRICH SCHWARZ, *The mirror in Art*, «Art Quarterly», 15, 1952, pp. 97-118; in particular for Dutch painting, see ARTHUR K. WHEELOCK, *Perspective, optics, and Delft artists around 1650*, New York & London, Garland Publishing, 1977. The use of a convex mirror as a drawing aid was not easy, because, for instance, the painter looking at a scene in the mirror would always be troubled by the mirror image of his own face that impeded a view of the scene he wished to paint. See MAGINI, *Breve instruttione*, p. 29.

[77] PORTA, *Natural Magick*, pp. 366-367, 370.

[78] E. DANTI, *Le Due Regole della Prospettiva Pratica: a reproduction of the copy in the British Library*, Alburgh, Archival Facsimiles Limited, 1987, pp. 94-96.

[79] JURGIS BALTRUSAITIS, *Anamorphic Art*, Cambridge, Chadwyck-Healey Ltd., 1977, pp. 131-169. For other reproductions of mirror anamorphoses, see FRED LEEMAN, JOOST ELFFERS & MIKE SCHUYT, *Hidden images: Games of Perception – Anamorphic Art – Illusion from the Renaissance to the present*, New York, Harry N. Abrams Inc., 1976; and MUSÉE DES ARTS DÉCORATIFS PARIS & RIJKSMUSEUM AMSTERDAM, *Anamorphoses: jeu de perspective/Anamorfosen: spel met perspectief*, Köln, M. Du Mont Schauberg, 1975. Niceron's perspective book is Niceron, «La Perspective Curieuse», 1638, but there are several editions, also in Latin, entitled the «Thaumaturgus Opticus». In the beginning of the seventeenth century, complete rooms full of playful effects with mirror anamorphoses were imagined in Dubreuil's «La Perspective Pratique» (1649).

and Piero della Francesca, and they became also quite popular in mid-sixteenth century Germany, where Schön, a pupil of Dürer, made his «Vexierbilder».[80] The Germans seem to have applied a correct method to make the «Vexierbilder», but this method was not represented in the sixteenth century manuals written for painters. In fact, the geometry of perspectival anamorphoses is mathematically equivalent to the rules for creating an ordinary picture in perspective.[81] Compared to the construction of an ordinary perspective picture, to make an anamorphic picture, there is only a need to bring the distance point closer to the central vanishing point and to heighten the horizon, or the central vanishing point.[82]

However, this equivalence was not observed by sixteenth century authors on linear perspective. Danti's construction of an anamorphosis was based on a parallel projection, that is the grid projected over the ordinary picture is simply lengthened.[83] The observation that a central projection was needed, happened in the direct research environment of Galileo. Cigoli was the first to record the correct geometrical rule to construct an anamorphosis, but his «Trattato pratico di prospettiva» never went to print until it was recently edited by Profumo.[84] (Fig. 5) Also, Cigoli invented an instrument to make an ordinary

[80] For Leonardo's anamorphoses, see VELTMAN, *Studies*, pp. 143-169; for Piero's anamorphic objects, see KIRSTI ANDERSEN, *The mathematical treatment of anamorphoses from Piero della Francesca to Nicéron*, in *History of mathematics – States of the art: Studies in honor of Christoph J. Scriba*, Joseph W. Dauben, Menso Folkerts, Eberhard Knobloch & Hans Wussing ed., San Diego, Academic Press, 1996, p. 7; for the German «Vexierbilder», see BALTRUSAITIS, *Anamorphic Art*, p. 11.

[81] For the mathematical equivalence, see ANDERSEN, *The mathematical treatment*, pp. 10-13. It's widely discussed why this mathematical equivalence went unnoticed until the seventeenth century, together with the related question why they were made, and whether or not they should be seen as a counter movement in painting. For these questions, see SALVATORE NAITZA, *Anamorfosi e legittimità prospettica tra Rinascimento e barocco*, «Annali della Facoltà di Lettere Filosofia e Magistero dell'Università di Cagliari», 23, 1970, pp. 173-239, criticizing Baltrusaitis' «surrealistic interpretation» of anamorphoses, and DIDIER BESSOT, *La perspective de Nicéron et ses rapports avec Maignan*, in *Geometria e atomismo nella scuola galileiana*, Massimo Bucciantini & Maurizio Torrini ed., Firenze, Leo S. Olschki, 1992, pp. 147-169.

[82] The distance point method was a geometrical construction method to make a picture in perspective. The distance between the distance point and the central vanishing point determined the ideal viewing distance of an observer. Herewith, an anamorphic picture determined an ideal lateral viewing point close to the picture plane.

