SAPIENZA
UNIVERSITÀ DI ROMA

Risk-mitigation techniques: from (re-)insurance to alternative risk transfer

Scuola di Scienze Statistiche
Dottorato di Ricerca in Scienze Attuariali – XXXIII Ciclo

Candidate
Antonio Pallaria
ID number 1724514

Thesis Advisor
Prof. Nino Savelli

2022/2023

Thesis defended on 14 febbraio 2023
in front of a Board of Examiners composed by:

Prof. N. Savelli (chairman)

Prof. M. Sibillo

Prof. L. Ballestra

Risk-mitigation techniques: from (re-)insurance to alternative risk transfer
Ph.D. thesis. Sapienza – University of Rome

This thesis has been typeset by LaTeX and the Sapthesis class.

Author's email: antonio.pallaria@uniroma1.it

Abstract

Insurance risks knowledge is becoming essential for both financial stability and social security purposes, moreover in a country with a very low insurance education like Italy.

In insurance industry, Solvency II requirements introduced new issues for actuarial risk management in non-life insurance, challenging the market to have a consciousness of its own risk profile, and also investigating the sensitivity of the solvency ratio depending on the insurance risks and technical results on either a short-term and medium-term perspective.

For this aim, in the present thesis, firstly a partial internal model for underwriting risk is developed for multi-line non-life insurers. Specifically, the risk-mitigation and profitability impacts of traditional reinsurance in the underwriting risk model are introduced, and a global framework for a feasible application of this model consistent with a medium-term analysis is provided. Reinsurance have to be considered in the assessment of Non-Life insurers risk profile, with particular regard to the Solvency II Underwriting Risk because of its impact on business and risk strategy.

Risk mitigation techniques appear as a key driver of Non-Life insurance business as they can change risk profile over either the short-term or medium-term perspective. They impact the technical result of the year in such a way that it is important to assess how reinsurance strategies decrease the volatility, reducing the capital requirements, but, on the other hand, they also change the mean of distributions in different ways according to the price for risk requested by reinsurers.

At the same time, risk mitigation also influences Non-Life insurance management actions as it can improve business strategy and capital allocation (also in potential capital recovery plans). Furthermore, the analysis a medium-term capital requirement would ask insurers to have more capital than in a one-year time horizon, and in this framework risk mitigation effects linked to reinsurance strategies must be assessed on either risk/return perspective trade-off.

On the other hand, (re)insurance can play an active role in mitigating physical risks, and in particular natural catastrophe risks. In this context, as well as in natural disasters, Alternative Risk Transfer (ART) is becoming a new significant actuarial and capital management tool for insurers and, potentially, for government measures in recovery actions of economic and social losses in case of natural disasters.

Catastrophe Bonds are insurance-linked securities that have been increasingly used as an alternative to traditional reinsurance for two decades.

In exchange for a Spread over to the risk-free rate, protection is provided against stated perils that could impact the insured portfolio. A broad literature has flourished to investigate what are the features that significantly influence the Spread, in addition to the portfolio's expected loss. Almost all proposed models are based on multivariate linear regression, that has provided satisfactory predictive performance

as well as easily interpretability.

This thesis also explores the use of Machine Learning models in modeling the determinant at issuances, contrasting both their predictive performance and their interpretability with respect to traditional models. An overview of the economics of CAT bonds, on current literature and on the statistical methodologies will be provided also.

Aim of this Thesis is to provide a solid framework of insurance risk transfer for both pure underwriting and catastrophe risks, investigating risk transfer practices from traditional to alternative and most innovative technique. In these fields, firstly a suitable risk model is used in order to describe main impacts on insurance business model. Then, the main innovative alternative risk transfer for catastrophe risks are illustrated and CAT Bond will be adequately described, investigating main pricing models using a machine learning approach.

Finally, a possible Italian CAT Bond issuance is provided in order to investigate an integrated solution with a traditional reinsurance underlying an alternative risk transfer in order to achieve a public-private partnership to natural catastrophe.

Contents

Chapter 1

Introduction

In the frame of actuarial science, non-life business is surely one of the main field where actuaries have mostly developed their techniques in insurance risk management. From 6 years, Solvency *II* is requiring strong skills for actuarial risk management in non-life insurers not traditionally necessary, even if adequately learned in every good actuarial science course, from advanced stochastic models to financial mathematics, data science and physical risk.

Nowadays, risk culture and knowledge of actuaries is going to be necessary in insurance business as well as in social sciences, where contrast of *Climate Change* would have to be the milestone of insurance market as well as government bodies.

In this process, the concept of insurance risk is fundamental in order to understand how it can be identified, analyzed and managed. Within every insurance risk management framework, reinsurance is the usual way to transfer risks that are over insurer's own capacity or large risks as those related to catastrophes.

Regards insurers' capacity, new solvency regime has disrupted insurance business and risk management as well as supervision. Risk-based prudential regime requires a deep knowledge of undertaking portfolio not only in a short term but also in a medium term view.

On the other hand, transfer of large risks would have by far a significant importance not only for insurers but also for governments, indifferently from rich to less developed countries, as a consequence of the well-known climate change.

Among european countries, Italian insurance market, and Italy in general for geographical reasons is one of the more exposed to natural disasters in terms of either frequency (exposure to most of natural risks and in many areas of the country) and either severity (where very often government refund significant economic and social losses of catastrophes, even if only partially, and increase in micro-cat).

Due to the characteristics of their business, insurance companies contribute to the implementation of adaptation strategies to climate change along two lines: providing businesses and households with protection from damage resulting from the materialization of physical risks; channeling, as long-term institutional investors, huge resources towards sustainable investments. In this way, they accelerate the achievement of the objectives of the Green Deal and thus help to mitigate transition risks.

There is undoubtedly room for a more important role for insurance companies in a strengthened framework of cooperation between the public and private sectors. In some European countries there are interesting experiences, variously articulated. Forms of public intervention could help redistribute the burden of insurance premiums to pursue solidarity objectives at the national level - for example by making the price of coverage highly mutual and undifferentiated across the country - or "reinsure" catastrophic damage. Many technical and political aspects, however, need to be carefully considered.

In Chapter 2 we briefly present basic aspects of actuarial techniques for insurance risks, focusing on non-life business, in lights of the new European regulation.

Classical underwriting risks models will be introduced and deeply illustrated. Furthermore, we provide a description of catastrophe risks in and out of Solvency II.

In Chapter 3 we introduce risk-mitigation techniques, form traditional reinsurance to alternative risk transfer, focusing on the most significant form for both premium and reserve risk. We also analyze the state of art of reinsurance for natural catastrophe, from traditional to alternative CAT coverages.

A complete treatment of insurance risk transfer is provided, suitable not only for actuaries but for every professionals involved in (re-)insurance business.

In Chapter 4 we summarize the risk-mitigation and profitability impact of reinsurance in the non-life underwriting risk model introduced, and a global framework for a feasible application of this model consistent with a medium-term analysis is provided. Numerical results are also figured out with evidence of various effects for several portfolios and reinsurance arrangements, pointing out the main reasons for these differences.

In Chapter 5 Catastrophe Bonds are deeply analyzed, as maybe the main alternative risk transfer instrument, with positive impacts for insurance market and governments in mitigating natural catastrophic. Specifically, we'll discuss financial as well as legal considerations regards CAT Bonds issuance.

In Chapter 6 methodological aspects of statistical learning are provided, either for linear and machine learning models.

This chapter provide a (short) technical guidance for pricing models used by actuaries, from classical linear models, eventually generalized, to machine learning ones. For the latter, interpretation of results will be analyzed, since possible *Black-box* usage is questioned in statistical world.

In Chapter 7 determinants of CAT Bonds pricing are investigated, comparing different statistical methodologies. Particularly, we'll use machine learning models, investigating explainability of statistical results.

Then, we'll use this model for pricing an Italian government CAT Bond, using data from internal property market, suggesting a systemic solution to the significant italian insurance protection GAP .

Chapter 2

Insurance risk management

2.1 Insurance risks in Solvency II

The new European solvency system (shortly Solvency II) has structured insurance solvency supervision according to a three-pillar approach as Basle II and for the first time an economic risk-based approach across all Member States has been introduced.

As stated in the Directive, Solvency Capital Requirement (shortly SCR) is calibrated in order to take into account all quantifiable risk to which insurance or reinsurance undertaking is exposed and its definition is based on a modular approach where risks they are compulsorily taken into account are classical insurance underwriting risks (Life, Non-Life and Health), market Risk, counterparty Default Risk and operational Risk.

New prudential regime is based on a prospective calculation to ensure accurate and timely intervention by supervisory authorities (the SCR), and a minimum level of security below which the amount of own funds should not fall (the Minimum Capital Requirement).

The SCR must be compared with undertakings' eligible basic Own Funds (OF), they are the available financial resources as the difference between assets and liabilities of the economic balance sheet imposed by the new regulation.

More specifically, Pillar I of Solvency II regime of insurance and reinsurance undertakings requires to "held an economic capital equal to an amount, the SCR, computed in order to ensure that ruin occurs no more often than one in every 200 cases over the following 12 months" (alternatively, a probability of default of 99.5% is considered). The European Parliament and the council of the European Union selected Value-at-Risk (VaR) of basic own fund of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period as measure of risk on basis of which SCR must be calculated.

In formula:

$$\mathrm{SCR}_t = VaR_{99.5\%}(\tilde{U}_{t+1} - U_t). \tag{2.1}$$

where $\tilde{U}_{t+1}$ represents Eligible Own Funds at the end of year t.

Obviously, when calculating the SCR, insurance and reinsurance undertakings takes account of any financial or insurance risk-mitigation techniques they have arranged (i.e., reinsurance contracts and special purpose vehicles or financial derivatives) provided that credit risk and other risk arising from the use of such techniques are properly reflected in the SCR calculation.

Solvency II, as a risk-based regulatory framework, aims to appropriate recognize all risk categories an insurance company faces and all the risk mitigation techniques that insurers use to reduce exposure and thereby lower their risk capital. This marked a departure from Solvency I and its limited recognition of risk mitigation techniques.

Pillar II allows insurance and reinsurance undertakings to calculate the SCR either in accordance with the (more general) Standard Formula (SF) provided by regulation, or using, partially or fully, an Internal Model (IM). The latter is an undertaking specific method in such a way that insurers must build a model based on the own undertaking risk profile, whereas SF is a standardized calculation method since it was calibrated by insurance regulation on market average basis. It is composed by factors reflective of the average size and performance of the portfolios of insurers in the European market.

Standard formula has been calibrated by EIOPA through the five Quantitative Impact Studies (QIS), carried on the whole EU insurance market on a voluntary basis by solo/group insurers.

The Directive describes requirements applying to insurance and reinsurance undertakings using, or willing to use, a full or partial internal model in the calculation of the SCR. Such models are subjected to prior supervisory approval on the basis of specific processes and standards, particularly regard the use test, statistical quality, calibration, validation and documentation. Actually, internal model development is one of the most important research areas in risk theory.

Finally, Pillar III has introduced some disclosure requirements that all European insurance companies have to provide to either market and supervisors. In particular, among others, they must report the Regular Supervisory Report (RSR) and the Own Risk and Solvency Assessment (ORSA) to the supervisors, incorporating some specific analysis of undertakings risk profile and a deep assessment of business strategy on the basis of company's industrial plan.

European Commission has adopted a Delegated Acts regarding implementing rules for the new solvency system, analytically providing new quantitative requirements and a proper methodology to evaluate them. Formula used by European Insurance and Occupational Pensions Agency (shortly EIOPA), as well as European National Competent Authorities (shortly NCA), in order to calculate the BasicSCR is given by the following equation:

$$\text{BasicSCR} = \sqrt{\sum_{i,j} Corr_{i,j} \cdot SCR_i \cdot SCR_j} + SCR_{intangibles} \tag{2.2}$$

where SCR_i represents risk module i and SCR_j represents risk module j, and where "*i,j*" means that the sum of the different terms should cover all possible combinations of i and j.

The factor $Corr_{i,j}$ denotes the item set out in row i and in column j of the correlation matrix given by EIOPA:

Table 2.1. Correlation matrix for BasicSCR Risk modules —Delegated Acts

module	market	default	life	non-life	health
market	1				
default	0,25	1			
life	0,25	0,25	1		
health	0,25	0,25	0,25	1	
non-life	0,25	0,5	0	0	1

As pointed out by Quantitative Impact Studies (QIS) exercise carried out from EIOPA during the decade before Solvency II definition and implementation, for non-life insurers the underwriting risk module has always the greatest impact on SCR. With regard to SF for non-life underwriting risk, delegated acts have established a unique sub-module for the joined valuation of risks related both to future claims arising during and after the period until the one-year time horizon for the solvency assessment (premium risk) and to a non-sufficient amount of technical provisions for old claims (reserve risk). The derived capital charge must be then aggregated to lapse and cat risk to quantify the capital requirement for non-life underwriting risk. Since the scope of this thesis focuses on reinsurance and CAT risks, hereafter only regulatory recall to Non-Life risk modules will be provided and commented.

For Non-Life Insurer the Underwriting Risk module has the greatest impact on SCR. It consists of three sub-modules:

- Premium&Reserve Risk
- Lapse Risk
- Cat Risk

Capital Requirement for Non-Life Underwriting Risk is given by the formula:

$$\mathrm{SCR}_{NL} = \sqrt{\sum_{i,j} CorrNL_{i,j} \cdot SCR_i \cdot SCR_j} \tag{2.3}$$

where the sum covers all possible combinations (i,j) of the sub-modules mentioned above. The term $CorrNL((i,j)$ denotes the correlation parameter for Non-Life Underwriting Risk for sub-modules i and j. Correlation between Premium&Reserve risk and CAT risk is equal to 25%, while lapse risk is uncorrelated with both:

Table 2.2. Correlation matrix for Non-Life Underwriting Risk sub-modules —Delegated Acts, Art. 114

sub-module	Premium&Reserve	CAT	Lapse
Premium&Reserve	1		
CAT	0,25	1	
Lapse	0	0	1

So a unique sub-module for the joined valuation of risks related both to future claims arising during and after the period until the one-year time horizon for the solvency assessment (Premium Risk) and to a non-sufficient amount of technical provisions for old claims (Reserve Risk) has been provided. For CAT risks, a unique sub-module is proposed.

In SCR evaluation, the risk mitigation effect of reinsurance is recognized. Reinsurance is defined as the transfer of insurance risk from one insurer to another through a contractual agreement under which one insurer (the reinsurer) agrees, in return for a reinsurance premium, to indemnify another insurer (the primary insurer or cedant) for some or all of the financial consequences of certain loss exposures covered by the primary insurer's policies.

Definitely, Solvency II came in force on 1 January 2016, and over the last decades introduced new issues for actuarial risk management in non-life insurance, so that also reinsurers design innovative solution and techniques in order to help insurance market facing those challenges. In this context, beyond traditional reinsurance for underwriting risks, Alternative Risk Transfer (ART) is becoming, specifically for natural disasters and CAT risks, a new significant actuarial and capital management tool for insurers.

2.2 (Pure) Underwriting risks

Focusing on for Premium&Reserve Risk under Delegated Acts SF, in order to later compare vary capital requirements evaluations, the SCR is equal to the following formula:

$$\mathrm{SCR}_t^{\mathrm{SF}} = 3 \cdot \sigma_{NL} \cdot V_{t,NL}, \tag{2.4}$$

where $V_{t,NL}$ is the volume measure (gross or net of reinsurance) that consists of the sum of the volume measures for premium and reserve risk of segments (LoB) involved in insurer's business eventually decreased for the geographical diversification and σ_{NL} is the standard deviation of ratio between aggregate claims amount of premium and reserve Risk and the volume measure, and then it is strictly related to the coefficient of variation (CoV) of their distributions.

Non-life insurance LoBs in Solvency II framework have been defined by Annex I of the delegated acts in a similar way as those already in force in European annual accounts legislation. For our purposes LoBs of interest, in according with EIOPA's labeling, Non-Life segments are defined by:

- 1) Motor Vehicle Liability insurance (MVL): obligations which cover all liabilities arising out of the use of motor vehicles operating on land;
- 2) Other Motor insurance (OM): obligations which cover all damage to or loss of land vehicles;
- 3) Marine, Aviation and Transport (MAT);
- 4) Property (Prop);
- 5) General Liability insurance (GL): obligations which cover all liabilities other than those in the LoB 1;
- 6) Credit and suretyship (CS);
- 7) Legal expenses (LE);
- 8) Assistance (AS);
- 9) Miscellaneous financial losses (MFL);
- 10) Non-Proportional reinsurance property (NP Reins P);
- 11) Non-Proportional reinsurance casualty (NP Reins C);
- 12) Non-Proportional reinsurance MAT (NP Reins MAT).

For a generic segment s, Volume measure is defined as:

$$\mathrm{V}_{t,s} = (V_{prem,s} + V_{res,s}) \cdot (0,75 + 0,25 \cdot DIV_s) \tag{2.5}$$

The standard deviation for premium and reserve risk is given by the aggregation of the L different LoBs $\mathbf{s}$=$[\sigma_1, ..., \sigma_h, ..., \sigma_L]$ using a fixed correlation matrix $\mathbf{R}$:

$$\sigma_{NL} = \frac{\sqrt{\mathbf{sRs'}}}{V_{t,NL}}. \tag{2.6}$$

Correlation parameters for two different segments are given by the correlation matrix set out in Annex IV of delegated acts.

The standard deviation for premium and reserve risk of a specific LoB s, under assumption of 0,5 correlation, is given by:

$$\sigma_s = \frac{\sqrt{\sigma^2_{s,prem} \cdot V_{s,Pre} + \sigma_{s,prem} \cdot V_{s,Pre} \cdot \sigma_{s,res} \cdot V_{s,res} + \sigma^2_{s,res} \cdot V_{s,res}}}{V_{s,Pre} + V_{s,res}} \tag{2.7}$$

Avoiding to reply similar formulas for reserve risk, we will only refer to premium risk, highlighting when and how those change in case of reserve risk. Formula (2.1) in case of premium risk only can be written as:

$$\text{SCR}^{\text{SF}}_{t,Pre} = 3 \cdot \sigma_{Pre} \cdot V_{t,Pre}. \tag{2.8}$$

Non-life insurance segmentation between LoBs and their standard deviation have been set out by delegated acts in Annexes I and II respectively. Actually, formula (2.8) assumes to measure the distance between the 99,5% quantile and the mean of the probability distribution of aggregate claims amount by using a fixed multiplier of the standard deviation equal to 3. It is worth mentioning that, under the assumption of a Log-Normal distribution for the aggregate claims cost, this multiplier holds only when σ_{Pre} is roughly 14.47%. It turns to overestimate capital requirement for big insurers they usually have smaller volatility coefficients.

The standard deviation for premium or reserve risk is given by the aggregation of the L different LoBs $\mathbf{s}$=$[\sigma_1, ..., \sigma_h, ..., \sigma_L]$ using a fixed correlation matrix $\mathbf{R}$:

$$\sigma_{Pre} = \frac{\sqrt{\mathbf{sRs'}}}{V_{t,Pre}}. \tag{2.9}$$

The net standard deviation of the h-th LoB in vector $\mathbf{s}$ of Equation (2.9) according to Solvency II is given by:

$$\sigma_h = \sigma_{Pre,h} \cdot NP_{\text{LoB}}, \tag{2.10}$$

where gross volatility factors are showed in the next Table 2.3:

Table 2.3. Premium and Reserve Risk Volatility factors for Lines of Business (LoBs)—Delegated Acts, Annex II.

LoB	σ_{prem}	σ_{res}
MVL	10%	9%
OM	8%	8%
MAT	15%	11%
Prop	8%	10%
GL	14%	11%
CS	19%	17,2%
LE	8,3%	5,5%
AS	6,4%	22%
MFL	13%	20%
NP Reins P	17%	20%
NP Reins C	17%	20%
NP Reins MAT	17%	20%

Formula (2.10) shows how values of gross standard deviation, only for premium risk in Table 2.3, can be multiplied by a proportional factor in order to take into account risk mitigation effect arising from existing particular XL treaties. The calculation of the premium risk SCR in non-life underwriting risk module of the SF is based on the principle of a correction factor for the disproportionate risk reduction arising out of non-proportional reinsurance treaties they shall be considered recognizable.

Notwithstanding, SF provides a factor model based on predefined standard deviations for each class of insurance and the premium volume of each LoB, a correction factor is needed to reduce the gross standard deviation to a level commensurate with the risk in case insurance undertaking has provided a suitable non-proportional treaty for its lines of business. The basic idea behind this approach is to calculate an adjustment factor which is designed to consider non-proportional reinsurance risk-absorbing effect. The non-life underwriting risk module of the SF is based on volume measures such as premiums and reserves and uses predefined standard deviations per LoB to calculate the SCR. In order to appropriately capture the risk-mitigating effect of non-proportional reinsurance, the adjustment factor is designed to lower the standard deviation. Delegated acts has set this factor out at 80% for Property, Motor Vehicle Liabilities (MVL), and General Liabilities (GL) and 100% for all other LoBs.

Following Solvency II directive, volume measure is equal to the maximum between last year and next year earned premiums plus the expected present value of future premiums after one-year for existing contracts and contracts of the following year. Delegated acts SF allows undertakings to take into account a geographical diversification of business held in different macro-geographical regions of the world, through the Herfindahl Index but it will not be taken into account in the model since it is assumed that standard insurer is representative of the Italian insurance market only. The volume measure for this risk component is based on the expected premiums earned and written during the following twelve months in order to consider new business, assuming a dynamic portfolio. Then gross premium volume which is relevant for risk capital valuation will be equal to:

$$V_{Pre,h} = \max[B_{t+1}; B_t], \tag{2.11}$$

while for the net volumes this value would be multiplied by a gross-to-net premium adjustment factor B^{NET}/B.

The net reserve volume which is relevant for risk capital valuation will be equal to:

$$V_{Res,h} = BEL_t - Recoverables_t, \tag{2.12}$$

where the net volume would be the Best Estimate of Liabilities net of Recoverables from reinsurance contracts.

According to the VaR risk measure at Solvency II confidence level in a one-year time horizon, the capital requirement (SCR) for premium and reserve risk could be derived through a simplified internal model, referring to the time horizon $[t, t+1]$, as:

$$\text{SCR}_t = VaR_{99.5\%}(\tilde{U}_{t+1} - U_t) = -Quantile_{0.5\%}(\tilde{Y}_{t+1}^{NET}). \tag{2.13}$$

Formula (2.13) is recognizing expected profit/losses in the capital requirement evaluation by considering safety loadings. This approach would be not conservative if a negative safety loadings were in force and a negative technical result would be expected, implying a consequent higher risk profile.

2.3 Natural catastrophes

EIOPA has designed and implemented a regulatory framework also in the complex field of natural catastrophes risk.

There is little "*natural*" about catastrophes, where it refers to the underlying peril, such as extremes of temperature, precipitation or wind, although even here the impact of humankind on climate is making an increasing contribution. Yet the impact of a catastrophe is ultimately determined by how exposed people and economic activity are to the peril, their vulnerability and which actions are taken beforehand and afterwards to mitigate the impact.

Natural catastrophes are substantially man-made, and their impact assessment can only be adequately processed by considering exposure and mitigating actions taken to bolster resilience. To better understand how insurance can help mitigate the impact of catastrophes, it is useful to first analyze how catastrophes affect the economy.

When natural disasters happen, they damage capital, crops, livestock, lives and livelihoods. This calamities reduces either productive and financial capacity, dependent on the type of natural peril (continued physical disruption until floodwaters recede) as well as economic disruption through supply chains and damaged infrastructure that can increase the initial perimeter of impact. Real cases include the March 2011 earthquake and tsunami in Japan that affected automobile production nationwide, the 2018 drought in Germany where low river levels disrupted transport of oil and other commodities, and the current pandemic.

After the initial phase of the disaster, a period of recovery can usually be observed, driven by the decrease of disruption effects and eventually by reconstruction, which can take years to complete. In short, the overall economic impact of catastrophes extends beyond the initial direct damage (often described as “economic damage” in the insurance literature), but include also business interruption as several secondary perils. The lost output in the months and years before full reconstruction, assuming it occurs, can far exceed the value of the initial direct damage.

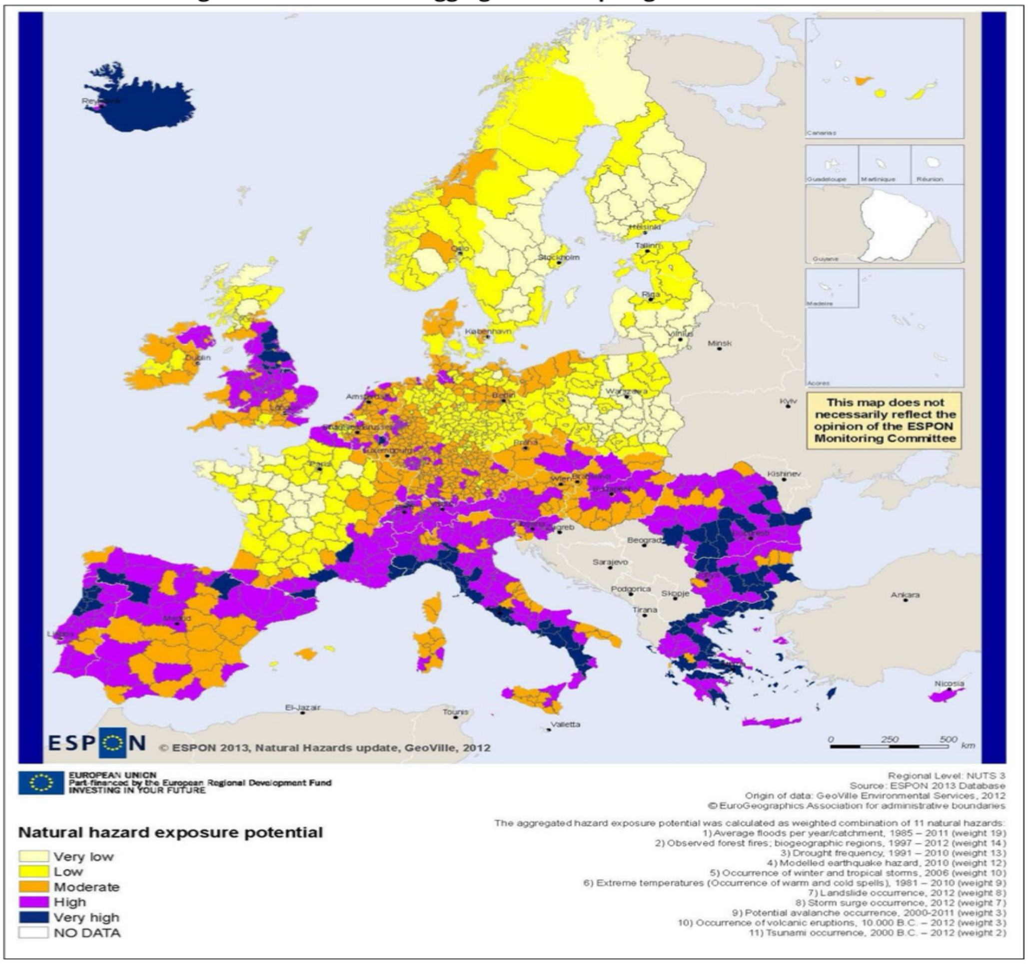
Global economic losses from natural disasters , either natural and man-made, were USD 202 billion last year, increasing from USD 150 billion of 2019 (see Swiss RE 2021). The magnitude of those losses continue to escalate from the 1970 to 2020, with an average annual growth rate of GDP-normalised losses of 1.3%, and a last 10-year average of direct losses of USD 222 billion. In accordance insured losses follow a similar trend, exceeding USD 89 billions last year from USD 69 billion of 2019, showing a 10-year average of USD 79 billion.

Hence, in recent years many reports that losses are out of control, urging public and private sector to reduce risk.

Italy is the fourth European country in terms of economy and population. A large part of its territory is exposed to some form of natural risk (earthquakes, floods, volcanic eruptions, landslides, etc.). These risks are widespread throughout the territory, with the exception of the volcanic risk, concentrated in the provinces of Naples (eruptive risks due to Campi Flegrei and Vesuvius), Catania (Etna) and the islands of Stromboli and Vulcano (see https://ingvvulcani.com).

The risks are high or very high for most of the provinces (Fig. 6.1), to a significantly greater extent than Spain and to a similar extent to Greece and Eastern European countries (see https://www.espon.eu/climate-2013).

Figura 3.1 – *Hazard risk* aggregato di 11 tipologie di rischi naturali



Fonte: Espon (2013).

Figure 2.1. Natural hazard exposures in Europe

Catastrophes represent significant financial hazards to an insurer, including the risk of insolvency, an immediate reduction in earnings and statutory surplus, the possibility of forced asset liquidation to meet cash needs, and the risk of a ratings downgrade.

Insurance of catastrophe risks has some characteristics and peculiarities that differentiates it significantly from the pure underwriting risks, making it more challenging to copy with and more prone to failure. In particular:

1. individual claims are correlated and insurers have to pay more clients at once, producing a liquidity pressure;

2. in the catastrophe insurance market losses are usually characterized by high peaks that relate to low-frequency high severity risks (LF-HS);

3. the correct prediction of loss probabilities of these low-frequency, high severity risks is a difficult task and require a different approach compared to the other high frequency, low severity risks (HF-LS) in the industry (e.g. car accidents).

Standard Formula and Internal models

As already mentioned at the beginning of this chapter, Solvency II takes into account Catastrophe Risk in calculation of the Solvency Capital Requirement. The SCR is calibrated using the VaR of the basic own funds of an insurance or reinsurance undertaking, subject to a confidence level of 99.5% over a one-year period.

This calibration objective is applied to each individual risk module and sub-module in Cat field. Standard Formula provides the following calculation method:

$$\mathrm{SCR}^{\mathrm{SF}}_{nlCAT} = \sqrt{(\mathrm{SCR}_{natCAT} + \mathrm{SCR}_{npproperty})^2 + \mathrm{SCR}^2_{mmCAT} + \mathrm{SCR}^2_{CATother}} \tag{2.14}$$

In this formula, four CAT risk sub-modules have been identified which in turn have several sub-risks (or perils):

- **Natural Catastrophe**: earthquake, windstorm, flood, hail and subsidence;
- **Catastrophe for Non-proportional Property reinsurance**, related to reinsurance business from non-proportional segment 28 of Annex I;
- **Man-made Catastrophe**: motor, fire, marine, aviation, liability, creditsuretyship;
- **Other Non-life Catastrophe**, for groups of obligations defined in Annex XII of Delegated Acts .

The Standard Formula, for NatCat risks provides tables showing, by peril, the gross loss damage ratio (Qcountry) for 1-in-200 year catastrophe events, within each CRESTA zone separated by country. The capital requirement for each CRESTA zone and each peril gross of reinsurance is Qcountry times the aggregated value of geographically weighted total insured value by peril for each country, where the weights are the zone relativity factors for each country provided by the Standard Formula.

In Solvency II, insurance companies have two choices for CAT risk. Either use a standard formula or run an internal model. Smaller companies that do not have the capacity/ability to run an internal model use the standard formula that usually requires higher capital requirements. On the other hand, an internal model requires the use of specialized staff. As a result, bigger companies tend to perform an internal model.

During ORSA through a risk profile they demonstrate that their model produces different results than the Standard Formula, so that they can justify the use of internal model. In most cases it is partial internal model since a major part goes outsourcing to vendors, but Solvency II requires that the company staff is well aware of its model, data input and each module.

Different CAT models in the same area and portfolio can provide significantly different results. In such a case, a conflict of interest can be created by favoring the model that leads towards the lowest capital requirements. Such a model could be more attractive to insurers. As a result, a good evaluation of the model is required.

Therefore, improving ability of the supervisory authorities to monitor the compliance to the Solvency II requirements is crucial and regulators need to understand and evaluate CAT models and their applications. Several gaps and weaknesses have been identified in the current regulation of Catastrophe risk. In particular:

- Lack of transparency regarding the calculation of the standard formula since there is no report that supports existing country risk factors;
- There could be a conflict of interest if some or all of the Q country factors have been proposed by CAT Risk companies, the insurance industry and/or vendors;
- Robustness of results and credibility concerns (e.g. some of the country factors are scientifically unjustifiable, either too high or too low, whereas some risks or processes are ignored);
- Aggregate country level exposure data are inadequate to properly reflect the high spatial and temporal variability in natural catastrophe risk;
- Inconsistencies and gaps in the peril correlation matrix. For example, subsidence and earthquakes not interrelated and independent to each other even if subsidence is one of the main earthquake environmental effects. In addition, subsidence phenomena can be triggered by weak or distant earthquakes and is regarded as a peril only in France even if several other countries that suffer from subsidence phenomena;
- Terminology issues can cause problems in lack of international CAT risks glossary;
- No specific regulations have been traced regarding the agricultural sector that would require a different approach.

Designing and implementing a regulatory framework in the complex field of CAT Risk is a lengthy and difficult process that requires a variability of expert advisers.

In addition, since scientific input and advances in catastrophe risk are rapidly evolving, regulators and insurers need to realise that modifications and updates will be a common process, implying that they need to incorporate these to their future plans. On the other hand, a common catastrophe culture is crucial to rely on people's risk consciousness.

Two landmark events in the 90ies (1992 Hurricane Andrew and 1994 Northridge earthquake California) that were unexpectedly destructive, firstly emphasized the importance of CAT risk modeling. Since then, CAT models have been advanced and now widely used, forming a tool of significant importance since:

- the quality of the model might determine the survival of the insurer, when a catastrophe occurs, by defining the pricing of risk;
- justify the Solvency Capital Requirements by running an internal CAT model;
- NatCat risk is a rating factor on corporate credit quality (e.g. S&P uses exposure and treatment of Natcat risk in downgrading many companies since 2005);
- assist decision making in risk diversification,
- suggest whether a transfer of risk (e.g. reinsurance) is required;
- set the policy conditions(e.g.deductibles);
- guide portfolio optimization (by determining the size and distribution of potential losses);
- controls the pricing of catastrophe bond market that is an emerging market;
- has a significant role in organizing the contingency plans immediately after the catastrophe.

CAT models

CAT Risk models lie outside “the traditional actuarial domain” and are more difficult to comprehend and evaluate both by insurers and regulators. Their construction relies heavily on the expertise of certain scientific disciplines (geology, meteorology, civil engineering), beyond statistics and actuarial analysis.

These models require a special treatment from insurers and regulators some of which have to be specialists, so as to monitor their compliance with Solvency II obligations and offer an adequate evaluation/supervision of the internal models. Currently the ability of the regulators to sufficiently supervise and evaluate internal cat models is questioned.

It is worth to notice that CAT models should be transparent, informing also the public in line with the proposals of the European Parliament.

Focusing on CAT models structure and evaluation, they usually have four main components:

1. the hazard module (location, frequency and severity of events);
2. the exposure module;
3. vulnerability/risk module (damage function/curves);
4. the financial module (loss).

The final output is the so-called *Exceedance Probability* curve that communicates the probability of any given financial loss being exceeded. CAT models are simplifications of complex natural processes and are highly dependent on data input. Therefore for their evaluation, it is crucial to evaluate the data input; the assumptions used and obtain a good understanding of the implied uncertainties. Completeness of data input and high spatial resolution are of major importance for assessing the risk.

However, completeness of data input for low frequency high severity events (e.g. earthquakes) is a major challenge and can not be dealt with the use of the historical records because they are too recent. The commonest constraint for building a reliable model is the lack of accurate and complete historical information about catastrophic events.

- If the data window is too short to identify a rare event, then missing an extreme loss will result in a much lower estimate of the average annual loss;
- On the other hand, if this narrow window happened to have recorded the extreme event, then the annual loss will be overestimated.

Recent scientific advances, can in several cases, eliminate the incompleteness problem by extending the history back in time and should be incorporated to modern CAT models. More specifically, simulations can be used in order to increase data available and machine learning models can improve accuracy of methods applied in CAT modelling.

2.4 Counterparty default risk

We have already mentioned the changes that Solvency II has introduced. The amount of capital regulatory system require insurers to hold strongly depend on the amount and quality of the risks they have assumed, and they included the risk of the reinsurer defaulting.

Under Solvency II, the Counterparty Default Risk on a reinsurer is the risk of a reinsurance partner no longer been able to meet its payment obligation in full and its calculation introduces credit rating provided that the reinsurer has a credit rating issued by a rating agency. The 2008 financial crisis has demonstrated that credit risk has been generally underestimated in the past.

As already mentioned, European Insurance regulation has recently provided a suitable formula in order to take into account reinsurance as a risk-mitigation technique in new Insurance solvency capital requirements established in Delegated Acts.

By using reinsurance, the SCR for the Underwriting Risk is reduced because part of the cedent's underwriting risk is transferred to the reinsurer. In the Solvency II standard model, reinsurance can have a risk-reducing impact on all of the underwriting risk modules.

Through Standard Formula Premium and Reserve Risk capital requirements are calculated based on a factor-based approach. Formula-based calculations allow capturing risks associated with new business expected to be written in the following 12 months, but to take the effect of risk mitigation techniques into account is more difficult.

Complex relationship between different risks could also give rise to dependencies in the risk profile. The circumstances that cause increased insurance losses, and therefore an increase in reinsurance recoveries, could in turn have a negative effect on the creditworthiness of the reinsurer. On the other hand, reinsurance transaction also adds risk, within the Counterparty Default Risk module. As it impacts the total effect on the capital requirement, a reinsurer's financial strength should be a key consideration.

Looking beyond the Solvency Capital requirement, reinsurance can also have positive effect on a cedent's own funds. Solvency II establishes an economic balance sheet that determines the value of own funds by subtracting the best estimate of liabilities, plus a risk margin, from the market value of an insurer's assets.

One of the main purposes of reinsurance is to reduce volatility of liabilities and this in turn leads to a smaller risk margin and an increase in the cedent's own funds as a result.

However, such complex relationships between Non-Life Underwriting Risk and Counterparty Default Risk or Market Risks have not been considered in the Premium and Reserve Risk module. They are implicitly taken up in the correlation parameters between the risk modules. Indeed, reinsurance contracts impact both on Non-Life Underwriting Risk (PremiumReserve) and on Counterparty Default risk.

For the former, Standard Formula says insurers must use premium and reserve volumes net of reinsurance contracts in order to compute the whole volume measure for non-life underwriting risk, the standard deviation of the Premium and Reserve Risk might be computed net of reinsurance when an undertaking-specific approach is used, whereas an adjustment factor for non-proportional reinsurance is introduced for the (gross) volatility coefficient of the Premium Risk.

As regard the latter, insurers must consider Counterparty Default Risk on those reinsurance contracts, defined as type 1 exposures, they respects some simple rules and consequently Companies must value the risk-mitigation effects of those contracts on solvency capital requirements in order to compute the SCR related to the reinsurer's default risk.

The Directive has defined Counterparty Default Risk module as to reflect "*the possible losses due to unexpected default, or deterioration in the credit rating, of the counterparties and debtors of the insurance or reinsurance undertaking over the following 12 months*".

It shall cover risk-mitigating contracts, such as reinsurance arrangements, securitisations and derivatives, and receivables from intermediaries, as well as any other credit exposures which are not covered in the spread risk sub-module.

Normally Non-Life insurer's Counterparty Default Risk represent moreover credit from the reinsurance arrangements it has provided for insurance portfolio and such insurers have smaller exposures with intermediaries or derivatives than Life insurers since for the former policies are usually arranged on annual basis and market risks on claims reserving are smaller than risks on investments of mathematical reserves.

Delegated Acts provide definitive method and assumptions to be used to assess the changes in the risk profile of the undertaking concerned and to adjust the calculation of the Solvency Capital Requirement in case insurance undertaking uses risk-mitigation techniques they fulfil qualitative criteria. These criteria have been introduced in order to ensure that risk has been effectively transferred to a third party.

