Damage Detection based on the Natural Frequency shifting of a clamped rectangular plate model

Damage Detection based on the Natural Frequency shifting of a clamped rectangular plate model

Irfan Hilmy[1], MM. Abdel Wahab[2], Erry Yullian T. Adesta[1], Tasnim Firdaus[1]

[1]International Islamic University Malaysia, Faculty of Engineering, Department of Manufacturing and Materials Engineering, P. O. Box 10, 50728 Kuala Lumpur, Malaysia
[2]Department of Mechanical Construction and Production, Ghent University, Belgium

E-mail: ihilmy@iium.edu.my

Abstract. Damage detection of any structure becomes the main concern in a failure analysis. Early failure detection is very important as it can prevent any catastrophic failure by replacing or repairing the damage part at early stage. One of the non-destructive methods of damage detection is using frequency based vibration analysis. Identification and comparison of a set of natural frequencies before and after damage is the main concern of this research. A rectangular plate clamped at all edges represented an initial undamaged structure. Based on Kachanov's definition, damage existence in a structure is introduced in the presence of some circular voids. The voids are generated randomly at different level of damage value. To obtain the Natural Frequencies, a Finite Element Model (FEM) of a clamped plate with the updated value of Young's Modulus is analyzed. From the FEM analysis result, it is found that the Natural Frequencies are shifted as the void existence increase. Using curve fitting, the model of Natural Frequency shifting as a function of damage evolution has been generated. It is found that the shifting of the Natural Frequency is greater at higher frequency value as indicated by the higher absolute gradient.

1. Introduction
The eigenvalue analysis technique is a fundamental tool and the most well-known for matrix computations. In vibration analysis, eigenvalue resemble natural frequency of an engineering structure. It is well known that any normal/perfect/undamaged structure has a set of natural frequency that will change due to the degradation of its mechanical properties. This status is called as damage or abnormal structure. The main aim of the damage detection is to avoid any catastrophic failure by replacing the damage part before it propagates to other main/vital part. To detect or identify the damage level of the structure based on the change of the natural frequency becomes the objective of this present paper.

The detection to determine a degree of damage in existing engineering structures is of great importance from the point of view of their safety and serviceability. An extensive testing and a visual inspection can be employed to measure and locate the degradation of structure by non-destructive techniques such as ultrasonic methods, thermography, acoustic emission, or the vibration testing. In vibration analysis, many methods have been developed, but the common method is Frequency Response Function (FRF) analysis. Using FRF, a shifting of natural frequency, mode shape, FRF or the existence of the new natural frequency can become an indication of the occurrence of damage

evolution [1]. The damage term in this paper relates with the damage evolution law theory as initiated by Kachanov [2]. To determine a failure of a structure, damage mechanics is considered a relatively new tool compare to the fracture mechanics that already established for quite some times. Fracture mechanics uses strain energy release rate and crack propagation phenomenon for its calculation while damage mechanics use the presence of large number of randomly distributed micro-cracks and voids. Damage mechanics deal with the crack initiation phase in a structural failure.

Kwak & Han [3] has developed a free vibration analysis of a rectangular plate with a rectangular or a circular hole based on the Independent Coordinate Coupling Method (ICCM). The latest publication related to the Eigen analysis of a plate with circular cut-out was generated by Hossain et al. [4]. The explicit closed form solution to natural frequencies and its mode shapes has been derived by Kang & Kim [5]. As far as the author's knowledge, none of the model as stated in the publications above incorporating the updated Young's Modulus as a function of material degradation in their numerical models. Figure 1 shows a model of structure/material contain damage in term of the existence of void/air bubble as proposed by Kachanov.
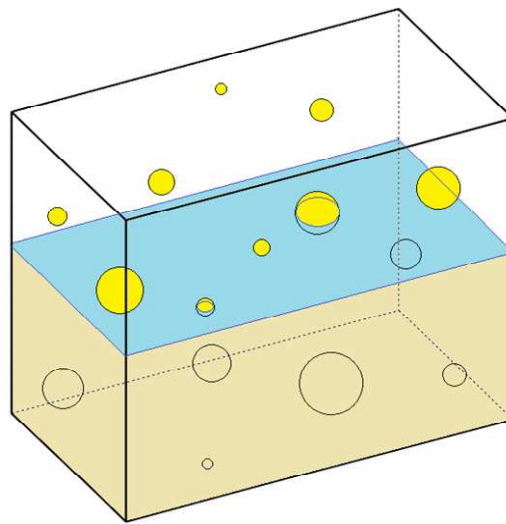
Figure 1. Damage definition, adapted from Kachanov [2]

In reality, this is the case where any real material always contains defect no matter how sophisticated any material forming process is. If from Figure 1, any imaginary plane was taken, then it can be modelled as a rectangular plane with any circles that represent void/air bubble as shown in Figure 2.