[83] To view the anamorphic picture, Danti made use of an instrument of Tomasso Laureti that forced the observer to take a lateral point of view close to the picture plane. Danti's proposal for a parallel projection was incorrect, but he did formulate the correct rule that the anamorphic picture has to be lengthened more when the observer's eye is closer, and less, when the eye is further away. See DANTI, *Le Due Regole*, p. 96.

[84] LODOVICO CIGOLI, *Trattato pratico di prospettiva di Lodovico Cardi detto il Cigoli*, Rodolfo Profumo ed., Roma, Bonsignori Editore, 1992, p. 159; edited from the MS 2660A in the «Gabinetto dei Disegni e delle Stampe degli Uffizi» in Florence. For a discussion of Cigoli's anamorphoses, see FILIPPO CAMEROTA, *Dalla finestra allo specchio: la «prospettiva pratica» di Lodovico Cigoli alle origini di una nuova concezione spaziale*, Ph. D. Università degli studi di Firenze, 1985-86.
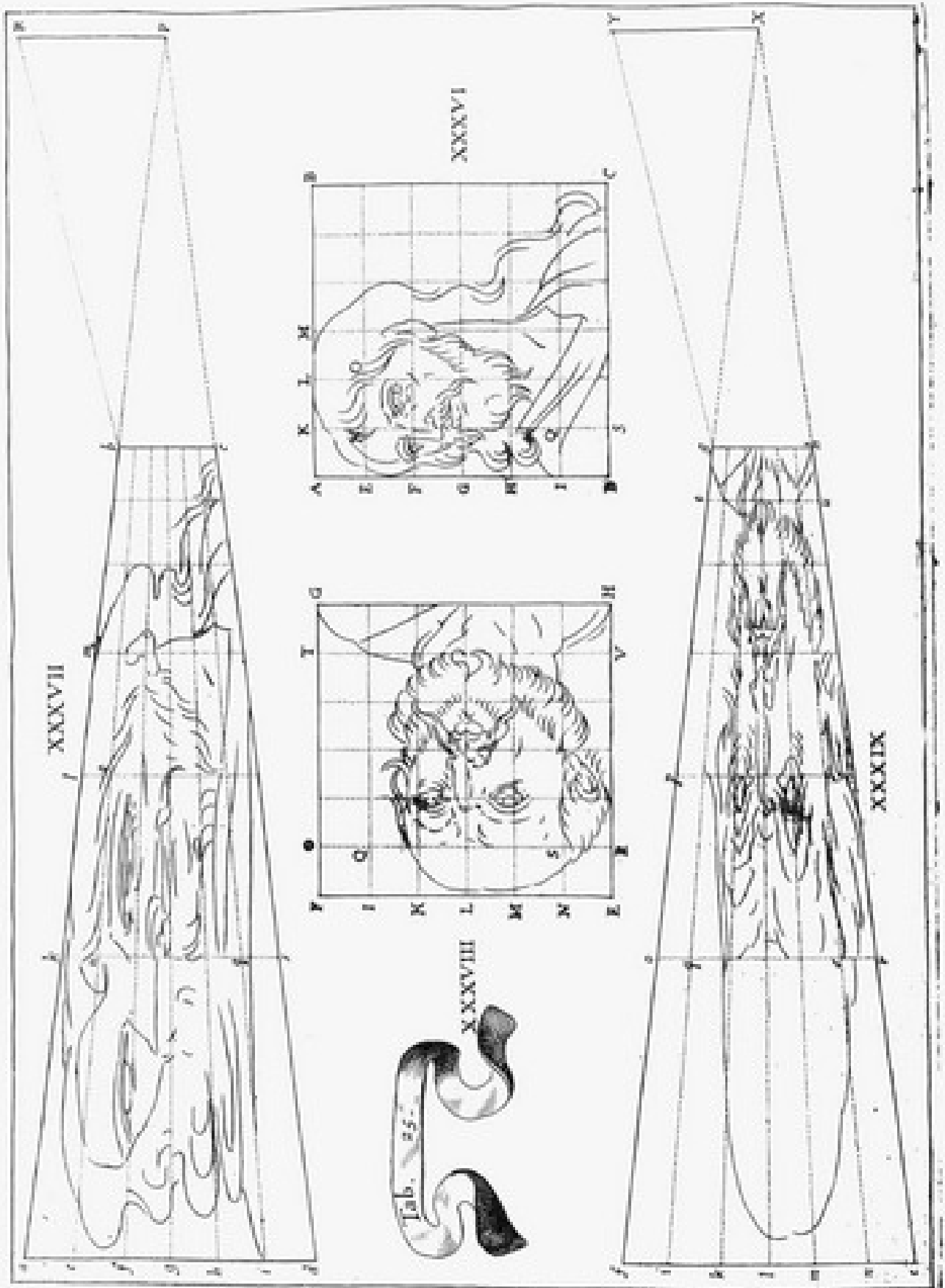
5. Geometrical construction of an anamorphosis, from: Niceron, La perspective curieuse, 1652. (Photo Courtesy of Ghent University Library).