As regard Counterparty Default Risk, it is given by the following formula:

$$\mathrm{SCR}_{def}^{\mathrm{SF}} = \sqrt{\mathrm{SCR}_{def,1}^2 + 1,5 \cdot \mathrm{SCR}_{def,1} \cdot \mathrm{SCR}_{def,2} + \mathrm{SCR}_{def,2}^2} \tag{2.15}$$

It is worth to emphasize how insurers has to apply a differentiation of two kinds of exposures, denoted by type 1 and type 2 exposures, and a different treatment according to their characteristics shall be used.

The class of type 1 exposures covers the exposures which may not be diversified and where counterparty is likely to be unrated (e.g. risk-mitigation contracts, cash at bank) whereas type 2 exposures are usually diversified and the counterparty is likely to be unrated (e.g. receivables from intermediaries, policy holder debtors, mortgage loans).

The capital requirements for type 1 and type 2 exposures shall be calculated separately and a low diversification (correlation coefficient of 0,75) has been allowed in their aggregation.

The main inputs of the calculation of $SCRdef, 12$ are the estimated loss-given-default (LGD) of an exposure and the probability of default of the counterparty (PD).

LGD

To calculate the market value of the loss given default, different recovery rates are used for the various risk-sharing instruments. “The recovery rates take account of the fact that, if the counterparty in a risk-sharing instrument defaults, a portion of the insurance obligations will still be satisfied.”

For Reinsurance, it is assumed that even following a default 50% of the LGD will be paid, so that only a half of the amount is included in the calculation of risk capital:

$$\text{LGD} = max[50\% \cdot (Recoverables + 50\% \cdot RM_{re}) - F \cdot Collateral; 0] \tag{2.16}$$

where:

- $Recoverables$ denotes the Best Estimate of amounts recoverable from reinsurance arrangements;
- RM_{re} denotes the risk mitigating effect on underwriting risk of the reinsurance arrangement;
- $Collateral$ denotes the risk-adjusted value of collateral;
- F denotes a factor to take into account the economic effect of collateral arrangement.

The market value of a loss on default of a reinsurer can so be derived from the relevant reinsurance recoverables. The amount of the reinsurance recoverables depends on the type of reinsurance cover. They are generally higher for proportional reinsurance contracts, such as Quota Share treaties. For non-proportional treaties, such as Stop-Loss or XL reinsurance, the reinsurance recoverables are usually lower than for proportional treaties that have the same impact on risk capital.

The reinsurance recoverables should be increased by the amount of the risk-mitigation effect no longer achieved due to a default, which equates to the difference between the hypothetical (gross) and the actual (net) capital requirement produced by the underwriting risk module. The hypothetical capital requirement is the capital that would be required according to the relevant risk module where no risk-sharing instruments are in place ($SCRgross$).

The actual net capital requirement takes the effect of risk-sharing instruments into account ($SCRnet$). For non-life reinsurance, a simplified method for this calculation can be applied in case reinsurance treaties with a unique counterparty affect only one non-life line of business. The risk-mitigating effect on underwriting risk is given by the difference between $SCRgross$ and $SCRnet$.

PD

The probability of default must also be established for the calculation of the capital requirement on single name exposure (rating class) basis. It shall be equal to the average of the probabilities of default on each of the exposures to counterparties that belong to the single name exposure, weighted by the loss-given-default in respect of those exposures.

Probability of default of single name exposure i for which a credit assessment by a nominated ECAI is available is given by the following Table:

Table 2.4. Probability of default in function of rating — Art. 199 comma 2.

Credit quality step	**0**	**1**	**2**	**3**	**4**	**5**	**6**
PD_i	0,002%	0,01%	0,05%	0,24%	1,20%	4,2%	4,2%

When a credit assessment by a nominated ECAI is not available for single name exposure to an insurance undertaking and where this undertaking meets its Minimum Capital requirement, probability of default depending on its solvency ratio, according with the solvency ratio:

Table 2.5. Probability of default in function of solvency ratio — Art. 199 comma 3.

Solvency ratio	**196%**	**175%**	**150%**	**125%**	**122%**	**100%**	**95%**	**75%**
PD_i	0,01%	0,05%	0,1%	0,2%	0,24%	0,5%	1,2%	4,2%

SCR for counterparty default risk

Given probability of default and loss-given-default of the counterparties in the portfolio of type 1 exposures, the capital requirement for type 1 exposures is calculated as follows:

$$\text{SCR}^2_{def,1} = \begin{cases} 3 \cdot \sigma & \frac{\sigma}{TLGD} <= 7\% \\ 5 \cdot \sigma & 7\% <= \frac{\sigma}{TLGD} <= 20\% \\ TLGD & \text{otherwise} \end{cases} . \tag{2.17}$$

where the sum is taken over all independent counterparties with type 1 exposures and σ is the standard deviation of the loss distribution of the type 1 exposure.

It given by the following formula:

$$\sigma = \sqrt{V} = \sqrt{V_{inter} + V_{intra}} \tag{2.18}$$

where V is the variance of the loss distribution of type 1 exposures and it is equal to the sum of:

$$V_{inter} = \sum_{j,k} \frac{PD_k \cdot (1 - PD_k) \cdot PD_j \cdot (1 - PD_j)}{1,25 \cdot (PD_k + PD_j) - PD_k \cdot PD_j} \cdot TLGD_j \cdot TLGD_k \tag{2.19}$$

where the sum covers all possible combinations (j,k) of different probabilities of default on single name exposures and $TLGD_i$ denotes the sum of loss-given-default on type 1 exposures from counterparties hearing probability of default PD_i.

and

$$V_{intra} = \sum_j \frac{1,5 \cdot PD_j \cdot (1 - PD_j)}{2,5 - PD_j} \cdot \sum_{PD_j} LGD_j \tag{2.20}$$

where the first sum covers all different probabilities of default on single name exposures whereas the second sum covers all single name exposures that have a probability of default equal to PD_j and it is applied on the loss-given-default on the single name exposure j $TLGD_j$.

As summarized above, introduction of reinsurance arrangements let insurers obviously face another source of risk, the Counterparty Default risk. In Reinsurance it represents the risk of default of reinsurers and consequently the risk that reinsurance Recoverables cannot be used by Insurers to pay their claims. In according with Delegated acts, Reinsurance turns to reduce non-life underwriting risk on contracts on which reinsurance treaty is applied, but this new source of risk has to be opportunely be taken into account.

All these news have a great impact on any insurer's risk management. Indeed, key issues of Solvency II include preparation for the ORSA process (Own Risk and Solvency Assessment) and hence the production of a risk strategy, which must incorporate considerations on risk-sharing instruments that the company might use in the future. The default probabilities for such instruments will play an important role, as they will determine the risk capital requirement under the Standard Formula (Counterparty Default Risk).

The effect of changes in business partners' creditworthiness on a company's own risk situation will also have to be assessed. Under Solvency II, the choice of risk-sharing instruments and financial strength of the business partner will have a considerable effect on the capital requirement. The rating determines the default probabilities, which is the prime factor in determining both the risk capital used in the SCR calculation and the adjustment factor for the reinsurance recoverables on the asset side of the Economic solvency Balance Sheet. Thus, a lower rating increases the adjustment factor, reducing the assets, whilst the risk margin on the liabilities side rises, leading to a fall in the economic capital.

On the other hand, other factors must be considered when measuring the Counterparty Default Risk for reinsurance receivables. Apart from financial strength, the number of reinsurance partners will be an important factor in the measurement of the capital requirement. Risk management can help make a company less vulnerable to losses resulting from reinsurer default by diversifying its risk on reinsurers.

However, it cannot be assumed that concentrating reinsurance on a single reinsurer with a good rating will result on a higher risk than spreading it across a number of reinsurers with worse ratings. Even if companies do not group reinsurers together and the diversification effect resulting from the use of reinsurers is taken into account, the Counterparty Default Risk on financially less robust companies is fundamentally higher and not necessarily compensated by the risk relief provided by diversification. Thus, spreading the risk across several reinsurers is not always beneficial, especially if the reinsures concerned have a variety of ratings.

At least on yearly basis rating agencies (See for example Fitch's *Global Reinsurance Guide* available online), publish an overview on worldwide Reinsurance industry, showing reinsurers' key variables they lead to this sector as well as rating outlooks and prices behaviour.

On the other hand, EIOPA is continuously examining how much reliance can be placed on information from rating agency. It is questionable, for example, whether the methods used by rating agencies are appropriate and sufficiently transparent and the speed at which they react to changes in creditworthiness is perceived as a problem. Whether rating information is sufficiently independent of companies and hence uninfluenced by conflict of interest is also open to doubt.

Chapter 3

Transfer of insurance risks

3.1 Basics of risk transfer

Focusing on actuarial aspects of reinsurance, any insurer usually takes out reinsurance cover for his insurance portfolio in order to keep the variation of the aggregate claim amount reasonable (i.e. he protects himself against losses arising from large, excessively numerous or catastrophic claims by reinsuring large claim amount or high claim frequency with one or more other insurance or reinsurance companies).

It's up to top management to determine the insurer's net retention that is suitable for the corporate planning. Fixing the insurer's limit of retained liability is probably the most important factor in planning the development of any underwriting portfolio. This limit of liability is known as the retention, and may be expressed either as a monetary amount or as a percentage share (usually linked with a monetary amount). An insurer must limit his liabilities in order to protect his assets, mostly his capital and reserves already established, which enable him to trade.

The actual profit achieved will turn on the level and form of retention. The retention may apply to a single risk or to a series of risks, or to a single loss or a series of losses. Retentions will usually be fixed on each line of business separately, with classes subjected to further sub-division, but there may be in addition an overall retention over the combined portfolios of the various classes. Factors which influence the retention are:

- Insurers' assets: the capital provided by shareholders and the solvency capital requirement;
- Size of portfolio: the larger the number of homogeneous risks in a portfolio, the lower is the probability of fluctuations in the expected claims experience, known that the larger the insurer's portfolio, the larger the retention which can be safely carried;
- Types and spread of risks: lines of business involved, technical basis, ect;
- Pattern of losses: the place, over the probability distribution, where the single losses picked up from the portfolio stay.

Generally reinsurance contracts are arranged by LoB, or for group of risks in the same LoB, since every LoB has its optimal reinsurance arrangements. Reinsurance business can be arranged by a treaty, on portfolio basis; this means that each separate claim of a LoB is divided between the cedant and the reinsurer. For specific and/or large risks reinsurance is usually arranged by a facultative coverage where both insurer and reinsurer has the faculty to regret the activation of the reinsurance contract.

Firstly we should consider each class separately; if the reinsurance for all classes is arranged on a claim by claim basis, it may be possible to construct a weighted claim size distribution net of reinsurance for the whole portfolio, using the additivity of compound variables, provided that assumptions required for additivity are met. On the other hand, a drawback of the claim by claim reinsurance arrangements is that they do not necessarily restrict the impact of exceptionally large values of the aggregate claim amount in all cases; “protection may be poor in situations where is large as a result of a high number of claims, rather than because of one or more very large claims”.

It is worth to emphasize that there is interconnectedness of insurers and reinsurers arising through reinsurance, and it could add systemic risk in the insurance industry. Theoretically, the default of one or a few names that are subsidiary to the industry could result in a chain of reinsurance or insurance company defaults. Under Financial Stability Board (FSB) point of view, exposure to natural catastrophe risk could be a potential source of systemic risk since reinsurers accumulate this risk for insurance industry. The timing and the size of catastrophe losses are, by nature, unpredictable and could cause balance-sheet shocks for both insurers and reinsurers. A big catastrophe can erode the capital base of a reinsurer, leading to default. Companies that have a large exposure to catastrophe risk relative to their capital bases will face it as their main risk.

Indeed, many reinsurers purchase retrocession protection, using this practise to limit the amount they would have to pay out after a major catastrophe event, but it introduces credit risk into the buyer’s equation. About 30% of reinsurers’ uses colletaral, whereby claim payments are made from the collateral account, even if the provider is in default, decreasing reinsurers’ credit risk.

FSB, contrary to expectations, has not added any reinsurance industry’s major players as globally systemically important insurers (G-SIIs) in their various updates; the designation would have involved more oversight, new capital requirements and resolution plans. The Board, together with the International Association of Insurance Supervisors (IAIS), have announced the development of a global insurance capital standard (ICS) similar to that for bank under Basel III, and its first application will be as a baseline for the capital loadings the G-SII regime requires.

Synthetically, an insurer takes out a reinsurance cover for his insurance portfolio in order to increase his underwriting capacity decreasing the variation of the aggregate claim amount and consequently his probability of ruin, in other words they adopt insurance risk-mitigation techniques in order to decrease its risk of insolvency.

On the other hand, reinsurance is increasingly becoming a risk-financing technique, alternative to subordinated bonds issuance, in order to have a positive effect on undertakings solvency position.

All these practises have been opportunely taken into account by new solvency system, especially regard reinsurance arrangements, which recognition has been opportunely regulated by Delegated Acts.

A technical guidance for the recognition of reinsurance as risk-mitigation technique under the Solvency II Standard Formula is provided by the following principles:

- *Economic effect takes precedence over legal form*: Risk-mitigation techniques should be recognised and handled consistently, regardless of their legal form or accounting treatment. Practically, if there is no or very little transfer of risk in a transaction it is recognised under accounting rules (eg IFRS), the transaction would not be considered as reducing risk and therefore offers no or very little capital relief. Additionally, insurers must identify and take into account all additionally and potentially new risks that arise from a reinsurance transaction. The main effect of this principle is to protect policyholders and ensure risks are valued on a real economical basis;

- *Legal certainty, effectiveness and enforceability*: The risk-transfer provided by the reinsurance arrangements must be clearly defined, legally effective and enforceable on an on-going basis and in all relevant jurisdictions. More practically, the reinsurance contract should put in evidence the scope of the cover and it must be enforceable. In order to protect the policyholder and ensure that the economic effect of the transfer of the risk to the reinsurer cannot be disputed, there must be a clearly defined and documented risk-transfer;

- *Liquidity and certainty of value*: The risk-transfer provided by the reinsurance contract shall be valued due to sound economic principles and at the real market value of assets and liabilities. The main effect of this principle is to ensure that the reinsurance arrangements is properly valued and will provide the required cash flow if triggered.

- *Credit quality of the provider of risk mitigation*: Reinsurer must meet the target solvency ratio of 100% and have at least a BBB credit rating, in order to ensure an insurer is buying reinsurance from a creditworthy party. Cedant must consider that the rating of the individual reinsurers marries more weight than having a more diversified panel of reinsurers;

- *Direct, explicit, irrevocable and unconditional features* It is exclusively connected with financial risk mitigation techniques and outlines when they can be used to reduce capital requirements. In other words, the buyer of reinsurance must have a direct claim on the reinsurer, the extent of cover must be clearly defined and must not contain clauses that are outside the control of the cedant.

Furthermore, on July 2021 EIOPA published an an Opinion on the use of risk mitigation techniques by insurance undertakings (`https://www.eiopa.europa.eu/media/news/opinion-use-of-risk-mitigation-techniques-insurance-undertakings_en`.

Since the implementation of Solvency II new risk mitigation techniques such as new reinsurance structures have appeared in the European market and some existing ones started to gain more relevance. The Opinion therefore addresses the use of risk mitigation techniques and includes a set of recommendations addressed to NSAs to ensure convergent supervision. The Opinion on the use of risk mitigation techniques by insurance undertakings is accompanied by an impact assessment and the feedback statement addressing the comments received during the public consultation.

Risk mitigation techniques and, in particular reinsurance, are efficient tools for insurance and reinsurance undertakings to manage their risks according to their strategy and capacity. They are used to mitigate risks and to enhance capital management by diversifying risks. The sound mitigation of risks is recognised in the calculation of the Solvency Capital Requirement.

This Opinion raises awareness about the importance to have a proper balance between the risk effectively transferred and the capital relief in the Solvency Capital Requirement. This balance is to be assessed following a case-by-case analysis to take into account the particularities of each reinsurance structure and its specific interaction with the risk profile of the undertaking.

NSAs are expected to coordinate and cooperate in the assessment of such structures going beyond a single Member State to ensure a convergent approach.

IVASS draws the attention on the EIOPA Opinion for the correct use of risk mitigation techiniques entirely adopting it with a Regulatory letter *Letter to the market of 28 July 2021* (https://www.ivass.it/normativa/nazionale/secondaria-ivass/lettere/2021/lm-28-07/Lettera_al_mercato_del_28_luglio_2021_Mitigazione_del_rischio_en.pdf?language_id=3)

3.2 Traditional reinsurance

Now it will be given a brief description of the so-called “traditional” reinsurance, the main business for any Non-Life Reinsurer. Reinsurance products are usually intermediate through a broker or directly by a primary insurer.

The two most reinsurance contracts classes are the proportional reinsurance, in which any claim is shared by the reinsured and the reinsurers proportionately to their respective sharing in the particular risk regardless of the size of the claim, and the non-proportional reinsurance, in which the shares of the reinsured and the reinsurers in each claim will vary according to the size of the individual claim on the overall quantum of losses. In practise, different types of reinsurance are often used for different classes of business and for different types of risk. Sometimes different types are combined.

These types of reinsurance can both be divided in individual, where the treaty impacts on every individual risk, and collective, where the treaty impacts on the whole portfolio. Each form of reinsurance tempers the insurer’s retained portfolio in its own way, and thus affects the choice of level of retention. Usually, a reinsurance contract has more then one reinsurers (e.g. one for each line of the reinsurance coverage), while an insurer combines more then one contract in a reinsurance program (e.g. Towers XL, Bouquets QS, XL on QS retention).

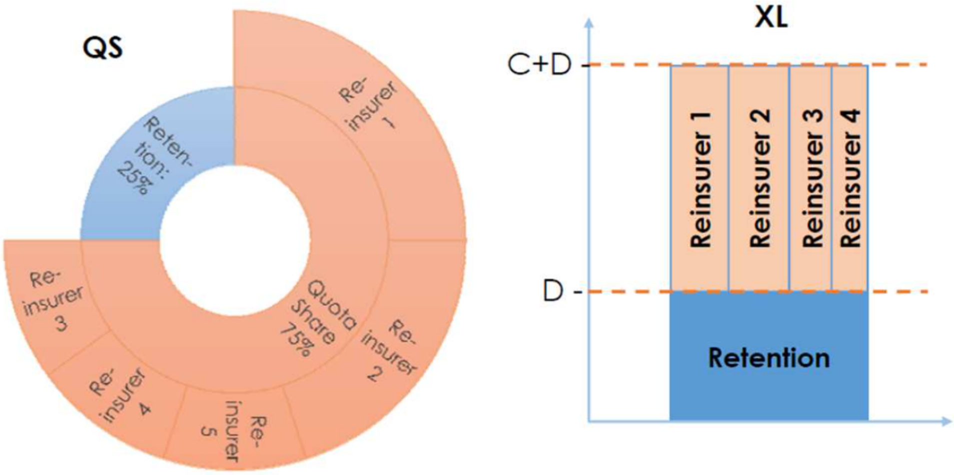


Figure 3.1. Classical Proportional and Non-Proportional structures.

Reinsurance contract basis can be agreed with the following clauses:

- Losses occurrence: under a loss occurrence reinsurance policy, a loss event that occurs during the contract period will be covered by the insurance contract, regardless of the length of time between the date of loss and the date reported to the insurance entity. Some contracts specify a "sunset clause", e.g. a specific date by which claims must be reported for the claim to be valid. This policy basis is very similar to the occurrence basis policies found in direct insurance.

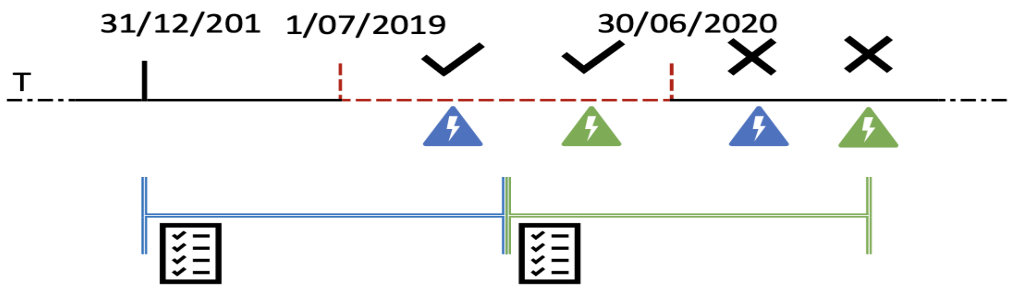


Figure 3.2. Losses occurrence clause example.

- Risk attaching: All losses arising from policies underwritten during the period of the reinsurance policy will be covered by the reinsurer, regardless of the date of occurrence and when the loss is reported. This type of basis is specific to reinsurance, as the policy period depends on the policy period of the underlying direct policies rather than on the timing (date of occurrence, date of reporting) of the claim itself.

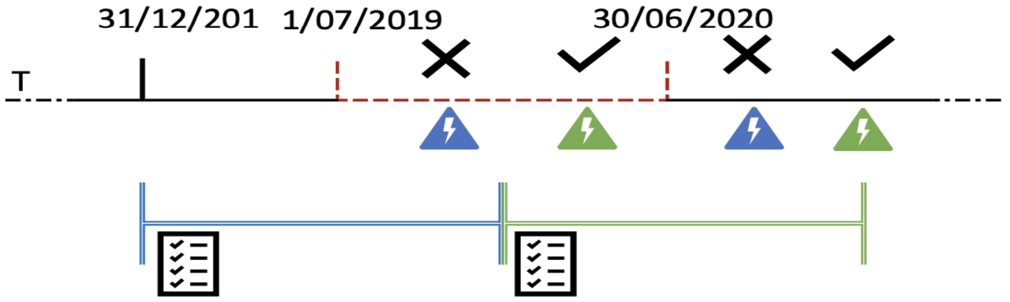


Figure 3.3. Risk Attaching clause example.

- Claims made: All losses reported during the term of the (reinsurance) policy will be covered, regardless of the date of occurrence and the start date of the policy. Note that some contracts will have a retroactive date, which in practice limits the applicability of the contract to losses occurring after a specific date.

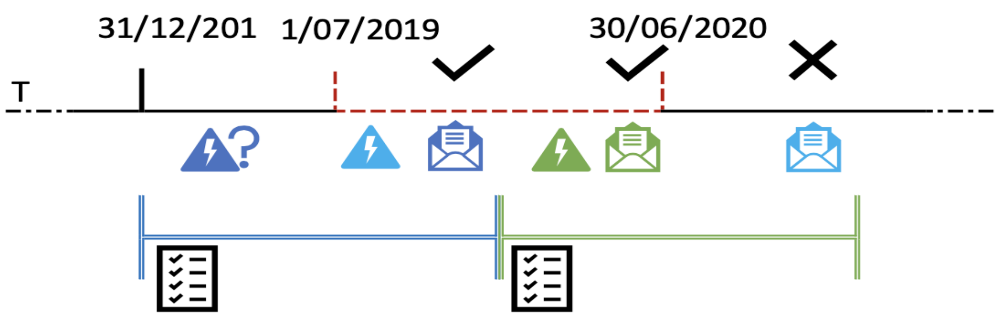


Figure 3.4. Claims made clause example.

Proportional reinsurance has the advantage that the reinsurance premium rating is easier than in the case of non-proportional reinsurance. Indeed, since the reinsurer pays a certain proportion of every claim, the reinsurance risk premium is the same proportion of the total risk premium.

This kind of reinsurance arrangements may be collectively made in a *Quota Share* treaty, in which any claim, irrespective of its size, is divided between the cedant and reinsurer in a fixed ratio; it does not change the cedant's claims pattern since the reinsurers pay their proportionate share of losses falling within the framework of the Quota Share treaty.

Such reinsurance basically reduces the cedant's overall loss cost and reduces or limit the loss on any one risk, without changing the balance of the portfolio – that is, the relative size of the potential claim fluctuations compared with the retained premium income.

Such reinsurance practises are usually negotiated for homogeneous portfolios in property insurance (i.e. homeowners policies); rather then to provide catastrophe capacity, in these lines of business insurers would moreover increase the solvency.

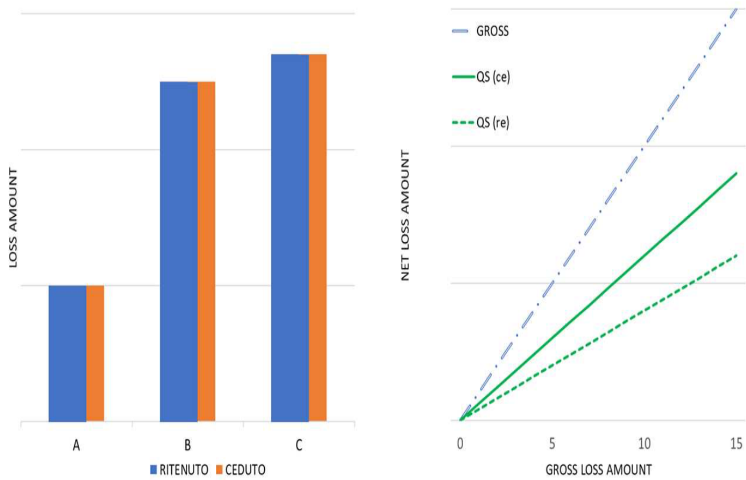


Figure 3.5. Quota Share example.

Generally in a Quota share treaty the safety loadings coefficient applied by both the insurer and the reinsurer is the same, and it is rationally true since this reinsurance arrangements operate a risk sharing between the two companies with no difference on the process randomness. In this case reinsurer's premium rating involves reinsurance commission loading coefficient, it can be equal or less than the cedant's expenses loading charged on the original risk premium volume.

Pricing of proportional reinsurance is usally represented by a fixed reinsurance commissions, paid to refund insurance companies of acquisition cost related to ceded portfolio of insurance policies. Another way to consider commission, that is more coherent with real proportional treaty, assumes that reinsurer pays to cedant sliding commission that rewards or penalizes primary insurer according to the ex-post profitability of portfolio protected by the treaty, i.e., a random commission rate whose value depends on the observed Loss Ratio is usually introduced

When reinsurance commission loading mismatches expenses loading ceded to reinsurer it may be dangerous for the primary insurer since it is univocally losing a part of its profit.

Thus *Surplus* reinsurance will reduce the retained loss cost on larger risk and limit the loss on any one risk, thereby providing a better balance for the cedant's net account.

Surplus treaty is useful to have a proportional reinsurance cover, with the reinsurer's share depending on the risk unit in such a way that, the larger the upper limit for the risk unit, the larger the reinsurer's share.

In a Surplus treaty, the insurer's retention quota is fixed for each risk of the insurer's portfolio using α_i, that can be obtained as the minimum between 1 and the ratio between the retention limit and the sum insured for the risk **i**. We later avoid developing more characteristics of this kind of reinsurance since it is not in our purposes to obtain a model for the Surplus treaty.

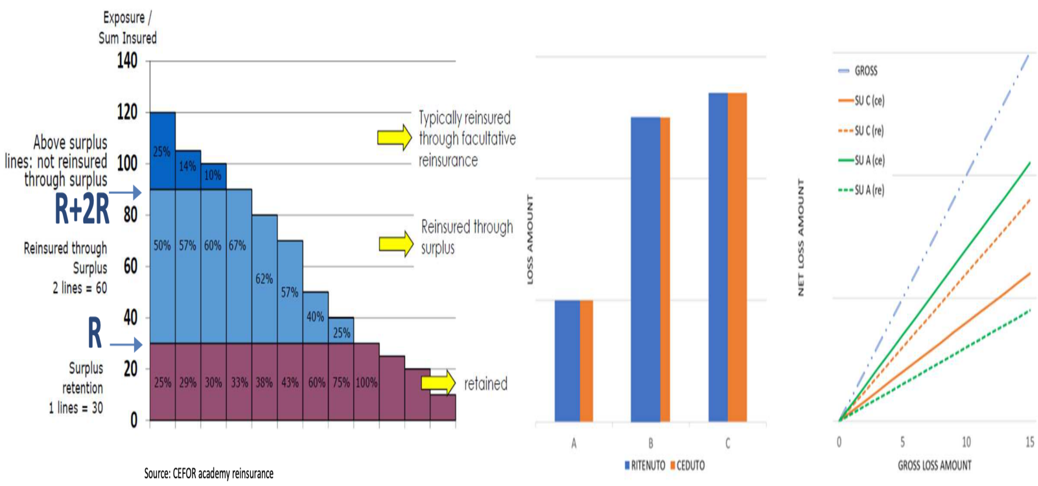


Figure 3.6. Surplus example

A Comparison between these two reinsurance treaties could be useful for comparing individual and collective form of proportional reinsurance:

- A Surplus treaty will provide the cedant with a larger retained premium income for the same monetary retention limit on any one risk and therefore better balance.
- On the Surplus basis the cedant will pay 100% of claims on smaller risks (i.e. those falling within its retention) with reducing proportions of claims to be retained as risks increase in size. The frequency of losses may vary considerably between smaller and larger risks.

- It is probable that the reinsurers will allow higher reinsurance commission on the Quota Share treaty. This treaty requires less administration since all risks falling within its framework are equally shared, whilst under Surplus reinsurance the shares of reinsurers vary from risk to risk according to the insured values. The type of reinsurance will therefore affect the expanse ratio of the retained account.

Non-Proportional reinsurance may be individually made in Excess of Loss treaty (shortly named *XL* treaty) and it protects individual risks and in this respect only is similar and therefore an alternative to proportional reinsurance. With Excess of Loss treaty reinsurer pays that part of each claim amount it exceeds an agreed limit M, the cedant's retention limit.

Of all individual losses which occur per risk, the reinsurer assumes those loss amounts exceeding the contractually fixed deductible (priority, retention) up to the upper limit of cover. The effect of such protection on the reinsured's retained claim pattern is to leave the frequency of claim unaltered (as with proportional reinsurance) but to limit the loss on any one risk and thus protect the retained account from an adverse fluctuation in individual claims amount.

In addition it aid to the reinsured's liquidity in the case of large loss amounts payable in the short term. However, as the cedant bears in full all losses up to the underlying deductible, the retained premium income will be smaller than under a comparable proportional treaty. Property insurance is the main area of application.

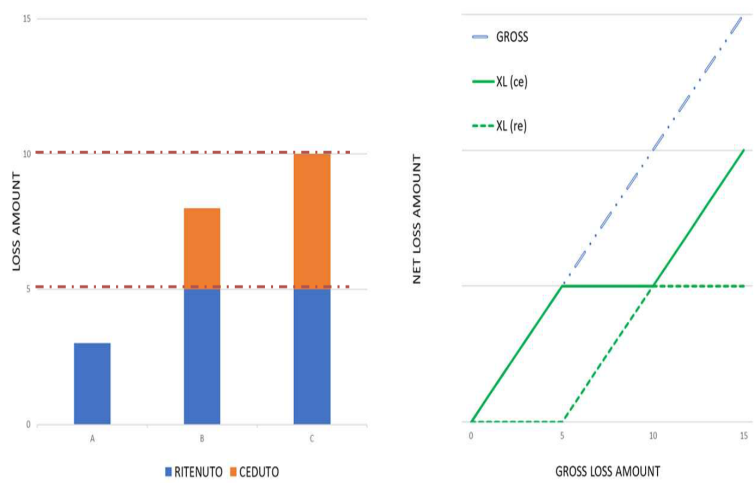


Figure 3.7. eXcess of Loss example

A per-event Excess of Loss (Often called Cat XL) cover is a non-proportional contract in which the layer is applied to the losses caused by the same event, i.e. the losses of different underlying policies caused by the same event are aggregated.

When the non-proportional treaty is made collectively, it may be a *Stop Loss* reinsurance or an *aggregate XL* reinsurance; they both limit or reduce the aggregation of claims over a given period of time, normally on an annual basis, providing protection not only against large individual claims but also against fluctuations in the number of claims.

In a Stop Loss treaty cedant stop to pay claims after a certain point (namely stop loss point) incurred within a certain period of time, consequently the reinsurer pays the excess over an agreed limit amount M of the cedant's aggregate claim amount accumulated during this time period. For this reason layer structure is usually based on underlying premium income. They, therefore, contain the Cedant's total retained loss cost (balance sheet protection):

- Under *Stop Loss*, this containment can be enjoyed by the Cedant when the deductible is exceeded either due to an increased frequency of loss or from the size of individual losses or a combination of the two.

- Under *aggregate XL* it is normal that only the individual claims in excess of certain figure are included in the deductible (*Stop Loss* for large claims portion of cedant portfolio).

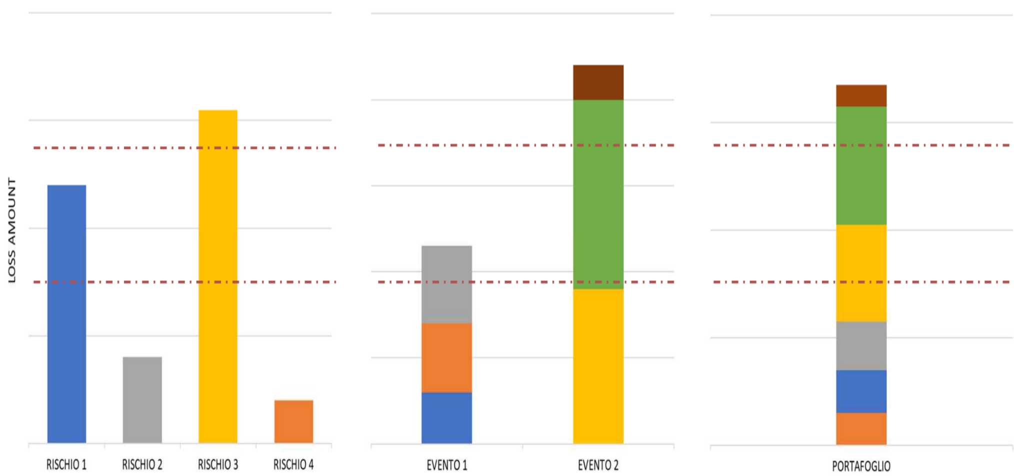


Figure 3.8. Comparison of Non-Proportional treaties (per-risk XL, per-event XL and SL).

Cumulative XL is a popular coverages in property, marine hull, motor hull, personal accident, and natural catastrophe insurance; it operates on the sum of several claims which cross the attachment point of single non-proportional coverages.

In reinsurance pricing, often non-proportional XL contracts specify an "Aggregate Annual Deductible" - Annual Aggregate Deductible (AAD) or an "Aggregate Annual Limit" - Annual Aggregate Limit (AAL).

These work as a layer applied to the reinsurer's aggregate losses.

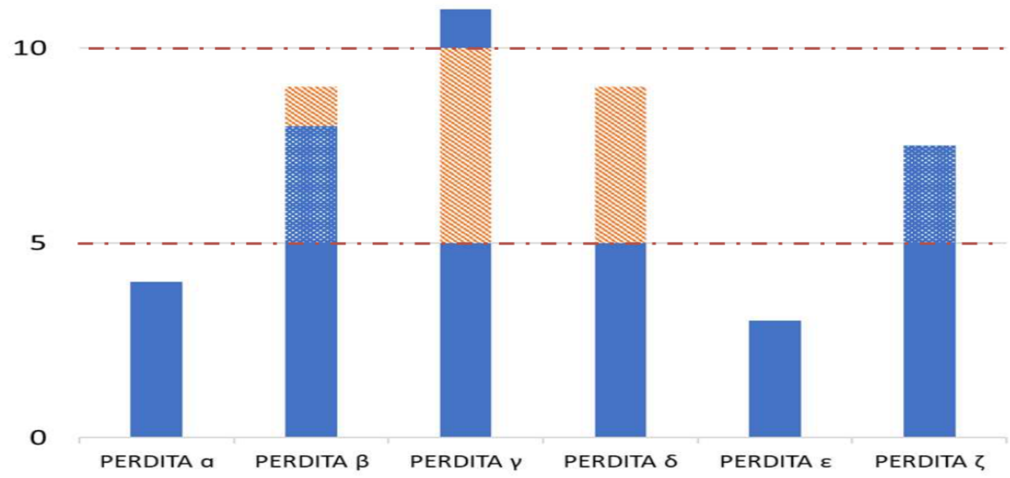


Figure 3.9. Aggregate deductibles

In stabilization clause, a multiplicative adjustment of deductible D and cover C, with the aim of making the insurer and reinsurer share the risk of an increase in the severity of claims due to claims inflation. A factor g is applied which transforms the pre-adjustment layer. The correction depends on an estimate of the claims inflation (e.g. the wage index in MVL) and the time of payment. There exist different calculation variants.

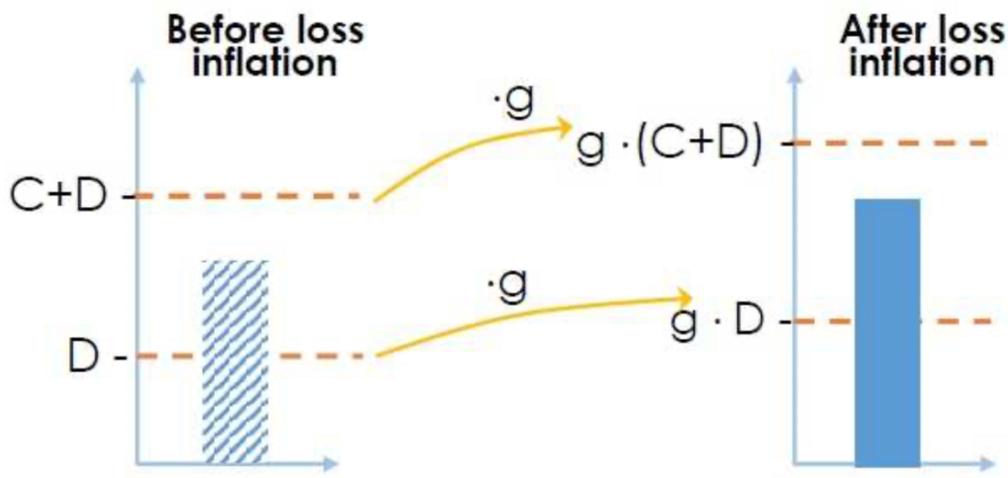


Figure 3.10. Stabilization clause

For non-proportional reinsurance contracts, there are different ways of contractually defining the reinsurance premiums:

- Fixed Premium Amount: fixed nominal amount;
- Fixed Premium Rate: fixed portion of the transferor's premium. The reinsurance premium increases / decreases with the primary insurer.

Some XL contracts specify a "Reintegration Premium", in which case the initial premium (called the premium base) provides coverage for limit C only once. After the coverage is exhausted, the payment of a 'reinstatement premium' replenishes the coverage. Typically we have:

- Full reinstatement premium: the reinstatement premium is equivalent to the premium base;
- Pro rata to amount: if the hedge is partially used, the reinstatement premium is deducted.

Often in non-proportional Reinsurance Programs such as XL towers, the various component layers provide aggregate conditions (AAD, and AAL). But what happens when one of the layers runs out?

In order to avoid "empty layers", ie lack of coverage for the transferor, the following clauses are often provided (for further details see the so-called "round-the-clock reinstatements"):

- Layer with Stretch-Down clause: if the underlying layer is missing, the layer extends downwards, lowering your deductible and increasing your coverage;
- Layer with Drop-Down clause: if the underlying layer is missing, the layer slides downwards, lowering the your deductible and keeping the coverage unchanged.