Analytical solutions for flexible plate case with some boundary conditions were developed by Brethee [6], Hashemi et al. [7] and Safizadeh & Mat [8]. Determination of the natural frequencies of flexible rectangular plates was developed by Geradin [9] using approximation method, which follows from the properties of deflection of the mid-plane. Some researchers have developed methods to improve and validate the analytical solution. Using asymptotic theory as proposed by Geveci & Walker [10], numerical solutions for plates with all-clamped edges are obtained and validated. Mochida & Ilanko [11] used the finite difference and superposition methods to obtain the natural frequencies of a free rectangular plate.
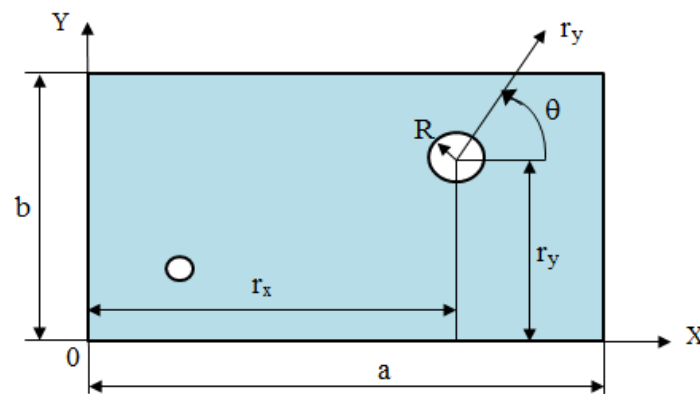
Figure 2. Rectangular plate with a circular hole as a model of damage, adapted from Moon [3]

2. Clamped plate model
Based on reference [11], the analytical formula of natural frequencies of all-clamped edges plate are obtained using the following formula:

$$f_{ij} = \frac{\lambda_{ij}^2}{2\pi\phi^2}\left[\frac{Eh^2}{12\rho(1-v^2)}\right]^{1/2} \qquad\qquad \text{Eq. 1}$$

where the natural frequency (f_{ij}) depends upon Young modulus (E), Poisson ratio (v), mass density (ρ) and the dimensionless natural frequency parameter (λ).

Refer to the notations stated in Figure 2, a fictitious plate model with the dimensions $a = 6$ inch, $b = 5$ inch and thickness=0.063 inch with all edges clamped was developed using ABAQUS Standard/Explicit model. A perfect model whereby no voids exist was validated with Moon's analytical equation. The overall flowchart is shown in Figure 3.
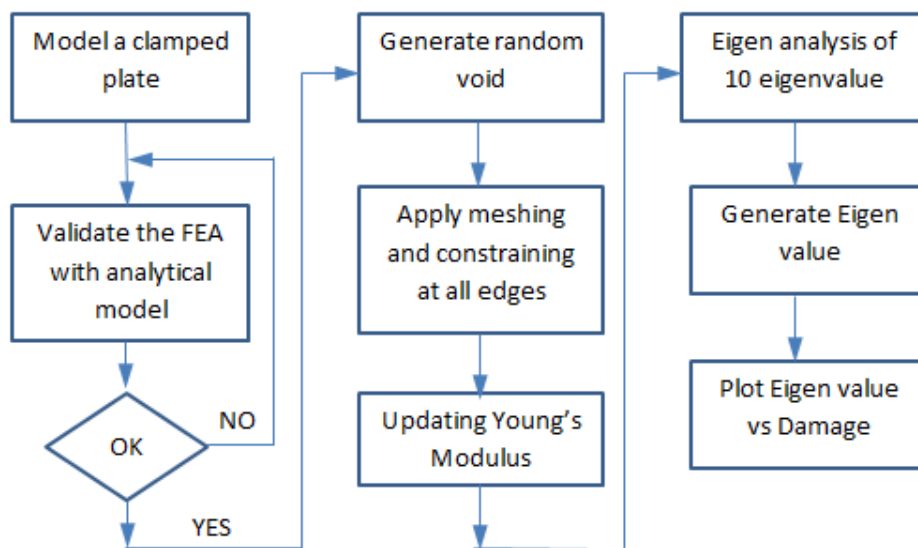