picture into an anamorphosis, that later would prove very influential in the French circles devoted to anamorphoses of Niceron and Maignan.[85] On engravings of the instrument to project anamorphic images, it is evident that in the anamorphosis left and right are reversed due to the left-right reverted orientation of the picture to be projected onto the wall. (Fig. 6) When an object is placed in front of a mirror, the orientation of the object is changed in the same way, so that the mirror to the unattentive observer seems to revert left and right. What is important, is that an anamorphosis is like an image seen in one of Porta's deforming mirrors.[86] Natural magic and manuals intended for painters focussed on the same problems and effects triggered by the same optical instruments, in particular non-plane mirrors.

Anamorphoses attracted the attention of Galileo, who was in contact with Cigoli, and, also, seems to have studied Porta's «Natural Magick».[87] In his «Considerazioni al Tasso», Galileo noted that anamorphoses "show a human figure when looked at sideways and from a uniquely determined point of view but, when observed frontally as we naturally and normally do with other pictures, display nothing but a welter of lines and colors which we can make out, if we try hard, semblances of rivers, bare beaches, clouds, or strange chimerical shapes".[88]

Galileo's description resembled the circulating images of powerful men, whose anamorphic images were hidden in what is a landscape-like picture when looked at under normal viewing conditions.[89] As Kemp and Hallyn have argued, Galileo showed his awareness of anamorphic effects at two

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[85] Cigoli's instrument was an adaptation of Dürer's sportello that could be used to project images, for example, on a curved vault. For Cigoli's instrument, see CIGOLI, Trattato, pp. 149-169. For a discussion of Niceron's instrument, derived from Cigoli's, see ISABELLA TRUCI, Le anamorfosi di Jean François Niceron all'Istituto e Museo di Storia della Scienza di Firenze, «Annali dell'Istituto e Museo di Storia della Scienza», 1, 1976, pp. 57-64.

[86] FILIPPO CAMEROTA, Il giardino anamorfico, in Il giardino delle muse: Arti e artifici nel barocco europeo, Maria Adriana Giusti & Alessandro Tagliolini ed., Firenze, Edifir Edizioni Firenze, 1995, p. 260. Although the geometry of cylindric and conic mirror anamorphoses was only recorded by Vaulezard in 1630, the effect of earlier perspectival anamorphoses was like that of deforming mirrors. It is no coincidence that Cigoli thought of a painting as a mirror and less as a window scene. See MARTIN KEMP, Lodovico Cigoli and the origins and ragione of painting, «Mitteilungen der Kunsthistorischen Institutes in Florenz», 35, 1991, pp. 133-152. For Vaulezard's «Perspective Cilindrique et Conique», see ANDERSEN, The mathematical treatment, pp. 17-25.

[87] At least, a copy of Porta's «Natural Magick» was present in Galileo's library, but it was a 1611 edition. See FAVARO, La libreria, p. 262.

[88] GALILEO, Considerazioni al Tasso, in GALILEO, Opere, 9, pp. 129-130. Translation in ERWIN PANOFSKY, Galileo as a critic of the arts: Aesthetic attitude and scientific thought, «Isis», 47, 1956, p. 6.