Classes often coexist in the same reinsurance program

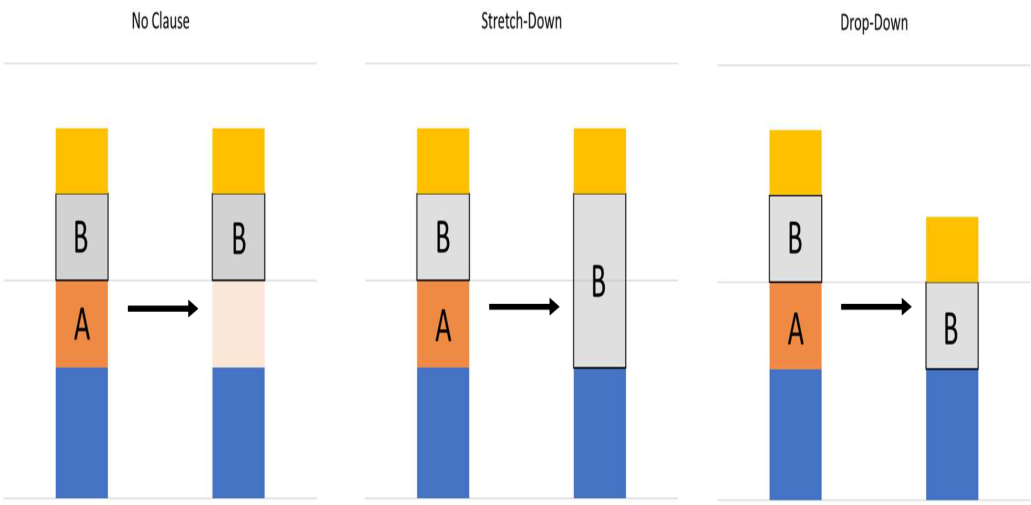


Figure 3.11. "round-the-clock" reinstatements

We finish with some more comments and references on the **reinsurance of natural catastrophes**. Specific reinsurance coverages for catastrophes are *XL per event*, *cumulative XL*, or *aggregate XL*. Specifically, natural disasters are usually reinsured because of their severity.

There is first the difficult task of describing the concept itself in a quantitative way. It is not straightforward to agree on a concrete definition of a catastrophic claim.

Clearly, a catastrophic claim falls into the category of large claims, where from the statistical side one may see an additional challenge in the fact that usually there are very few data points for a systematic study available, and the available ones are often only rough estimates of the true value, even long after the catastrophe has occurred. At the same time, it will often be difficult to make the data points comparable.

Insurability of natural catastrophes can only be achieved in a sustainable way if there is an equilibrium between losses and premium income, over both time and in space. However, the geographical distribution of the claims is often difficult to assess.

Insurance companies therefore typically use a bottom-up approach in which they use scientific/expert knowledge in connection with the time and size of a natural catastrophe (e.g., with high-resolution physical models for weather parameters), and calibrated in terms of risk exposure (this can involve very detailed information from engineering on building structures).

Missing data are often estimated by expert knowledge, and parameters in the model are sometimes hard to determine in the presence of sparse and inaccurate loss data. Nevertheless, in recent decades scientists have built up an impressive toolkit to quantify respective risks, and nowadays there exist several commercial firms who professionally assess the risk for certain natural catastrophes in specific regions and who offer their services to the (re)insurance industry.

When studying the patterns of natural catastrophes and building models, one also needs to carefully consider and incorporate systematic changes in risks due to climate change.

There are several examples of models or collections of claim data for specific types of natural disasters, from catastrophic wind losses and connected XL cover, to Hurricanes, Hail, Flood (which is impacted from climate change), earthquakes and business interruptions

Man-made and other types of catastrophes are equally challenging to deal with are terrorism, cyber risk and actuarial risks related to pandemic diseases.

Some reinsurance companies offer interesting illustrative material to catastrophes, see, for instance, the web-sites http://www.swissre.com and http://www.munichre.com, where information on recent catastrophes and their actuarial consequences is regularly updated.

3.3 Alternative Risk Transfer

There are several alternatives to traditional reinsurance available in the market, summarized under the term alternative risk transfer (ART), that can be defined as all risk-transfer methods where ultimate counterparties are the capital markets players from (re-)insurers to pension funds, banks and hedge funds.

They increase the efficiency of the marketplace and can also be particularly helpful at times when traditional reinsurance capacity is limited (e.g., after major natural catastrophes such as Hurricane Andrew in 1992 or Hurricane Katrina in 2005, or following the event of Sept. 11 2001).

In terms of volume, ART nowadays constitutes about 20-25% of the total reinsurance business, measured in terms of dedicated capital.

Some ART solutions take a more integrated approach to reduce the risk over time (rather than treating the various types of risks separately), providing additional diversification over time in connection with underwriting cycles, counterparty risk, and coverage of large catastrophes. Others provide alternatives in the nature of the relationship between insurer and reinsurer by creating a secondary market where one can enter or leave coverage in a more flexible way than in classical reinsurance treaties.

In this section we briefly discuss some of these alternative possibilities. Actually, a segmentation of reinsurance products is needed in order to make some comparisons between traditional reinsurance products already mentioned above and alternative reinsurance market. Such more complex reinsurance products, usually linked with capital market, go into reinsurance business as a form of coverage that underlines quotations on financial markets.

In general, the risk transfer can be via alternative risk carriers and alternative products. Obviously, it is always possible *self-insurance retention* (SIR), which can be both regulated and non-regulated and is particularly popular in the USA.

Alternative risk carriers

These non-traditional structures appear in times when traditional reinsurance premiums are very high or coverage is not available at all (which happened after Hurricane Katrina), and are often established by traditional reinsurers in order to tap into the external capacity offered by the capital markets from hedge funds, investment banks, private equity and other opportunistic investors and increase the efficiency and diversification of the company's reinsurance program.

They usually have a limited life expectancy and are broken up when market conditions deteriorate, after which any remaining capital funds are returned to investors and the sponsor. The investors' exposure is limited to the amount of the deposit, which equals the reinsurance contract limit (so it is a fully collateralized reinsurance form where counterparty risk is partially or totally eliminated).

ILS are the most important example of insurance securitization. They are financial instruments whose values are driven by insurance loss events, enabling insurance risk to be (directly) placed on the capital market .

Among the most important examples are catastrophe bonds (CAT bonds), which are bonds with the additional feature that the investor will not receive the coupon (or even not the principal) if a certain trigger related to the occurrence of natural catastrophes is hit. In turn, the coupon in the absence of that trigger is substantially higher. Such bonds are traded over the counter and can increase the insurance capacity for risks for which it is difficult to find traditional reinsurance. The initiator of the CAT bond is the insurance company that seeks this protection.

The trigger can be the individual loss experience of the issuer due to a catastrophe or (increasingly more often) an index representing the average catastrophe loss experienced by the insurance sector in a prespecified time interval and business line. Finally, it is very common nowadays to have parametric triggers, that is, a physical measurement (such as wind speed, magnitude of an earthquake etc.), as this is a reliable, "objective", and usually easily accessible trigger for investors (whereas for a trigger linked to the individual loss experience of the issuer there are obvious moral hazard issues and the final settlement can take much longer). One then speaks of an index transaction (instead of an indemnity transaction). However, triggers based on an index or parametric triggers again introduce basis risk for the issuer (on the plus side, the insurer does not have to pass on the actual claim data to the outside in this case).

In practice, there is usually a special-purpose vehicle (SPV) that acts as an intermediary (located in a tax-friendly environment), which then issues a conventional reinsurance policy to the insurance company. The SPV uses the premium payments of the insurance company to pay the coupons to the investor, and if the event is triggered (or maturity of the bond is reached without the trigger), the amount in the SPV is paid out to the insurer and the investors according to the bond specifications. One particular advantage of the CAT bond is that – in contrast to traditional reinsurance – there is no counterparty risk for the insurer (unless the construction involves further parties like swap providers), since the reinsured amount is already available in the SPV (or the trust account). At the same time, for investors this can be an attractive product, since the underlying trigger event will typically be independent of other investments and the excess coupon can be considerable (in the long run, it will of course be determined by demand and supply, as well as the concrete loss experience of previous years). Conceptually, CAT bonds represent a secondary reinsurance market,where the investor (who takes the reinsurer's role here) has more flexibility to leave or enter the "contract" along the way. In recent years the CAT bond market has increased considerably in size.

Also CB were designed after the already mentioned two particularly significant natural disasters - in the United States - in terms of both economic and social impact: Hurricane Andrew in 1992 and the Northridge earthquake in 1994, which struck the San Fernando valley in California. Both events caused extensive damage in terms of loss of property, life and business continuity According to some sources, hurricane Andrew caused in the excess of 25 billion USD worth of damage while the Northridge earthquake caused around property damage amounting to an estimated 49 billion USD.

Following these events, awareness about the inadequacy of traditional systems in the global insurance industry increases, leading to develop more appropriate measures to protect insurers' assets through the capital market.

In more details, the Chicago Board of Trade (CBOT)in 1992 introduced a new type of derivative insurance contract, CAT Futures, taking the first steps in the catastrophic reinsurance market. In 1995 catastrophe options, insurance loss warranties based on Property Claim Services (PCS) estimate of insurance losses from natural catastrophes, replaced this contract. This provided an opportunity to expand the capacity of the reinsurance market. In 1997 (see [20]), illustrated catastrophic risk management focusing on different strategies, where the problem of moral hazard is not negligible, showing how this can be reduced by parametric insurance at the price of an increase in basis risk. On the other side, (see [9]) underlined the importance of the development and of the use of financial instruments related to natural risks.

[25] in 1999 analyzed several financial instruments used to transfer catastrophic risk to capital market, as CAT bonds, CAT futures and CAT options traded on CBOT, catastrophe risk exchanges, catastrophe swaps and hedging instruments. Furthermore, he analyses the low diffusion of alternative instruments of catastrophic risk transfer which have not yet replaced traditional forms of insurance despite market growth

Struck by the enormous policy claims, several insurance companies defaulted. This affected the ability of the insurance market to provide proper coverage to those who suffered the effects of both events. Furthermore, following to the events of 1992 and 1994, insurance providers became increasingly adverse to insurance policies covering certain events in specific areas (especially coast regions exposed to hurricanes) and additionally, where insurance coverage could actually be bought, the premiums rose quickly to reflect to account for the possibility of significant losses.

The events of 1992 and 1994 thus spurred undesirable changes in the insurance market ultimately leading to suboptimal coverage against certain risks. To offset the risks borne by insurance companies, additional reinsurance was bought on the market and State-funded insurance programs were put in place.

For all such initiatives, however, an intrinsic limitation was identified in the available capital in the insurance industry which could absorb potential losses. Insurance operators turned to the financial markets which, by sheer size, could potentially provide additional risk-absorption capability.

CAT bonds were therefore devised to transfer insurance risk borne by insurance and reinsurance companies to the financial markets.

For a general understanding of CAT bonds refers to website ARTEMIS (https://www.artemis.bm/library/what-is-a-Catastrophe-bond/) and Edesses's book *Catastrophe bonds: an important new financial instrument* (https://risk.edhec.edu/publications/Catastrophe-bonds-important-new-financial-instrument).

Further ILS products (with, however, a much smaller market) are, for example, longevity swaps and products related to embedded-value securitization and extreme mortality securitization. Altogether, the global ILS risk capital outstanding in 2020 exceeded 35 billion US dollars.

Buying *Industry loss warranties (ILW)* an insurer obtains protection based on the total loss arising from an event to the entire insurance industry rather than their own losses, so through a kind of derivative contract or private reinsurance.

The insurer pays a premium to who writes the ILW cover and in return receives coverage for a specified limit if industry losses exceed the predefined amount under the ILW trigger.

These contracts can be viewed as a resemble of reinsurance contracts, but the trigger is the loss of the entire insurance industry arising from an event (measured through some index) rather than the individual loss experience.

The market for such contracts has considerably grown over the last years, with reinsurance companies and hedge funds being typical protection providers.

A disadvantage for the insurer is the eventual discrepancy (referred to as *basis risk*) that can arise when the reinsured amount is not based on cedant own loss experience, even if the industry index will usually be reasonably correlated. This basis risk may lead to protection inefficiencies.

Reinsurance pools act like a mutual, where each insurance company can cede its risk and its premiums of a specific LoB, for instance for very large risks. In Italy there is Pool Ambiente, previously Pool Inquinamento, providing reinsurance and co-insurance for environmental risks.

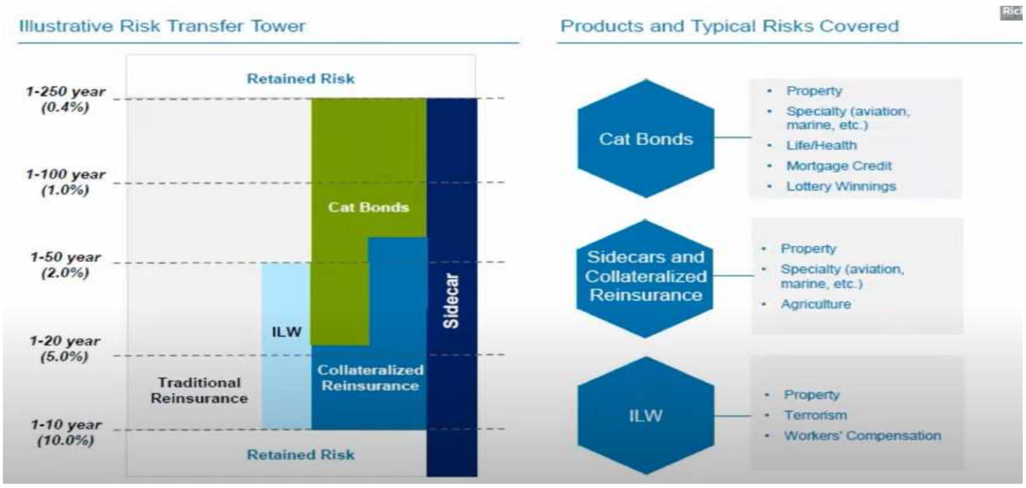
As another example, *captives* are insurance or reinsurance companies which insure the risks of their parent company in a cost-effective manner, becoming popular vehicles in insurance industry and usually located in a tax-friendly country. In that way, access to the global reinsurance market can be obtained, which due to the larger diversification possibilities of reinsurers may lower premiums through reduced capital cost.

In addition, this allows a certain degree of time diversification (captives are usually allowed to hold equalization reserves, whereas according to the current accounting standards insurance companies are not). For smaller captives, it is also popular to operate with its parent as a reinsurer to which the insurance risks of a first-line insurance company were preliminary transferred. A particular advantage of such a construction is that from a regulatory perspective it will then be treated as a reinsurer.

An alternative are so-called reinsurance *Side-cars*, where investors deposit capital and in turn participate in the premiums and claims of the insurer. They are typically QS-type treaty for a line of business, with a retention for the company that ensures alignment of interests between insurer and investors.

Side-cars are special-purpose reinsurers that provide dedicated collateralized quota-share reinsurance, often for a single ceding company that transfers a portion of its underwriting risk (and related capital investment), and in turn receives a ceding commission. They also can be source of fee income for the reinsurers that underwrite or provide management services to such third-party risk vehicles.

On the other hand, Sidecars are only one way for capital market investors to gain exposure to insurance risk, since the capital market is a major alternative risk carrier, particularly through Insurance-linked securities (ILS).



Source: Aon Securities LLC, copyright Allianz ACRTS – 15-sep-20

Figure 3.12. Alternative Risk Transfer tower

Alternative risk products

In multi-year/multi-line products several business lines are bundled together and/or over a longer time horizon, which leads to a smoothing of the aggregate risk and hence to lower premiums. A disadvantage is counterparty risk for the insurer, and it can also be non-trivial to cooperate across business lines within the insurance company.

It is worth to mention that the concrete implementation of reinsurance is moving to an higher use of financial function for the ceding company.

As an example, arbitrage-free pricing is increasingly applied also in the context of reinsurance contracts. As highlighted by Albrecher ([1]), the Fundamental Theorem of Asset Pricing asserts that in the absence of arbitrage possibilities, the pricing functional must be a positive and linear functional defined in the Hilbert space.

Then, by virtue of the Riesz representation theorem, one can express as an expectation with respect to a modified (distorted) random variable. The reinsurance market is incomplete (illiquid), and so there is no unique choice for such an adjustment of the physical probability measure.

In this direction moves *Contingent capital*, that refers the option for the insurance company to raise debt or equity capital for pre-specified conditions, in case there is a severe aggregate insurance loss experience or another pre-specified event occurs. It can be seen as a put option on the own shares with a predefined strike value.

By setting up the conditions before financial distress, fresh capital can in such a case be acquired in a much cheaper way than on the market. As already commented for finite reinsurance, this is a means of financing rather than a transfer of insurance risk.

As a variant of this solution, contingent convertible bonds ("CoCo" bonds) have become increasingly used (particularly since from many NCAs this is now considered as regulatory-efficient capital), and in recent years products combined trigger (the occurrence of natural catastrophes and the solvency ratio of the company) appeared.

In *Multi-trigger products* the reinsurer pays losses only subordinated to a second event, usually correlated to the insurer's financial result. For instance, reinsurance treaty activates only when the losses exceed a certain threshold and simultaneously a stock index, commodity price, exchange rate etc. is below a prespecified level.

These solutions aim to considerably lower premiums, but still may serve the overall financial result of the insurer well. At the same time, as capital requirement for reinsurers decreases, it result for them in benefit from this variant particularly if there are several such contracts with independent triggers in reinsurers portfolio.

Finite risk reinsurance is a particular example of structured reinsurance, which involves solution where, in addition to insurance risk transfer there is also a significant weight on other goals in a product.

Finite reinsurance is defined as a contractual form between an insurer and a reinsurer that is tailored towards the concrete needs of the insurer, providing a combination of risk transfer and risk financing. While a substantial goal of the transfer is to enhance the insurer's financial results, for tax reasons it has to contain a (limited) amount of insurance risk transferred to the reinsurer in order to be classified as reinsurance.

Such contracts have a duration of several years and combine loss experience and investment returns. They formalize a longer-term relationship between the two parties which has a time diversification component, as the reinsurer can count on incoming premiums and the insurer on agreed coverage to known conditions over a longer time horizon. There are retrospective and prospective variants.

Definitely, alternative reinsurance can be defined as any form of managing and transferring insurance risks through the use of the capital markets rather than the traditional reinsurance market.

The capital structure of alternative capital providers is typically on managed funds from institutional investors, with a mandate to invest in instruments related to natural catastrophe events. These funds invest in non-traditional structures, they can generally be divided in:

- Catastrophe bonds (CAT bonds);
- collateralized quota-share reinsurance vehicles (Side-cars);
- Industry loss warranties (ILWs).

Alternative reinsurance market is substantially located in U.S. Insurance and Capital markets. Until 2014, when non-traditional capital showed a continuous growth, increasing by approximately 25 percent, with high growth for collateralized reinsurance (over 25%) and a less intense growth of CAT bonds emission, while sidecar and ILW saw significant percentages increases, growing 50 percent and near 100 percent respectively.

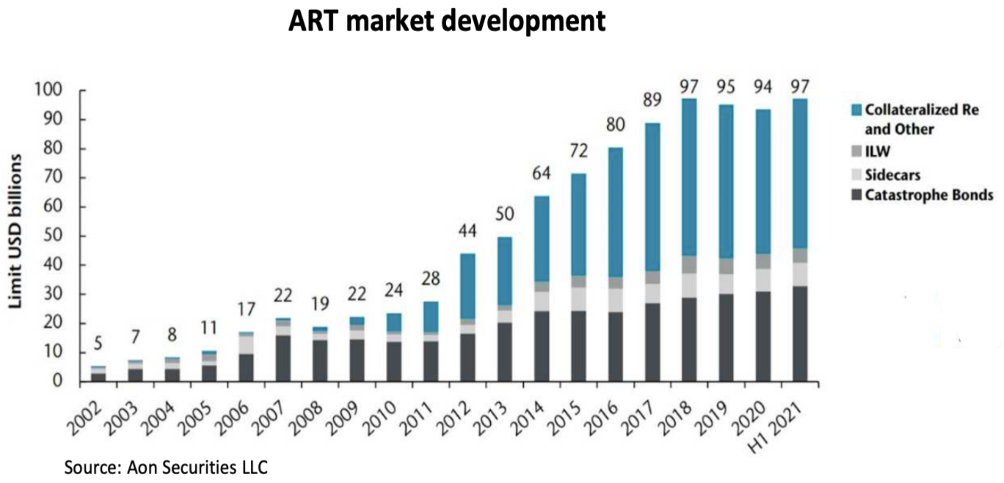


Figure 3.13. ART market development

ART market has the higher growth in 2005, following Hurricane Katrina (re)insurers were essentially forced to increase issuances of catastrophe bonds and expand the use of sidecars in order to absorb underwriting capacity as retrocession availability became more scares and expensive.

These solutions are typically collateralized to the contract limit, minimizing the counterparty default risk to cedants. These investments also have pre-defined maturities (from one to five years). This provides funds with distinct exit strategies and the ability to better manage their capital positions.

Then in the last ten years market is reaching its top near 100 dollar billions, three times the value at 2006.

Summarizing, due to down pricing insurers have previously bought more protection, but this time many large insurers are buying less reinsurance protection. Cedants are purchasing less reinsurance, rationalizing their reinsurance panels, or turning to alternative capital, making it more difficult for smaller, less-diversified reinsurers to compete.

Ceding companies are also using fewer reinsurers for protection. Many large, global reinsurance companies are choosing to do business with a select few reinsurers that can offer significant capacity across a range of lines and regions, marginalizing less diversified small and midsize reinsurers. Therefore, these kind of reinsurers are likely to consider merging or forming consortia to gain size.

During last years, companies' risk management and capital modelling capabilities have improved, obviously after that, following global financial and European sovereign debt crises, a recovery of the balance sheets of many large reinsurers' buyers has been needed. Then, they can assess their risk profile on a portfolio basis, rather than at an individual entity or regional level, optimizing their reinsurance arrangements, which typically means that they buy less reinsurance.

Loss Portfolio Transfer and Adverse Development Cover

As already mentioned before, if in addition to insurance risk transfer there is also a significant weight on other goals in a product we speaks about finite risk reinsurance. In case of Premium&Reserve risks, there are retrospective and prospective variants of finite reinsurance.

Retrospective ones include Loss Portfolio Transfer (LPT), where the insurer transfers outstanding claims from some long-tailed business of previous years to the reinsurer, and in turn pays a premium consisting of the net present value of these claims plus fees. In that way he passes on risks related not only to amount but also to the timing of loss development. LPT can also be defined as a (retrospective) Quota Share coverage limited to Best Estimate of Lioabilities. The premium naturally depends on the level of premiums ceded and by the insurance portfolio volume.

In Adverse Development Covers (ADC) the incurred but not reported (IBNR) losses are also included. Here the claims reserves are not transferred to the reinsurer, but the reinsurer only covers losses that exceed the reserves which the insurer has already built up, and for this transfer a premium is paid (this can be set up like an XL or SL contract on the adverse loss development and is also very convenient in cases of mergers or takeovers of a company). ADC can be defined as a (retrospective) Stop Loss coverage limited to Best Estimate of Liabilities. Usually a retention value of the best estimates, above which the reinsurer pays, is contractually fixed.

LPT/ADC covers have an immediate impact on Reserve risk and Market risk.

The premium naturally depends on the level of retention chosen and by the insurance portfolio volume. The treaty capacity is calculated based on the value $3 \cdot \sigma_{Res}^2 \cdot V_{res}$ used in the standard formula, with an immediate impact on Reserve Risk.

The market for Adverse Development Covers and Loss Portfolio Transfer has been growing in the past few years. Despite this growth, reinsurers are still struggling to define a standard method for pricing such covers. In this context, this article aims at providing an innovative method for pricing such contracts.

The proposed method is based on the famous Mack model and fits a Constant Elasticity of Variance (CEV) model to the Mack results (expected value and standard deviation) on each future development year of each accident/underwriting year. Having fitted the CEV model, it is possible to estimate the value of the Adverse Development Covers for each accident/underwriting year using standard European option pricing techniques and to compare this valuation with usual Non-Life Insurance valuation techniques.

Also EIOPA used LPT/ADC example in the public consultation process of Opinion on risk-mitigation techniques mentioned above. In particular, it has been called ‘Bifurcated (split) cover for long tail business’.

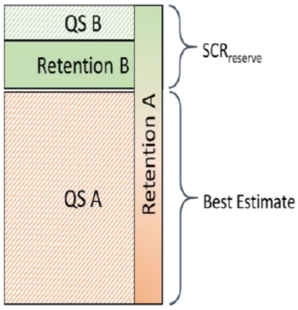


Figure 3.14. Illustration of an Adverse Development Cover where "QS B" mitigates reserve risk and "QS A" generates recoverables and thus considerably reduces the capital requirement for reserve risk (source: EIOPA

In order to reduce the capital requirement due to non-life reserve risk, a reinsurance arrangement consisting of two parts is tailored. It consists of an adverse development cover (upper part) that mitigates the loss development risk, but with a retention well above the best estimate, and a finite reinsurance type of cover (lower part) that generates reinsurance recoverables, although not beyond the best estimate.

By generating recoverables, the lower part reduces the volume measure for the standard formula SCR calculation of premium and reserve risk.

Although the reinsurance arrangement is given as one single contract, it actually can be seen to combine two completely independent contracts: an upper layer that transfers real risk but does not come with any significant SCR relief and a lower layer leading to a considerable SCR reduction without mitigating any of the loss development risk. The reduction in the SCR can be materially greater than the risk mitigation of the arrangement.

In a situation like this an undertaking may consider the appropriateness of applying the standard formula. Key Findings that can be identified are:

- It is possible to replicate the Mack model estimating the ultimate non–life insurance reserves with a CEV model and to find a good fit for the CEV model;
- The proposed CEV model seem to provide better results than models based solely on the ultimate view of the non–life insurance reserves;
- It is important to take into account not only the ultimate volatility of the insurance reserves but also the way in which the volatility develops. Such conclusion matches the usual question of the volatility smile for option pricing techniques.

Prospective variants of finite risk reinsurance include spread loss treaties, where for the transfer of specified losses (with annual and overall limits) the insurer pays premiums to the reinsurer onto an "experience account" and these premiums (minus fees and expenses) are then invested. At the end of the contract period, the balance is settled with the insurer, exposing the reinsurer to the counterparty risk that the insurer may not be able to pay a potential negative balance. Finally, finite quota share arrangements can include over- or undercompensation of claims over prespecified periods of time.

Chapter 4

Underwriting risk net of reinsurance

4.1 Premium & Reserve risk in CRM

As showed by Pallaria and Savelli ([48]), in classical risk theory literature stochastic risk reserve $\tilde{U}$ (net of reinsurance) at the end of year t using the Collective Risk Model (CRM) is given by the relation:

$$\tilde{U}_t = U_{t-1} + \left[\sum_{h=1}^{L} \left[(B_{t,h} - E_{t,h} - \tilde{X}_{t,h}) - (B_{t,h}^{RE} - C_{t,h}^{RE} - \tilde{X}_{t,h}^{RE}) \right] \right], \tag{4.1}$$

where the big square bracket represents the random variable (r.v.) of a one-year technical result $\tilde{Y}_t^{NET}$ for the period $(t-1, t)$, evaluated at the end of year $t-1$, as the difference between insurance and reinsurance results. When a tilde is taken on a character, then it will mean that it is a random variable.

Gross earned premiums $(B_{t,h})$, the stochastic aggregate claims amount of year t related to new business $\tilde{X}_{t,h}$ and general and acquisition expenses $E_{t,h}$, with $h = 1, ..., L$ Lines of Business (LoB), are realised at the end of the year.

Claims cost of the year incorporate both payments for claims incurred during the year and the provisions for new claims $(\tilde{X}_t = \tilde{X}_{t,h}^{paid,CY} + \tilde{V}_{t,h}^{S})$.

Regarding not only premium but also reserve risk, claims development result given by last year claims reserve net of payments for claims incurred in previous year can lead to profit or loss.

Actually, in a calendar year insurers must take into account both earned and written premiums in their market-consistent valuation of assets and liabilities introduced by Solvency II with the new Economic Balance Sheet, but for practical purpose it is here assumed that earned premiums are equal to the written premiums. Neither dividends nor taxation are considered in the model.

Earned premiums are the difference between written premium of the year and the one-year change in premium reserve for unearned premiums and unexpired risk $(B_{t,h} = B_{t,h}^{written} - \tilde{V}_{t,h}^{P} + V_{t-1,h}^{P})$.

With regard to the original insurer's portfolio, for each LoB the initial gross written premiums $B_{t,h}$ (GWP) are composed by gross risk premiums ($P_{t,h} = \mathbf{E}(\tilde{X}_{t,h})$), safety loadings applied as a quota (constant) of the gross risk premiums ($\lambda_h \cdot P_{t,h}$) and the expenses loading as a (constant) coefficient applied on the GWP ($c_h \cdot B_{t,h}$):

$$B_{t,h} = P_{t,h} + \lambda_h \cdot P_{t,h} + c_h \cdot B_{t,h}. \tag{4.2}$$

Underlying assumption about expenses is that they are deterministic and then they are affected by a stochastic behaviour. In other words, in premium risk might be implicitly included also the expense risk (together with claims risk), linked to the volatility of the expense amount, whatever method is used to the standard deviation of premium risk evaluation, but here we neglect this component also because for many non-life segments it is rather negligible (because of the short term duration). It is also assumed that safety loading coefficient λ_h is kept constant over the time horizon.

In Equation (4.1) $B^{RE}_{t,h}$ denotes the written premium volume ceded to reinsurer in the h-th LoB whereas $\tilde{X}^{RE}_{t,h}$ and $C^{RE}_{t,h}$ are respectively the amount of claim refunded by reinsurer and the reinsurance commissions, the latter is supposed to be equal to a (constant) coefficient applied on the gross premium volume ceded to reinsurer ($C^{RE}_{t,h} = c^{RE}_{t,h} \cdot B^{RE}_{t,h}$). Reinsurance commissions can also be stochastic: $\tilde{C}^{RE}_{t,h} = \tilde{c}^{RE}_{t,h} \cdot B^{RE}_{t,h}$.

Note that either $B^{RE}_{t,h}$ and $c^{RE}_{t,h}$ depend on the type of reinsurance arranged as well as the reinsurer's share $\tilde{X}^{RE}_{t,h}$. As for original portfolio, also for reinsurance written premiums composition we can obtain:

$$B^{RE}_{t,h} = P^{RE}_{t,h} + \lambda^{RE}_h \cdot P^{RE}_{t,h} + c^{RE}_h \cdot B^{RE}_{t,h}. \tag{4.3}$$

To evaluate characteristics of $\tilde{Y}^{NET}_t$, we can make some assumptions about total claims cost for the h-th LoB gross and net of reinsurance. The aggregate claim amount $\tilde{X}_{t,h}$ for each LoB in original insured portfolio is given by a collective approach where $\tilde{X}_{t,h}$ is assumed to be a mixed compound Poisson process as in CRM developed by [19] and [36], where claim size $\tilde{Z}$ have expected value m and coefficient of variation (CoV) $c_{\tilde{Z}}$.

Regarding premium risk only, both payments and reserves for claims incurred in previous years are necessarily covered by initial claims reserve and their volatility attains to reserve risk. Actually, reserving for outstanding claims is one of the central topics of modern actuarial theory and practise but it is here assumed that claims occurred in accounting year t they are finally settled in the same year and therefore no claims provision at the end of the year is needed.

As for the original aggregate claim amount, as well as for reinsurer's share $\tilde{X}^{RE}_{t,h}$, we can assume a mixed compound Poisson process:

$$\tilde{X}^{RE}_{t,h} = \sum_{i=1}^{\tilde{K}^{RE}_{t,h}} \tilde{Z}^{RE}_{i,t,h} \tag{4.4}$$

depending on the type and the parameters of the reinsurance treaty in force.

Reinsurance treaties are often arranged on a claim by claim basis (i.e., Excess of Loss coverages); this means that each separate claim is divided between cedant and reinsurer and we have no impact on claims count ($\tilde{K}_{t,h}^{RE} = \tilde{K}_{t,h}$), where clearly the claim size distribution must be an unconditional expression. With regard to severity, the size of an individual claim net of reinsurance can be written as

$$\tilde{Z}_{i,t,h}^{NET} = \tilde{Z}_{i,t,h} - \tilde{Z}_{i,t,h}^{RE}. \tag{4.5}$$

It is possible to model gross premiums and claims separately from the impact of reinsurance on them, allowing direct assessment of the effects of different reinsurance strategies both on a company's expected performance and solvency. Then we consider the aggregate claim amount net of reinsurance:

$$\tilde{\Gamma}_{t,h} = \tilde{X}_{t,h} - \tilde{X}_{t,h}^{RE} = \sum_{i=1}^{\tilde{K}_{t,h}} \tilde{Z}_{i,t,h}^{NET}, \tag{4.6}$$

that is defined as stochastic insurer's net retention.

We can find main characteristics and probability distributions of claim variables net of reinsurance, and of reinsurer's share, in order to assess risk profile of the "reinsured" insurer. The probability distribution function and the risk premium of the net aggregate claims cost can be obtained from a function of original aggregate claims amount and reinsurer's share, given the type and the parameters of the reinsurance treaty in force.

Introducing the new notation for reinsurance arrangements, risk reserve Equation (4.1) can be rewritten as:

$$\tilde{U}_t = U_{t-1} + \left[\sum_{h=1}^{L} \Big[(B_{t,h} - E_{t,h}) - (B_{t,h}^{RE} - C_{t,h}^{RE}) - \tilde{\Gamma}_{t,h}\Big]\right], \tag{4.7}$$

where technical result net of reinsurance for the period $(t-1, t)$ in the square bracket can be also written as:

$$\tilde{Y}_t^{NET} = \sum_{h=1}^{L} \Big[(B_{t,h} - B_{t,h}^{RE}) - (E_{t,h} - C_{t,h}^{RE}) - \tilde{\Gamma}_{t,h}\Big]. \tag{4.8}$$

For their natural implementation in CRM applied on insurance business, we will moreover concentrate on two classical reinsurance strategies, the Quota Share (QS) and the Excess of Loss (XL) treaties, showing how these arrangements impact on non-life insurers' risk profile. This kind of analysis is needed accordingly with new European insurance regulation that ask to European insurers to calculate their solvency capital requirements considering contracts arranged with reinsurers, which in turn are asked to fulfill minimum capital requirements introduced with Solvency II directive.

***Quota Share* and Collective Risk Model**

In QS treaties, moments of cedant's share ($\tilde{\Gamma}_{t,h}$) directly depends on initial aggregate claims amount since the former is a linear transformation of the latter, with proportional factor equal to the retained quota α_h. Then basic characteristics of the cedant's share can be computed by transforming the characteristics of the initial aggregate claim amount as in [18].

In the case of Quota Share reinsurance treaty, with insurer's retention quota for the h-th LoB α_h (with $\alpha_h \epsilon [0,1]$) fixed for each claim ($\tilde{Z}^{NET}_{i,t,h} = \alpha_h \cdot \tilde{Z}_{i,t,h}$), reinsurer's share for each claim and reinsurer total claims cost are $\tilde{Z}^{RE}_{i,t,h} = (1-\alpha_h) \cdot \tilde{Z}_{i,t,h}$ and $\tilde{X}^{RE}_{t,h} = (1-\alpha_h) \cdot \tilde{X}_{t,h}$ respectively, so that we obtain in a very trival way for the retained claims cost (4.6):

$$\tilde{\Gamma}_{t,h} = \alpha_h \cdot \tilde{X}_{t,h}. \tag{4.9}$$

Then basic characteristics of the cedant's share is computed by transforming the characteristics of the initial aggregate claim amount in order to obtain:

$$\begin{aligned} \mathbf{E}(\tilde{\Gamma}_{t,h}) &= n_t \cdot \alpha \cdot m_t = \alpha \cdot \mathbf{E}(\tilde{X}_{t,h}). \\ \sigma^2(\tilde{\Gamma}_{t,h}) &= \sigma^2(\tilde{X}_{t,h}) \cdot \alpha^2 \\ \gamma(\tilde{\Gamma}_{t,h}) &= \gamma(\tilde{X}_{t,h}) \end{aligned} \tag{4.10}$$

In term of solvency it is important to know how Quota Share reinsurance changes the risk profile of the primary insurer. Notwithstanding that both the mean and the standard deviation of the net aggregate claim cost are less than in the original situation, they are decreasing by the same proportion α, then the coefficient of variation is not changed from the original situation.

On the other hand, the skewness of the net aggregate claims amount is not affected by quota share reinsurance since skewness operator is invariant to linear transformations. In other words Quota Share reinsurance introduced here has no impact on the right tail of the initial aggregate claim cost so that the risk profile of the insurer is practically the same.

Written premium volume ceded to reinsurer (4.3) is:

$$B^{RE}_{t,h} = (1-\alpha_h) \cdot B_{t,h} = (1-\alpha_h) \cdot \Big[(1+\lambda_h) \cdot P_{t,h} + c_h \cdot B_{t,h}\Big], \tag{4.11}$$

whereas reinsurer commission paid by reinsurer to primary insurer is $C^{RE}_{t,h} = c^{RE}_{t,h} \cdot B^{RE}_{t,h}$ with a constant resinsurance expenses loading coefficient commonly established by reinsurer.

Reinsurer's safety loadings coefficient is usually fixed at the same value of the coefficient charged by the cedant ($\lambda^{RE}_h = \lambda_h$). It implies that the reinsurer may agree to fix reinsurance commission to pay back to the primary insurer which provided the business, but he accepts not to charge a different safety loadings coefficient than cedant; in this way the real reinsurance pricing process is nested in the commission rate mechanism.

In other words, in Quota Share it is usually assumed that both insurer and reinsurer agree to charge the same safety loadings coefficient on risk premiums so the latter is accepting the underwriting policy of primary insurer. Indeed, they are simply sharing initial portfolio variability as well as profit achieved from the contracts in force with original policyholders. Then it is implicitly assumed that insurer and reinsurer are using same technical basis, i.e., frequency, severity, etc.., in premium rating. Gross-to-net adjustment factor for premium volume of a single LoB in the case of QS reinsurance is given by $B^{NET}/B = \alpha_h$.

Consequently, equation of the insurance technical result (4.8) after QS reinsurance is given by:

$$\tilde{Y}_t^{NET} = \sum_{h=1}^{L} [[\mathbf{E}(\tilde{\Gamma}_{t,h}) + (\lambda_h \cdot P_{t,h}^{NET} - \Delta c_{t,h}^{RE} \cdot B_{t,h}^{RE})] - \tilde{\Gamma}_{t,h}], \tag{4.12}$$

where the term $\Delta c_{t,h}^{RE} = c_{t,h} - c_{t,h}^{RE}$ represents reinsurance pricing effect on QS treaty, applied directly on premium volume ceded to reinsurer. Usually the difference is non-negative and it assumes the value zero when reinsurer refunds entirely to primary insurer the same expenses loading coefficient charged on the original risk premium volume.

With regards to safety loading coefficient, it's decreasing by retained quota only and it derives from assumption that both cedant and reinsurer use same technical basis as described above. On the other hand, pure premium is decreasing by retention quota as well as the aggregate claims cost.

***eXcess of Loss* and Collective Risk Model**

In a per risk eXcess-of-Loss (XL) treaty, signed for a single LoB[7], for the projected year t reinsurer pays the excess $\tilde{Z}_{i,t}^{RE} = \max[0; \tilde{Z}_{i,t} - M_t]$ over an agreed amount in respect of each claim M_t, called priority, projected over the following years by the claim inflation rate (in a similar way as $\tilde{Z}_t$ is rescaled over each year as afore mentioned). In the next part of this section, for sake of simplicity, we will disregard the time index t.

In this paper, we are concerned with per claim excess of loss reinsurance, where the cedant's retention is defined for each claim in a certain group of risks, reinsurer paying the excess if a claim exceeds the retention level M, and there is no limit to reinsurer exposure. Cedant's share of the claim is $\tilde{Z}_{i,t}^{NET} = \min[\tilde{Z}_{i,t}; M_t]$.

It is worth emphasizing that, under assumptions that the aggregate claim amount $\tilde{X}$ is a compound variable with claim size distribution function $S(\tilde{Z})$ and that all risks are reinsured using the same retention limit M, the net aggregate claim amount $\tilde{\Gamma}$ is also a compound variable having the same claim number variable $\tilde{K}$ but with claim size d.f. S_M that depends from the d.f. of the gross claim size:

$$S_M(\tilde{Z}) = \begin{cases} S(\tilde{Z}) & \tilde{Z} < M \\ 1 & \text{otherwise} \end{cases} . \tag{4.13}$$

More generally, k-th moments of the cedant's share of a claim are given by:

$$a_{k,\tilde{Z}^{NET}} = \mathbf{E}(\tilde{Z}^{NET^k}) = \int_{-\infty}^{M} Z^k \, dS(Z) + M^k \cdot (1 - S(M)) \tag{4.14}$$

consequently, risk indices of claim size distribution (see S2002) are $r_{k,\tilde{Z}^{NET}} = a_{k,\tilde{Z}^{NET}}/(a_{1,\tilde{Z}^{NET}})^k$.