Figure 3. Flowchart of the research

Using Matlab, the coordinate of the voids were randomly generated, while maintaining the size of the void constant. The models were generated near two conditions: very low damage value (0 until 0.034) and near critical damage value (0.1 until 0.134). The geometry with the holes generation at different level of damage is shown in Figure 4.

(a)   (b)   (c)
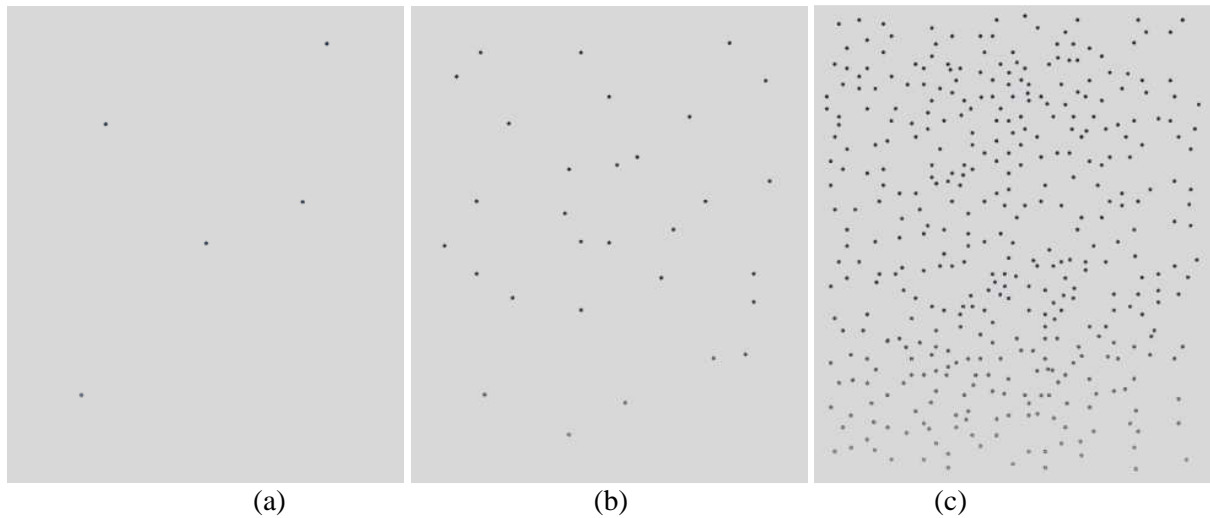
**Figure 4.** Generation of void model with (a) 5, (b) 30, and (c) 100 holes

Figure 5 shows meshing status for different damage level. It can be seen that the number of elements rising rapidly to accommodate finer meshing near hole.