[89] In Florence, Pietro Accolti claimed to have made such an anamorphic picture of Cosimo II, Grand Duke of Tuscany. PIETRO ACCOLTI, Lo inganno degl'occhi, «The Printed Sources of Western Art», 14, Portland (Oregon), United Academic Press, 1972, p. 49.
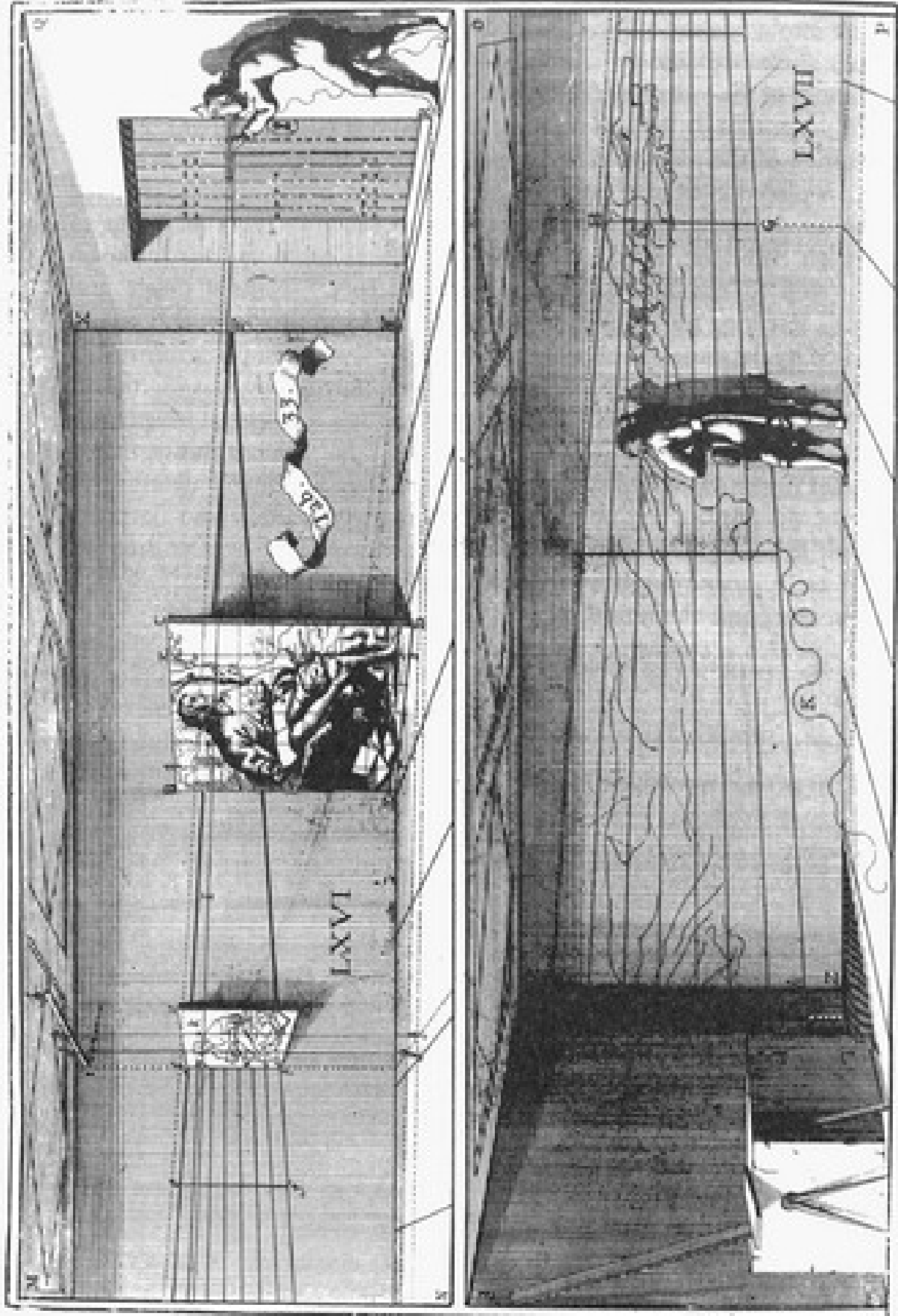
6. Instrument to project an anamorphic picture, from: Niceron, La perspective curieuse, 1652. (Photo Courtesy of Ghent University Library).

occasions when making astronomical observations, once during moon observations, and once observing sunspots. In both cases, he used a terminology of what was seen «in faccia» and what was seen «in scorcio» or «delineate» similar to the words with which he had described anamorphoses in the «Considerazioni al Tasso».

In the «Sidereus Nuncius», Galileo had argued that the moon had a mountainous surface. However, if this was true, then what with the moon's circumference? Did it not have to look like a jagged wheel? Galileo postulated that the moon was surrounded by an atmosphere, just like the earth. To reach the surface of the moon, our visual rays had to traverse more of this atmosphere at the moon's periphery, than in the middle of it, because they cut the atmosphere more obliquely. Galileo argued that, inhibited by the moon's atmosphere, the mountains at its periphery were not seen. Thus, the moon did not appear with an irregular circumference.[90] The argument was convincing, if one was willing to accept a lunar atmosphere. However, Galileo soon learned how dangerous it was to postulate a lunar atmosphere. Adversaries of lunar irregularity used the lunar atmosphere to accommodate Galileo's moon observations to Aristotelian cosmology by proposing a mountainous lunar surface surrounded by a perfectly regular crystal sphere, invisible, but completely smooth.[91]