Moments of the reinsurer's share can be obtained from Equation (4.13) as:

$$a_{k,\tilde{Z}^{RE}} = \sum_{i=1}^{k} \binom{k}{i} \cdot (-M)^{k-i} \cdot [a_i - a_{i,\tilde{Z}^{NET}}], \tag{4.15}$$

where $a_i = a_i(\infty) = a_{k,\tilde{Z}}$. Note that value of $\tilde{Z}^{RE}$ is zero whenever the size of a claim is smaller than the retention limit M.

The main characteristics of cedant's share in an excess of loss treaty can now be easily computed through basic characteristics of the cedant's share of individual claims net of reinsurance $\tilde{Z}^{NET}$.

With regard to severity, many distributions may be used for modeling cost of a single claim in actuarial practice and, among others, LogNormal and Pareto distribution are the most popular; the latter usually better fits heavy tail distribution whereas the former often represent so-called attritional claim[8]. On the other hand, the Solvency II framework has defined a standardized capital requirement assuming a LogNormal distribution not only for premium risk, one of the main risk factors

[7] We avoid to refer to h-th LoB in some formulas of this subsection for practical purpose

[8] See CSZ2014 and EIOPA Calibration Paper on Premium and Reserve Risk

of non-life insurance, but also for reinsurance mitigation recognition. Anyhow, the lognormal assumption is taken in our simulation model but it is not clearly failing the consistency of our general framework.

Under the assumption of LogNormality for the claim size distribution, the k-th moments of the net claim size are given by:

$$a_{k,\tilde{Z}^{NET}} = e^{k\mu + \frac{1}{2}k^2\sigma^2} \cdot F_{\mu+k\sigma^2,\sigma}(M) + M^k \cdot [1 - F_{\mu,\sigma}(M)], \tag{4.16}$$

where $F_{\mu,\sigma}(M)$ is the cumulative distribution function of a r.v. LogNormal distributed with generic parameters μ and σ computed in point M. Parameters are obtained by the well-known Normal-to-LogNormal parameters changing formulas $\mu = 2 \cdot \ln m - \frac{\ln a_2}{2}$ and $\sigma^2 = \sqrt{\ln a_2 - 2 \cdot \ln m}$.

Then net expected claim size can be obtained as:

$$m_M = m \cdot F_{\mu+\sigma^2,\sigma}(M) + M \cdot [1 - F_{\mu,\sigma}(M)], \tag{4.17}$$

in which we can fix the retention limit as $M = \mathbf{E}(\tilde{Z}) + k \cdot \sigma(\tilde{Z})$ where k is a multiplier of claim size standard deviation. Therefore, as for expected claim size, also net risk premium given by the formula $P_M = n \cdot m_M$ is a concave, continuous, increasing function of retention limit M (See Figure 4.1).

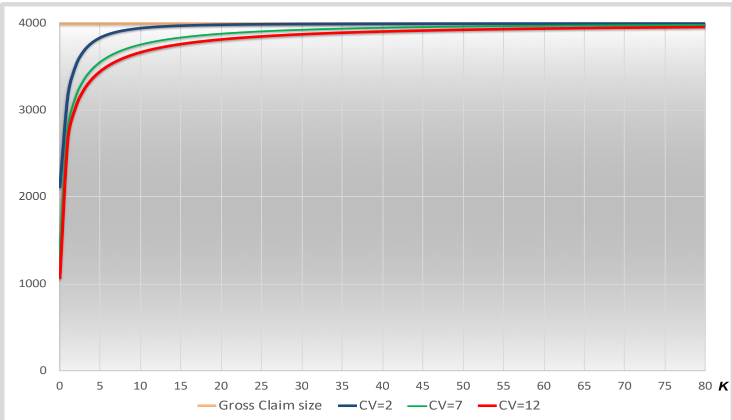


Figure 4.1. Expected single claim size m_M for different $c_{\tilde{Z}}$ and retention limit (by multiplicative factor k).

With regard to reinsurer risk premium P^{RE}, it is given by the well-known relationship:

$$P^{RE} = \mathbf{E}(\tilde{X}^{RE}) = \mathbf{E}(\tilde{K}) \cdot \mathbf{E}(\tilde{Z}^{RE}) = n \cdot (m - m_M). \tag{4.18}$$

In the present model, expected value of $\tilde{Z}^{RE}$ is easily computed taking into account the LogNormal distribution with given initial parameters rescaled every year by the inflation rate i only. Reinsurance risk premium will be relatively small compared to the original risk premium if retention limit is growing up.

We compute the second moment of single claim cost in order to study its retained volatility, from formula (4.16) we obtain:

$$a_{2,\tilde{Z}^{NET}} = (\sigma_Z^2 + m^2) \cdot F_{\mu+2\sigma^2,\sigma}(M) + M^2 \cdot [1 - F_{\mu,\sigma}(M)] \tag{4.19}$$

and consequently also net risk indices change (See Figure 4.2)

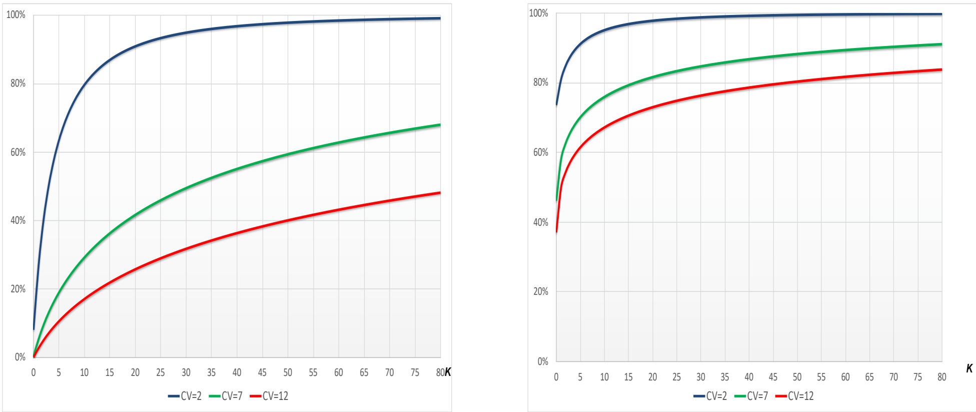


Figure 4.2. Gross-to-net $a_{2,\tilde{Z}}$ (**Left**) and $r_{2,\tilde{Z}}$ (**Right**) for different $c_{\tilde{Z}}$ and retention limit (by multiplicative factor k).

Then, in the case of per risk XL treaty the reinsurer's aggregate claim amount is also a compound variable, but the number of non-zero claims is usually much smaller for a reinsurer, then stochastic claim amount charged to the reinsurer for year t is:

$$\tilde{X}^{RE} = \sum_{i=1}^{\tilde{K}} \max[0; \tilde{Z}_i - M]. \tag{4.20}$$

The retained claims cost (4.6) in excess of loss reinsurance is given by the following formula:

$$\tilde{\Gamma} = \sum_{i=1}^{\tilde{K}} \min[M; \tilde{Z}_i]. \tag{4.21}$$

Its basic characteristics directly depend by the lowest three net retained moments about the origin $a_k(\tilde{Z}^{NET})$ introduced above:

$$\begin{gathered}
\mathbf{E}(\tilde{\Gamma}) = n \cdot m_M = \mathbf{E}(\tilde{X}) \cdot \left(\frac{m_M}{m}\right), \\
\sigma(\tilde{\Gamma}) = n \cdot a_{2,\tilde{Z}^{NET}} + n^2 \cdot m_M^2 \cdot \sigma_q^2 = (n \cdot m)^2 \cdot \left(\frac{r_{2,Z}}{n} \cdot \frac{a_{2,\tilde{Z}^{NET}}}{a_{2,Z}} + \sigma_q^2 \cdot \left(\frac{m_M}{m}\right)^2\right), \\
\gamma(\tilde{\Gamma}) = \frac{n \cdot a_{3,\tilde{Z}^{NET}} + 3n^2 \cdot m_M \cdot a_{2,\tilde{Z}^{NET}} \cdot \sigma_q^2 + n^3 \cdot m_M^3 \cdot \mu_{3,q}}{(n \cdot a_{2,\tilde{Z}^{NET}} + n^2 \cdot m_M^2 \cdot \sigma_q^2)^{\frac{3}{2}}}.
\end{gathered} \tag{4.22}$$

It can be easily shown how this kind of reinsurance changes primary insurer risk profile. Under assumptions of the CRM made above, the ratio between coefficient of variation of the aggregate net and gross claims cost is:

$$\frac{\mathrm{CoV}(\tilde{\Gamma})}{\mathrm{CoV}(\tilde{X})} = \sqrt{\frac{\frac{1+c_{\tilde{Z^{NET}}}}{n} + \sigma_q^2}{\frac{1+c_{\tilde{Z}}}{n} + \sigma_q^2}}. \tag{4.23}$$

Equation (4.23) for different c_Z and retention limit (by the multiplicative factor k) is reported in the next Figure 4.3, where all ratios between net and gross CoV of the aggregate claims cost stay between 80% and 100%, decreasing as the CoV of single claims cost of the LoB grows.

It emphasizes that adjustment factor for non-proportional reinsurance provided by Solvency II delegated regulation—it is 80% for some LoBs, potentially decreased by using USP for this factor—has been fixed as the lower bound of the range of y-values in order to emphasize this potential shortfall.

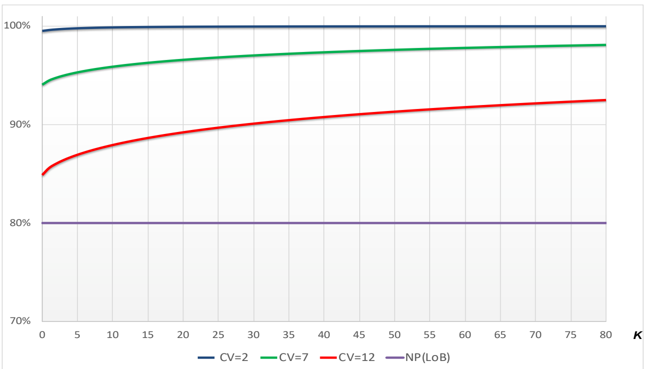


Figure 4.3. Gross-to-net coefficient of variation (CoV) for different $c_{\tilde{Z}}$ and retention limit (by the multiplicative factor k).

Now we analyze impacts of XL reinsurance on both solvency and profitability terms in Equation (4.8) by reintroducing segmentation between several LoBs. In the case of XL reinsurance treaty, for each LoB the safety loading coefficient λ_h^{RE} is kept constant over the full time horizon and it is assumed deterministic.

Safety loading coefficient in non-proportional treaties is usually greater than the one applied by insurer, and it increases as insurer's retention limit is growing up. No explicit commissions are usually provided in case of excess of loss coverage, so that for reinsurance written premiums (4.3) we get:

$$B_{t,h}^{RE} = (1 + \lambda_h^{RE}) \cdot P_t^{RE} \qquad \text{and} \qquad c_{t,h}^{RE} = 0. \tag{4.24}$$

Under the assumptions made above equation of insurance net technical result

(4.8) after XL reinsurance can be rewritten as:

$$\tilde{Y}_t^{NET} = \sum_{h=1}^{L} \Big[[\mathbf{E}(\tilde{\Gamma}_{t,h}) + (\lambda_h \cdot P_{t,h} - \lambda_h^{RE} \cdot P_t^{RE}] - \tilde{\Gamma}_{t,h} \Big]. \tag{4.25}$$

Reserve risk

On a general basis, the claim reimbursements that have not yet been paid at the end of the financial year imply the claims reserve. The nature of such a balance-sheet item estimate is a major risk source for non-life undertakings, due to the problems that its potential underestimation can bring about.

In order to get a proper quantification, actuarial methodologies have already been an integral part of the specific estimation process for a long time. This is often due to the lacking of the case by case assessment of each claim file adopted by companies to calculate the ultimate cost representation for the long tail branches.

Despite, in comparison with the traditional methods, stochastic methodologies are less ready-to-use, they have several advantages: they are based on explicit and coherent statistical hypothesis; they get the ad hoc adjustments and discretionary estimates to the minimum; beyond the very accurate best estimate of reserve, they provide confidence intervals of the reserve itself in line with fixed probability levels. Notably, through these methodologies, it is always possible to get to an estimate of the first order-moments mean and the second-order moments variance of the reserve distribution. Of course, also the overall probability distribution can be deducted, either through analytical methods—if further appropriate hypothesis are adopted—or through simulation techniques.

Regards reserve risk in Solvency II framework, it is defined as fluctuations in technical result of the Claims Development Result (CDR) in a one-year time-framework. The CDR is the technical result of the evolution of the claim settlement process. In other words, it calculates if the claims reserve $V^S_{t,h}$ — set aside in the generic t balance-sheet year, for the $h-th$ LoB — is enough to pay the claims $\tilde{X}^{paid,PY}_{t,h}$, between t and $t+1$ and to set aside the new claims reserve $\tilde{V}^S_{t+1,h}$ in $t+1$. It is a random variable if the observation moment is t, while it is a deterministic value if the observation moment is $t+1$.

In the risk estimate and solvency capital calculation framework, we are interested in t observation random variable, while, in the balance-sheet analysis framework, we are interested in the deterministic aspect observed in $t+1$:

$$C\tilde{D}R_{t,h} = V^{S,PY}_{t,h} - (\tilde{X}^{paid,PY}_{t,h} + \tilde{V}^{S,PY}_{t+1,h}) \tag{4.26}$$

We can then enrich equation of the insurance technical result (4.8) as:

$$\tilde{Y}^{NET}_t = \sum_{h=1}^{L} \Big[(B_{t,h} - B^{RE}_{t,h}) - (E_{t,h} - C^{RE}_{t,h}) - \tilde{\Gamma}_{t,h} + C\tilde{D}R_{t,h} \Big]. \tag{4.27}$$

Particularly, we have a loss if $CDR_{t+1,h} < 0$, while we have a gain with a positive result. Claims paid for previous years portfolio $\tilde{X}^{paid,PY}_{t,h}$ and the new claims reserve in $t+1$ for open claims $\tilde{V}^S_{t+1,h}$, are assumed to be Compound Mixed Poisson Process as for premium risk.

The use of stochastic methods has been consolidated through the Solvency II project, by reaching a co-ordinated target in a probabilistic key (best estimate added to risk margin) as a prescribed requisite to estimate the claims reserve and the reserve risk capital. Indeed, in such a framework, an exact definition of best estimate, risk margin and reserve risk capital can only be provided by the application of a stochastic model of estimate to the historical time-series of the claims. Here, the risk margin is additional and aimed at clearly quantifying the risk capital yield according to the uncertainty level of the cash flows to come.

In stochastic estimates—beyond the financial kind of uncertainty, linked to the investments yields and to the legal aspects connected with the paying-off delay—three kinds of risk must be taken into account: model risk, estimation risk, and process risk. Model risk means the risk that an unfitting model could be used to represent the phenomenon; the estimation risk is linked to the volatility of the estimator used in order to infer on the model parameters; the process risk is linked to the variance of the phenomenon under scrutiny.

In order to create a connection with the previous practice many of the stochastic models for the reserving have been built by widening the traditional deterministic techniques, particularly the well-known chain ladder methodology, based on the development of the cumulative payments. Keeping this in mind, we need to emphasise that some of the most used stochastic methods—Mack and ODP just allow to make automatic estimates of the reserve and only apply when the basic chain ladder hypothesis are met. The claims reserving working party of 2002 British actuaries has spotted nothing short of 26 qualitative factors to be taken into account in the claims reserving. Nonetheless, this limit is more neglectable in determining the capital requisite because it is function of a volatility quantification.

As to the estimate of cash flows of future payments which have already happened, and to estimate the different kinds of risk that have to be taken into account in the risk margin assessment, this work uses the stochastic models evolution included in the GLM class. It is known that such models allow for using different distributions for the response variable and the explicative variable's parameters which are estimated to be linked to the response variable.

Therefore, different traditional methods to estimate the claims reserve can be reviewed under this light; as we have already said, claims reserve estimates resulting from particular generalized linear methods indeed match with the ones resulting from deterministic methods to estimate the claims reserve, such as the chain ladder and the separation methods.

In non-life undertakings, in order to estimate the claims reserve for accidents still to be paid generated by an insured risk portfolio at the end of the financial year, we generally make reference to the historical payments triangle, updated at the estimate date.

Notably, we assume that the observations concerning payments already made are connected to accidents happened in a limited previous time-framework; thus, sums paid for accidents happened or generated in previous years are available in this kind of diagram. For each accident year, data are divided into development years, a variable which quantifies the claim payment year.

The idea underlying the chain ladder method is that there is a proportion between the cumulative payments of two close development years, except for an erratic component with a null mean:

$$\tilde{C}^{t}_{i,j+1} = C_{i,j} \cdot f^{t}_{j} \cdot \tilde{\epsilon}_{i,j}. \tag{4.28}$$

looking at Equation above, we conclude that, in the chain ladder model, the cumulative payment are showed by a line through the origin for each j development year.

4.2 SCR: one-year view evaluation

To show the effect of a Partial Internal Model (PIM) for premium risk in the case of reinsurance based on a CRM consistent with the framework introduced in previous Section, three non-life insurance companies with a different size are considered (their figures are summed up in Table 4.1). It is assumed that all insurers underwrite contracts in same three LoBs (MVL, OM, and GL) with the same mix of portfolio (proportions used are approximately the real proportions in the Italian insurance market for these three LoBs). In the beginning, comparison of results gross of reinsurance will allow us to describe the effects of a different portfolio dimension on the aggregate claim amount distribution and so on the capital requirement. Then, it will be possible to assess also impacts of reinsurance strategies on insurers risk profile.

Table 4.1. Baseline Portfolio Mix in terms of Gross premium volumes (amounts in million of Euro).

LoBs	**Omega**		**Tau**		**Epsilon**		**All Insurers**
	B_t	B_{t+1}	B_t	B_{t+1}	B_t	B_{t+1}	$B_{t,h}/\sum_h B_{t,h}$
MVL	600	630	300	315	60	63	60%
OM	200	210	100	105	20	21	20%
GL	200	210	100	105	20	21	20%
TOTAL	1000	1050	500	525	100	105	100%

The main parameters of CRM are in Table 4.2. Insurers have the same characteristics apart from expected number of claims denoting the size of Insurer.

Omega is assumed to be five times larger than Tau, and ten times larger than Epsilon in terms of GWP. Omega, Tau, and Epsilon are assumed to be hypothetical insurance companies representative of the Italian insurance market, whose parameters have been calibrated by historical results observed during the last 12 years (time horizon 2006–2017) and contained in Year End balance sheets of non-life Italian insurers reported every year by Italian insurers Association (ANIA) on its website, also on an aggregate basis.

In particular, parameters calibration have been based on data from some specific sheets required by Italian insurers supervision (IVASS) and very similar to EIOPA's Quantitative Reporting Templates related to claim processes. Standard deviation of systematic volatility (σ_q) and the safety loading coefficient (λ) are obtained by Italian market Loss Ratios and Combined Ratios.

Regarding the latter, it depends by the average of empirical combined ratios observed during the previous 12 years; negative values of this coefficient represent LoBs where an average of combined ratios greater than 100% (e.g., in GL) is registered. The CoV of claim size c_Z is fixed, for each LoB, and calibrated on the basis of volatility of single claim size observed in empirical datasets of non-life insurance companies in the previous calendar years.

Furthermore, a dynamic portfolio is assumed and then n_t and m_t, reported in Table below for the initial year for each LoB considered, will increase accordingly with annual rate of real growth g as to frequency and annual claim inflation rate i as to severity, assumed to be almost 2% and 3% respectively for all LoBs in the simulations.

Table 4.2. Parameter for Collective Risk Model (CRM) analysis of Premium Risk.

Insurer	LoBs	n_t	σ_q	g	m_t	c_Z	i	λ	c
Omega	MVL	114,846.03	7.9%	1.95%	4,000	7	3%	2.8%	21.3%
	OM	51,594.74	12.1%	1.95%	2,500	2	3%	8.9%	29.8%
	GL	14,260.81	14.7%	1.95%	10,000	12	3%	−4.4%	31.8%
Tau	MVL	57,423.74	7.9%	1.95%	4,000	7	3%	2.8%	21.3%
	OM	25,797.01	12.1%	1.95%	2,500	2	3%	8.9%	29.8%
	GL	7,130.40	14.7%	1.95%	10,000	12	3%	−4.4%	31.8%
Epsilon	MVL	11,484.60	7.9%	1.95%	4,000	7	3%	2.8%	21.3%
	OM	5,159.47	12.1%	1.95%	2,500	2	3%	8.9%	29.8%
	GL	1,426.08	14.7%	1.95%	10,000	12	3%	−4.4%	31.8%

Table 4.3 shows some features of reinsurance structures as retained quota and reinsurance commission rate for QS and attachment point, net single claim cost, and safety loading coefficient applied on reinsurer premium for XL. Furthermore, for each strategy we assume an alternative reinsurance pricing that is unfavorable for the primary insurer but more realistic.

With respect to QS treaty, we assume that retention quota is fixed for each LoB and we investigate on the different impact given by deterministic reinsurance commissions in case they are less or equal to insurer's expense loading. For the bottom, loss in expenses loading is represented as a percentage of expenses loading coefficient of primary insurer ($\Delta c_h^{RE,2} \geq 0$) in (4.12)), it has been fixed at $20\% \cdot c_h$ for each LoB.

For XL treaty, retention limits per LoB $M_{t,h}$ are equal to the sum of average claim cost and a fixed multiplier k of the standard deviation: $M_{t,h} = \mathbf{E}(\tilde{Z}_{t,h}) + k_h \cdot \sigma(Z_{t,h})$ with $k_h = 15$ for MVL and GL and $k_h = 5$ for OM, independently from insurer's size. Usually this multiplier is greater when a bigger insurer is considered but at the same time a greater multiplier has been assumed for long tails LoBs as MVL and GL.

Furthermore, a more realistic scenario with a reinsurer safety loading coefficient greater than primary insurer safety loading coefficient is considered taking into account the reduction in variability of the ceding company, and then for LoBs with higher (lower) $c_{\tilde{Z}}$ we will assume an higher (lower) λ_h^{RE}, this is particularly important for OM and GL.

For the bottom, a favourable reinsurance pricing can be assumed because of the low reduction of variability given by XL on risks of this LoB; for the latter, where a negative safety loading coefficient has been assumed in original portfolio, we will consider a very high reinsurance safety loading coefficient.

Second scenario for both reinsurance strategies would represent a more realistic arrangement that can be achieved in reinsurance market, and we named it High scenario.

Table 4.3. Parameters of reinsurance strategies.

LoBs	**Quota Share**			**eXcess of Loss**			
	α_h	$\Delta c_h^{RE,1}$	$\Delta c_h^{RE,2}$	$M_{t,h}$	m_M	$\lambda_h^{RE,1}$	$\lambda_h^{RE,2}$
MVL	95%	0	$20\% \cdot c_{MVL}$	424,000	3,831	2.8%	5%
OM	90%	0	$20\% \cdot c_{OM}$	27,500	2,398	8.9%	1%
GL	85%	0	$20\% \cdot c_{GL}$	1,810,000	9,393	−4.4%	10%

The next Table 4.4 reports characteristics of simulated distribution of losses for total portfolio and for each LoB, obtained using software R (R Core Team 2018, Vienna, Austria) and applying 100,000 simulations so that a stable convergence of results with exact moments can be assured. CoV and skewness of gross and net of reinsurance aggregate cost of next-year claims are figured out.

Table 4.4. Coefficient of variation (CoV) and skewness of simulated distribution for each LoB (Baseline Portfolio Mix).

Insurers	**LoBs**	$\tilde{X}_{t,h}$		$\tilde{\Gamma}_{t,h}^{QS}$		$\tilde{\Gamma}_{t,h}^{XL}$	
		CoV	γ	**CoV**	γ	**CoV**	γ
Omega	MVL	8.1%	0.17	8.1%	0.17	7.9%	0.14
	OM	12.0%	0.23	12.0%	0.23	11.9%	0.22
	GL	16.3%	1.15	16.3%	1.15	13.9%	0.28
	TOTAL	8.4%	0.32	8.4%	0.30	8.0%	0.15
Tau	MVL	8.4%	0.19	8.4%	0.19	8.1%	0.14
	OM	12.0%	0.23	12.0%	0.23	12.0%	0.23
	GL	19.7%	3.17	19.7%	3.17	14.8%	0.28
	TOTAL	8.8%	0.37	8.7%	0.34	8.1%	0.15
Epsilon	MVL	10.2%	0.82	10.2%	0.82	8.9%	0.16
	OM	12.4%	0.25	12.4%	0.25	12.3%	0.25
	GL	32.4%	6.02	32.4%	6.02	20.4%	0.46
	TOTAL	11.9%	1.93	11.6%	1.74	9.3%	0.25

We have that the bigger insurer shows for several LoB a CoV not so far from value of the standard deviation of the structure variable $\tilde{q}$ due to a relevant diversification effect given by the size, and results put in evidence that effect of non-pooling risk is significant for OM. The presence of a size factor can be noticed by the greater increase of variability showed by LoB with high coefficient c_Z as GL of smaller companies with respect to bigger insurers.

Focusing on the reinsurance effect, proportional and non-proportional treaties affect distribution of the total claims cost in a very different way. As can be expected, QS reinsurance intuitively does not change characteristics of total claims distributions as CoV and skewness for single LoBs, while mean and standard deviation are simply scaled by the retention quota α_h. XL reinsurance, as well-known, has different impacts on characteristics of aggregate claims cost (for each LoB) according to the magnitude of $c_{\tilde{Z}}$ of LoB considered and it hardly changes aggregate claims cost distributions of LoBs with higher $c_{\tilde{Z}}$ as MVL and GL (See Figure 4.4). Particularly, CoV and skewness of net distributions for each LoB decrease thanks to this kind of reinsurance. Nevertheless, the relative effect of XL is higher as the size of the company decreases, because of a higher pooling risk of smaller insurers.

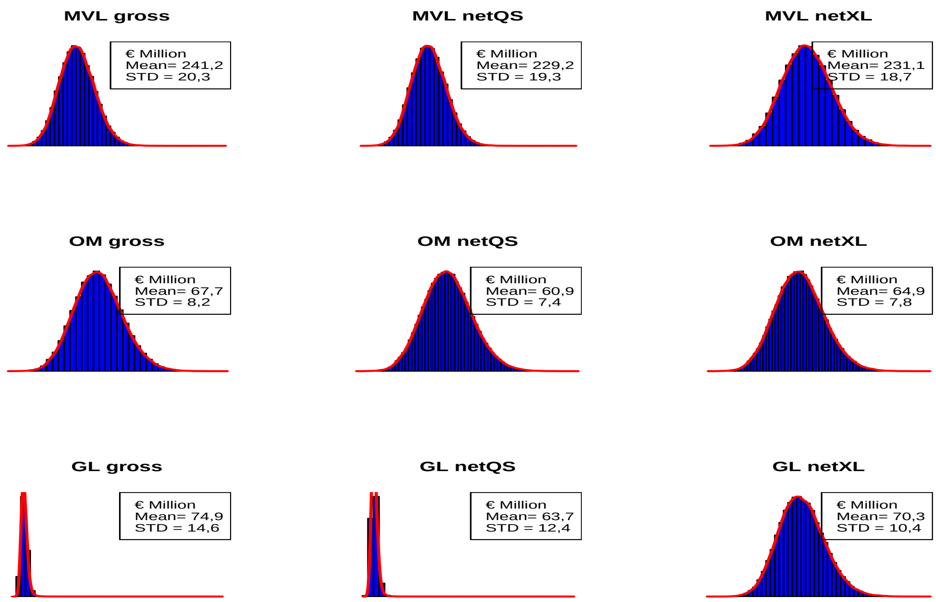


Figure 4.4. Simulated distributions of Aggregate Claims Cost for Tau (for each Line of Business (LoBs)).

Aggregated distributions have characteristics very similar to MVL because of the high weight of this segment in insurers' portfolios, and differences arise as the size of the insurer decreases, but the same comments on reinsurance effects made for a single LoB hold here. Total claims cost distribution from gross to net of QS is affected by only differences arising from applying different retentions between LoBs, and the relative effect decreases as the size of insurer grows. XL reinsurance effects on volatility and shape of aggregated distribution on single LoB basis apply also on aggregate basis. As can be seen from Table 4.5, the decrease in either CoV and skewness is higher for Epsilon.

The aggregation between LoBs has been derived with a simple Gaussian copula, where parameters of the copula function have been calibrated by using the correlation matrix proposed by SF in delegated acts (see Figure 4.5). Alternatively, we have to consider a hierarchical structure based on Archimedean copulas, more properly considering significant tail dependency between several LoBs (see CV2019, SC2009, and SC2011).

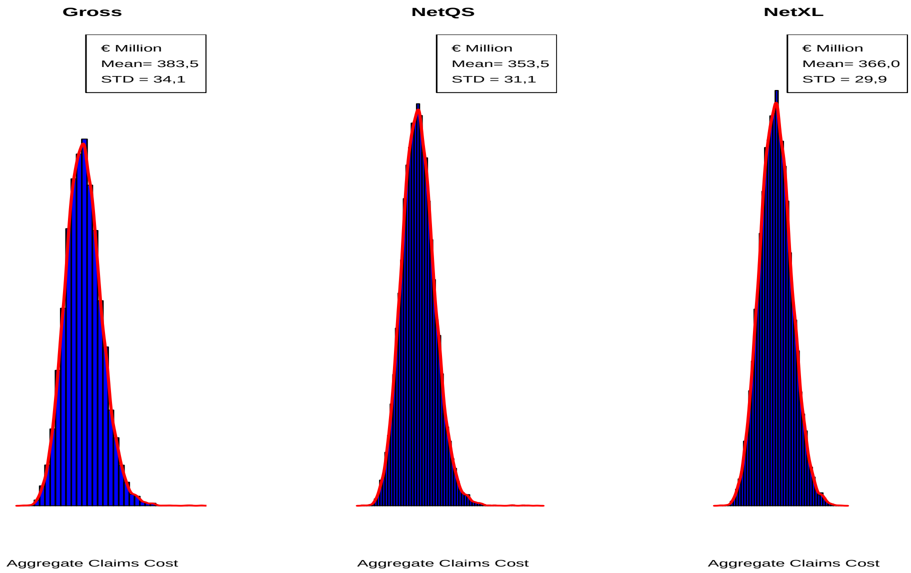


Figure 4.5. Simulated distributions of Total Portfolio Aggregate Claims Cost for Tau.

Distributions of gross and net technical results are strictly related to distributions of total losses of the portfolio (see Figure 4.6). Mean and volatility of technical result decrease because of reinsurance, whereas distributions are characterized by a negative skewness since the r.v. $\tilde{Y}^{NET}$ is negatively affected by the aggregate claim amount distribution. It is worth mentioning that in proportional reinsurance pricing can have a significant impact on capital requirement when an IM is applied since it affects the mean of technical result distribution in such a way that an higher pricing can hardly move the whole distribution, and so the quantile 0.50%, left.

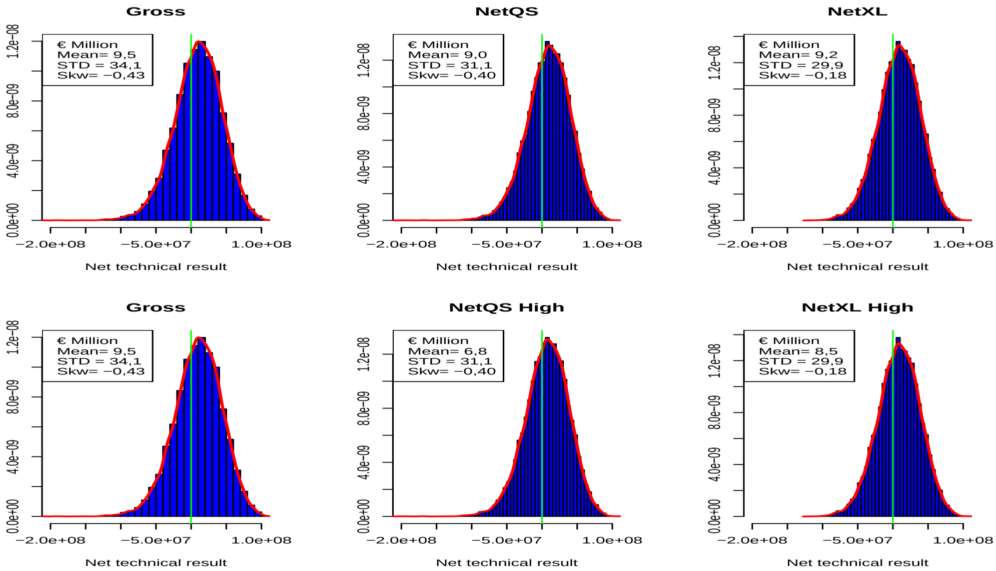


Figure 4.6. Simulated Net technical result distributions for Tau (green lines on x-value 0), according to different reinsurance strategies.

4.3 From Short-term to Medium-term assessment

One of the key indicators of insurance business after Solvency II is the Solvency Ratio (SR), defined as the ratio between the OF and the SCR at time of evaluation. The former represents the available financial resources of an insurance company at the end of a calendar year as the sum of the initial capital and the technical result, the latter has been already commented and analyzed.

In the methodological framework given in the previous sections the SR at the time t can be computed as:

$$\mathrm{SR}_t = \frac{U_t}{\mathrm{SCR}_t}, \tag{4.29}$$

where we assume that the capital at the end of the year U_t has been observed and so it is not stochastic, whereas SCR_t is assumed to be at the time of its calculation.

As is well-known, SCR computed by either SF or IM considers only one year of new business. In the consideration of the overall solvency needs required by Solvency 2 Pillar II to be reported in the ORSA report, the risks resulting from the perspective business shall be taken into account, and then Solvency II ratio projections are made on the basis of the strategic business plan for a time horizon of 3 or 5 years.

So we need a methodology in order to consider capital adequacy over the planning period, and the dynamic context as the evolution of assessment framework is provided by the projection of future financial position including capital requirements (SCR_t) and Own funds (U_t), fixing a target capital ratio that insurers must not breach in order to have an appropriate capital planning. These amounts are based on multiple assumptions and parameters (combined ratios, expected dividends, etc.) and the LoB evolution shall be considered.

It is worth mentioning that this method is appropriate for the business plan where there are no material change in the risk profile during the considered time horizon.

With regard to Own Funds projection, it considers at least the following elements: run-off of the inforce business, new business expected to be written over the strategic plan horizon and the market evolution (including underwriting cycles).

Then the r.v. $\tilde{U}_t$ is estimated in each year t as the sum of initial own funds and the net expected profit of the year:

$$\mathbf{E}(\tilde{U}_{t+1}) = U_t + \mathbf{E}(\tilde{Y}_{t+1}^{NET}). \tag{4.30}$$

In the projections of the Solvency Capital Requirement over the following 3 or 5 years, the results of the last year calculation are the starting point of the calculation of the following years. SCR figures are then projected for each sub-risk and aggregated to the total SCR, using assumptions on the run-off of the inforce business and new business mix consistent with the ones used for the projection of own funds/strategic plan.

We can use Equation (4.29), where over the following years the key drivers used to project the SCR are the premium volume and the business mix development.

The probability distributions of gross and net technical results of the valuation year and over the following 3 years are simulated and reported in the next Figure 4.7.

Reinsurance affects yearly the gross technical result in a similar way as it does at the first year, reducing the mean and the volatility.

On the other hand, distributions are characterized by a less negative skewness during the following years, because of the size factor effect given by the volume growth.

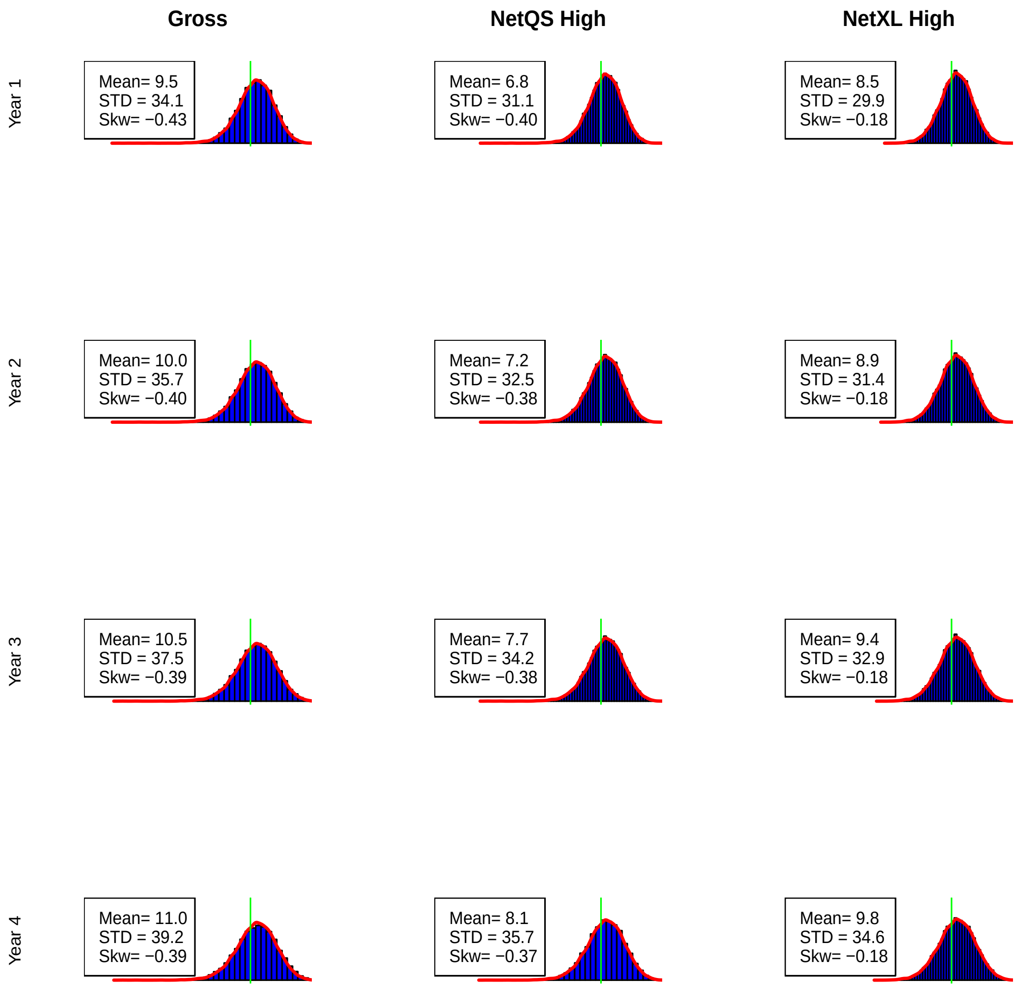


Figure 4.7. Simulated Gross and Net technical result distributions for Tau (green lines on value 0), during the following 4 years—Baseline Portfolio mix (the k-th row represents $\tilde{Y}_k$ distribution).

Once both OF and SCR (using IM) have been projected, the Solvency II ratio (4.29) for the years over the first can be easily derived by calculating the ratio between the two amounts as given above for $k = 2, 3$:

$$\mathbf{E}(\tilde{\mathrm{SR}}_{t+k})_{\mathrm{ORSA}} = \frac{\mathbf{E}(\tilde{U}_{t+k})}{\mathrm{SCR}_{t+k}} = \frac{\mathbf{E}(\tilde{U}_{t+k-1}) + \mathbf{E}(\tilde{Y}^{NET}_{t+k})}{\mathrm{SCR}_{t+k}}. \tag{4.31}$$

When this ratio is higher than the target capital ratio over the plan period, this allows potential dividend payments each year. This is related with the Risk Appetite Framework, one of the key pillar of the risk management system, which provides risk governance tool to set risk limits and monitor risk positions.