(a)   (b)   (c)
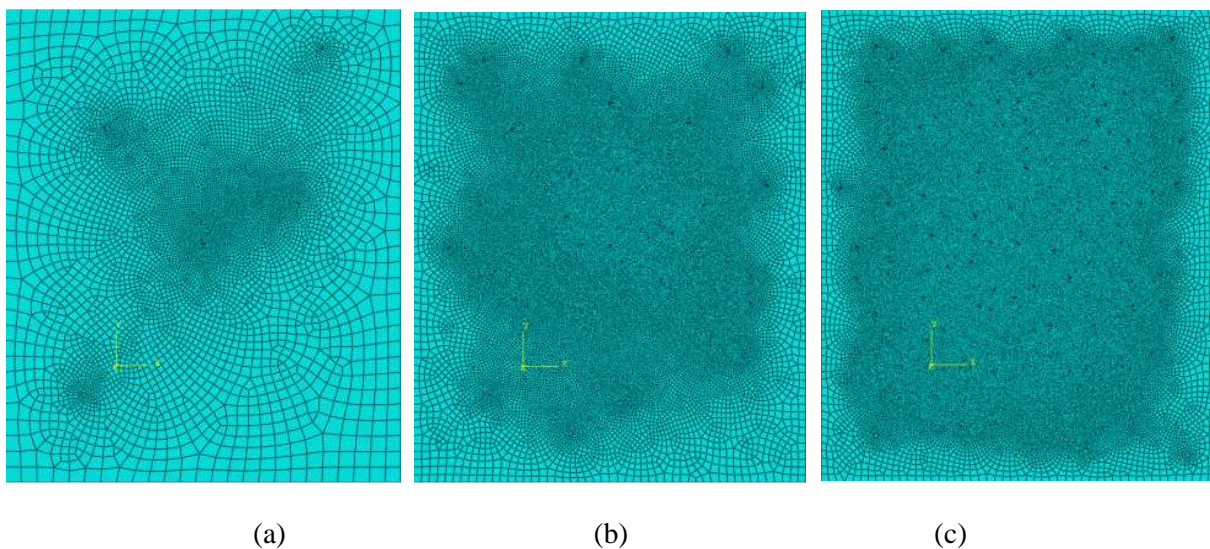
Figure 5. Generation of meshing for different hole number respective to the Figure 3
(a) 7,118 (b) 39,065, and (c) 77,496 elements

During the development of Damage, the Young's Modulus value degrades. The relation of Damage as a function of Young's Modulus degradation has been developed by Wahab et al. [12] and shown in Figure 6. Based on this phenomenon, the value of Young's Modulus was updated respective to the damage value. It can also be seen that after the damage reach the critical value (beyond 200 number of cycles), damage value increases rapidly that eventually it leads to a catastrophic failure.
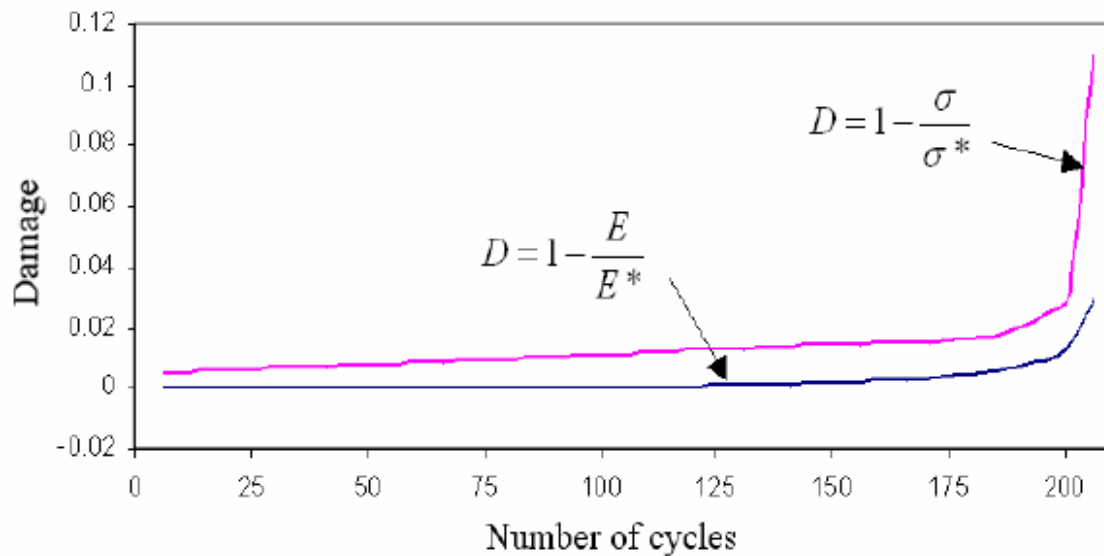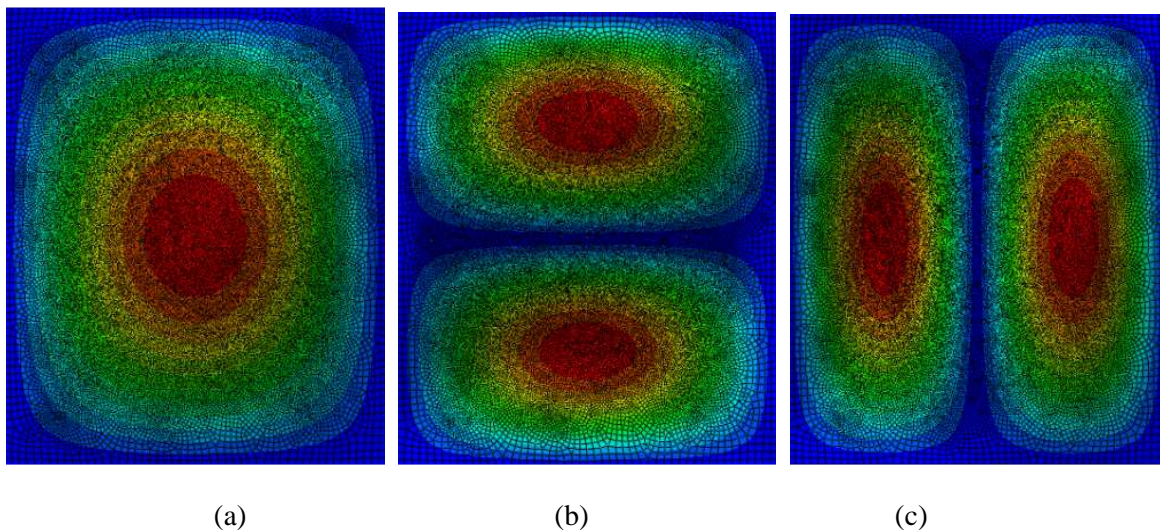
Figure 6. Damage calculation using different methods, adapted from Wahab et al. [12]