Aware of this danger, Galileo tried to account for the regular circumference of the moon, without postulating a lunar atmosphere in a letter to Grienberger of September 1611. A difference was observed between the terminator of the moon when in quadrature and when in a gibbous phase. In the former case, the terminator appeared «in faccia», while in the latter case it appeared «in scorcio». (Fig. 7) Due to effects of foreshortening, near the circumference, the sinuous terminator lost some of its irregularity. As Galileo observed, near the circumference, due to the lateral viewpoint, the terminator "loses much of its width and appears, although long, compressed and narrow, because the visual ray is little elevated above the boundary".[92] The closer to the circumference of the moon, the more the terminator would lose its irregularity, until, at the visible circumference of the moon, it would appear circular without any irregularity at all. Exploiting the effect of anamorphosis, Galileo had an

[90] GALILEO, *Sidereus Nuncius*, in GALILEO, *Opere*, 3, p. 70. Translation in VAN HELDEN, *Sidereus Nuncius*, p. 50.

[91] Colombe's and Brengger's arguments are discussed in REEVES, *Painting the heavens*, pp. 155-158.

[92] Galileo to Grienberger, September 1, 1611, in GALILEO, *Opere*, 11, p. 187. FERNAND HALLYN, *Le regard pictural de Galilée sur la lune*, «Créis», 2, 1994, p. 34.
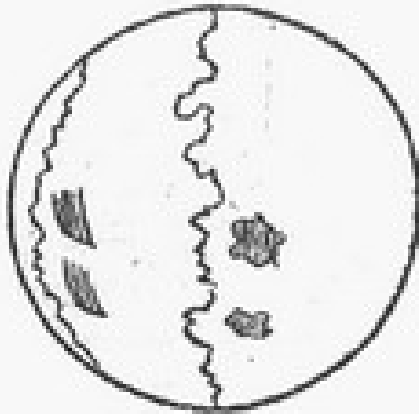
7. Galileo, Moon Drawing, Letter to Grienberger, September 1, 1611.

argument for a circular circumference, without postulating a lunar atmosphere, and still allowing a lunar surface fully covered with mountains.

Shortly thereafter, Galileo became involved in a controversy on sunspots with Scheiner.[93] Scheiner had argued that the spots were satellites circulating in an orbit around the sun. Galileo disagreed because of the spots' irregularity. He thought it more likely that the spots were very near to the sun. Again, one of his arguments was based on differences between the appearance of the spots when near the circumference of the sun and when in the middle of the solar body. The most evident was that the spots became thinner at the periphery of the sun.[94] Galileo concluded:

For those who understand, in virtue of perspective, what is meant by a spherical surface retreating near the periphery of the observed hemisphere, this will be a manifest argument that the sun is a globe, that the spots are close to the solar surface, and that as they are carried on that surface toward the center a growth in length is always discovered, while they preserve the same breadth.[95]

Again, the anamorphic effect gave the argument away. Thus, Galileo's use of anamorphosis, his studying of Porta and his copying of Ausonio's «Theorica» show his acquaintance with a tradition that focussed on optical effects obtained with instruments.

What the manuals, intended for painters, and natural magic shared, were optical instruments. One focus of Ausonio's «Theorica» was the use of a concave mirror as a burning mirror. Also, Porta spent a lot of words on burning mirrors, explaining what could be attained by them, but also how

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[93] On the sunspot controversy, see A. VAN HELDEN, Galileo and Scheiner on sunspots: a case study in the visual language of astronomy, «Proceedings of the American Philosophical Society», 140, 1996, pp. 357-395.

[94] GALILEO, Istoria e dimostrazioni intorno alle macchie solari, in GALILEO, Opere, 5, p. 119. KEMP, The science of art, p. 97.

[95] GALILEO, Istoria e dimostrazioni intorno alle macchie solari e loro accidenti, in GALILEO, Opere, 5, p. 119. Translation adapted from STILLMAN DRAKE, Discoveries and opinions of Galileo, New York, Doubleday Anchor Books, 1957, p. 107. For a full account, see KEMP, The science of art, p. 97.

to make them, either in metal or in glass, by cutting a parabolic section, including the instruments needed for that task.[96] For Porta, research on burning mirrors was triggered by the legendary story of Archimedes who is claimed to have used a burning mirror to defeat the ships of the enemy at Syracuse. The reference to Archimedes, who was during most of the sixteenth century known as a builder of ingenious instruments, was common among sixteenth century mathematicians, who tried to build such a mirror without taking the legendary story at face value.[97] Galileo made the same reference when he discussed the burning effects of parabolic and spherical concave mirrors in his last work, the «Two New Sciences» of 1638.