The next Table 4.5 reports SR for company Tau over the following three years according to the two different reinsurance strategies already used in the paper.

The initial capital U_0 has been fixed at 25% of the initial GWP B_0 (500 million euro in any scenario), so that the starting SR in the gross of reinsurance case is 135% and this value can be interpreted as the initial Risk Appetite of Tau in the Baseline scenario.

In the baseline scenario insurer shows stable results in terms of both OF and SCR and so on SR, because of the high weight on portfolio of a stable LoB as MVL.

SR increases over the years, remaining over 135% in any case, whereas net of reinsurance cases show higher values of the ratio than can be used when an higher SR would be achieved (i.e., in capital management perspective).

Table 4.5. Solvency Ratio (SR) for Tau during the following 3 years (amounts in mln of Euro) (Baseline Ptf—20% GL).

	t	**0**	**1**	**2**	**3**
Gross	OF	125.0	134.5	144.5	154.9
	SCR	92.8	96.1	101.4	105.7
	SR	135%	140%	142%	147%
Net QS High	OF	125.0	131.8	141.7	152.1
	SCR	85.9	89.5	93.4	98.2
	SR	145%	147%	152%	155%
Net XL High	OF	125.0	133.5	143.4	153.8
	SCR	75.8	79.2	82.9	87.5
	SR	165%	169%	173%	176%

As can be seen in the next Tables 4.6 and 4.7, business strategy hardly influences the solvency position of an Insurer, in such a way that a different portfolio mix impacts not only the magnitude of OF and SCR but also the behaviour of the SR over the years.

Table 4.6. SR for Tau during the following 3 years (amounts in mln of Euro) (Motor Ptf—10% GL).

	t	**0**	**1**	**2**	**3**
Gross	OF	125.0	141.4	158.3	175.9
	SCR	77.6	81.0	84.6	88.5
	SR	161%	174%	187%	199%
Net QS High	OF	125.0	137.8	151.2	165.1
	SCR	73.0	76.4	78.3	82.6
	SR	171%	180%	193%	200%
Net XL High	OF	125.0	140.6	156.9	173.7
	SCR	66.4	68.9	71.6	74.3
	SR	188%	204%	219%	234%

In particular, a portfolio oriented on the Motor segment shows better results than in the Baseline scenario, with a significant increase in the OF is only partially offset by the increase of the SCR, leading to a SR from 161% at the starting year to 200% at the end of the projection.

This value increases when other reinsurance strategies are also considered, with a significant impact of XL because of an important decrease in the SCR only partially attenuated by the decrease in the OF due to the cost of the coverage.

Table 4.7. SR for Tau during the following 3 years (amounts in mln of Euro) (Liabilities Ptf—40% GL).

	t	**0**	**1**	**2**	**3**
Gross	OF	125.0	126.9	129.0	131.3
	SCR	114.6	118.6	120.8	124.9
	SR	109%	107%	107%	105%
Net QS High	OF	125.0	124.6	124.3	124.1
	SCR	103.2	107.0	107.8	113.7
	SR	121%	116%	115%	109%
Net XL High	OF	125.0	125.5	126.2	127.0
	SCR	92.9	95.8	98.5	101.8
	SR	135%	131%	128%	125%

On the other hand, a portfolio oriented on Liabilities segments shows an hard decrease of the SR over the time horizon because the lower technical results over the years do not let OF to grow as the SCR does.

The SR moves from 109% at the starting year to 105% at the end of projections. Reinsurance arrangements partially attenuate this behaviour but they amplify their effects over the years of projection.

Furthermore, the QS strategy negatively impacts a company's OF that decreases in the planning period while the SCR grows, and then the fall in SR is more significant than in the other two strategies (gross and net of XL reinsurance).

It is important to note that the XL strategy already helps an insurer to improve its solvency position at the starting year and over the planning period, with an increase of both OF and SCR lower than in the gross case.

Our results show how the behaviours of the SR drivers are amplified by the presence of XL and QS reinsurance, either in the positive or negative directions.

They obviously achieve a reduction of the SCR because of the lower volatility of the Aggregated distribution of claims due to reinsurance. On the other hand, insurers must pay attention to the pricing structures of the reinsurance arrangements in force, where higher pricing can lead to a reduction of the technical result and so of the OF, if compared with either gross of reinsurance case and a multi-year time horizon.

Now we consider risk reserve equation over several years. Starting from time 0 we develop Equation (4.1) in the gross of reinsurance case fixing a number of years larger than 1, generally here defined T:

$$\tilde{U}_T = U_0 + \left[\sum_{h=1}^{L} \sum_{s=1}^{T} \Big[(1+\lambda_h) \cdot P_{h,s} \Big] - \sum_{h=1}^{L} \Big[\sum_{s=1}^{T} \tilde{X}_{h,s} \Big] \right], \tag{4.32}$$

where all assumptions made in the first chapter about classical risk reserve ratio

Equation (4.1) hold. In the big square bracket we can find the difference between the sum of the gross premium volume (net of expenses) over T years and the total claims cost over T years.

The latter value has been defined as the aggregation over the T years of the sum of total claims cost of the h-th LoB, assumed to be independent.

Some accuracy must be taken on the aggregation between LoBs in the case of a multi-year distribution and its impact on the solvency analysis and capital requirements in case of time dependency of claim costs.

Reinsurance effects are taken into stochastic Equation (4.32) in order to show how these treaties impact over insurer's expected profit as well as its risk profile in the medium term because, usually, reinsurance treaties also use a multi-year approach.

With regard to net of QS treaty, the following risk reserve equation is obtained:

$$\tilde{U}_T^{NET,\mathrm{QS}} = U_0 + \left[\sum_{h=1}^{L} \sum_{s=1}^{T} \Big[(1+\lambda_h) \cdot \alpha_h \cdot P_{h,s} \Big] - \Big[\sum_{h=1}^{L} \sum_{s=1}^{t} \cdot (1-\alpha_h) \cdot B_{h,s} \cdot \Delta c_h^{RE} \Big] - \sum_{h=1}^{L} \Big[\sum_{s=1}^{t} \tilde{\Gamma}_{h,s} \Big] \right]. \tag{4.33}$$

In case of XL reinsurance, the net risk reserve Equation (**??**) turns into:

$$\tilde{U}_T^{NET,\mathrm{XL}} = U_0 + \left[\sum_{h=1}^{L} \sum_{s=1}^{T} \Big[(1+\lambda_h) - (1+\lambda_h^{RE}) \cdot \pi_h^{RE} \Big] \cdot P_{h,s} \Big] - \sum_{h=1}^{L} \Big[\sum_{s=1}^{T} \tilde{\Gamma}_{h,s} \Big] \right], \tag{4.34}$$

where π_h^{RE} is the ratio between original and reinsurance risk premium (P_h^{RE}/P_h).

In the next Figures 4.8, 4.9 and 4.10 the net risk reserve ratio in a three-years analysis is reported both gross and net of Reinsurance in the Baseline portfolio mix case.

With regard to reinsurance pricing structures, we analyze the more realistic situation that has been defined as Higher pricing in the previous Section **??** where starting values have been fixed at 25%.

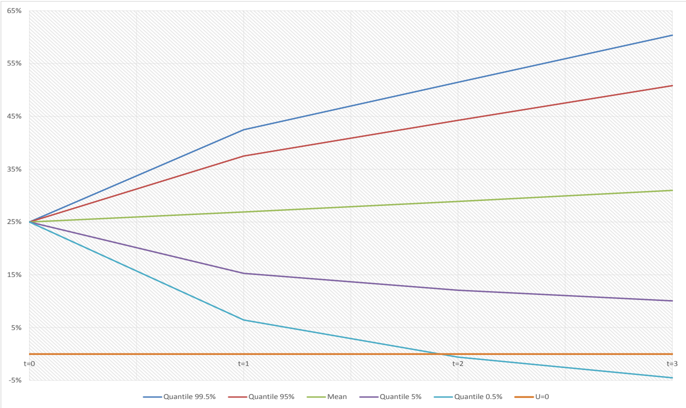


Figure 4.8. $\tilde{u}_t$, for $t = 0$, 1, 2, and 3 Gross of Reinsurance-Baseline portfolio mix ($u_0 = 25\%$).

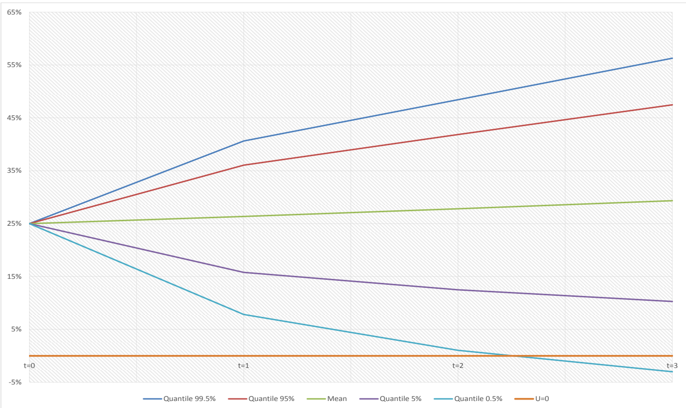


Figure 4.9. $\tilde{u}_t$, for $t = 0$, 1, 2, and 3 Net of Quota Share (QS) Reinsurance-Baseline portfolio mix ($u_0 = 25\%$).

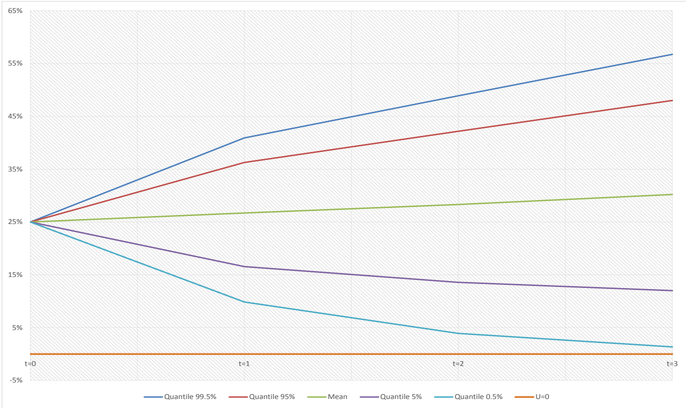


Figure 4.10. $\tilde{u}_t$, for $t = 0$, 1, 2, and 3 net of Excess of Loss (XL) Reinsurance-Baseline portfolio mix ($u_0 = 25\%$).

The quantiles of risk reserve are mitigated on both sides by the presence of reinsurance, more so with XL than with QS because of the reduction of the volatility.

It is worth noticing that in the last two years the 0.50% quantiles are lower than 0 either for Gross and Net QS, while it is always positive over the planning period in the case of XL. On the other hand, the mean increases during the time horizon for all the distributions considered because of the positive technical results of the years.

Once we have fixed the methodological framework underlying our medium-term model, it needs to be developed into a capital requirement calculation technique. Such analysis can be alternative as well as additional to the one carried on the previous paragraph for a medium-term assessment of insurance risks.

Nevertheless, we can improve the methodology of the previous Section (one-year approach for the SCR) by introducing a multi-year approach to the SCR consistent with Solvency 2 framework provided by Delegated Regulation.

Equation (2.13) can be generalized accordingly with a confidence level ϵ in a time horizon $[0, T]$, and then we obtain for the multi-year SCR Premium Risk:

$$\mathrm{SCR}(0, T|\epsilon) = VaR_\epsilon(\tilde{U}_T - U_0) = U_0 - Quantile_{1-\epsilon}(\tilde{U}_T), \tag{4.35}$$

where

$$Quantile_{1-\epsilon}(\tilde{U}_T) = -Quantile_{1-\epsilon}(\sum_{s=1}^{T} \tilde{Y}_s^{NET}). \tag{4.36}$$

Formulas (4.35) and (4.36) as in Formula (2.13) are recognizing expected profit/losses in the capital requirement evaluation by considering safety loadings over all of the years considered.

In practice, we can introduce an alternative SCR that considers not only a one-year but also a multi-year time horizon, taking the maximum between these quantities:

$$\mathrm{SCR}_0^* = \max[\mathrm{SCR}(0,1|99.5\%); \mathrm{SCR}(0,2|99\%); \mathrm{SCR}(0,3|95\%)]. \tag{4.37}$$

This approach permits to consider the business evolution over the company's Industrial Plan in the calculation of the solvency capital requirement, and it would likely change consistently with business strategy the amount of capital required to insurance companies.

To show the effect of this different capital requirement, three years with a different confidence level are considered and the corresponding quantiles of risk reserve distribution for Tau are reported in Table 4.8. The comparison of results gross and net of reinsurance will allow to describe the effect of different capital threshold on the solvency position, while a more deep analysis on differences with other possible portfolio mix, especially with regards to quantiles, let us justify the applicability of this rule in a more realistic market context. SCR computed through Equation (4.37) for $T = 1$ with $\epsilon = 99.5\%$ and $T = 3$ with $\epsilon = 95\%$ are indicated in bold.

Baseline scenario results show that in three years the company would improve its solvency position and the $\mathrm{SCR}(0,1|99.5\%)$ is greater than $\mathrm{SCR}(0,3|95\%)$. This behaviour is smoothed in case of reinsurance, where QS is more effective than XL since they reduce the confidence intervals of the risk reserve distribution and they are applied on a more balanced portfolio.

A portfolio oriented on Motor segments shows better results in terms of solvency, reducing SCR and increasing differences between one-year and three-year quantiles. Reinsurance effects in terms of lower SCRs decrease if compared with the baseline portfolio mix because of the lower weight of MVL.

On the other hand, an higher weight of "long tail" LoBs in the portfolio shows the most interesting results. Here we have that $\mathrm{SCR}(0,1|99.5\%)$ is greater than $\mathrm{SCR}(0,3|95\%)$ only in the gross of reinsurance case. QS results are very close to them while in the case of XL we have an $\mathrm{SCR}(0,1|99.5\%)$ lower than $\mathrm{SCR}(0,3|95\%)$, even if they are lower than the gross SCRs.

Table 4.8. SCR for Tau according to different T and ϵ (amounts in mln of Euro).

Mix		**Gross**		**Net QSH**		**Net XLH**	
	$1-\epsilon$	**0.5%**	**5%**	**0.5%**	**5%**	**0.5%**	**5%**
Baseline	$T = 1$	92.8	48.6	85.9	46.1	75.8	42.3
	$T = 2$	127.9	64.6	119.8	62.6	105.5	57.1
	$T = 3$	147.5	74.7	140.1	73.6	118.3	65.0
Motor	$T = 1$	77.6	40.8	73.0	39.5	66.4	36.1
	$T = 2$	102.5	49.4	98.5	49.6	89.0	45.6
	$T = 3$	115.5	49.8	111.7	52.0	101.3	46.0
Liabilities	$T = 1$	114.5	63.4	103.2	58.8	92.9	55.7
	$T = 2$	148.7	81.5	136.9	74.4	114.4	69.2
	$T = 3$	186.7	108.7	172.7	103.3	151.3	94.6

The results reported above show the effectiveness of different reinsurance strategies considered.

QS reinsurance can be relevant for short-term to medium-term assessment of insurers' solvency profile as it can hardly change the mean of the technical result distribution. Furthermore, differences by row between quantiles increase as the retention quota decreases, and significant attention must be paid to the pricing structure of the proportional treaties in force, even in a less risky portfolio as Motor mix.

In the XL reinsurance results it has to be emphasized how this kind of reinsurance operates on lower quantiles of technical result distribution on either a one-year or multi-year time horizon. Nevertheless, it works significantly on more skewed distributions as in Liabilities oriented portfolio.

Chapter 5

Alternative natural catastrophe Risk Transfer

5.1 Economics of CAT Bond

Complexity in actuarial risk management is increasing during last years in light of innovations in many areas of insurance sector. Solvency II regime, Advanced Data Analytics and Climate change is challenging insurance and reinsurance companies to have an on-going understanding of actual and potential risks they faces.

As already mentioned, *CaTastrophe Bonds*, henceforth CAT Bonds or CB, pertains to the event - linked securities class, representing one of the main innovative insurance risk transfer available for both private insurance and public sectors.

A significant increase in the CAT bond market was recorded in the aftermath of the 2005 Katrina Hurricane, leading to a flow of capital of 4.7 billion USD in 2006 and 7.1 billion USD in 2007. Following the surge of 2007, the 2008-2009 crisis (including the collapse of Lehman Brothers) showed the risk of using industry-tied players to invest in pooled premiums and principals.

Ultimately, the choice of more secure counterparties reignited the CAT bond market. Between 2010 and 2017 the CAT bond grew significantly, in part as a result of unremarkable interest rates available in the market, consequently with a better pricing and modelling of CAT bonds. In the first half of 2018, an author notes, the CAT bond market witnessed positive growth despite the occurrence of several triggering events including several hurricanes.

CBs have known substantial growth rate during the last two decades, but lower than growth of reinsurance capital, confirming insurers choice of reinsurance despite securitization. From the investors' side, appealing features are their risk-return profile, their low correlation with traditional asset classes and their little credit risk, thanks to their full collateralization. Meanwhile the ceding insurers appreciate the typical duration of such securities, that shelters from the cyclical price fluctuations, and their credit risk exposure, both much more favorable the sponsors than the ones typical of the traditional reinsurance market.

It is worth to mention that new Solvency II regime fully recognize those derivatives in Solvency Capital Requirements calculation if some conditions are met, so CB can also be used as a risk-mitigation technique for solvency purposes. In addition, CBs allow the underlying exposures to be ceded to capital markets whose capacity is much bigger than the one of the sole Insurance Industry. It is possible therefore to obtain Spread values for CB much lower than the hypothetical corresponding Rate on Line (RoL). As of the fourth 2020 quarter, more than 45 Bln dollars CBs are outstanding and more than 115 Bln have been issued since 2020, providing protection from catastrophic events to their sponsors (see 5.1).

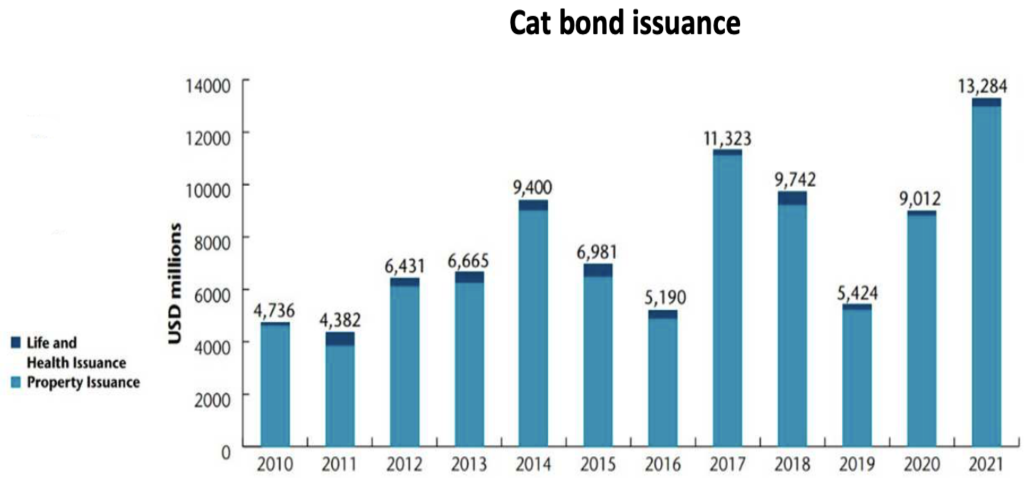


Figure 5.1. CAT Bond issuances in USD million dollars - last 10 years

CAT bonds are essentially securities which allow insurance companies to transfer the risks borne as a result of issuing insurance policies to the financial markets. The process normally entails the creation of an SPV (a special purpose vehicle) which acts as a bridge between the insurance and the financial market.

The issuer/insurance company (or, as the case may be, the reinsurance company) assigns part of the premiums collected through the insurance policies to the SPV, often incorporated in a jurisdiction which affords tax exemption, which undertakes to compensate the issuer in the event of catastrophe-based loss or another insurance event (i.e. a parameter).

Depending on the jurisdiction of incorporation and the characteristics of the deal, an SPRV may be used (special purpose reinsurance vehicle) as opposed to a generic, unregulated SPV. Empirically favored jurisdictions are the Bermuda, the Cayman Islands or Ireland if the target market is Europe.

CAT bond triggers are often either indemnity-based, industry-loss-based and parametric. The choice of trigger will materially affect the characteristic of the bond including the risk profile.

The SPV will issue CAT bonds to the market thereby collecting from bond-purchasers. The bond's principal is pooled with the premiums and normally reinvested in risk-free products. In the U.S. this role could be played by the U.S. Treasury money market fund.

In the event the loss (or event) does not occur, the SPV will pay out the agreed principal with interests to the bond holders in accordance with the bond terms. However, in the event of a loss or event (which triggers the obligation upon the SPV to compensate the issuer) bond holders will witness either a complete or a partial loss of their principal and interest. Effectively, the ultimate loss is transferred from the issuer (an insurance or reinsurance company) to the bond holders.

Proceed are therefore held in a Trust account and invested in highly rated securities in order to earn the risk-free rate (e.g. LIBOR), that flows through the SPV to the investors. In addition, the Sponsor pays a Spread (S) to the Investors through the SPV, that is set at issuance. A call option is embedded in the CB that may be triggered if a specified catastrophic event occurs, as further specified. If the call option is triggered, proceeds are released from the SPV to help the insurer pay claims arising from the event. Otherwise the investors will receive the full principal upon bond term expiration.

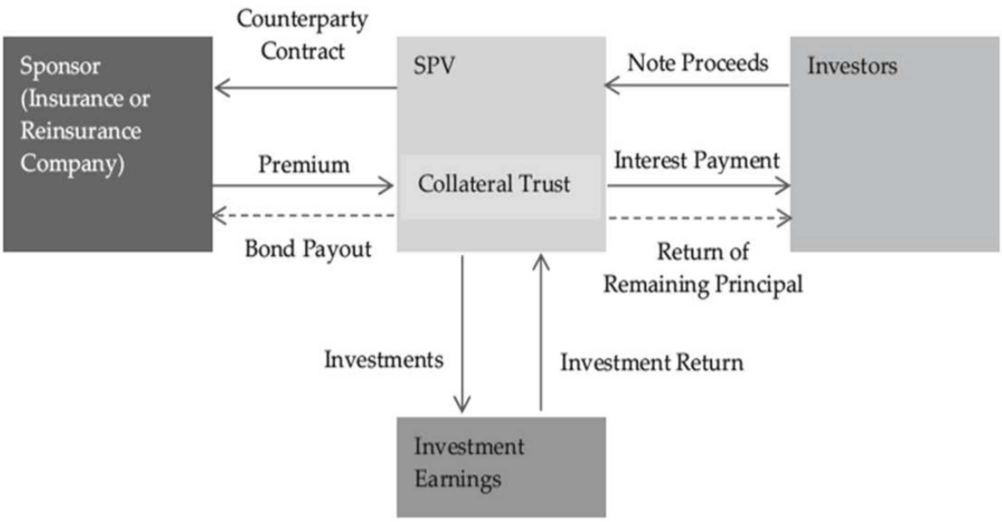


Figure 5.2. Cat Bond Structure.

CAT bonds provide certain advantages to both insurers (or risk-bearing issuers) as well as investors:

- the former can benefit from the multi-year term of CAT bonds as opposed to limited, one-year long reinsurance agreements and can rely on the bond's complete collateralization. Furthermore, CAT bonds effectively allow other types of players to provide reinsurance-like loss absorption capacity which, in turn, exerts downward pressure on the reinsurance market to lower reinsurance costs.
- the latter may benefit from the fact that CAT bonds are largely uncorrelated with the behavior of traditional markets Reliance on financial service providers as Lehman Brothers to provide risk-free investment options for pooled premiums

and CAT bond principals caused adverse correlation between CAT bonds and the financial markets. Reliance on public entities such as the U.S. Treasury money market fund or analogous entities largely offsets this specific form of correlation. On a very different level, correlation between a single triggering event and the financial instrument designed to allow investments in CAT bonds as an asset class can also be offset. This is the case of funds which invest in several different CAT bonds. In such cases multiple investments in different CAT bonds may reduce the correlation between risks and performance.

Consider, for example, the Tenax UCITS ICAV authorized by the Central Bank of Ireland as an ICAV. The Tenax prospectus is available online (http://www.tenaxcapital.com/UserFiles/Files/Prospectus_Tenax_UCITS_ICAV_5_May_2017_including_First_and_Second_Addendum.pdf), and thus provide safe harbor in the event of financial downturns. Furthermore, while complex and not without risk, CAT bonds provide strong returns, often higher than the average return rate of comparable purely financial instruments.

Generally, CAT bonds are issued by insurers, reinsurers and insurance-providing State funds. Occasionally, CAT bonds can be issued by other companies (e.g. utility companies, etc.). Structuring agents support issuing entities in modeling the bond (assisting in determining, inter alia, attachment and exhaustion points) and the placement is generally done by investment banks. It should be noted that in the U.S. CAT bonds are customarily privately placed to a limited number of prospective investors. For this reason, a placement of a CAT bond is really similar to a club deal. Indeed, there are no CAT bonds publicly offered or traded in the US and a secondary market exists wherein transactions are supported by a market maker.

Other key agents involved in CAT bond issuance are modeling agents tasked with estimating the risk of CAT bond, rating agents which rate the bonds (generally below investment grade), performance index compilers and investors. The latter are generally institutional investors such as pension and hedge funds owing to the complex nature and level of risk associated with CAT bonds, consistently with issuers exemption from the registration requirements for certain sales to qualified institutional buyers, generally institutional investors which respect certain requirements. Individuals are not eligible.

Many studies on risk transfer equilibria agree that large losses are still primarily either retained or transferred through traditional reinsurance, and despite securitization has a significant advantage over reinsurance and retention because of the substantially higher available capital and risk-bearing capacity of capital markets, traditional reinsurance is still the dominant risk transfer mechanism for catastrophe risks.

Under an economic perspective, the threshold risk level above which the insurer chooses securitization increases with the magnitude of potential losses in its portfolio. Given that catastrophe risk is typically associated with "low probability-high severity" losses, an insurer is more likely to choose retention or reinsurance to transfer catastrophe risk. Further, because an insurable risk is only transferred via securitization if the probability of potential distress-triggering losses is high, catastrophe bonds have high premia (relative to the ex ante expected losses) and a majority of them have ratings below investment grade.

[58] suggests that the high costs of catastrophe securities reflect the rational incorporation of their risks by capital markets based on insurers' observed risk transfer choices. Furthermore, in increase in the loss size increases the marginal cost of subsidizing higher risks as well as the marginal cost associated with the reinsurance markup. Consequently, the trigger risk level above which insurers choose securitization increases. In other words, the interval of risks that are securitized shrinks as the loss size increases. As catastrophe exposures are characterized by low probabilities and large magnitudes of potential losses, they are more likely to fall in the intervals of risks described above where retention or reinsurance rather than securitization are chosen. Hence, the volume of securitization is low relative to retention and reinsurance in catastrophe risk transfer.

Under financial mathematics perspective, CBs pay regular coupons to investors unless a predetermined event(s) occur, leading to a full or partial loss of capital. The coupon rate C received by the investor is composed both by a risk-free component, that reflects the time value of the money, and an additional coupon rate, namely the S, that reflects the compensation for bearing the insurance risk. The S of any transaction naturally exceeds the modeled Expected Loss (EL), to allow a positive rate of return as compensation for risk.

Therefore

$$S_i = EL_i + margin_i \tag{5.1}$$

being $EL_i = E[L_i]$.

Spreads and Expected Loss are customarily expressed as a percentage of the CB face value. It is worth to mention that EL is the actuarially fair premium of insurance risk underlying CB, representing a proxy of the best estimate of reinsurance recoveries in case of full reinsurance, and Spreads are composed by an additional margin that is comparable with the safety loading applied by reinsurer in traditional reinsurance.

Modeling CBs spread at issuance is important for the perspective of the sponsors and investors to support the deal arrangements' negotiations. Bodoff and al. ([5]) discuss various formulation of the relationship between the Spread, S and EL, from the practictioners' multiple approach $S = \cdot EL$ to the standard deviation margin one $S = EL + \sigma(L)$. Then, their paper presented many separated simple linear regressions (by clusters of issuances defined by selected predictors) as well as a multivariate one, that allowed for territory, peak multiperil features.

In 2000, Bantwal e Kunreuther (*A CAT bond premium puzzle?, Journal of Psychology and Financial Markets*) tried to explain, with an excessive risk adversion of investors and a high adversion to loss, rise in level of CAT bond prices. Niehaus (*The allocation of catastrophe risk, Journal of Banking & Finance 26 (2002) 585–596*) investigated the limitations of CB market, concluding that the reason for this event was the need to develop more accurate pricing models capable of better measuring catastrophic risk exposures.

As possible solutions that would increase investors' interest in financial instruments dedicated at catastrophic risk management, there are investment tranches, instruments based on predetermined indexes and the development of new products based on equity volatility dispersion. Cummins e Trainar, in 2009 (*Securitization, insurance, and reinsurance*, Journal of Risk and Insurance 76 (3): 463–492) carried out a study on the main disadvantages, but, above all, the advantages of using CAT Bonds, mostly their natural correlation with economic cycle trends.

Advantages of CAT bonds

CAT bonds have some of advantages. If compared to traditional reinsurance - where there is a residual risk of insurer default – they are characterized by a collateral, which at least partially counterparty risk. In addition to this, unlike one-year protection of traditional forms of insurance - which provide for a one-year term - CAT bonds are characterized by multi-year maturity, thus giving the issuer the possibility to set prices for a longer period of time.

In the light of their particular characteristics, the investment in CAT bonds allows insurance or reinsurance companies to have an additional source of financing and, at the same time, represents a valid opportunity to diversify the overall risk of the portfolio for the individual investor at a much lower cost than if it were necessary to diversify alternatively and, above all, ensuring higher returns.

In fact, one of the main advantages of investing in catastrophe bonds is the positive effect that the addition of this financial instrument has on the diversification of the portfolio held by the investor, as it is by nature interrelated to the evolution of the economic cycle and, consequently, makes the revenues independent from the performance of the remaining financial markets.

If an investment portfolio is held, the addition of CAT bonds makes it possible to obtain gains interrelated to those of the other assets of a different nature held in the portfolio, as well as an improvement in the investor's overall position, since two different scenarios could occur and both would imply potential benefits:

- a first scenario envisages that, with the addition of a CAT bond, there is a reduction in the overall risk assumed by the portfolio at the same level of return (determined before addition);
- a second scenario, on the other hand, envisages an improvement in the overall return through the upward shift of the efficient frontier (following the addition of a CAT bond to the portfolio) at the same level of risk assumed before addition.

Moreover, the use of CAT bonds makes it possible to overcome certain limits concerning the insurance industry (e.g. moral hazard or adverse selection) through the guarantee of greater flexibility and the possibility of modelling these solutions on the basis of all the individual and specific needs of companies or through an improvement in the diversification of the investment portfolio.

A further advantage that makes CAT bonds particularly advantageous for companies is the possibility of including such instruments in the calculation of the Solvency Ratio. Under Solvency II regulations, in fact, companies are required to carry out assessments of the potential impact that natural disasters would have on the company itself.

Since all the underwriting, credit, financial and operational risks to which a company is exposed are taken into account in this calculation, the minimization of the credit risk obtained through the use of CAT bonds also entails a reduction in Solvency ratio and, therefore, a lower capital provision.

This effect, therefore, is the reason why the catastrophe bond market is particularly attractive in the second and fourth quarter of the year with issues in these periods that are greater than in the remaining months.

Factors blocking CAT bonds'

Although the use of such instruments offers the possibility to benefit from various advantages, both from the investor's and the reinsurer's point of view, CAT bonds are not exempt from limitations: they also have an excessively complex structure that requires the involvement of many parties as well as careful diligence prior to the issue of the bonds. One of the main disadvantages of CAT bonds that contributes to a limitation on the demand side is that the costs are excessively high, not only on the issuer's, but also on the investor's side, especially if a comparison is made with the low level of the costs of traditional forms of insurance.

All this is a consequence of the lack of standardization in contracts. While this implies several advantages, at the same time it may lead to a very high level of fixed costs during the issuance phase. In order to counterbalance this high costs, the level of risk transferred has to be high. However, this scenario does not occur in the case of small companies which, despite a high potential loss in the event of a Catastrophic event, decide to retain the risk or choose alternative forms of insurance.

Moreover, CAT bonds are a relatively young insurance financial instrument, the birth and diffusion of which can be traced back to just over two decades ago. So there is still a lack of knowledge on the subject and CAT bonds were sometimes seen as too "exotic" financial instrument to be judged suitable for investment. For all these reasons, together with a lack of information about the market transparency and a contractual autonomy left to issuers and subscribers – which makes each individual contract a unique case – there is a high difficulty in finding plausible data, a factor which increases investor mistrust or, at least, does not encourage the spread of such instruments.

To limit the spread of CAT bonds there is also the problem of the heterogeneity of the triggers that can be chosen during the conclusion of the contract. While this allows a choice to be made on the basis of the specific needs of the parties involved, a lack of standardization can lead to exposure to high moral hazard or basic risk.

Specifically, even if the basis risk is low, as the trigger has the advantage of having a high probability that the compensation is actually equal to the loss incurred, although, at the same time, it requires specific knowledge of the actual risk exposure of the Sponsor, which may involve high costs.

A trigger of a parametric type or calculated on the basis of a specific index (such as, for example, the losses of an entire industry) has the highest degree of transparency for investors, but also the highest level of basis risk for the Sponsor, since the estimates made on objective parameters may not necessarily be the same of the losses actually incurred. This implies the need to carry out periodic reviews of the portfolio in order to always ensure the maximum satisfaction of the needs of the firm transferring the risk to the desired level of exposure.

The industry and geographical area in which a company operates also play a major role in deciding whether or not to use CAT bonds.

If a company faces the risk of business interruption due to its complex infrastructure or its position in areas more exposed to events of a certain magnitude, if a catastrophic event occurs, it will be more inclined to use alternative forms of risk coverages.

On the contrary, companies exposed to lower risks may consider the need for catastrophic risk cover as secondary, although worldwide the frequency and intensity of catastrophic events has increased exponentially.

An appropriate risk culture is essential. In recent years, however, there has been a growing awareness of the need for preventive catastrophe coverage on a global scale, making the CAT bond one of the most important financial instruments in the entire insurance landscape.

In this context, intervention by the authorities and therefore public-private cooperation is crucial as they could introduce solutions that make catastrophe risk coverage mandatory for companies or, alternatively, could provide incentives and thus facilitate the market penetration of these insurance instruments.

5.2 ILS regulation

From a regulatory prospective, additional efforts were made by other jurisdictions with the ILS market , the larger family to which CAT bonds belong, suggesting increased attractiveness for CAT bonds and other ILS instruments.

An example are the UK ILS regulations which entered into force on 8 December 2017 (i.e. the Risk Transformation Regulations 2017 and the Risk Transformation Tax Regulations 2017, https://www.nortonrosefulbright.com/en/knowledge/publications/898f8503/new-united-kingdom-insurance-linked-securities-ils-regime).

These set forth the regulatory and tax environment in which UK-domiciled insurance special purposed vehicles (ISPVs) may operate. The regime is consistent with Solvency II Delegated Regulation 2015/35, but provides for radical changes in corporate and insolvency law, including the provision which allows a single ILS to be used in several different deals (multi arrangement ISPV or mISPV).

Therefore, this allows the creation of a new protected cell company regime (PCC). The new regime is designed to reduce costs, regulatory burdens and incentivize ILS arrangements.

The experience of the U.S. market in the offering of CAT bonds, as well as the recent amendments to the rules of other jurisdictions, suggest that the ILS market could grow significantly in the future and provide additional risk-absorption capacity. In turn, this could provide a beneficial increase in insurance coverage in areas prone to Catastrophic events.

Traditionally, primary insurers turned entirely to the reinsurance market to offset the risks exceeding their own capacity. Now insurers see the CAT Bonds as a way to diversify their reinsurance capabilities and strengthen the access to capital markets; CAT Bonds are used in order to complement the reinsurance options available (with many reinsurers offering CAT-bond-like products). Moreover, there are substantial differences between reinsurance products and CAT Bonds.

Traditional reinsurance contracts offer reinstatement provisions, which reset coverage when the reinsurance limit is exceeded. No such provision is generally found in a CAT bond structure.

Reinsurance also tended (at least until the recent past) to be annual contracts, whereas CAT bond coverage typically covers events that may occur over a 3- to 4-year period. Primary insurers hence have the chance to renegotiate more favorable annual contracts or lock-in capacity via a CAT bond for a longer period of time at a fixed price.

Furthermore, contracts of insurance and reinsurance are subjected to the principle of utmost good faith.

On the contrary, CAT Bonds market is purely cross-borders: no good faith issue may be conceivable, and this is a relevant factor in increasing certainty. It may be critical in fact for an insurer selling this risk to be able to claim full credit on its financial statements for all risk transfers, including CAT Bonds.

Since CAT Bonds are fully collateralized with the bond proceeds protected by a reinsurance trust for the benefit of the ceding insurer, provided that the documentation is properly and accurately drafted, the ceding insurer should rely on a relatively smooth and quick collection of reinsurance.

CAT bonds were also issued in the European market by establishing the issuer in one of the Member States. Ireland is most often the target jurisdiction due to an attractive legal and tax environment as well as a favorable regulatory process.

In such deals generally the issuer is authorized as an SPRV by the Central Bank of Ireland pursuant to applicable reinsurance regulations and issues the CAT bond in accordance with the relevant rules which may include the requirement to draft a prospectus/offering circular, if applicable

For an example, consider the offering circular for the issuance of the CAT bond notes issued by Atlas Reinsurance VII Limited, an authorized SPRV established in Ireland.

With respect to the SPRV incorporation process in Ireland, guidance has been provided in 2012 by the Irish Central Bank in the form of a document on Special Purpose Reinsurance Vehicles.

While a detailed analysis of the document or current requirements is not the purpose of this thesis, it is noteworthy that the authorization process for SPRV in the 2012 document required specific documentation including, but not limited to, detailed risk assessments, pre-sale rating agency reports, details on the envisaged use of any hedging instruments such as interest rate swaps or currency contracts and prospectus/offering circular or private placement memorandums, where applicable. In accordance with Solvency requirements, Irish SPRV must be fully funded.

Compare parametric

The use of parametric products is notoriously on the rise and is taking on a larger role in insuring Catastrophic events.

Although parametric solutions have been available since the late 1990s, they have recently found their way into corporate insurance. Parametric insurance solutions offer a means to guarantee direct payout after a qualifying event and protect against devastating risks in ways traditional insurance packages cannot.

If compared to traditional insurance coverage, such products, in fact, do not require claims adjusting: that can bring significant time and costs savings. The policy can be triggered not by the calamity itself, but also by the weather conditions which had brought to the calamity. The payment is generated by the receipt of specific weather data; a storm or weather event must go over a defined point and events may also refer to an index-based trigger (for instance crop shortfall) or within a defined area.

A policy might be for instance structured to pay out 50%, 75%, or 100% of a predefined limit for a category 3, 4, or 5 hurricanes, respectively, happening within a 60-mile radius around a specific location or an earthquake with minimum magnitude in a defined geographic area.

It is well known however that parametric insurance products may cause legal or regulatory uncertainty in jurisdictions where the insured must have an insurable interest at the time the policy is underwritten, or at the time the loss takes place.

Furthermore, the amount of the insurance pay-out must correspond to the actual loss incurred by the insured, pursuant to the indemnity principle, according to which the insured is not - and will never be – entitled to make a profit out of the insurance, but just to recover the damages incurred.