3. Numerical results

Typical result of the Eigen analysis is shown in Figure 7, where the mode shapes of the first three natural frequencies are plotted.

(a)   (b)   (c)

Figure 7. Mode shape of the first four natural frequencies (case damage value = 0.034):
(a) 19.83, (b) 44.15, and (c) 54.97 (Hz)

Figure 8 shows the result of the Eigen analysis. Ten natural frequencies were extracted for different damage level up to Damage value = 0.134. The reason behind this maximum value is based on the assumption that at that level, the damage already reaches its critical value. In theory, above critical value, the damage will increase rapidly that eventually it will lead to a catastrophic failure.
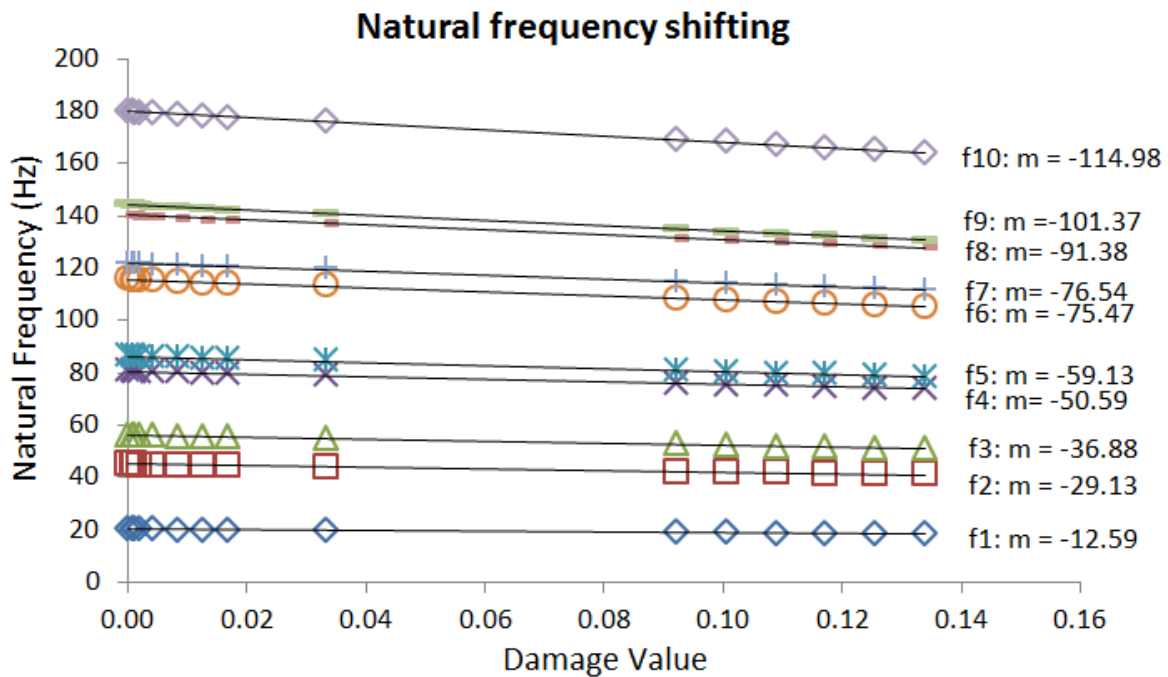
Figure 8. Natural frequency shifting vs Damage evolution