I have seen lead instantly liquefied by a concave mirror three spans in diameter, and am of the opinion that if the mirror were very large, smooth, and of parabolic shape, it would liquefy any metal in short time. For we see that a spherically concave mirror, neither very large nor well polished, liquefies lead with great power and burns every combustible material – effects that give credibility to the wonders of the mirrors of Archimedes.[98]

Moreover, after seeing the recently finished book on the burning mirror of Bonaventura Cavalieri, a student of Galileo's pupil Castelli, Galileo believed that Archimedes' burning mirror had actually existed at the time.

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[96] PORTA, *Natural Magick*, pp. 371-378. Glass technology to make mirrors only matured in the sixteenth century. During the Middle Ages, most mirrors were small. They were made either in metal or in glass. The technology to make large sheets of glass did not yet exist. The size of the well known mirror in Van Eyck's «Arnolfini» is rather exceptional, but it was exaggerated by Van Eyck, maybe by using a convex mirror to paint the scene. See DAVID L. CARLETON, *A Mathematical Analysis of the Perspective of the Arnolfini Portrait and other similar interior scenes by Jan Van Eyck*, «Art Bulletin», 64, 1982, p. 122. At the beginning of the sixteenth century, Venetian crystal revolutionized the making of mirrors, when, from Flanders, the tin-amalgam process was introduced. This process consisted in laying a tin-foil on to glass by means of mercury as a cementing medium, that is cold metal was applied to glass avoiding the difficulties connected with hot materials. Also, in principle, any size of mirror was possible to make now, but it would take a while before metal mirrors disappeared. Magini still made large metal mirrors at the beginning of the next century. For the Venetian crystal mirrors of the sixteenth century, the technology involved and their invention, see BRUNO SCHWEIG, *Mirrors*, «Antiquity», 15, 1941, pp. 266-267; and LUIGI ZECCHIN, *Vetro e vetrai di Murano*, 3, Venezia, Arsenale Editrice, 1990, pp. 165-169, pp. 368-371.

[97] PORTA, *Natural Magick*, pp. 355, 371-372, 375, about the «secrets of the most eminent Artificer, Archimedes». The standard approach of sixteenth-century mathematicians was to try to find the way Archimedes would have built these burning mirrors. Porta claimed to have invented «a parabolic section that may burn to infinite distance». For the legendary story of Archimedes, see KNORR, *The geometry of burning-mirrors*, pp. 53-55, and D.L. SIMMS, *Archimedes and burning mirrors*, «Physics Education», 10, 1975, pp. 517-521. On the perception of Archimedes in the sixteenth century as a designer of ingenious instruments, see W. R. LAIRD, *Archimedes among the humanists*, «Isis», 82, 1991, pp. 629-638.

[98] GALILEO, *Discorsi*, in GALILEO, *Opere*, p. 86. Translation in STILLMAN DRAKE, *Galileo Galilei: Two New Sciences*, Madison, The University of Wisconsin Press, 1974, p. 48.

As to Archimedes and the effects of his mirrors, all the miracles that are read in other authors are rendered credible to me by reading the books of Archimedes himself, long ago studied by me with infinite astonishment. And if any doubt lingered, the book lately published about the burning glass by Father Buonaventura Cavalieri, which I read with admiration, is enough to put a stop to all difficulties for me.[99]

Galileo studied Archimedes as a young student with Ostilio Ricci. Again, this was a training in mathematical instrument making that eventually would be the starting point of Galileo's own invention of the geometric and military compass. The study of burning mirrors was a legitimate research subject for these mathematicians of the sixteenth century.

One of the most popular mathematical works of the sixteenth century was written by Oronce Finé. His «Protomathesis», published in 1532, was a prototypical work of the tradition of mathematical instrument making, applied to practical activities as dialling, map-making and surveying. To measure by sight depths, heights and distances, Finé discussed the use of already mentioned instruments and procedures like a mirror placed on the ground between the measurer and a tower or the geometrical quadrant, but also the *squadra* and the Jacob's staff.[100] What's more important, the Italian translation of Finé's «Protomathesis» by Cosimo Bartoli, and dedicated to Guidobaldo del Monte, was published together with Finé's work on burning mirrors, translated by Ercole Bottrigaro.[101] Finé's way of

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[99] GALILEO, *Discorsi*, in GALILEO, *Opere*, pp. 86-87. Translation in DRAKE, *Two New Sciences*, pp. 48-49. The work of Cavalieri referred to is BONAVENTURA CAVALIERI, *Lo specchio ustorio, overo, Trattato delle settioni coniche: et alcuni loro mirabili effetti entorno al lume, caldo, freddo, suono, e moto ancora*, Bologna, Presso Clemente Ferroni, 1632. After criticizing Porta's «parabolic section that burns at an infinite distance», also, Cavalieri deviced a way to make the burning mirror of Archimedes.