The application of principles of insurance law and contracts, including the indemnity principle, is alien to CAT Bonds. It is also worth noting that early CAT bonds were largely based upon parametric or non-indemnity triggers, but the experience of the recent years shows that the majority of recent CAT bonds provide an indemnity-based reinsurance cover.

The legal nature of CAT Bonds: structure and function
The rules governing issuance

As explained above, CAT bonds are financial instruments — or bonds — that offer a guarantee linked to the occurrence of a natural event of a predetermined scale. They are an important catastrophe risk management tool for the insurance industry, where risk is transferred from the insurance market to the capital market.

CAT bonds are financial solutions with also a clear economic-social function — especially for insurance and reinsurance companies, and anyone else exposed to financial risks deriving from natural disasters — as they cover potential losses arising from the occurrence of Acts of God.

However, the diffusion of CAT bonds and similar products was restricted by particular legal constraints. Such limitation is evident going back to the history of their evolution and considering the phenomenon from a comparative perspective, as illustrated in the previous paragraphs.

It is possible to underline two legal criticalities on the side of public regulation:

- in the Italian legal system there is currently no organic discipline governing the operation of that instrument. In fact, as noted earlier, the TUF regulatory discipline of funds and secondary markets applies to CAT bonds, an alternative investment instrument. The question is if this rules framework is adequate or not to CAT bonds and to weather derivatives market;

- Moreover, in order to guarantee the insurance company's solvency, measuring the level of capitalization, international governmental regulations were issued in order to limit all the risks it takes on, including the Catastrophic risk. In particular, Solvency II (Directive 2009/138/EC), which extended the Basel II regulations to the insurance sector, pushed Member States to legislate so as to establish control and risk assessment mechanisms within insurance companies. In fact, this type of risk is taken into account only in the calculation of the Basic Solvency Capital Requirement (See Article 105 of Solvency II). As a result, the insurance company is asked to carry out an analysis and quantification of the extent of the catastrophic risk in order to assess its ability to meet all its commitments.

The other question is if insurance companies have adequate guidelines for quantifying Catastrophic risks and for quantifying mitigation through the issue of CAT bonds.

Considering the benefits of Catastrophe bond issuance as an alternative to traditional reinsurance, EIOPA dealt with this issue in 2014 by involving several multidisciplinary experts with the aim of finding solutions to the different issues related to insolvency risk, a work in progress, with the intention of providing guidelines in Final Report on Public Consultation No. 14/036.

Specifically, Guidelines 6 and 7 on contract boundaries (Guideline 6 - Identification of a discernible effect on the economics of a contract: 1.19-1.20; Guideline 7 - Estimation of obligations: 1.21-1.22) aim to enhance the use of CAT bonds in risk management by companies — at the level of reserve building — so that companies issuing catastrophic risk products can lower their policy costs and increase the demand for catastrophic risk coverage.

In view of the above, it is clear that there is currently no ad hoc organic regulatory framework for CAT bonds. Additionally, on the public offering side, they are subject to the rules provided for bonds, which may raise questions considering that they are characterized by the presence of the rational risk, as mentioned above, that is different from the business risk of bonds where the loss of capital is linked to the default of the issuer.

On the demand side, it is instead necessary to regulate transparency and the information intended for the reference market, which is so complicated that it requires an ad hoc structure that explains and updates the final client on the content of the instrument under discussion. Such disclosure should be different from the formalism provided for by Consob regulations in the pre-contractual phase, and more in conformity with the description of the rational risk proper to CAT bonds.

In recent years, the CAT bond market has significantly increased in size and some large Italian groups have also made use of it. The supervision experience, however, was focused on the assessments relating to the transfer plan, the effects on the capital requirements and did not concern the phase of incorporation and authorization of the vehicle without which the product cannot be fully defined.

For historical and "ecosystem" reasons in which the SPVs operate (note: the Private Insurance Code, while providing for the discipline relating to the authorization and exercise of the activity of SPVs, refers to a MEF Regulation not yet issued - see Article 57 of the CAP), the vehicle companies have always been located in Ireland.

The sector Authority of that country is therefore the agency responsible for assessing compliance with the stringent requirements of the Solvency II directive for the operation of SPVs (in the case of sponsors located in the territory of the Republic, a prior consultation by the Competent authority - Article 9 of the Implementing Regulation (EU) 2015/462).

The role of Supervisor for access and operation of SPV

The Solvency II legislation establishes the conditions of access and exercise of insurance and reinsurance activities for SPVs, providing for the issue of an authorization subject to compliance with a series of conditions. The Supervisor must consequently verify:

- A. the presence of contractual conditions aimed at defining:

  - full financing (the SPV must be "full funded");
  - the actual transfer of risk;
  - the rights of debt holders, considering that:

    the claims of debt holders or providers of financing mechanisms are constantly subordinated to the reinsurance obligations of the vehicle company towards the insurance or reinsurance company;

    the debt holders or financing providers have no right of recourse on the assets of the insurance or reinsurance undertaking;

- B. the implementation of an adequate governance system to meet the requirements:
  - the competence and integrity of the people who manage the SPV;
  - of competence and integrity for shareholders or members who hold a qualified shareholding;
  - internal control and risk management.
- C. the solvency of the SPVs, which to be considered fully financed must meet all the following requirements:
  - the assets are valued according to the market consistency principle pursuant to art. 75 of directive 2009/138 / EC;
  - the SPV has at all times assets whose value is equal to or greater than the aggregate maximum exposure to risk and is able to pay the amounts for which it is responsible at their due date;
  - all proceeds from the issue of securities or other financial instruments have been paid.
- D. the investment criteria:
  - invest only in assets and tools of which they can adequately identify, measure, monitor, manage, control and report risks; they must guarantee the safety, quality, liquidity and profitability of the portfolio as a whole;
  - must invest the assets in an appropriate manner to the nature and duration of the liabilities of the special purpose vehicle. All assets are invested in the best interest of the insurance companies;
  - may use derivative instruments to the extent that they contribute to a reduction in risk or facilitate effective portfolio management;
  - must keep investments and assets not admitted to trading on a regulated financial market at prudent levels;
  - must be adequately diversified;
  - do not expose the vehicle company to an excessive concentration of risks.

In the last part of this section we will limit ourselves to considering Italian legislation, although other systems also place legal constraints on the distribution of financial instruments.

Framing difficulties represent one of the primary legal constraints with regard to the diffusion of CAT bonds in the Italian system, as bond issuers might fear these contracts will be declared "illegal" (and therefore invalid) by an Italian Court, resulting in lengthy appeal proceedings.

Potential contracting parties are also subject to legal restrictions:

- On the issuers' side (offers side), although the State runs the risk of financial losses deriving from the occurrence of natural Catastrophes (in particular, damage to cultural heritage), its issuance of financial instruments is limited by public law (e.g. Legislative Decree 267/2000). As regards businesses and, in particular, SMEs exposed to the risk of Catastrophic events such as companies in the agrofood industry, limitations are found in the regulation on the issue of bonds pursuant to the civil code (art. 2412 cc limited to joint stock companies) and the regulation of sector (TUF Legislative Decree 58/1998 and Consob Regulation on emission of financial instruments) which dictates specific limits of form and quantities to protect investors and the market;
- On the investors' side (demands side), restrictions are placed on possible participation by institutional investors. With regard to banks, companies and pension funds, the governance rules in Basel III, Solvency II Directive 138/2009/EC, and the so-called IORP II Directive 2341/2016 establish restrictions on the purchase of derivatives. The possibility for the State to invest in derivatives or bonds with risk of capital loss is limited by the rules of accounting and public finance, as well as by the principle of "patrimonial stability" of the State referred to in Article 119 of the Italian Constitution.

Consequently, parties of such contracts are typically insurance or reinsurance companies (on the offer side) and retail investors (on the demand side). As regards the content of the contracts (which are instrumental to carry out the operation), it is first necessary to frame the phenomenon before assessing their validity.

In order to achieve the aforementioned economic objective and attract potential counterparties, the content of the contract is that of the sale of a bond issued, for example, by the insurance company. Specifically, the issuer undertakes to pay a periodic premium for a certain period of time and to return the capital paid at the end of a certain term if no predetermined event (earthquake, flood, etc.) occurred during the duration of the contract that exposes the issuer to over-indebtedness with respect to the coverage for risk of Catastrophic events of the insured parties.

Under the TUF regulation, these would be subordinated bonds whereby the payment amount by the subscribers depends on the payment of the issuer's non-subordinated creditors, e.g. those insured against the risk of a catastrophic event that has yet to have occurred. The particular nature of this bond is noteworthy as its "subordination" is not linked to a business risk (typical default risk of bond loans), but rather to a catastrophic risk. This, in turn, affects the subsumption of the risk within models that allow for its predictability, within certain time limits.

In the event of climatic events (excessive rain, excessive drought, etc.), derivative contracts are used, given the particular agility in translating risk into indexes to which cash flows can be linked. It is possible, for example, to hedge the risk of negative impacts of lower and higher temperatures on economic activities through the purchase of derivatives that provide for the payment of a premium in exchange for the right to obtain a certain amount in case of rising or falling temperatures compared to a certain level/index.

As a general rule, a reduction or exclusion of the premium is given where the "insured" business acquires the right to receive a certain amount if temperatures exceed a certain limit, but, at the same time, undertook to pay a given sum if the temperatures fall below a certain limit or vice versa. The major dealers are banks, financial companies, and insurance companies that cover both their own risks and those related to the insured, e.g. a company that also insures damage from adverse weather events to farmers, and debtors, e.g. a bank that provides credit to companies whose activity is sensitive to weather events.

5.3 CAT Bonds pricing literature review

As stated above CAT bonds are the most important ART in insurance market, representing financial instruments issued by an insurer with the condition that if the issuer suffers a catastrophe loss greater than an amount specified on the contract, the obligation to pay interest/principal is deferred or forgiven, thus effectively prompting a default on the bond. These bonds allow sponsors, most often a (re)insurer, to transfer a portion of its catastrophe risk to capital markets through securities purchased by investors and actively traded in the secondary market. Favourably for the sponsor, CAT bonds offer collateralized (most often invested in U.S. Treasury Money Market Funds) protection that is locked in at a fixed cost over multiple years (typically two to four years). This allows the (re)insurer to be less subject to changing reinsurance market conditions. For the investor, CAT bonds offer a comparatively high yield and an opportunity to diversify their portfolios.

Over time, in actuarial sciences numerous attempts have also been made to calculate a fair price for these financial instruments. The pricing of reinsurance products is, generally, more complex than the one for traditional insurance contracts, since the covered risks, from a statistical point of view, have a distribution characterized by heavy queues - and, therefore, by a greater probability that the values deviate from the average – also being affected by systemic components (all these reasons make the management difficult). Finally, their grouping aimed at risk-sharing is complex.

In order to establish an appropriate price, Tilley (1997) and Cox and Pedersen (2000) took as a starting point the cash flows originated from the investment in catastrophic bonds, showing how they were related exclusively to natural disasters-linked variables instead of economic and financial variables. Another point of view was given by Embrechts and Meister. In 1997, in "Pricing insurance derivatives, the case of CAT-futures", they proposed a criterion based on the maximisation of utility.

Virginia R. Young (2004) focused on the concept of expected utility and indifference pricing, built around investor preferences and introduced through utility functions. The advantage of the following models is that they are valid even in the case of an incomplete market. Young used the utility principle to price a catastrophe bond, in which the utility function is represented exponentially. In 2007, Masahiko Egami and Virginia Young herself proposed a further approach, also based on indifference prices, which is an extension of the previous model, assuming, however, a more articulated structure of the underlying catastrophe bonds.

In 2000 Morton Lane proposed a model whose aim was to calculate the premium to be paid by the sponsor in order to receive adequate coverage from the transferred risk. In this perspective, the price of a CAT bond can be expressed through a financial part, the LIBOR, increased by a part of a purely insurance nature in the form of a spread, a function of the riskiness of the insured event, but also of the propensity to risk of the investors themselves.

One of the most successful techniques used to price a specific risk is the use of distortion operators. A leading figure of this theoretical current is Shaun Wang. In the 2000s, through his studies on the pricing of CAT bonds, he proposed a model capable of explaining the behaviour of CAT bonds' spreads through the determination of a specific premium for each risk, thus allowing the risk incurred to be included in the calculation of the premium: at the same level of risk, in fact, the premium paid is higher for individuals who show adversion to the risk itself.

In addition to the multivariate linear regression models, other regression frameworks have been proposed in the literature. In particular, Papachristou used Generalized Additive Models (GAM) as an alternative to classical linear regression, while powers and logarithmic functions were presented in Major and Lane Financial ([44]; [40]. Finally, Carayannopoulos and colleagues ([10]) proposed time-series regression approach. It is worth to underline that very few papers have considered the impact of features of the loss distribution additional to the EL, the much less availability is a possible reason for. Among these, Ciumas and Coca ([15]) considered also the attachment and exhaustion probability when modeling CBs' Spread using a multivariate linear regression approach.

Although a multitude of different approaches were developed over time, there has been no real comparison between the different models to determine the ideal one. The lack of market transparency, in fact, - together with the contractual autonomy left to issuers and subscribers which makes each individual contract a unique case - led to a high difficulty in finding plausible data and, therefore, excessive diversification in model development, making it hard to determine a universal pricing strategy.

All studies agreed that the relationship between Spread and EL appears almost linear, simple or multivariate, so the attention has been focused on how other factors may explain the Spread at issuance further. The multivariate linear regression in Braun study ([6]) suggested the following predictors for the S: the Size, Term, Idemnity type, Peril, Territory, Peak - Perils, the issuance period ROL index and BBSpread and whether the issue was given an investment grade (BBB or more). When compared to previous approaches, the Braun model offered higher predictive performance.

A classical econometric approach has been generally used in cited works. That means that the Ordinary Least Squares (OLS) regression modeling is the reference model; statistical inference assesses the variable significance and special econometric techniques, like Heteroscedasticity Consistent Errors, are used to overcome deviations from the normality of error assumptions.

The Braun study is, according to the author's opinion, the most comprehensive study at the date, when the supporting data set size and the number of predictors are considered in a regression framework.

The S value is the feature that mostly matters Investors and Sponsors. In fact, it represents the compensation for the expected loss (EL), so it can be considered a proxy for the reinsurance premium for the exposures at risk. The estimation of the EL is usually performed by a third party modeling firm (TPMF), a CAT modeler or a Broker ([62]) for example. Often, the TPMF provides the investors not only the

EL estimate, but also the attachment and exhaustion probability, all expressed as a percentage of the notional amount.

Literature has pointed out that the EL is the most important predictor of the Spread, and that a linear function provides a good approximation of their relationship ([5]; [26]). Nevertheless supplementary CB features could also influence the price of CBs; the research has striven to identify which were and what was the best functional relationship to approximate the observed ratio between spread and EL. Comprehensive studies from Braun, Lane and Bodoff (41) identified other features that could influence CBs prices, the most important follow:

- the trigger: an Indemnity trigger would require higher spread due to moral hazard;
- the competing financial environment, represented for example by the Spread on comparably rated corporate bonds. The spread on BB bonds has been identified as reference proxy by the literature;
- the status of the competing reinsurance market. Lower capacity in the traditional reinsurance market would also increase the number of sponsoring companies looking for capacity in the financial markets, thus increasing the offer and consequently increasing the Spread; the reverse is expected in soft market environments; at this regard the Guy Carpenter ROL index has been identified as reference proxy;
- the hazard the portfolio is insured for: the type of peril(s) covered, their geographic location, whether “peak” perils or zones are in those covered by the bond. In particular, peak risks typically demand higher margins due to concentrations in investors’ ILS portfolios compared to other zones or perils ([49]).

Historically CBs have been used to reinsure property exposures from natural catastrophe perils, where US hurricanes dominate the market. Other “peak perils” include US earthquakes, European windstorms, Japanese earthquakes and typhoons. Non peak natural catastrophe perils include Australian cyclone, Mediterranean earthquake and Mexican earthquake. Eventually the market has been expanded to cover LH portfolios as well as non - traditional insurance risks, like Lottery hedging ([3]). As anticipated, an occurrence of a CAT event may trigger the CB coverage, that pays off depending by the trigger that was specified in the contract:

- Idemmity: recoveries directly depend by actual losses of the sponsor;
- Parametric and Parametric Indexed: actual reported parameters (e.g., wind speed, earthquake magnitude or location, etc) determine the pay-off, e.g. depending on whether the parameter has exceeded a threshold set in advance;
- Industry-index: the payoff are linked to the overall industry losses;
- Modeled loss triggers: the payoff is determined simulating losses that are generated by setting some actually experienced physical parameters of the CAT event into the CAT model maintained by the modeling agent.

Hybrid triggers have been proposed as well. The multiplicity of triggers on the market is due to the effort to reconcile opposing needs. Sponsor companies (and regulators) would prefer to have a pay-off as close as possible to actual portfolio's losses, avoiding basic risk. Investors would like to avoid moral hazard and to get an assessment of their possible losses as quickly and objectively as possible. Therefore, Sponsors' clear preference for indemnity triggers contrasts with investors' preference for different trigger categories.

Other features that can be considered are:

- the duration (it is expected that Spread increases with duration due to liquidity preference theory);
- size of the issue (higher sizes reduce the impact of transaction costs);
- whether an occurrence or aggregate trigger is set;
- the credit rating (both of the Sponsor and of the bond itself).

Relevant ML algorithms

The use of machine learning models has been very little explored as of the second half of 2020 in ILS literature. In fact the Eggert and colleagues' paper ([21]) underlines the lack of research about how to exploit "data-intensive analytics" tools to improve the estimation of both loss prediction and risk premium.

Their work analyzes all the business components of managing a CAT Bond issuances (portfolio exposures and claims data bases, CAT loss modeling and investors' portfolio optimization) from a business analysis perspective and it discusses how high-performance computing methods could benefit the process. In addition, various ML approaches have instead been compared in ([2]) to model the trigger of parametric-index based earthquake CBs.

The first work where traditional approaches and ML ones were compared in estimating S versus EL and other predictors is the technical report of Lane. Lane work discusses earlier research pointing out that currently proposed linear models show a relatively high RMSE despite good R^2s. It compared the predictive performance of the Braun model and others versus a Random Forest model's one.

The comparison was performed both on sample data, represented by CBs issuances until 2014, and out of sample data consisting of all CBs issued in subsequent three years. The paper concludes that despite the RF model moderately improves the predictive performance at the cost of losing interpretability, a significant level of variability remains unexplained. Recently Götze et al. have used basic ML methods in order to estimate spread at issuance ([30]), coming to the same results of Lane without giving a complete explanation of ML results.

Many ML models are being proposed in the scientific literature. We limited our comparison to the following one: Elasticnet, Random Forests (RF), Deep Neural Networks (also known as Deep Learning, DL), Gradient Boosting Machines (GBM), Stacked Ensembles (StkEns) and Cubist. All the considered algorithms belong to the supervised learning family, since they aim to maximize the predictive accuracy of a dependent variable y, the CB Spread, given a set of predictors X.

All ML aforementioned but Cubist have been applied thanks to the H2O library ([42]) within the R statistical software. The H2O software is an open source project, that well interfaces with R and Python programming languages, containing strongly optimized implementations of most relevant ML algorithms (by parallelization, GPU exploiting, modeling facilities), providing a common interface to perform hyper-parameters' tuning, model assessment and scoring. At this regard, the R caret package ([39]) provides also a unified interface of many ML algorithm and it has been used to manage Cubist models.

Aim of this thesis is to see whether the patterns captured in our data set can provide material for new explanatory-driven studies in the future. The consistency of the introduced machine learning method in explaining results regards risk transfer choice of insurers has been investigated, confirming results of Subramanian and Wang ([58]).

Finally, the potential of the introduced machine learning method in facilitating investors' activity in the catastrophe bond market is also of interest.

Interpretability is important for developing a forecasting model because the modeler can identify the causes for the poor performance of the model relatively easily. Our study contributes extensively to the literature on asset pricing and empirical studies on machine learning. This paper is the first to compare different machine learning methods for forecasting CAT bond premia and to provide an approach for tuning hyperparameters in enhanced machine learning methods.

Besides these novelties, it can be emphasized that the CAT bond market is much smaller than other asset markets and, therefore, provides a more challenging environment for the application of machine learning methods ([34]; [31]).

Our analysis may help practitioners to assess the potential of machine learning methods for asset pricing in general and, more specifically, in the context of pricing CAT bonds in insurance industry. An important finding we obtain in this context is that machine learning can already perform quite well on a relatively small data set.

Chapter 6

Statistical learning approach for pricing

In this chapter we will illustrate the main models used by actuaries not only for pricing but recently also in claims reserve evaluation, offering an introduction of the theoretical foundation of statistical learning and developments of Machine Learning.

Particularly, we provide methodological aspects of those two attempts to describe the functional relationship (e.g. f, can be linear, quadratic, cubic, etc.) between an output (e.g. y can be expected premium for a group of policyholders, link ratios in Loss Development Methods of claims reserving, price of an ILS, etc.) and one ore more inputs (e.g. $\boldsymbol{X}$ covariates representing policyholder's behaviour, claims dynamics for accident and development years, ILS characteristics, etc.):

$$\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{X}) \tag{6.1}$$

6.1 Generalized and other regression models

We will start with Generalised Linear Models which represent the benchmark of any actuarial analysis.

For many years, models used by actuaries were limited to the simple linear ones. Given the increasing sophistication in the insurance market and a growing competitiveness, actuaries moved to the more complete generalized linear models and, in the last 40 years, the GLM have been the industry standard for Non-Life insurance.
For this reason, GLM are usually the base to test and compare the applicability and performance of other models to the different lines of business.

The target variable's result is influenced by both a random and a deterministic component.
The systematic component is the fraction of the variance in the outcomes that is attributable to the predictors' values. The random component is the portion of the

result that is determined by factors other than our model's predictors.

This contains both "real randomness", i.e. the fraction driven by conditions that are unexpected even in theory, and that which may be predicted using extra factors not included in our model.

Our objective when modeling with GLMs is to use our predictors to "explain" as much of the variability in the result as possible.
To put it another way, we want to move as much variability as possible from the random component to the deterministic component. Both the random and deterministic components of GLMs are explicitly assumed in GLMs.

The variable to explain is

$$\boldsymbol{Y} = (\boldsymbol{Y_1} \ \dots \ \boldsymbol{Y_n})' \tag{6.2}$$

whose densities belong to the exponential family law. We say that $\boldsymbol{f_{y_i}}$ belongs to the exponential family law if and only if we can find $\boldsymbol{\theta} \in \boldsymbol{R}$ (canonical parameter), $\boldsymbol{\phi} \in \boldsymbol{R}$ (parameter of dispersion), $\boldsymbol{a}$ non-null function defined on $\boldsymbol{R}$, $\boldsymbol{b}$ defined on $\boldsymbol{R}$ and twice derivable, $\boldsymbol{c}$ function defined on $\boldsymbol{R^2}$, such as:

$$\boldsymbol{f(y;\theta,\phi)} = \mathbf{exp}\,\frac{\boldsymbol{y\theta - b(\theta)}}{\boldsymbol{a(\phi)}} + \boldsymbol{c(y;\phi)}\} \tag{6.3}$$

The following is how GLMs model the connection between $\boldsymbol{\mu_i}$ (the model prediction) and the predictors:

$$\boldsymbol{g(\mu_i)} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1 x_1} + \boldsymbol{\beta_2 x_2} + \dots + \boldsymbol{\beta_p x_p} \tag{6.4}$$

meaning that some specific transformation of $\boldsymbol{\mu_i}$ to the intercept ($\boldsymbol{\beta_0}$) plus a linear combination of the predictors and the coefficients.

The link function may be thought of as an $\boldsymbol{E(Y)}$ transformation that connects the real $\boldsymbol{Y}$ values. In this way $\boldsymbol{g(\cdot)}$ is a one-to-one continuous differentiable transformation of $\boldsymbol{\mu_i}$

$$\boldsymbol{\eta_i} = \boldsymbol{g(\mu_i)} \tag{6.5}$$

that is called a link function.
We may be able to simulate a variety of response variables depending on the link function specification, may be count data response, continuous linear response, binary response or Multinomial.
The transformed mean is assumed to follow a linear model:

$$\boldsymbol{\eta_i} = \mathbf{X}_i\boldsymbol{\beta} \tag{6.6}$$

that we can invert in:

$$\boldsymbol{\mu_i} = \boldsymbol{g^{-1}}\left(\mathbf{X}_i\boldsymbol{\beta}\right) \tag{6.7}$$

If the target variable is distributed as a Bernoulli with parameter $\boldsymbol{p_i}$, like it's the case in modeling a cancellation risk, the link function $g : (0,1) \to R$ should be used. To begin, we can define the Bernoulli distribution as

$$f_B(y_i; p_i) := p_i^{y_i}(1-p_i)^{1-y_i} = e^{y_i ln p_i + (1-y_i) ln(1-p_i)} = \tag{6.8}$$

$$= e^{y_i ln(\frac{p_i}{1-p_i}) + ln(1-p_i)} = e^{y_1 ln(\frac{p_i}{1-p_i}) - \left[ln\ (\frac{p_i}{1-p_1}) - ln\ p_i\right]} \tag{6.9}$$

and if we define $\theta_i := ln(\frac{p_i}{1-p_i})$ and $\phi := 1$, then

$$f_B(y_i; \theta_i) = e^{y_i\theta_i - \left[\theta_i - ln\left(\frac{1}{l+e^{-\theta_i}}\right)\right]} \tag{6.10}$$

which belongs to the exponential family, with

$$c(\theta_i) := \theta_i - ln\left(\frac{1}{1+e^{-\theta_i}}\right),\ h\left(y_{i,\phi}\right) := 0. \tag{6.11}$$

This implies that

$$E\left[y_i\right] = \frac{dc\left(\theta_i\right)}{d\theta_i} = \frac{d}{d\theta_i}\left[\theta_i - ln\left(\frac{1}{1+e^{-\theta_i}}\right)\right] = \frac{1}{1+e^{-\theta_i}} = p_i \tag{6.12}$$

$$Var\left(y_i\right) = \phi\frac{d^2c\left(\theta_i\right)}{d\theta_i^2} = \frac{d}{d\theta_i}\frac{1}{1+e^{-\theta_i}} = \frac{e^{-\theta_i}}{\left(1+e^{-\theta_i}\right)^2} = p_i\left(1-p_i\right) \tag{6.13}$$

which is what we expect from a Bernoulli distribution. Using the canonical link function of the logistic regression is such that

$$g\left(p_i\right) = ln\left(\frac{p_i}{1-p_i}\right) \tag{6.14}$$

which is the logit function. This meaning that the probability equals

$$E\left[y_i\right] = p_i = g^{-1}\left(x_i^T\beta\right) = \frac{1}{1+e^{-x_i^T\beta}} = \frac{1}{1+e^{-\beta_0-\sum_{j=1}^{n}\beta_j x_{ij}}} \tag{6.15}$$

defining the logistic function.

The Multivariate Adaptive Regression Spline (MARS) algorithm is a multiple piecewise linear regression technique in which each break point provides an area for a specific linear regression equation.

Breakpoints for classifying continuous variables are often selected using this method.

Before utilizing it in a GLM, MARS can assist in determining the breakpoints that will be used to categorize the amount of insurance component. MARS can also aid in the detection of variable interactions.

Each data point for each predictor is analyzed as a knot, and a linear regression model with the candidate feature is created.

Penalized Regression A common scenario in statistics is that we wish to do some sort of inference and/or prediction on some output $\boldsymbol{Y}$ based on possibly many inputs $\boldsymbol{X_1, X_2, \cdots, X_p}$.

Traditionally, the number of predictors/inputs p is relatively small and the sample size is certainly larger than p, usually several times larger.

Fitting a least squares regression model with a continuous response $\boldsymbol{Y}$ is a standard example. We could fit a full model using all available inputs.

$$\boldsymbol{Y = \beta_0 + \sum_{i=1}^{p} \beta_p X_p + \epsilon} \tag{6.16}$$

The principle of parsimony guides us to consider not using all possible inputs $\boldsymbol{X_i}$. There are several reasons for this.

- Model interpretability
- Losing degrees of freedom for error (particularly when n is small)
- Multicollinearity (i.e. the inputs $\boldsymbol{X_1, X_2, \cdots, X_p}$ are heavily correlated with each other

In least squares, we estimate the parameter vector $\boldsymbol{\beta}$ using the matrix equation

$$\boldsymbol{\hat{\beta} = (X^T X)^{-1} X^T Y} \tag{6.17}$$

When that design matrix X is nearly singular, this can lead to inaccurate estimates of the parameters and their standard errors, which is a good reason to not always use all of the X variables.

Ridge, LASSO and Elastic Net

One of the oldest alternatives to least squares regression is a technique called ridge regression. Unlike ordinary least squares, it will use biased estimates of the regression parameters.

The regularization function used in ridge regression is the $\boldsymbol{\updownarrow^2 - norm}$

$$\boldsymbol{R(f) = \sum_{i=1}^{p} \beta_i^2 = ||\beta||_2} \tag{6.18}$$

Recall that the ordinary least squares estimator is found by

$$\boldsymbol{\hat{\beta} = (X^T X)^{-1} X^T Y = S^{-1} X^T Y} \tag{6.19}$$

where

$$\boldsymbol{S = X^T X} \tag{6.20}$$

The ridge regression estimator is

$$\boldsymbol{\hat{\beta}^*_{(\lambda)} = (S + \lambda I)^{-1} X^T Y = (S + \lambda I)^{-1} S \hat{\beta}} \tag{6.21}$$

So the ridge regression estimator $\hat{\boldsymbol{\beta}}^*_{(\lambda)}$ is said to be a shrunken estimator of $\boldsymbol{\beta}$. What happens is that by adding the $\boldsymbol{l^2 - norm}$ as a regularizer, we are balancing minimizing the sum of squared residuals with minimizing the sum of squared beta coefficients, and thus many of the individual $\hat{\boldsymbol{\beta}}^*\boldsymbol{s}$ will get much smaller, close to (but not equal to) zero.

Note that while ridge regression has a regularization component, it does NOT have a model selection component that will remove some $\boldsymbol{X_i}$ inputs altogether from the model.

A more recent alternative to OLS and ridge regression is a technique called Least Absolute Shrinkage and Selection Operator, usually called the LASSO. Like ridge regression and some other variations, it is a form of penalized regression, that puts a constraint on the size of the beta coefficients.

Unlike ridge regression, the LASSO utilizes the $\boldsymbol{l^1 - norm}$ (i.e. absolute values rather than squares), so

$$\boldsymbol{R(f)} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{p}} |\beta_{\boldsymbol{i}}| = ||\beta||_{\boldsymbol{1}} \tag{6.22}$$

Estimation of the LASSO coefficients is considerably more involved that the matrix equations given above for ridge regression, involving convex optimization. What happens is that many of the $\hat{\boldsymbol{\beta}}^*$ will be shrunken, as in ridge regression, but many of the estimates will shrink all the way to $\hat{\boldsymbol{\beta}}^*_{\boldsymbol{j}} = \boldsymbol{0}$. This effectively means that the input $\boldsymbol{X_j}$ is removed from the model, and thus the LASSO has both a regularization AND a model selection component.

It has become a popular choice to replace ridge regression for those looking to go beyond ordinary least squares when fitting a multiple regression model.

Elasticnet

Generalized linear models (GLM) extend the classical linear regression framework, by relaxing the assumption of marginal gaussianity and instead allowing the dependent variable to belong to any of the exponential family distribution. The expected value of the response variable, y is till partially modeled using additive terms, $\boldsymbol{g(\mu) = E[y] = f(x)}$ being $\boldsymbol{g}$ the link function specific for the marginal distribution and f a function containing additive terms.

Currently GLMs represent the gold standard in personal lines pricing, as the monograph of Golburd and colleagues ([**GKT16**]) well describes. GLMs share some relevant limitations with the traditional multivariate linear regression framework.

First, non - linear relationships between the predictors and the outcome can be handled only partially and need to be explicitly hypothesized in the equation (as for predictors interactions). Also, there is no "built-in" approach to exclude non relevant features or multi-collinear ones.

The elasticnet regression extends the traditional GLM to overcome the two issues mentioned last. While traditional GLMs are estimated maximizing the loglikelihood (logLik), the coefficients of the elasticnet regression are found by optimizing max(logLik)-Penalty being $\boldsymbol{Penalty = *(*|||ŕ||| + (1-)||ŕ(2)||)}$.

The first term, the ridge penalty throws off non - significant features, whilst the second one, the lasso penalty allows coping with multi - collinearity. Vehicles' clustering is a known application of Elasticnet to actuarial practice, as described in ([**QYZ16**]).

A method called elastic net regression combines the use of $\boldsymbol{l^2}$ and $\boldsymbol{l^1}$ regularization into a single procedure. An advantage is that the LASSO cannot fit more than n variables into a model, which is limiting in situations where the number of $\boldsymbol{X_i}$ inputs p exceeds the number of data case n.
Also, when a group of inputs are highly correlated, the LASSO tends to only want to pick up one of them.

The estimate for an elastic net regression can be found as:

$$\hat{\beta}^* = \underset{\beta}{\boldsymbol{argmin}}(||\boldsymbol{y} - \boldsymbol{X}\beta||^2 + \boldsymbol{\lambda_1}||\beta||_1 + \boldsymbol{\lambda_2}||\beta||_2) \tag{6.23}$$

Basically we have the sum of squared residuals piece, the $\boldsymbol{l^1 - norm}$ piece, and the $\boldsymbol{l^2 - norm}$ piece. Ordinary least squares is the special case where both $\boldsymbol{\lambda_1 = \lambda_2 = 0}$ (i.e. no regularization), ridge regression is when $\boldsymbol{\lambda_1 = 0}$ (i.e. only $\boldsymbol{l^2}$ regularization), and the LASSO is when $\boldsymbol{\lambda_2 = 0}$ (i.e. only $\boldsymbol{l^1}$ regularization).

6.2 Machine Learning approach

The currently "standard" definition of "Machine Learning" (ML) is the use of algorithms to solve specific problems that are evaluated using a given metric as such algorithm can improve as more data are given ([**mitchell1997machine**]).

In other words, the statistical problem to find a model that best describes a phenomenon is solved not giving an *a priori* determined equation, but instead providing an algorithm that allows improving its fitting to data as more experience (data) is given. The increasing availability of a wide amount of data as well as of computing power has made possible the use of such models and the increasing importance of Data Science across all industries.

Two are the main broad categories of ML models: unsupervised learning and supervised learning.

Unsupervised learning models work on unlabeled data, classified or categorized by finding commonalities and dissimilarities. Notable algorithms within this category are the k-means clustering and dimensionality reduction techniques like the principal component analysis.

Instead, supervised learning (also known as predictive modeling) algorithms try to learn functions that map input vectors to output one, that consists in regression or classification models according to the nature of the dependent variable. Few examples of predictive modeling algorithm are multivariate regression, classification trees, gradient boosting algorithm, deep neural networks.

Chollet ([**Cho18**]) cites two other categories of ML problems that are acquiring increasing importance: semi-supervised learning, that differs from supervised learning since labels are generated from input data and reinforcement learning, where the algorithm chooses actions that maximize a form of reward.

Data science has known increasing popularity in actuarial science both from practitioners and Academia. See ([**FDM14**]) for a general overview of supervised learning models in actuarial practice, whilst the Kuhn's book ([**KJ13**]) is an excellent reference for a broader presentation to general principles of predictive modeling.

The use of ML models generally comprises the following steps: data collection and pre - processing; models fitting and tuning; selection of the final model; performance assessment, deployment and (when possible) interpretation of the modeling results.

The first step is common to almost all statistical analysis: it involves collecting available data relevant to the process, performing preliminary descriptive statistics that help in gathering a preliminary understanding of the relationships between the variables, coping with data quality issues (missing values and outliers assessment) and performing some feature - engineering if suitable.

Feature engineering basically means creating additional variables by transforming original ones; it typically requires strong specific-domain knowledge and often creativity; it is usually the key ingredient of winning ML competitions (as Kaggle).

All the ML models this paper applies allow some automatic feature selections and engineering within their algorithm; nevertheless adding some "human-driven" feature engineering often helps ML algorithms to obtain better results.

After data have been pre - processed, various candidate ML models are fitted and compared with respect to a performance metric. Relevant performance metrics are the root mean squared error (RMSE) of prediction for continuous outcomes, the logarithmic loss for binary of multinomial classification problems.

The main interest of a predictive modeling workflow is to select the model that maximizes predictive performance. Great care is given in avoiding over-fitting, that occurs when a model does not generalize well on unseen data.

To overcome over-fitting, the original data is split into a training set: the train set is used to estimate models' parameter, while the test set is used to assess the model's performance. Moreover, most ML models are defined by the values of some hyper-parameters whose optimal configuration cannot be known in advance or determined analytically.

The hyper-parameter tuning process is often performed choosing the hyper-parameters' combination that maximizes the performance metrics over a grid of possible values.

A separate validation set is often used to rank the hyper-parameters' configuration. When the data set is not excessively large the k-fold cross-validation is an alternative to distinct validation and test sets. The CV approach partitions the train data into k folds; k-1 folds are actually used to train the model, and the remaining one is used to compute the performance metrics. The process is repeated k times, one time for each fold. The performance metric is obtained averaging the k scores.

Classification Tree

Originally developed by Leo Breiman [**Bre13**], classification and regression trees (CART) use a simple but intuitive approach to form a regression surface.

Rather than using least squares to estimate the intercept and slope parameters for a regression equation, a technique called recursive partitioning is used to split the data into several groups based on values of the predictor variables $\boldsymbol{X_i}$, where the predicted value $\boldsymbol{\hat{Y}}$ for each group is the mean of that response variable $\boldsymbol{Y}$ for all data points placed n that group.

How does this work? The sample of N cases is split into two groups:

- the first group (let's call it the "left" group) are those cases where $\boldsymbol{X_i \geq t_i}$
- the second group (the "right" group) are the other cases where $\boldsymbol{X_i < t_i}$

There will be subsequent such binary splits based on these groups and the variables $\boldsymbol{X_i}$, and when the process terminates we are left with j such groups or terminal nodes $\boldsymbol{R_j}$, where there is a predicted value $\boldsymbol{\hat{Y}_{R_j}}$ for each node that is the mean response for all cases in that node (for a regression tree where $\boldsymbol{Y}$ is numerical) or the most common value of the response variable (for a classification tree where $\boldsymbol{Y}$ is categorical).

Various algorithms are used to decide which variable $\boldsymbol{X_i}$ to split at each step and which splitting value or threshold $\boldsymbol{t_i}$ is used at each step of the tree construction.

The most common method (the default) is this: at step $\boldsymbol{k}$, $\boldsymbol{group_k}$ of $\boldsymbol{N_k}$ cases remain to be split, where those cases have mean

$$m_k = \sum \frac{y_i}{N_k} \tag{6.24}$$

and sum of squares

$$s_k^2 = \sum (y_i - m_k)^2 \tag{6.25}$$

The threshold value $\boldsymbol{t_i}$ is chosen to minimize the sum of the sum of squares for the left and right groups

$$s_{k,left}^2 + s_{k,right}^2 \tag{6.26}$$

The idea is for these two groups formed by the split or partition to be as "different" as possible. The ultimate goal is to minimize the residual sum of squares for the entire tree

$$RSS = \sum (y - \hat{y})^2 \tag{6.27}$$

CART and the Cubist model

Classification and regression trees (CARTs) are popular supervised learning algorithms that permit to easily handle non - linear relationships between the outcome and candidate independent variables for both regression and classification problems ([**breiman2017classification**]).

Basically, a CART recursively searches through available predictors to find the value that best splits the data into two or more groups (partitions), according to a given loss metrics. The process is repeated across subsequent partitions and a hierarchical tree-like structure with branches and final leaves is therefore created; the process ends when subsequent splits give no significant gain in reducing the prediction error.

Tree–based models can alternatively be seen as a set of nested if-then statements ("rules") on the predictors that partition the data ([**Kuh16**]).

The ease of interpretability and communicability to non - technical audience, their fast computability and their inherent feature selection capability contribute to their popularity.