It also can be seen from Figure 8 that the absolute value of the gradient of the trend line increases (from 12.59 to 114.98) as the Natural frequency rises (from f_1 up to f_{10}). It means that the reduction of Natural frequency due to material's property degradation is higher at high Natural frequency number.

4. Conclusions

The present paper is concerned with the shifting of Natural frequency as a function of damage evolution. Based on this finding, damage value is treated as the value of the whole structure and not refers to any particular place in the structure. This is due to the assumption that the void exist in a randomly manner. For future works, the location of void will be determined as a function of space. The FEA model also will be upgraded into a solid model with the void randomly scatter in a 3D space. Therefore, the shifting of the natural frequency can be determined also as a function of the location of the void.

5. Acknowledgement

6. Reference

[1] K. Dems, J. Turant 2011 Structural damage identification using frequency and modal changes, *Bulletin Of The Polish Academy Of Sciences Technical Sciences*, Vol. 59, No. 1

[2] L. M. Kachanov 1986 Introduction to continuum damage mechanics *Mechanics of Elastic Stability* Volume 10 ISBN: 978-90-481-8296-1

[3] Moon K. Kwak, Sangbo Han 2007, Free vibration analysis of rectangular plate with a hole by means of independent coordinate coupling method, *Journal of Sound and Vibration*, 306:12-30

[4] Nazmul Hossain, KH. Nazmul Ahshan, Md. Zahid Hossain and Md. Shahriar Islam 2015 Effect on Natural Frequency of a Simply Supported Plate Due to Circular Cutouts, *Proceedings of 10th Global Engineering, Science and Technology* ISBN: 978-1-922069-69-6

[5] Sang Wook Kang & Sang-Hyun Kim 2008 Vibration analysis of simply supported rectangular plates with unidirectionally, arbitrarily varying thickness, *Jour. of Sound & Vibr*. 312,551-562.

[6] Khaldoon F. Brethee 2009 Free Vibration Analysis of A Symmetric and Anti-Symmetric Laminated Composite Plates With a Cutout at The Center *Al-Qadisiya Journal For Engineering Sciences* Vol. 2 No. 2,

[7] Shahrokh Hosseini-Hashemi, Mahmoud Karimi, Hossein Rokni 2012 Natural frequencies of rectangular Mindlin plates coupled with stationary fluid *Applied Mathematical Modelling* 36 764–778.

[8] Safizadeh, M. R.; Darus, I. Z. Mat May 2010 Natural Frequency Analysis of All Edges Clamped Flexible Thin Plate, *International Review of Mechanical Engineering* Vol. 4 Issue 4, p433

[9] Michel Geradin 2006, *Mechanical Vibrations: Theory and Application to Structural Dynamics*, Sons Inc & John Wiley, USA,

[10] Berk Geveci, Walker, J. D. A. May 2009 Nonlinear Resonance of Rectangular Plates *Mathematical, Physical and Engineering Sciences*. 457, pp. 1215-1240, ,

[11] Mochida, Y, Ilanko, S 2008 Bounded natural frequencies of completely free rectangular plates *Journal of Sound and Vibration* 311 pp. 1–8,

[12] MM Abdel Wahab, I Hilmy, IA Ashcroft and AD Crocombe, Damage Modelling of Adhesive Bonding, *4th International Conference on Fracture and Damage Mechanics*, Mallorca, Spain, 2005