[100] ORONCE FINÉ, *Protomathesis, seu opera mathematica*, 1532. References are to the Italian edition, ORONCE FINÉ, *Opere di Orontio Fineo del Delfinato, divise in cinque parti; aritmetica, geometria, cosmografia, e oriuoli; tradotte da Cosimo Bartoli, Gentilhuomo, & Academico Fiorentino; et gli specchi, tradotti dal Cavalier Ercole Bottrigaro, Gentilhuomo Bolognese*, Venetia, Presso F. Franceschi, 1587, pp. 27-49. Finé dealt with the Jacob's staff in FINÉ, *Opere*, pp. 33-35. The Jacob's staff, first described by Levi ben Gerson, with later variations by Apian and Gemma Frisius, consisted of a longitudinal staff equipped with a traverse staff as long as one of the segments into which the longitudinal staff was divided. Again, measurement was based on the construction of similar triangles. The traversal staff and the segment of the longitudinal staff between the latter and the eye of the measurer made, respectively, the base and the height of a triangle of which the visual rays of the measurer formed the sides. This triangle was similar to the one described by the visual rays with the unknown size as its base. Simple triangulation procedures then allowed to determine the unknown size, for example the breadth of a building or the height of a mountain. See FILIPPO CAMEROTA, *Giorgio Vasari*, pp. 141-142, 171-173; TURNER, *Early scientific instruments*, pp. 20-21.

[101] Ercole Bottrigaro (1531-1612) was a humanist, who besides a translation of Finé's «De

proceeding was identical with Porta's in the «Natural Magick», that is he explained how to construct a parabola, described the instruments to cut a parabolic section and how to make a burning mirror, and discussed the effects obtained by burning mirrors. Thus, the mathematicians who focussed on mathematical instrument making, like Finé, developed an interest in optical instruments. Their treatment of the subject was similar, that is it focussed on the construction of the instrument and the possibilities it had for its user. In the case of Porta and Magini, this also involved an actual designing of mirrors, in the same way that these mathematicians had provided Renaissance society with mathematical measuring instruments for practical activities.[102]

Even the medieval optical tradition was reinterpreted by mathematicians with this focus during the sixteenth century. There was no particular focus on burning mirrors in the work of Alhazen and Witelo, but the frontispieces of sixteenth century editions of their works, by Tanstetter and Apian in 1535, and by Risner in 1572, showed an emphasis on Archimedes' burning mirrors.[103] (Fig. 8) Galileo absorbed the sixteenth century interpretation of medieval optics. One of the few instances he referred to Alhazen and Witelo, that is in his annotations to Sizzi's «Dianoia», he referred to what they said about the construction of mirrors.[104] Also, his interest in mirrors

Specchio Ustorio», also edited Ptolemy's «Geography». He lived most of his life in Bologna, where he was known to Magini, see Magini's letter to Galileo of 11 January 1611, in GALILEO, Opere, 11, p. 19. On Ercole Bottrigaro, see the Dizionario biografico degli italiani, pp. 491-495.

[102] Magini is known to have designed two quadrants in the 1590s and to have published a treatise «De Dimetiendi Ratione per Quadrantem, & Geometricum Quadratum» (1592) to explain the use of this instrument for topographical measurement. See FABRIZIO BONOLI & MARINA ZUCCOLI, On two sixteenth-century instruments by Giovanni Antonio Magini (1555-1617), «Nuncius», 14, 1999, pp. 201-212. Turner's analysis of the Danti-Giusti case shows that it is better to refer to these mathematicians' designing their instruments instead of actually making them, because the latter would have been the job of a craftsman. The «Theorica», in Aussonio's and Galileo's version, stresses the need for a good craftsman to make the mirrors: «Hoc apparet per comparationem apparentiarum horum Speculorum, et illorum, qui ab artifice quodam factae siunt sine mensura». In GALILEO, Opere, 3, p. 865. On Danti and Giusti, see TURNER, The Florentine Workshop, p. 133.

[103] For Risner's edition, see the reprint by Lindberg in WITELO, Opticae Thesaurus.