Nevertheless, known limitations of CART are: the low robustness to small changes on data and, at least for the standard implementation, the propensity to over-fit unless special arrangements are taken (pruning). Also, since their prediction is constant at the end of the leaf, they tend to fail at extremes and other ML models offer better performances often.

The "Model trees" represents an evolution of the original CART approach and its most notorious implementation is the M5 algorithm ([**WW96**]). When the model trees make a split of the data, the M5 algorithm fits a linear model to the current subset using all the predictors involved in the splits along the path ([**KJ16**]).

The Cubist model improvements the M5 approach on the following key points: how linear models smooth results at terminal nodes; the ability to adjust prediction using nearby points from the training set; the potential to use boosting.

In synthesis, the Cubist algorithm finds optimal partitions of the training data determined by a specified range of the predictors' value on which separate linear regressions are fit.

More in detail, a Cubist model consists of trees where terminal leaves (and intermediate nodes) are characterized by linear regressions that employ the predictors used in previous splits within their equations. The final predictions are also determined by the outcome regressions within intermediate nodes.

he predictive performance of the Cubist algorithm is also enforced by ideas coming from boosting (the "commitees") and k-nearest neighbors. In particular, the "commitees" are trees created in sequence where the more recent tree tries to correct the errors of previous ones. Finally, Cubist models allow smoothing the prediction of a new sample according to the k nearest neighbors values.

Therefore the Cubist model blends ideas both from the CART and from the linear regression approaches, the latter having been traditionally the reference model for CB analysis. As already mentioned, the Bodoff paper's idea was to fit separate linear regressions in the partitions defined by peril and territory ([**BG2012**]) that could be thought as a "manual" implementation of the Cubist approach.

Aggregation Methods

To improve performance of CARTs, different aggregations methods are used. Bootstrap aggregation, or bagging represents the most natural way to reduce variance in a machine learning tool, consisting in running the model on $\boldsymbol{N}$ random samples, and average the $\boldsymbol{N}$ predictions.

The most powerful methodologies of aggregation applied to CARTs are the Random Forest and the GBM, that we will use in the next chapter to estimate the frequency of the cancellation cover.

Random Forest

A simple enhancement of bagging is represented by Random forests [**Bre01**]. The algorithm was originally created to aggregate trees and can be easily extended to other tools.

In bagging, the main drawback is that the most significant predictors in the database will be always used for the first splits in any bagged tree. It produces a lot of strongly associated bagged trees, which makes it difficult to reduce variance much. Each split in every tree in a random forest is compelled to take into consideration just a subset of the $\boldsymbol{m}$ predictors. Before performing any split, the algorithm chooses $\boldsymbol{p} < \boldsymbol{m}$ predictors at random and splits using one of them.

This is actually a simple approach to add more variation to the process.
As a result, the random forest can create a variety of trees, and if the strongest

predictors aren't chosen for the initial splits, there's a good probability that the bagging method couldn't have generated the final tree.

A standard choice in statistical software to set $\boldsymbol{p}$ is $\boldsymbol{p = \sqrt{m}}$ rounded down. This results in a very small number of viable predictors at each split, ensuring a decent amount of variability throughout the forest.

In extremely randomized trees, or extra trees, both the subset of predictors and the splitting rule are random, in order to introduce an additional source of variance.

RFs try to overcome CART limitations joining blending ideas from CART to the bagging framework ([**Bre01**]).

The bagging framework stands for "bootstrap aggregating"; it basically means partitioning the training data into k independent samples, fitting distinct models on each sample and averaging their predictions. RFs bags simple CARTs. As for most ML algorithms, hyper-parameters must be tuned to optimize performance. The ones most relevant for RFs are: the number of independent trees (the aforementioned bagged models), the depth and a minimum number of obs in trees' final leaves, and the fraction of available predictors considered when fitting a single tree.

A known actuarial application of RFs is the ranking of suspicious claims ([**FDM16**]). Also, in the ILS literature, the RF is the sole ML model currently tested to predict Spread at issuance in Morton's 2018 paper ([**Lan18**]).

Gradient Boosting Machine

Boosting is not built on bootstrapped samples, although it is still based on trees aggregation. Both bagging and random forests work on the same principle: the larger the sample, the better the estimation.

Set the interaction depth $\boldsymbol{d}$, or the number of splits in each tree, first. Because we wish to group together weak learners, d should be a modest number, such as 1 or 2.

Let's use it for a first, weak prediction for the $\boldsymbol{j^{th}}$ training record:

$$\boldsymbol{\hat{\theta}_{0j}^{boost} := \hat{\theta}_{0j} = \theta_j + \varepsilon_{1j}, \quad \forall_j = 1, ..., n} \tag{6.28}$$

where $\boldsymbol{\varepsilon_{1j}}$ is the estimation error from the initial step. Using the same predictors, the algorithm then builds a new tree with d splits to predict $\boldsymbol{\varepsilon_{1j}}$, so that the boosting estimator at the subsequent step is

$$\boldsymbol{\hat{\theta}_{1j}^{boost} := \hat{\theta}_{0j} + \hat{\varepsilon}_{1j} = \theta_{0j} + \varepsilon_{2j}, \quad \forall_j = 1, ..., n} \tag{6.29}$$

Once again, the algorithm builds a tree to predict $\boldsymbol{\varepsilon_{2j}}$, and obtain the new estimation:

$$\boldsymbol{\hat{\theta}_{1j}^{boost} := \hat{\theta}_{0j} + \hat{\varepsilon}_{1j} + \hat{\varepsilon}_{2j} = \theta_{0j} + \varepsilon_{3j}, \quad \forall_j = 1, ..., n} \tag{6.30}$$

The iteration goes on until the N^{th} estimation:

$$\hat{\theta}_{1j}^{boost} := \hat{\theta}_{0j} + \sum_{i=1}^{N} \hat{\varepsilon}_{ij}, \quad \forall_j = 1, ..., n \tag{6.31}$$

Weak learners are therefore taught to utilize the residuals of prior (weak) estimations instead of the target variable.

It's worth noting that when N grows, this approach is prone to over t. Even if d is really tiny, we don't know how fragile the single trees are. We can also choose a learning rate λ to better regulate the learning process. This way the final estimation is:

$$\hat{\theta}_{1j}^{boost}(\lambda) := \hat{\theta}_{0j} + \lambda \sum_{i=1}^{N} \hat{\varepsilon}_{ij}, \quad \forall_j = 1, ..., n \tag{6.32}$$

The boosting approach is an algorithm were "weak" models (learners, t (x)) are sequentially added in order to build a "very strong" learner. In detail, the prediction at stage t, is given by the predictions at the previous stage plus a contribution of a weak learner that models the residuals of the t-th stage: $F_t(x) = F_(t-1)(x) + *h_t(x)$; the parameter is known as learning rate and it helps in avoiding over-fitting, while the $h_t(x)$ term models the residuals of prediction at time t and its contribution is partially added to the whole model F_t.

Typical algorithms used for h_t terms are CARTs, thus many ABMs' tuning parameters are shared with RFs models, apart . The stochastic descendant gradient is used to train these models, the acronym GBM comes from.

Boosted trees have been known great popularity in the analytic community for their exceptional predictive performance; most winning Kaggle competitions are based on GBMs at least in part.

As for other algorithms, tuning is needed to find the optimal values of the models' hyper-parameter as Malohlava and Candel detail ([**MC18**]). Many recent evolutions of the original GBM algorithm have been proposed; XGBoost ([**CG16**]) and LightGBM ([**Ke17**]) the most promising ones at the moment this paper being drafted.

Ensemble learning and stacked ensemble

The Ensemble learning framework belongs to the so - called "meta learning" algorithms. A two stages approach is used to perform the predictions: algorithms of different nature $1, 2, \ldots, J$ are trained in the first stage to obtain a set of $y_{i,j}$ of initial predictions.

Such predictions represent the inputs for a supplementary model, the "Super Learner", on the true y that aims to optimally combine the base learners as detailed in ([**VP07**]). For example, an elastic-net GLM can be used as meta learner to combine GBMs and RFs models.

Most top-performing Kaggle competitions are ensembles of many base models. The H2O ML suite provides an efficient implementation of the Ensemble learning algorithm, the Stacked Ensemble (StkEns) that is used in this stage.

Deep Learning

Artificial neural networks (ANN) are ML algorithms that aim to mimic the brain functioning: input data flows into layers of networks were their linear combinations are properly weighted and output to subsequent layers using nonlinear activation functions.

Deep neural network (DNN) are ANN characterized by more than one layer. ANN have been formulated since the 50s, but innovations in computing power, advances in methodologies that improve generalization avoiding over-fitting and data availability has determined a renewed interest in ANN, also because they have proved to be very effective in many artificial intelligence applications like image recognition, natural language processing and pattern classification.

Many algorithms compose the Deep Learning family. One of the most simple that has been used in our work is the multi layer perceptron, whose typical applications lie in supervised learning problems.

Defining a DNN means configuring its architecture: the number of layers and units (neurons) for each layer, the activation functions and other parameters that regulate the training as well specified in ([**Cho18**]).

Few applications of Deep Learning have been proposed until now in actuarial science; for example Schelldolfer and Wutrict ([**SW19**]) propose the use of Deep Neural network to strength classical non-life pricing.

6.3 Pricing model interpretation

Probably, the most important reason that has limited a widespread adoption of ML modeling is the lack of models' output interpretability. Interpretability is not an issue when using multivariate regressions financial econometricians are familiar with. Models' interpretability significantly supports the adoption of innovative approaches and often is a fundamental requirement especially in regulated environment like the Insurance Industry.

Significant research efforts have been recently devoted to developing tools that ease the understanding of ML models. One of this approach, the LIME model ([**RSG16**]), locally approximates a prediction surface provided by the ML model by a linear regression on which the interpretation of results are based.

Another approach, the Interaction-based Method of Explanation ([**Kon10**]), determines the contribution of each feature in the final prediction using a technique based on the game theory (the Shapley Value).

([**MBC18**]) describes a more statistical approach to interactions were their strength in ML models can also be investigated by using H-statistic. One way to estimate the interaction strength is to measure how much of the variation of the prediction depends on the interaction of the features.

This measurement is called H-statistic, introduced by Friedman and Popescu ([**FP08**]), a total interaction measure that tells us whether and to what extent a feature interacts in the model with all the other features.

If a feature does not interact with any of the other features, we can express the prediction function $\hat{\boldsymbol{f}}(\boldsymbol{x})$ as a sum of partial dependence functions, where the first summand depends only on j and the second on all other features except $\boldsymbol{j}$:

$$\hat{\boldsymbol{f}}(\boldsymbol{x}) = \boldsymbol{PD_j(x_j) + PD_{-j}(x_{-j})} \tag{6.33}$$

where $\boldsymbol{PD_{-j}(x_{-j}}$ is the partial dependence function that depends on all features except the $\boldsymbol{j}$-th feature.

This decomposition expresses the partial dependence (or full prediction) function without interactions (between features $\boldsymbol{j}$ and $\boldsymbol{k}$, or respectively $\boldsymbol{j}$ and all other features).

In a next step, we measure the difference between the observed partial dependence function and the decomposed one without interactions. We calculate the variance of the output of the partial dependence (to measure the interaction between two features) or of the entire function (to measure the interaction between a feature and all other features).

The amount of the variance explained by the interaction (difference between observed and no-interaction PD) is used as interaction strength statistic.

The statistic is **0** if there is no interaction at all and **1** if all of the variance of the $\boldsymbol{PD_{j,k}}$ or $\hat{\boldsymbol{f}}(\boldsymbol{x}))$ is explained by the sum of the partial dependence functions. An interaction statistic of 1 between two features means that each single PD function is constant and the effect on the prediction only comes through the interaction.

The H-statistic can also be larger than 1, which is more difficult to interpret. This can happen when the variance of the 2-way interaction is larger than the variance of the 2-dimensional partial dependence plot.

Mathematically, the H-statistic proposed by Friedman and Popescu for the interaction between feature $\boldsymbol{j}$ and $\boldsymbol{k}$ is:

$$\boldsymbol{H^2_{j,k} = \sum_{i=1}^{N} [PD_{j,k}(x_j^{(i)}, x_k^{(i)}) - PD_j(x_j^{(i)}) - PD_k(x_k^{(i)})] / \sum_{i=1}^{N} PD^2_{j,k}(x_j^{(i)}, x_k^{(i)})} \quad (6.34)$$

The same applies to measuring whether a feature j interacts with any other feature:

$$\boldsymbol{H^2_{j,k} = \sum_{i=1}^{N} [\hat{f}(x)^{(i)} - PD_j(x_j^{(i)}) - PD_{-j}(x_{-j}] / \sum_{i=1}^{N} \hat{f}(x)^{(i)}} \quad (6.35)$$

The H-statistic is expensive to evaluate, because it iterates over all data points and at each point the partial dependence has to be evaluated which in turn is done with all n data points.

In the worst case, we need 2n 2 calls to the machine learning models predict function to compute the two-way H-statistic (j vs. k) and 3n 2 for the total H-statistic (j vs. all).

Chapter 7

Case study: issuance of an italian CAT Bond

7.1 Estimating CAT Bond pricing

The first part of this chapter will follow approach proposed by Spedicato and Pallaria ([56]). This thesis will use the DALEX package ([**LeD18**]), a recent R library that provides ready-to-use statistical routines also based on the aforementioned approaches, improved by iml package ([46]). The routines contained in those libraries allow to:

- *evaluate model performance*, that basically means contrasting the residual distributions of competing predictive models;
- *variables' response analysis*, that allows investigating the impact of each independent variable on the modeled outcome.

  Some tools that have been developed at this regard are the Partial Dependence Plots (PDP), that shows the expected value of the outcome $\boldsymbol{y_i}$, given other predictors. The package's documentation describes other tools, the Accumulated Local Effects and the Merging Paths approach, that are alternative ways to investigate the contribution of continuous and categorical predictors to the final outcome;
- *discover interactions between predictors*, defined as the share of variance that is explained by the interaction (package:iml);
- *predictions' breakdown with interactions*, that allows understanding how each variable marginally contributes to the final predicted outcome for a given single observation.

  In addition, it can nicely represent the contributions of the variables using a waterfall-plot style.

All ML aforementioned but Cubist have been applied thanks to the H2O library within the R statistical software (R Core Team 2018).

The H2O software is an open source project, that well interfaces with R and Python programming languages, containing strongly optimized implementations of most relevant ML algorithms (by parallelization, GPU exploiting, modeling facilities), providing a common interface to perform hyper-parameters' tuning, model assessment and scoring.

At this regard, the R caret package provides also a unified interface of many ML algorithm and it has been used to manage Cubist models.

We have chosen not to explicitly include time (e.g. by the issue year) in the model, as also in the Braun paper. The reason is two-fold:

- when dealing with a linear model, using different models for different periods would certainly improve the predictive performance, but would include a significant subjectivity in defining the time windows;
- including time as predictor implies setting methodological choices on predictions extrapolations on "future" periods (not seen in the training set).

7.1.1 Statistical data analysis

The collected data set records the following characteristics of CBs, all known at issuance:

- Spread, $\boldsymbol{S}$;
- Expected Loss, $\boldsymbol{EL}$;
- Maturity (i.e. bond's term);
- Size (in Million US Dollar);
- Trigger type (Indemnity, Index, Parametric, ...);
- Main territory; in addition, a dummy variable named "Multiterritory" indicates whether the portfolio at risk covers more than one territory;
- Perils covered: dummy variables named "RiskWind", "RiskEQ", "RiskOthers" and "Multirisk" track the peril's covered as well as whether more than one peril is covered.
- The rating of the Sponsor (four classes);
- Coverage type: "occurrence", "aggregate", not recorded (this variable has been consistently recorded in primary sources only since the second half 2017);
- Guy Carpenter's yearly Rate on Line (RoL) index, a proxy of the price levels in the property traditional Reinsurance Market, for which CBs represent an alternative.

  The observation of its trend allows identifying soft and hard market cycles.
- Thomson Reuters BBSpread, that is the yield in excess to risk-free assets of a reference portfolios of corporate bonds BB rated.

  As many CB are BB rated, such index is representative of the yield that securities similar to CBs are priced in financial markets.

Collecting CB characteristics required to manually gather the data and to perform cross-checks across the following sources:

- Artemis (Artemis 2020): Bermuda on-line newsletter focused on ILS topics.

  It collects information on CAT Bond issuances (since 1997) both in structured and not form. It is the leading source for CB and ILS data sources;

- Lane Financial (Morton N., Lane and Beckwitt, Roger 2020) quarterly market reviews: Lane Financial is a consulting firm specialized in the ILS market, for which it publishes quarterly reviews.

  New CB issuances key features (e.g. Spread, Expected Loss and Term) are tabulated in their periodic technical reports;

- Aon Benfield and Willis Towers Watson quarterly reports focused on CAT Bond issuances.

  Aon's earlier publication dates back to 2009; it has been enriched with summary tables containing key features of previous quarter's issuances since 2011: EL, Perils Covered, Spread. Since 2016, Willis Towers Watson has drafted a quarterly report similar to the one from AON;

- Bloomberg (Bloomberg 2014), the well known financial data provider. It has been used to cross-check the data from previously mentioned sources and to retrieve CUSIP - ISIN identifiers.

Next figure show an extract of Database used.

Issuer	IssueYear	DataEmission	Maturity	Size	ExpectedLoss
Sanders Re II Ltd. (Series 2020-1) CL B	2020	01/03/20	4	100	0,0092
Integrity Re II Pte. Ltd. (Series 2020-1)	2020	01/03/20	3	125	0,0198
Akibare Re Pte. Ltd. (Series 2020-1)	2020	01/03/20	4	75	0,0081
Atlas Capital Reinsurance 2020 DAC (Series 2020-1)	2020	01/04/20	4	200	0,0284
Azzurro Re II	2020	11/05/20	4	150	0,02685
	Spread	Trigger	US	JP	EU
Sanders Re II Ltd. (Series 2020-1) CL B	0,13	I	1	0	0
Integrity Re II Pte. Ltd. (Series 2020-1)	0,0725	I	1	0	0
Akibare Re Pte. Ltd. (Series 2020-1)	0,0275	I	0	1	0
Atlas Capital Reinsurance 2020 DAC (Series 2020-1)	0,075	IL	1	0	0
Azzurro Re II	0,0425	I	0	0	1
	Territory	RiskEQ	RiskWind	RiskOthers	MultiriskYN
Sanders Re II Ltd. (Series 2020-1) CL B	US	1	1	1	Y
Integrity Re II Pte. Ltd. (Series 2020-1)	US	0	1	0	N
Akibare Re Pte. Ltd. (Series 2020-1)	JP	0	1	1	Y
Atlas Capital Reinsurance 2020 DAC (Series 2020-1)	US	1	1	0	Y
Azzurro Re II	EU	1	0	0	N
	RatingCompo	RoLindex	RatingCatBon	BBSpread	Occurrence_A
Sanders Re II Ltd. (Series 2020-1) CL B	1	191,34		2,82	A_O
Integrity Re II Pte. Ltd. (Series 2020-1)	1	191,34		2,82	O
Akibare Re Pte. Ltd. (Series 2020-1)	1	191,34		2,82	O
Atlas Capital Reinsurance 2020 DAC (Series 2020-1)	1	191,34		2,82	A
Azzurro Re II	1	191,34		2,82	O

Figure 7.1. Extract of DataBase used

Descriptive analysis

The table below reports the key statistics of the continuous variables from the study data set:

Table 7.1. Continuous variables regarding CBs dataset

Variable	num	mean	std.dv.
Expected Loss	730	0.0248	0.0270
Maturity	730	3.1920	1.0715
Size	730	131.9003	113.5486
Spread	730	0.0741	0.0518

Over 730 CB issuances reported, expected spread is **7.4%** with a **5.2%** standard deviation.

The following scatter-plot depicts the relationship between $\boldsymbol{S}$ and $\boldsymbol{EL}$, split by territory:

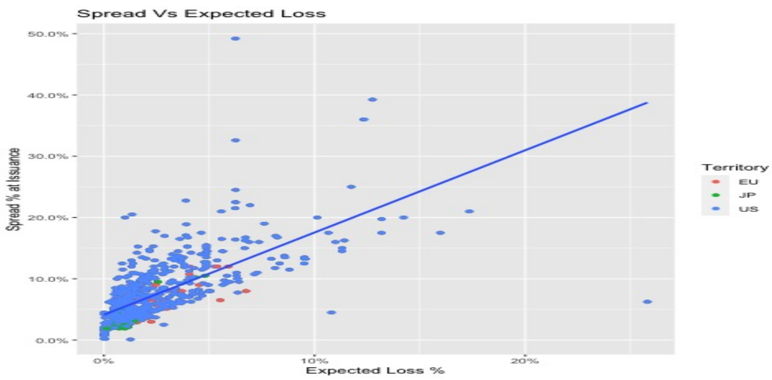


Figure 7.2. Spread vs Expected Loss

The number and amount issued by year is shown in the graph below:

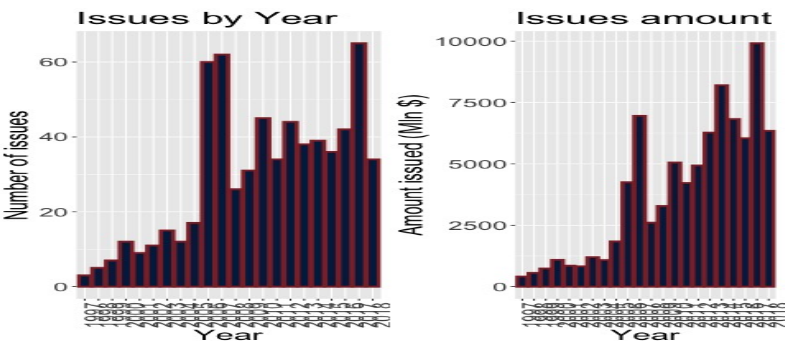


Figure 7.3. CAT Bonds by number and amount issued by year

The two forthcoming figures display the RoI Index and BB Spread time series:

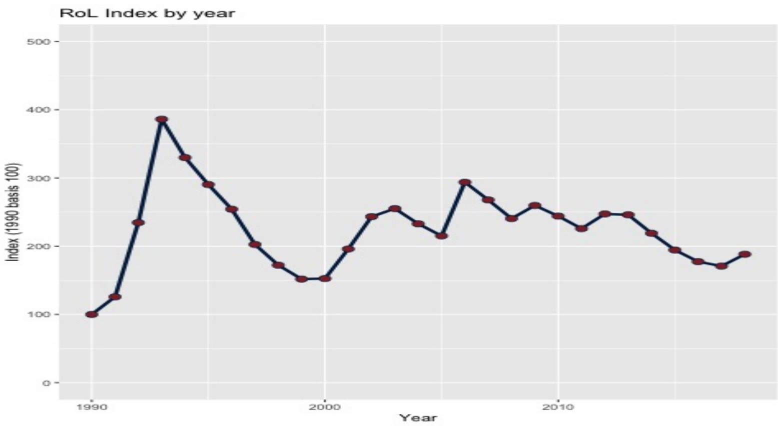


Figure 7.4. ROL Index

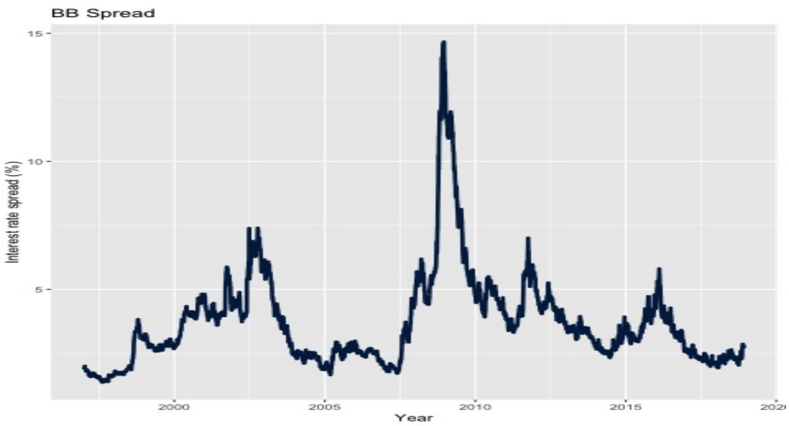


Figure 7.5. BB Spread

Bivariate Analysis

The relationship between trigger type, S and EL is shown below. Idemnity loss issues represent the most represented trigged among the CB issued until 2020.

Table 7.2. Spread and EL by Trigger type

Trigger	num	Spread	Expected Loss
ID = Indemnity	327	0.0652	0.0240
IL = Index-linked	194	0.0892	0.0288
PA = parametric	110	0.0682	0.0221
OT = other	99	0.0801	0.0228

When issue's size and maturity are considered, EL and Spread appear to decrease when maturity increases, as well as when size increases

Table 7.3. Spread and EL by Maturity

Maturity	num	Spread	Expected Loss
[0.417, 2.91)	146	0.1038	0.0310
[2.908, 3.02)	153	0.0729	0.0235
[3.019, 3.10)	139	0.0720	0.0207
[3.103, 4.02)	158	0.0703	0.0287
[4.019,10.00]	134	0.0495	0.0194

Table 7.4. Spread and EL by Size

Size	num	Spread	Expected Loss
[3.0, 50.2)	165	0.1018	0.0366
[50.2, 85.0)	129	0.0807	0.0288
[85.0, 126.6))	148	0.0688	0.0200
[126.6, 205.0)	175	0.0595	0.0187
[205.0,1500.0]	113	0.0554	0.0190

US territory covers the great majority of historical CB issues and it shows the highest value of Spread and Expected Loss; the lowest is observed in Japan issues.

Table 7.5. Spread and EL by Territory

Territory	num	Spread	Expected Loss
EU	53	0.0528	0.0202
JP	53	0.0381	0.0117
US	624	0.0789	0.0264

Issues covering multiple perils shows higher Spread and EL, even if the increase of EL is lower than the increase of Spread.

Table 7.6. Spread and EL by Multirisk

Multirisk: Y/N	num	Spread	Expected Loss
Y	367	0.0652	0.0240
N	363	0.0892	0.0288

The bivariate relationship between Spread and EL as a function of Sponsors' rating is somewhat unexpected: unrated sponsors' exhibit the lowest spreads and the higher expected losses; no definite pattern emerges as the sponsor's rating increases.

Spread appears to generally increase as the RoL index increases:

Table 7.7. Spread and EL by ROL index

ROL index	num	Spread	Expected Loss
[152, 182)	174	0.0628	0.0340
[182, 219)	132	0.0650	0.0240
[219, 246)	168	0.0714	0.0197
[246, 268)	132	0.0862	0.0222
[268,294]	124	0.0903	0.0226

The bivariate relationship between the KPIs and BBSpread appears unclear.

Table 7.8. Spread and EL by BB Spread

BB Spread	num	Spread	Expected Loss
[1.53, 2.38)	150	0.0674	0.0292
[2.38, 2.84)	203	0.0770	0.0262
[2.84, 3.45))	85	0.0688	0.0200
[3.45, 4.44)	147	0.0752	0.0223
[4.44,11.92]	145	0.0863	0.0225

Issues covering aggregate occurrences exhibit higher Spread and losses:

Table 7.9. Spread and EL by Multirisk

Occurrence aggregate	num	Spread	Expected Loss
A	190	0.0725	0.0312
O	252	0.0706	0.0273
M	288	0.0782	0.0185

7.1.2 Model results and interpretation exercise

The purpose of this second susection is to understand whether ML approaches are superior to classical ones, both in predictive accuracy and in terms of models' explicability, also taking into account the size of the data set, relatively small when compared to those of typical ML applications.

At this regard ML models will be compared to linear regression approaches in order to predict the Spread versus the EL and other predictors known at the issuance of the security.

Data will be split in a train containing 80% of issuance and a test set that collects the 20% most recent issuances. The predictive performance will be evaluated according to the Root Mean Square Error of prediction (RMSE), for actual and estimated $\boldsymbol{S}$.

The following models have been estimated and compared:

- A simple linear regression in the form

$$\boldsymbol{S} = \boldsymbol{\alpha} + \boldsymbol{\beta} \cdot \boldsymbol{EL} \tag{7.1}$$

- A multivariate linear regression containing almost all predictors of the full Braun model, thenceforth the "Econometric Model" (EM): EL, size, maturity, trigger, territory, peril, RoI index, BB Spread. Also the following interaction terms between territory and peril have been included: US and Wind, US and EQ, EU and Wind and JP and EQ.

  Differently from the EM, the Swiss Re issue dummy variable has not been included in the model to improve generality as well as the investment grade status of the CB, since it was not collected in our data set and, also, because it was found not significant in the Braun paper.

  A major additional difference with the aforementioned article is the inclusion of coverage type and the Sponsors' rating, thus including all the available predictors recorded in our dataset in a linear model. All models herewith presented share the EM's set of predictors;

- An ElasticNet regression including the same predictors of the EM; the optimal weight of rigde and lasso penalties (the $\boldsymbol{\alpha}$ hyper-parameters) has been determined using random grid search;

- A GBM where a random grid search was used to optimize various hyper-parameters the most important ones follow: the boosting learning weight, the minimum number of observations in trees' terminal nodes, the predictors' and rows' sampling percentages and the maximum depth of each tree;

- A DNN using the multilayer perception approach. The interested reader could oversee [**(Candel et al. 2016)**] for further details on tuning these models.

  As for the aforementioned parameter, hyper-parameters' tuning was used with some judgmental choices. In particular, the candidate architectures of the hidden layers were set between one and two layer and the number of neurons within each layers could vary between 2 3 and 2 6;

- A RF tuned on the following parameters: the maximum depth of each tree, the number of rows and columns to sample for each tree and the total number of independent trees;
- A stacked ensemble model (StkEns) where the Elasticnet regression, the GBM and the RF (the base learners) were combined using a GBM model as meta-learner (with hyper-parameters set as default values).

  The DNN model was excluded from the base learners' list since its inclusion empirically reduced the predictive performance;
- A Cubist model, where the number of boosting trees ("commitees") and nearest–neighbors to adjust predictions were tuned.

All the models were trained using the 10-fold CV approach and the optimal hyper-parameters' set has been found minimizing RMSE on the validation set by Random Grid Search. Eventually, data on **730** issues were collected, sorted by issuance date and split into a train (the 80% least recent ones) and a test set (20% most recent ones).

As far as the author's knowledge, the dataset under study is the widest collected for CB research to date. The training set was used to fit the different models whose prediction in the test set have been ranked in terms of RMSE.

The R Software (R Core Team 2018) has been used as well as statistical libraries dedicated to machine learning.

In particular, the caret package (Jed Wing et al. 2018) and Cubist (Kuhn and Quinlan 2018) was used to fit and optimize the Cubist and the EM models, while remaining ones were modeled using the h2o (LeDell et al. 2018) R package.

Models' performance comparison

The following table displays predictive performance of selected models on test set CAT Bonds.

Table 7.10. Models' rmse on test set data

model	rmse
RF	0.0278
Stacked Ensemble	0.0293
GBM	0.0295
Cubist	0.0318
DNN	0.0322
EC	0.0456
elasticnet	0.0458
lm	0.0566

The following considerations can be drawn:

- The accuracies of linear models (lm, EC and Elasticnet) is the lowest; ML models can more than halve the prediction error of linear ones;
- Combining more ML models (the stacked ensemble) is confirmed as an effective approach to gain predictive performance. In fact, the StackedEnsemble model is the one with the lowest prediction error;
- The Cubist model, that blends features from CART and linear regression models, shows very good performance as its RSME is just a little higher than the StkEns's one.

  A possible reason for this behaviors lies in using linear regressions as "local" models that effectively tracks the "natural" relation between S and EL;
- ML models appear to be an effective tool in terms of predictive accuracy even with "small data" set as the one modeled in our analysis.

Model explainability

The interpretation of models' output has been performed using the algorithms contained in the DALEX and iml R packages to which the interested reader is referenced for methodological details.

The survival plot indicates that the residuals of the StkEns and of the Cubist models are almost always smaller than those obtained by the EM one. The plot also confirms the strong similarity of StkEns and Cubist errors of prediction.

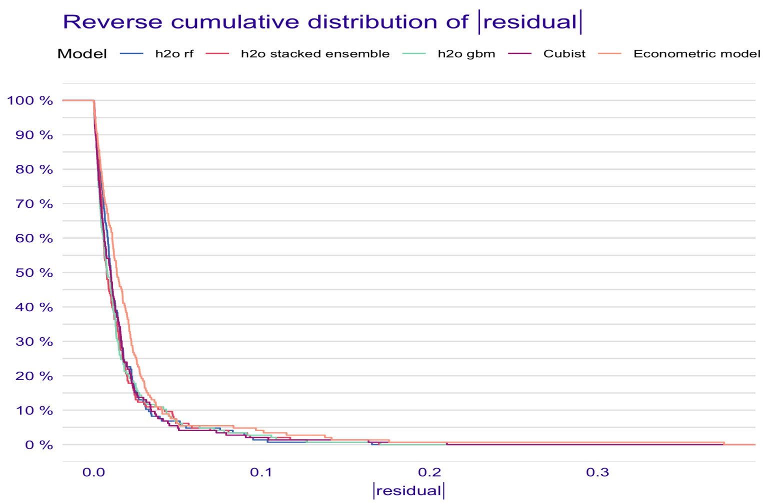


Figure 7.6. Prediction errors analysis

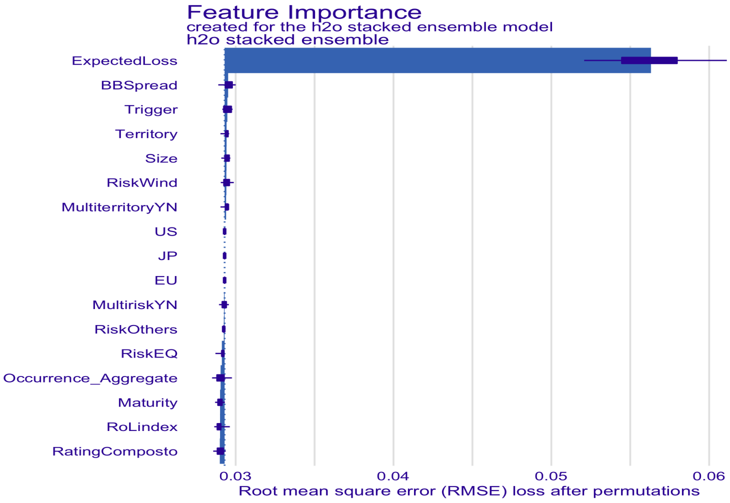


Figure 7.7. StackedEnsemble model's variable importance

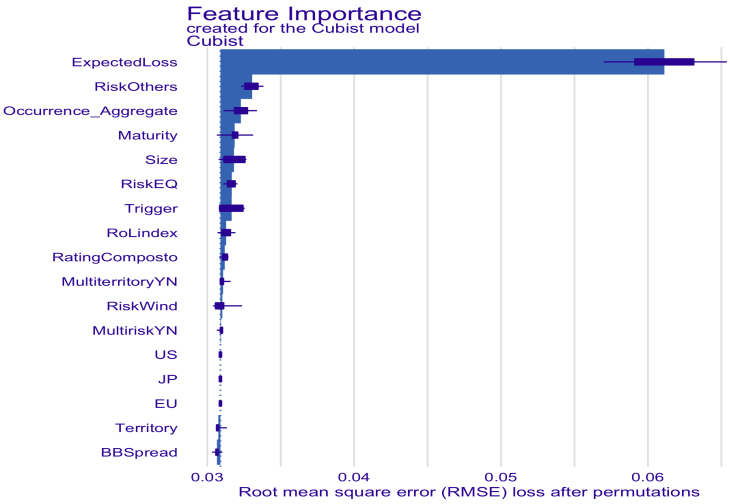


Figure 7.8. Cubist model's variable importance

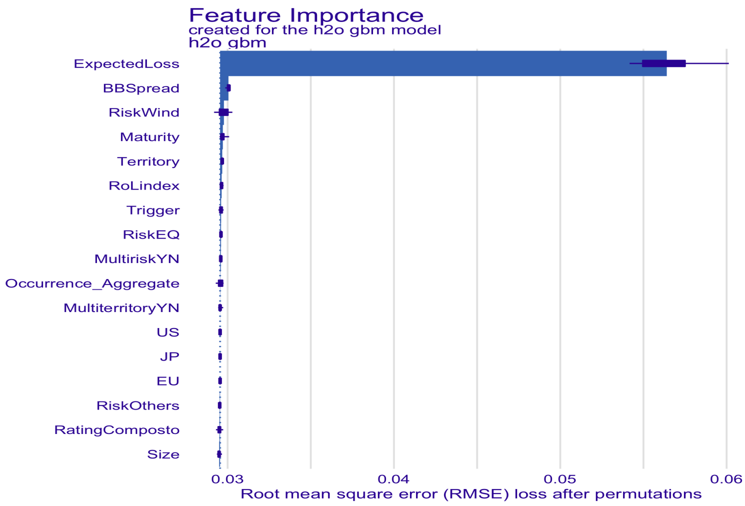


Figure 7.9. GBM model's variable importance

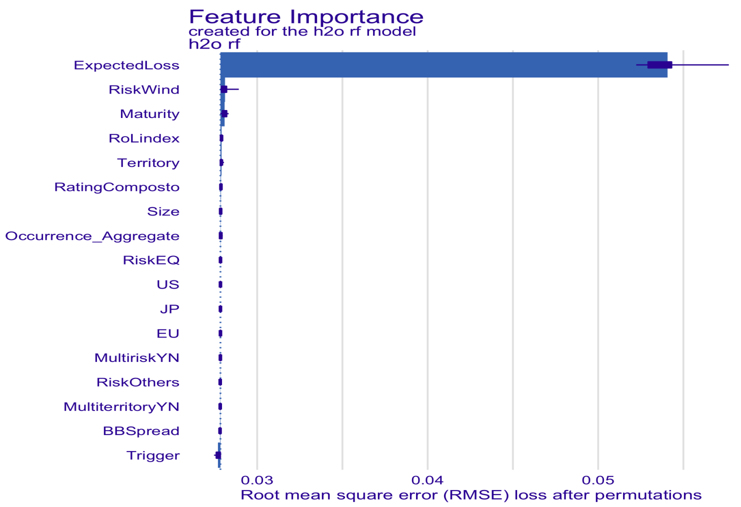


Figure 7.10. Random forest model's variable importance

The variable importance analysis stresses the overwhelming importance of the EL as the main predictor of Spread, for both selected ML models and in agreement with existing literature.

The second and third variables in order of importance, the Cover (Occurrence-aggregate) and the Trigger type show a much smaller relative importance compared to the EL, even if the two ML approaches give them slightly different weights.

It is worth to underline that, having the cover being consistently provided only for recent issuances, it could mask a time effect. Only future research on a wider and updated data set might provide more insight on this point.

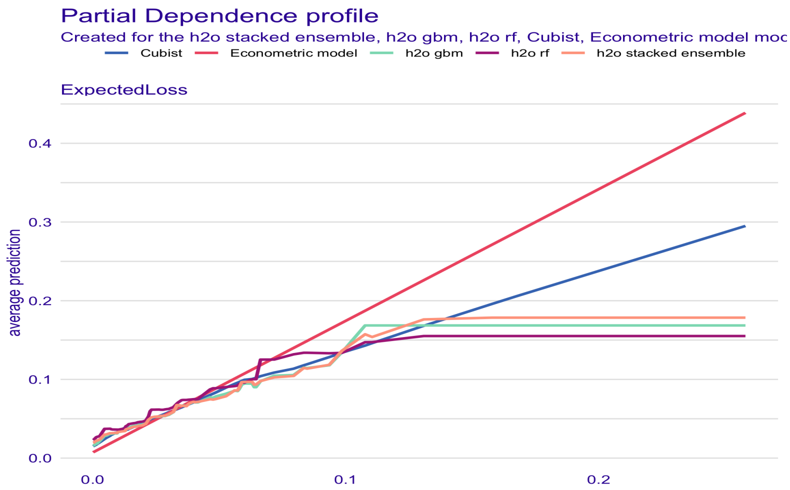


Figure 7.11. Expected Loss marginal effect plot

The marginal effects plot, according to both the StkEns and the Cubist approaches, indicates that the marginal relationship between spread and expected loss is better approximated by a concave down relation.