[104] «Alhazen and Witelo teach the construction of a parabolic mirror». Sizzi, Dianoia astronomica, optica, physica, in GALILEO, Opere, 3, p. 239. See also VASCO RONCHI, Scritti di ottica, Milano, Edizioni Il Polifilo, 1968, p. 409, with an Italian translation of the «Dianoia». It is difficult to determine whether or not Galileo was ever involved in the actual process of mirror making. He was in contact with Girolamo Magagnati, who was a poet, but also active in the Venetian glass industry. Magagnati is credited with the invention of a certain colored glass to imitate precious stones, but he also invented a procedure that allowed to make mirrors larger and faster than before. On Magagnati, see ANTONIO FAVARO, Girolamo Magagnati, in Amici e corrispondenti di Galileo, Paolo Galluzzi ed., Firenze, Libreria Editrice Salimbeni, 1983, pp. 65-92; STILLMAN DRAKE, Galileo Gleanings XIV: Galileo and Girolamo Magagnati, «Physis», 6, 1957, pp. 269-286; ZECCHIN, Vetro e vetrai, pp. 356-364. A shopping list of November 1609 shows Galileo

8. Frontispiece, from: Risner, Opticae Thesaurus, 1572.

in contact with mirror makers of Murano. See GALILEO, *Opere*, 10, p. 270 (Eileen Reeves, personal communication). It is known that Galileo did purchase telescope lenses from mirror-makers, who seem to have been better up to the job than spectacle-makers. Thus, contacts with mirror-makers in Murano might have proved useful to Galileo when he began working on the telescope. After his move to Florence, he was informed about developments in glass technology in Venice by Sagredo. However, it is not clear whether Galileo ever did any manual labor on glass, mirrors or lenses himself, although a letter to Giuliano de' Medici shows that, at least in 1610, Galileo had a glass polishing facility of his own. See Sagredo to Galileo, August 4, 1618, in GALILEO, *Opere*, 12, p. 405; Galileo to Giuliano de' Medici, October 1, 1610, in GALILEO, *Opere*,

was connected to the designing of mathematical instruments, that is his geometric an military compass. The knowledge on which it was based came out of the same tradition. Thus, Ausonio's «Theorica» does not need to be marginalized, as it has been before, by either not discussing it or leaving it outside the picture of Galileo's science. Galileo being a mathematician, it has its place in Galileo's scientific activity.

5. CONCLUSION

Why has the «Theorica», or its date, not been appreciated? The underestimation of mathematical and optical instrument designing has been responsible for Galileo's optics being caught within an anachronistic «two cultures» concept of art and science. However, attention to mathematical and optical instrument designing shows the ubiquity of perspective in all the mathematical «arts». When using a «two cultures» concept of art and science, Galileo's «artistic» training is considered not to be related to his copying of Ausonio's «Theorica». When the big picture of art and science is considered, it is evident that with the invention of the geometric and military compass and Ausonio's «Theorica», Galileo continued an optical tradition, unaffected by modern disciplinary boundaries, which stressed mathematical and optical instrument designing and the skill and operative knowledge that went along with it.

Ricci introduced Galileo to a mathematical tradition that was primarily interested in developing instruments that were used for a wide variety of measuring tasks. At the turn of the sixteenth century, the most straightforward motivations of this research, out of which Galileo's geometric and military compass developed, were the military ambitions of the mathematicians' patrons. Even less emphasis has received the fact that these mathematicians were also interested in the designing of mirrors. Again, a military context was provided by the legendary story of Archimedes. At the end of the sixteenth century, Ausonio's «Theorica», Porta's «Natural Magick» and, also, manuals intended for painters took up

10, pp. 440-441; SILVIO A. BEDINI, *The makers of Galileo's scientific instruments*, in *Atti del Simposio Internazionale di Storia, Metodologia, Logica e Filosofia della Scienza: Galileo Galilei nella storia e nella filosofia della scienza*, Florence, Gruppo italiano di storia della scienza, 1967, p. 94, reprinted in SILVIO A. BEDINI, *Science and instruments in seventeenth-century Italy*, Aldershot (Hampshire), Variorum, 1994, pp. 89-115; SILVIO A. BEDINI, *«A Treatise on Optics» by Giovanni Christoforo Bolantio*, «Annals of Science», 52, 1995, p. 105, reprinted in SILVIO A. BEDINI, *Patrons, artisans and instruments of science, 1600-1750*, Aldershot, Ashgate Variorum, 1999, pp. 103-126.

these instruments, while emphasizing the perceptual tricks that could be played with them. When Galileo copied Ausonio's «Theorica», around the same time he invented the geometric and military compass, it is argued that he stayed within the same mathematical tradition that previously has been neglected by Lindberg, Drake and others who were not able to appreciate the «Theorica».

Acknowledgments

SUMMARY

This paper will deal with Galileo's optics prior to his involvement with the telescope. It will be argued that an anachronistic distinction between art and science has obscured the scope and the interconnections of Galileo's optics. The paper will focus on an analysis of the «Theorica speculi concavi sphaerici», a less known document in the MS Gal. 83, that always has resisted interpretation, because its connection with Galileo's involvement with a tradition of mathematical and optical instrument designing has not been understood.