It is worth to remember that early papers on CBs as the one from Kreps, Major and Lane proposed similar functional forms.

The graphs present an overall concave structure, and, not surprisingly, a positive relationship between the premium and EL.

However, while the slope of the graphs is seen decreasing toward the higher values of the EL, locally, it exhibits convex areas, where the premium increases more sharply with the increasing EL.

Thus, the non-trivial relationship between the premium and EL suggests that it is challenging to identify a parametric representation of the premium as a function of the EL as would be required by a linear regression. Therefore, it provides a potential explanation for the ML, and in particular random forest. outperforming the other considered methods.

Furthermore, it confirms how an increase in the loss size increases the trigger risk level above which securitization is chosen, as underlined by some recent researches about CAT Bond market prices (subramanian2018reinsurance). Hence, catastrophe exposures, which are characterized by lower probabilities and higher severities, are more likely to be retained or reinsured rather than securitized.

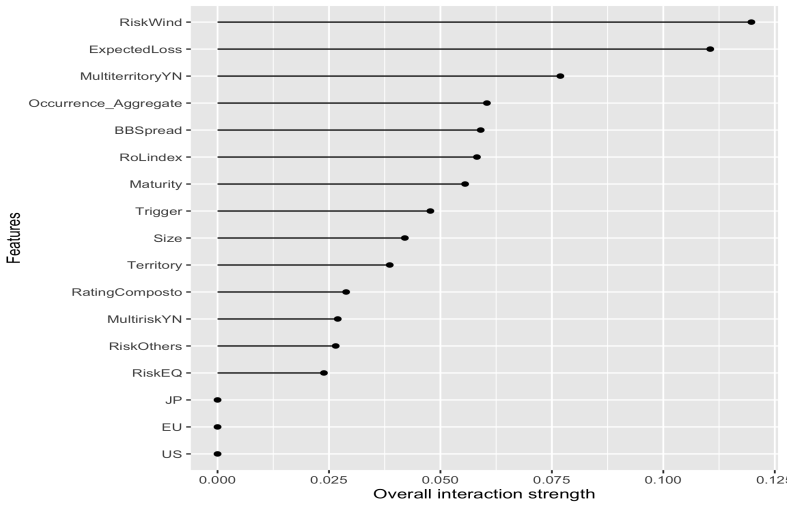


Figure 7.12. StackedEnsemble model's interactions

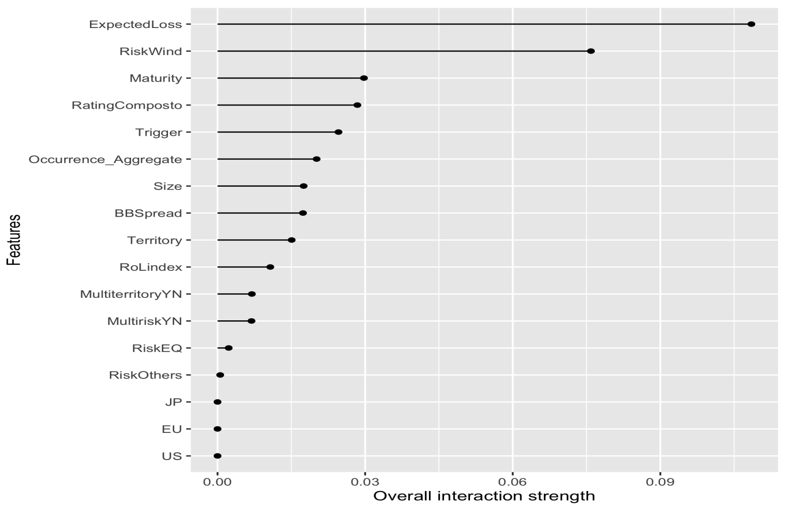


Figure 7.13. Random forest model's interactions

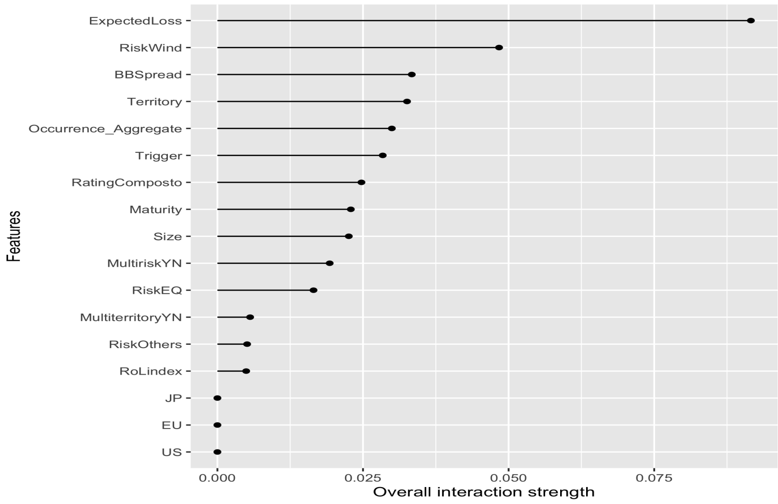


Figure 7.14. GBM model's interactions

The interactions analysis stresses the overwhelming importance of selected ML models in market prices dynamics interpretation. The second and third variables in order of importance, the Cover (Occurrence-aggregate) and the Trigger type show a much smaller relative importance compared to the EL, even if the two ML approaches give them slightly different weights.

It is worth to underline that, having the cover being consistently provided only for recent issuances, it could mask a time effect. Only future research on a wider and updated data set might provide more insight on this point.

Finally, the DALEX software provides an algorithm to generate predictions breakdown, that is to decompose the final prediction of a given CB issue in the contributions of single predictions.

For illustrative purpose, the below reported figure depicts the marginal contribution of each predictor to the final predicted Spread for the 2016 Akibare Re CB (CUSIP US00973XAA81).

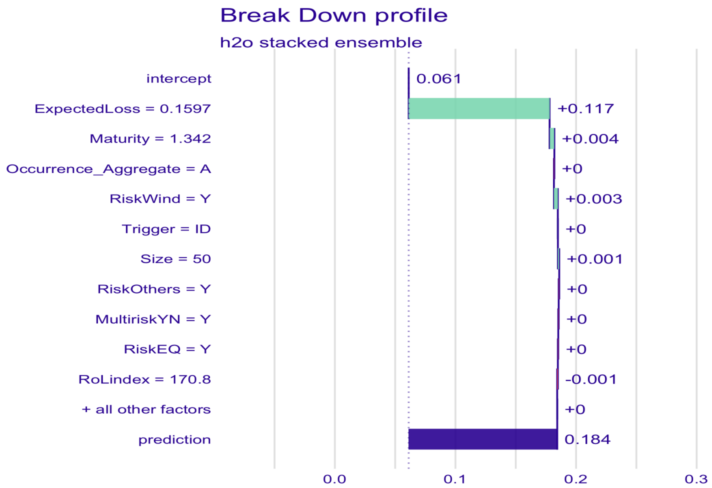


Figure 7.15. Staked ensemble model's breakdown

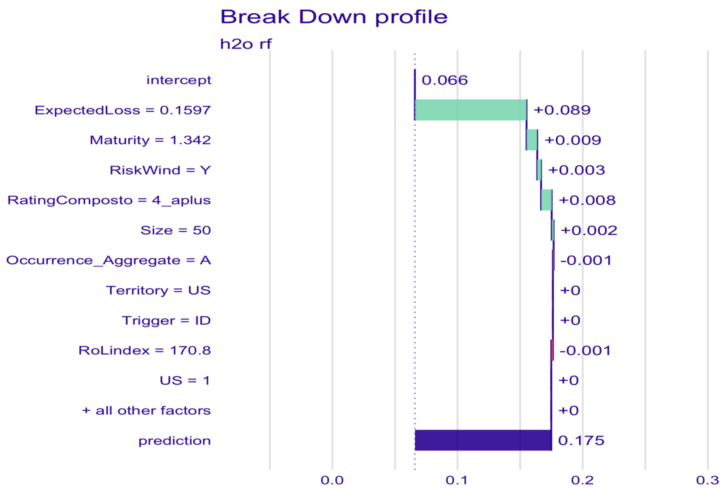


Figure 7.16. Random forest model's breakdown

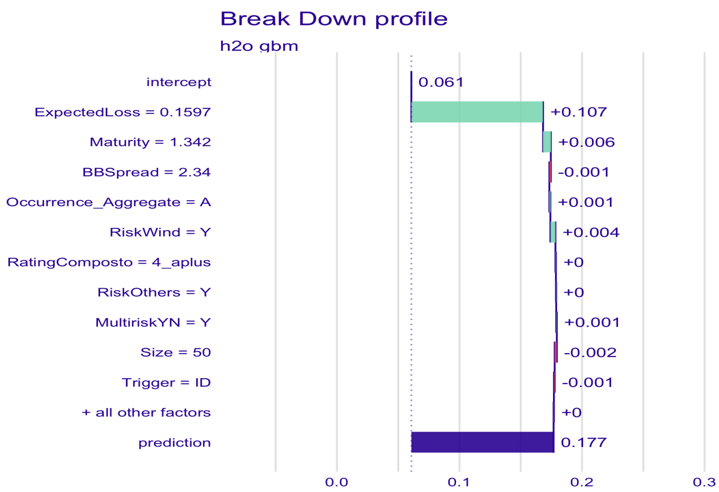


Figure 7.17. GBM model's breakdown

Such kind of representations may help non - technical stakeholder to understand the drivers of Spread at issuance and the outputs of ML models.

The following conclusions can be drawn from our analysis.

First of all, it confirms the EL as the most relevant predictor of CBs Spread at issuances also using ML approaches. The variable importance analysis shows that the relative importance of second and third predictors are a small fraction of the EL one.

In addition, ML approaches appear to outperform classical liner models in predicting S when prediction error is used to rank models even on small data set as of our study. Therefore they can be successfully employed to support pricing guidance when placing CB and similar ILS, thus they explain a significant portion of variability that is left not modeled by traditional models.

Further considerations supporting the use of ML models as an alternative to classical econometric approaches is the reduced computational timing (the CB issues dataset is very small compared to the ones when ML models are typically used) and the no need of features selection and engineering, that is “manually” performed when classical econometric approaches are employed. Also this helps in the periodic model’s update to take into account data from quarterly new issuances.

A relevant point of attention, at the author’s judgment, is that a concave down function better approximates the relation between S and EL than a linear one. Strong support for such finding is that two models, the Stacked Ensemble and the Cubist approach, that use different learning algorithm, consistently support this result. As future research directions, repeating the exercise with increased data as new CB are issues could help in confirming the validity of ML alternatives compared to classical approaches. Finally, a formal treatment of the period of issuance would certainly be a positive contribution.

7.2 Reinsurance solution design of *Sisma Italia* Bond

In the last 10 years natural disasters, including atmospheric events and earthquakes, have caused damage in Italy for over 50 billion. Expenses that burden general taxation, with often delayed and partial interventions. And every time there is a disaster, with massive damage and losses of human lives, the discussion on a possible intervention reopens system through collaboration between the public and private insurance sectors.

A specific feature of Italy is the very high exposure to earthquake risk (first country in Europe and eighth in the world in terms of potential damage measured as a share of GDP), combined with a strong exposure to flood risk, made more insidious by an evolution adverse to climate change underway.

From 1950 to today, earthquakes have caused 5,000 victims and those that occurred between 1968 and 2017 have produced direct damage for 108 billion euros. The state spent 122 billion on earthquakes that occurred between 1968 and 2012, often paid out years after the event.

In addition to direct damage and human losses, natural disasters also produce indirect damage in terms of non-development, of a high but difficult to quantify amount. Natural hazards are aggravated by some man-made factors: - high soil consumption, which amplifies flood damage; - the coexistence in many areas of the country (especially in the South) of housing degradation and seismic danger.

The risk for the Italian housing stock is material: 14.2 million homes are in areas at high or very high seismic risk, 7.4 million in medium risk areas, out of a total of 34.7 million homes with a value of 5,400 billion euros.

The risk for Italian families is amplified by the concentration of their wealth in the possession of homes and by the fact that the 70% of the families own the house they live in.

The seismic risk is very intense on the Apennine ridge, which has the highest frequency and intensity of earthquakes both in the historical perspective (fig. 7.18) and in the perspective one, given that the high danger of the area, geologically very active due to the continuous movements of the faults and the probable subduction of the Eurasian plate compared to the African one (Lovett, 2011).

The seismic risk is aggravated by the greater diffusion of conservative degradation of residential buildings in the areas of the country most subject to this risk, in particular the South and Sicily. From a historical point of view, between 1950 and 2017 earthquakes caused over 5,000 victims, 97% of which were caused by the 5 major seismic events.

In the face of these risks, the use of the insurance tool is very scarce, given that policies for damage from natural disasters protect just over 2% of homes.

Under-insurance for this type of risk is a worldwide phenomenon, due to the inability of individuals to make correct decisions and for the excessive premiums demanded by insurance companies in the face of reduced demand. These problems are accentuated in Italy by the reduced insurance culture and by the trust in public interventions.

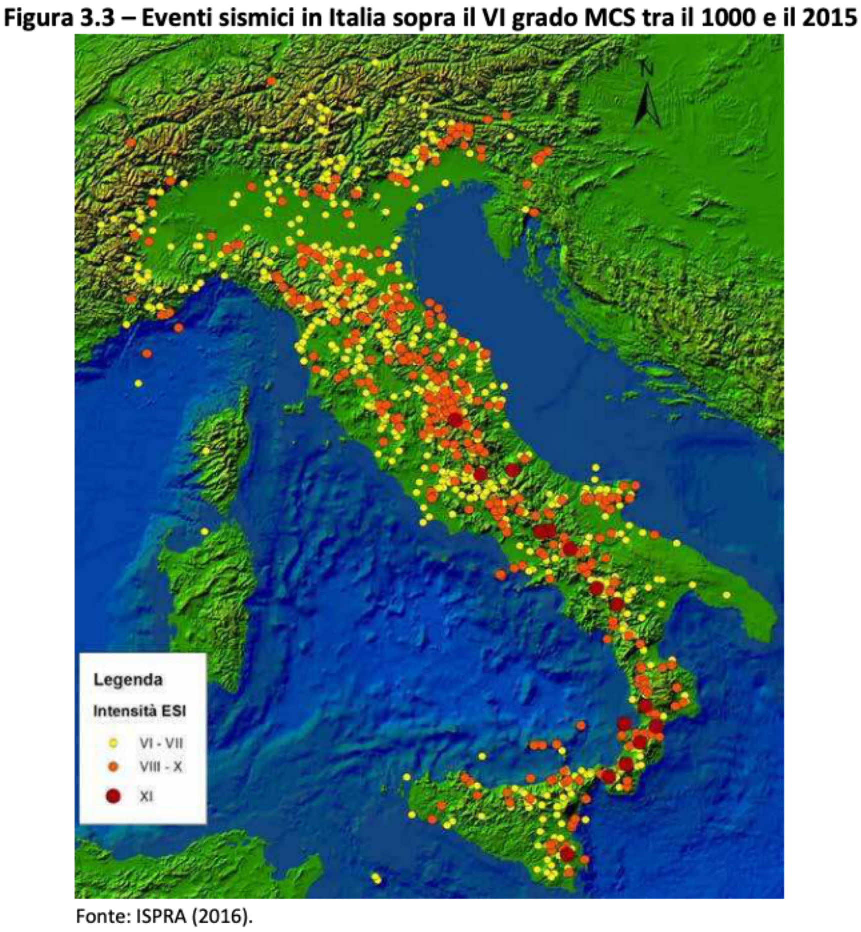


Figure 7.18. Earthquake risk in Italy

The design and pricing of policies tailored to cope with the consequences of the risks associated with the climate transition cannot ignore the understanding, measurement and modeling of rapidly changing climate scenarios. The mere use of extrapolations based on historical data is no longer able to predict - as in the past - a scenario of future risks.

The lack of robust data, the difficulty of estimating both the frequency and severity of events and the vulnerability of the activities to the occurrence of such events have so far led to a scarce diffusion - at world, European and Italian level - of insurance products that offer protection against damage caused. from natural events. There is now talk of a double gap: the Climate Risk Data Gap that affects all economic sectors, financial and otherwise, is accompanied by the Insurance Protection Gap (see figure 7.19).

It is therefore evident that the data necessary for an adequate assessment of the climate risk by the insurance sector are many and complex.

In order to support Italian insurance companies in the virtuous process of in-

Figure 7.19. European Earthquake risk protection GAP

cluding climate change in risk estimation models, reducing the protection gap and supporting Italian families and companies in the decarbonisation process, IVASS, during the *COP26* of 2021, undertook to launch a new information platform on physical and transition risks towards a sustainable economy of the Italian insurance sector. The project started on July 2022.

The extreme concentration of the damage deriving from the statistical distribution of natural events of significant magnitude makes coverage through the private insurance sector alone problematic.

On the other hand, private coverage does not lend itself to a redistribution of the risk of a solidarity nature; it places a higher cost for insurance coverage on those who live in higher-risk areas.

The comparison with the experiences and solutions adopted by many countries highlights the programming shortcomings of our country.

The international framework shows examples of cooperation between the public actor and insurance companies, which lead to high levels of insurance protection for homes, with different regulatory solutions, functional to the different degree of economic and social development of the various countries.

The construction of a protection system with organic characteristics similar to that of other countries cannot neglect reinsurance mechanisms, an indispensable support when insurance companies manage unpredictable risks such as natural ones. Some countries provide for publicly guaranteed reinsurance, with the state acting as the reinsurer of last resort.

Recently, Italian insurance association (ANIA) is continuously asking a government intervention for improving protection of italian property market, as for example mandatory insurance for Italian households as for motor insurance. Usually, government avoid such a intervention for political reasons because it can be seen as a "*new tax*" with profits for already big players as insurers.

ANIA suggests a public-private partnership like in France, where insurance companies intervene alongside the State in the event of natural disasters. Indeed, the ex-post interventions by the State represent an expense that indirectly weighs on the taxpayers and the insurance solution would have the advantage of guaranteeing rapid compensation times, reconstruction and restart of the affected activities.

ANIA's proposal is based on the assumption of compulsory adherence to insurance coverage by homeowners and SMEs. According to the companies, only this hypothesis would be able to guarantee an adequate mutualization of risks, managing to keep costs low for everyone and avoiding that only those who live in the areas at the highest risk, where the probability of an earthquake or flood is much higher, are insured.

Specifically, an ANIA's analysis stated that the annual premium for a policy to protect homes and businesses from the risk of an earthquake or flood could be between 70 and 100 euros for residential homes, while for a small and medium-sized business the outlay could be between 100 and 400 euros based on the riskiness of the territory.

The scheme developed by ANIA excludes in particular large companies (those with more than 250 employees), since today more than 90% of them are already insured against the risk of both earthquakes and floods. Insurance companies will cover losses up to the maximum sum of 10 billion euros, and then it would be up to the state to intervene again.

A specific attention must be paid at what would be covered by these policies, any exclusions and deductibles. The hypothesis put forward by ANIA provides for a reimbursement based on the reconstruction value of the property of 150 thousand euros for residential buildings and of 370 thousand euros for small and medium-sized enterprises (of which about 40% for the content).

The premiums paid by the policyholders which would flow into a fund to draw from in case of need. In practice, what already happens for the *Fondo Vittime della Strada*, which is managed by Consap and fed with a percentage of the MVL premiums, would be replicated for natural disasters.

Even in the case of a fund for natural disasters, there would therefore be no direct gain for the insurance companies, which could, however, be remunerated for claims management (just as happens for the *Fondo Vittime della Strada*, which mandates individual insurance companies for each Italian Region) and obviously take advantage of any additional clauses that customers voluntarily decide to purchase to expand coverage.

Furthermore, italian government could, for example, add additional tax breaks for large companies (as requested by ANIA itself) but also for the weakest segments of the population. Already today, the premium paid for a home policy that provides coverage against earthquake and flood can be deducted by 19% and the benefit rises to 90% in the event that anti-seismic works have been carried out on the property using the 110% bonus.

According to ANIA's calculations, considering the average annual outlay of the State of 7 billion euros for reconstructions, with compulsory policies the net savings for the public accounts would be between 1-2 billion euros, which at least for the first years could in part be used precisely to start the new system.

Starting from an analysis of Natural disasters protection gap in Italy, published by Italian supervisor (see Cesari and D'Aurizio), a public-private partnership in line with ANIA's proposal is a possible solution for these issue.

In this context, financial innovations introduced by Italian CAT Bond issuance can increase applicability of such a reinsurance solution. In the suggested echo-system, CAT Bonds mechanism can overcome many issues related to data quality, underwriting capacity and financial sustainability.

Reinsurance solution proposed in this thesis consist of a public-private reinsurance partnership designed as:

- *Sponsor* can be the Italian insurance industry, acquiring a mandatory premium for property protection with natural catastrophe coverages as in ANIA's proposal;
- *SPV* can be an *ad-hoc* insurance company created by a government's financial institution (e.g., *Cassa Depositi e Prestiti, SACE* who can fit the role of national reinsurer and invest in Italian government bonds (e.g., BTP);
- *Depository Bank* can be the an Italian bank admitted to trade in Monetary Markets;
- *Investors* can be limited to institutional investors, also other (re-)insurers.

that involves coordination of supervision from many Authorities:

- *IVASS* competent for Italian insurance market, generally sponsor of CAT bonds;
- *MEF* as CDP's shareholder;
- *Bank of Italy* competent for the supervision of Depository bank;
- *Consob* competent for financial instruments supervision, specifically on investors' protection.
- *Covip* competent for pension funds, as potential investors.

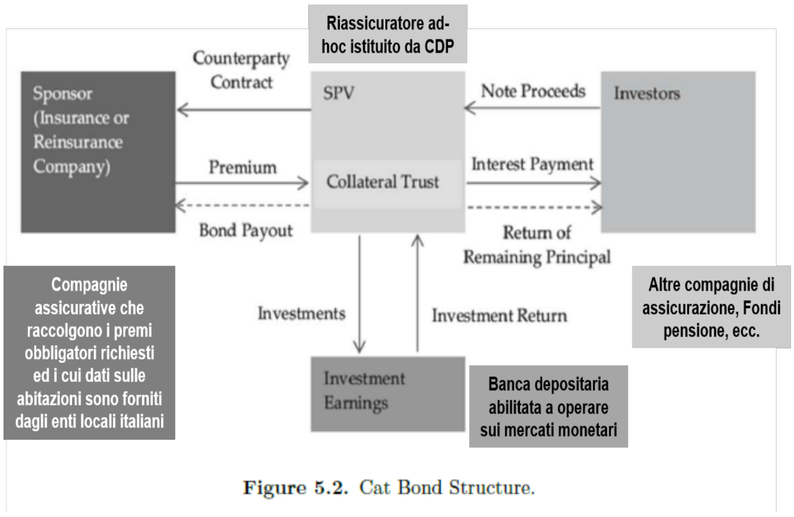


Figure 7.20. Sisma Italia Bond design

Following those data sources, modeling strategy and results we can use the following characteristics of CBs, all known at issuance:

- Expected Loss, EL, equal to € 4 billion;
- Maturity (i.e. bond's term) 5 years;
- Trigger type Indemnity;
- Main territory Italy;
- Peril covered "RiskEQ";
- The rating of the Sponsor is A;
- Coverage type "occurrence";
- Guy Carpenter's yearly Rate on Line (RoL) index at time of issuance
- Thomson Reuters BBSpread medium-low.

Chapter 8

Conclusions

In this thesis, first a short-term to medium-term assessment of a Non-Life insurance risk management have been provided, focusing on insurers risk profile with particular regard to the Solvency II Underwriting Risk. In this evaluation, reinsurance as a risk-mitigation technique has been considered in our model and its impact on our assessment opportunely reported and explained.

As numerical results evidence, business and risk strategy, as well as capital requirement methodology, may have a huge impact on the assessment of financial position of insurers under the new prudent regime of the European insurance industry. Risk mitigation techniques appear as a key driver of Non-Life insurance business as they can change risk profile over either the short-term or medium-term perspective. They impact the technical result of the year in such a way that it is important to assess how reinsurance strategies decrease the volatility, reducing the SCR given by the lower VaR, but, on the other hand, they also change the mean of distributions in different ways according to the price for risk requested by reinsurers.

At the same time, risk mitigation also appears as a key driver of Non-Life insurance management actions as it can improve business strategy and capital allocation (also in potential capital recovery plans). Different portfolio mixes change insurers' solvency positions, and the relative effect depends on the insurer dimension and the type of reinsurance arranged. From a medium-term perspective, a non-life insurer risk profile hardly depends on the business and risk strategy, particularly in terms of insurance portfolio mix. On that point, the reinsurance again plays an active role in letting smaller insurers survive in a dynamic and competitive market context, compensating the lower level of dimension diversification in the case of XL treaty or reducing the full range of the amount at risk in the case of a Quota Share.

On the other hand, a medium-term capital requirement would ask insurers to have more capital than in a one-year time horizon. Furthermore, this effect can vary a lot between insurers with not balanced portfolios, where more volatile LoBs can deteriorate solvency position of the companies. In this framework, risk mitigation effects linked to reinsurance strategies must be assessed on either risk/return perspective trade-off as figured out in the previous section.

The development of alternative risk management, understood as the process of identifying, analyzing, evaluating and integrated management of insurance and financial risks by companies has strongly influenced insurance and reinsurance "way of doing".

For years now, traditional insurance/reinsurance products have been accompanied by alternative risk transfer solutions or techniques other than insurance and reinsurance through which an instrument is identified to cover the risks of losses.

The development of these products is attributable to the limited ability of the (re) insurance market, especially in the most negative cyclical phases, to offer traditional risk mitigation techniques in a market increasingly characterized by complex risks such as those deriving from natural disasters (earthquakes, floods, etc.) or by business interruption, also in light of the growing instability of weather conditions due to climate change.

The need of insurance companies to reduce the volatility of the solvency position deriving from the market consistent principles of Solvency II, has also stimulated the search in the reinsurance market for products with a specific or mainly financial content intended to mitigate both insurance and market risks.

This phenomenon, called Alternative Risk Transfer (ART), however, remains a not easily defined concept. In general, it can be said that we are witnessing an alternative risk management when we are faced with instruments, of a combined insurance and financial nature, characterized by the following common elements:

- they are tailor-made for specific problems of the risk taker;
- offer multi-dimensional coverage: multi-year and / or multi-risk;
- replace the pure transfer of risk with risk financing;
- allow the assumption of risks by non (re) insurers;
- they incorporate financial instruments such as derivatives.

Among them, CAT Bonds provide some desirable aspects in increasing insurance capacity and resilience of property insurance market as well as improving protection against natural catastrophes.

Pricing of CAT Bonds blend ideas from actuarial and financial mathematical worlds, where machine learning approach can increase robustness of such kind of pricing models.

In this regard, the availability and accessibility of data and models for climate risk assessment are crucial to improve risk assessment and management by all the players involved in: policy-makers, industry, supervisors and academia.

In this thesis we tried to concretely and quickly contribute to bridging the insurance protection gap.

In particular, we design an innovative collaboration scheme between the public and private sectors in order to strengthen the resilience of our country with respect to harmful events connected to environmental risks and climatic.

Specifically, a governmental SPV can reinsure all catastrophe coverages, with characteristics adequately fixed in order to offset insurance profits, issuing Italian CAT Bonds in order to transfer a quota of natural catastrophes risks to capital markets.

List of abbreviations

ADC Adverse development cover
ANIA Italian insurance companies association
ART Alternative risk trasfer
CAT Catastrophe
CONSAP National insurance general agent
CONSOB Italian securities market commission
CoV Coefficient of variation
CRESTA Catastrophe Risk Evaluation and Standardizing Target Accumulations
CRM Collective Risk Model
EL Expected Loss
EIOPA European Insurance and Occupational Pension Authority
EU European Union
CAT Catastrophe
FSB Financial Stability Board
GDP Gross Domestic Product
GLM Generalized Linear Model
GWP Gross Written Premiums
IAIS International Association of Insurance Supervisors
ILS Insurance-Linked Securities
IM Internal Model
IVASS Italian insurance supervisory authority
LoB Lines of Business
LPT Loss Portfolio Transfer
ML Machine Learning
NET Net of reinsurance
NSA National Supervisory Authority
OF Own Funds
ORSA Own Risk and Solvency Assessment
QIS Quantitative Impact Study
QS Quota Share
RE Reinsurance (or reinsurer)
RSR Regulatory Supervisory Report
SCR Solvency Capital Requirement
SF Standard Formula
SPV Special Purpose Vehicle
XL eXcess of Loss
VaR Value at Risk

Bibliography

[1] Albrecher, Hansjörg, Jozef L. Teugels, and Jan Beirlant. 2017. *Reinsurance: Actuarial and Statistical Aspects*. New York: Jhon Wiley & Sons.

[2] Jesica de Armas et al. "Minimizing trigger error in parametric earth-quake catastrophe bonds via statistical approaches". In: *Modeling and Simulation in Engineering, Economics and Management.*, Springer, 2016, pp. 167–175.

[3] Artemis. Artemis.bm. http://www.artemis.bm/, 2021.

[4] Przemyslaw Biecek. "DALEX: Explainers for Complex Predictive Models in R". In: Journal of Machine Learning Research 19.84 (2018), pp. 1–5. https://jmlr.org/papers/v19/18-416.html.

[5] Neil Bodoff and Yunbo Gan. "An analysis of the market price of cat bonds". In: *Variance* 6.2 (2012), pp. 161–177.

[6] Alexander Braun. "Pricing in the Primary Market for Cat Bonds: New Empirical Evidence". In: *Journal of Risk and Insurance* 83.4 (2016), pp. 811–847.

[7] Leo Breiman. "Random Forests". In: *Machine Learning* 45.1 (Oct. 2001), pp. 5–32. issn: 1573-0565. https://doi.org/10.1023/A:1010933404324

[8] Leo Breiman. *Classification and Regression Trees*, CRC Press, 2017. isbn: 9781351460484.

[9] Michael S Canter, Joseph B Cole, and Richard L Sandor. "Insurance derivatives: a new asset class for the capital markets and a new hedg- ing tool for the insurance industry". In: *Journal of Applied Corporate Finance.* 10.3 (1997), pp. 69–81.

[10] Peter Carayannopoulos, Olga Kanj, and Marcos Fabricio Perez. *Pricing Dynamics in the Market for Catastrophe Bonds.*, SSRN. 2018.

[11] Cesari, Riccardo, and D'Aurizio Leandro. 2019. Quaderno IVASS n. 13 *Calamità naturali e coperture assicurative: valutazione dei rischi e policy options per il caso italiano*. Rome: Aracne.

[12] Cesari, Riccardo, and Valerio Arturo. 2019. *Risk Management E Imprese Di Assicurazione. Aggregazione Dei Rischi E Requisiti Di Capitale Nella Nuova Vigilanza Prudenziale (Con Esempi in R)*. Rome: Aracne.

[13] Tianqi Chen and Carlos Guestrin. "Xgboost: A scalable tree boosting system". In: *Proceedings of the 22nd acm sigkdd international conference on knowledge discovery and data mining*. ACM. 2016, pp. 785–794.

[14] Francois Chollet. *Deep Learning with Python*. Ed. by Manning publications. Manning, 2018.

[15] Cristina Ciumaş and Ramona Alexandrina Coca. "Analysis of Risk Premium Determinants on Cat Bonds". In: *Procedia Economics and Finance* 32 (2015). Emerging Markets Queries in Finance and Business 2014, EMQFB 2014, 24-25 October 2014, Bucharest, Romania, pp. 1487– 1493. issn: 2212-5671.

[16] Clemente, Gian, Savelli Nino, and Zappa Diego. 2015. The impact of Reinsurance Strategies on Capital Requirements for Premium Risk in Insurance. *Risks* 3: 164–82.

[Clemente and Savelli(2017)] Clemente, Gian Paolo, and Nino Savelli. 2017. Actuarial Improvements of Standard Formula for Non-Life Underwriting Risk. In *Insurance Regulation in the European Union Solvency II and Beyond*. Basingstoke: Palgrave Macmillan.

[17] Clemente, Gian. 2018. The effect of Non-Proportional Reinsurance: A Revision of Solvency II Standard Formula. *Risks* 6: 50.

[18] Daykin, Chris D., Teivo Pentikaïnen, and Martti Pesonen. 1994. Practical Risk Theory for actuaries. In *Monographs on Statistics and Applied Probability 53*. London: Chapman & Hall.

[19] Neil A Doherty. "Financial innovation in the management of catastrophe risk". In: *Journal of Applied Corporate Finance* 10.3 (1997), pp. 84–95.

[20] Linda Eggert, Yingjie Fan, and Stefan Voss. Data-Intensive Analytics for Cat Bonds by Considering Supply Chain Risks. July 2016.

[21] Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) (recast) (Text with EEA relevance) https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A02009L0138-20210630.

[22] European Commission. 2015. European Commission Delegated Regulation (EU) 2015/35 Supplementing Directive 2009/138/EC(Solvency II), 10 of October 2014. *Official Journal of the EU* 58: 1–797.

[23] Jerome H Friedman, Bogdan E Popescu, et al. "Predictive learning via rule ensembles". In: *Annals of Applied Statistics* 2.3 (2008), pp. 916–954.

[24] Kenneth Froot et al. The financing of catastrophe risk. University of Chicago Press Chicago, 1999.

[25] Marcello Galeotti, Marc Gürtler, and Christine Winkelvos. "Accuracy of premium calculation models for CAT bonds An empirical analysis". In: *Journal of Risk and Insurance* 80.2 (2013), pp. 401–421.

[26] George, Guillaume. 2013. *Insurance Risk Management & Reinsurance*. Paris: ENSAE.

[27] Gisler, Alois. 2009. *The Insurance Risk in the SST and in Solvency II: Modelling and Parameter Estimation*. Helsinki: Astin Colloquium.

[28] Mark Goldburd, Anand Khare, and Dan Tevet. *Generalized Linear Models for Insurance Rating*. Ed. by Casualty Actuarial Society. 5. Casualty Actuarial Society, 2016, pp. 89–95. isbn: 9780996889728.

[29] obias Götze, Marc Gürtler, and Eileen Witowski. "Improving CAT bond pricing models via machine learning". In: *Journal of Asset Management* 21.5 (2020), pp. 428–446.

[30] Shihao Gu, Bryan Kelly, and Dacheng Xiu. "Empirical asset pricing via machine learning". In: *The Review of Financial Studies* 33.5 (2020), pp. 2223–2273.

[31] International Actuarial Association. 2004. *A Global Framework for Insurer Solvency Assessment*. Ottawa: International Actuarial Association.

[32] Guolin Ke et al. "Lightgbm: A highly efficient gradient boosting decision tree". In: *Advances in Neural Information Processing Systems*. 2017, pp. 3146–3154.

[33] Amir E Khandani, Adlar J Kim, and Andrew W Lo. "Consumer credit- risk models via machine-learning algorithms". In: *Journal of Banking Finance* 34.11 (2010), pp. 2767–2787.

[34] gor Kononenko et al. "An efficient explanation of individual classifications using game theory". In: *Journal of Machine Learning Research* 11.Jan (2010), pp. 1–18.

[35] Klugman, Stuart A., Harry H. Panjer, and Gordon E. Willmot. 2008. *Loss Models: From Data to Decisions*, 3rd ed. New York: Jhon Wiley & Sons.

[36] Max Kuhn and Kjell Johnson. Applied predictive modeling. Springer, 2013.

[37] Max Kuhn. "Rules Rules Rules! Cubist Regression Models". In: Boston R User Group. 2016.

[38] Max Kuhn. Contributions from Jed Wing et al. caret: Classification and Regression Training. R package version 6.0-81. 2018. https://cran.r-project.org/package=caret.

[39] Morton N Lane. "Pricing risk transfer transactions". In: *Astin Bulletin* 30.2 (2000), pp. 259–293.

[40] Morton N. Lane. Regressions and Machine Learning - Some observations, some lessons. Tech. rep. Lane Financial, Aug. 2018.

[41] Erin LeDell et al. h2o: R Interface for 'H2O'. R package version 3.20.0.8. 2018.

[42] Michal Malohlava and Arno Candel. Gradient boosting machine with H2O. 2018.

[43] John A Major and Rodney E Kreps. "Catastrophe Risk Pricing in the Traditional Market". In: Alternative Risk Strategies (2002).

[44] Tom Michael Mitchell. Machine Learning. McGraw-Hill international editions - computer science series. McGraw-Hill Education, 1998. isbn: 9780070428072.

[45] Christoph Molnar, Bernd Bischl, and Giuseppe Casalicchio. "iml: An R package for Interpretable Machine Learning". In: JOSS 3.26 (2018), p. 786.

[46] Christoph Molnar. Interpretable machine learning. Lulu. com, 2020.

[47] Pallaria A., Savelli N., 2019. *Premium risk net of Reinsurance: from short term to medium term assessment.* Risks

[48] Niraj Patel. *The Drivers of Catastrophe Bond Pricing.* Tech. rep. Partner Reviews, 2015.

[49] Wei Qian, Yi Yang, and Hui Zou. "Tweedie's Compound Poisson Model With Grouped Elastic Net". In: Journal of Computational and Graphical Statistics (2016). issn: 15372715.

[50] Marco Túlio Ribeiro, Sameer Singh, and Carlos Guestrin. ""Why Should I Trust You?": Explaining the Predictions of Any Classifier". In: CoRRabs/1602.04938 (2016).

[51] Sandstrom, Arne. 2011. *Handbook of Solvency for Actuaries and Risk Managers: Theory and Practice.* Boca Raton: CRC Press.

[52] Savelli, Nino. 2002. Risk analysis of a non-life insurer and traditional reinsurance effects on the solvency profile. Paper presented at 6th International Congress on Insurance: Mathematics and Economics, Lisbon, Portugal, July 15–17.

[53] Savelli, Nino, and Gian Paolo Clemente. 2009. Modelling Aggregate Non-Life Underwriting Risk: Standard Formula vs. Internal Model. *Giornale dell'Istituto Italiano degli Attuari 72:* 301–38.

[54] Savelli, N., and Gian Paolo Clemente. 2011. Hierarchical Structures in the aggregation of Premium Risk for Insurance Underwriting. *Scandinavian Actuarial Journal* 3: 193–213.

[55] Jürg Schelldorfer and Mario V Wuthrich. "Nesting Classical Actuarial Models into Neural Networks". In: Available at SSRN 3320525 (2019).

[56] Pallaria A Spedicato G. "Estimating CB pricing using ML approach". To be submitted to: *North American Actuarial Journal* (2021), p. 20.

[57] Ajay Subramanian and Jinjing Wang. "Reinsurance versus securitization of catastrophe risk". In: *Insurance: Mathematics and Economics* 82 (2018), pp. 55–72.

[58] Mark J Van der Laan, Eric C Polley, and Alan E Hubbard. "Super learner". In: Statistical applications in genetics and molecular biology 6.1 (2007).

[59] Venter, Gary G. 2001. Measuring value in reinsurance. *Casualty Actuarial Society Forum*. Available online: https://www.casact.org/pubs/forum/01sforum/01sf179.pdf (accessed on 13 June 2019).

[60] Yong Wang and Ian Witten. *Induction of model trees for predicting continuous classes.* Ed. by University of Waikato. Department of Computer Science, 1996.

[61] Willis Capital Markets Advisory. ILS Glossary. Tech. rep. Willis, 2015.

[62] Willis Towers Watson. 2018. Global Reinsurance and Risk Appetite Survey Report. Available online: https://www.willistowerswatson.com/-/media/WTW/PDF/Insights/2018/07/risk-appetite-gains-momentum-in-a-changing-world.pdf (accessed on 13 June 2019).

[63] Induction of model trees for predicting continuous classes. *Ed. by University of Waikato. Department of Computer Science*, 1996.