A New Generation of the Fuzzy-set-theoretical Representation of Modified Linguistic Terms

Martine De Cock
Fuzziness and Uncertainty Modelling Research Unit
Department of Applied Mathematics and Computer Science
Ghent University, Krijgslaan 281 (S9), 9000 Gent, Belgium
e-mail: Martine.DeCock@rug.ac.be

Abstract

Fuzzy set theory provides a framework in which natural language expressions can be modelled mathematically, thereby even taking the vagueness of linguistic terms into account. In fuzzy systems a linguistic term (e.g. fast) is represented by a fuzzy set, while a linguistic modifier (e.g. very) is modelled by an operator (also known as *hedge*) transforming a fuzzy set into another. In the paper first we give a short overview of the traditional fuzzy set theoretical approaches to this problem. We point out that these "hedges of the first generation" are in general technical tools without meaning of their own, which explains their most important shortcomings. To overcome this, we present two new approaches in which the representation of linguistic modifiers is endowed with an inherent semantics ("hedges of the second generation"): the framework of fuzzy modifiers based on fuzzy relations (recently developed by us) and the so-called horizon approach (an extension of the research initiated by Novák).

1 Introduction

Computational semantics is concerned with computing the meanings of linguistic objects [3]. To be able to compute with meanings, first they have to be represented in some suitable way. For this traditional approaches often rely on a (first order) logic which is binary. A sentence like "the car is cheap" is for instance represented by a formula *cheap(car)* which, given some valuation, has either true or false as its truth value. However in real life it is not easy to divide the class of cars into those that are cheap and those that are not. The traditional mathematical approach to this problem is to choose one price, say, 2500 euro, to call every car that does not exceed this price cheap, and all the others not. However no intuitively acceptable motivation can be given why one should still consider 2500 euro to be cheap and 2500,01 euro not anymore. The essence of the issue is that cheap is a vague linguistic term, meaning that the transition from being cheap to not being cheap is not abrupt but gradual. Applying linguistic modifiers to them, such as very, more or less, rather,... sheds light on the gradualness of vague expressions [16]. These little words play an important part in human communication [15]. In an attempt to bring logic nearer to colloquial language, in 1960 Kubinski [15] succeeded to smuggle some of the vagueness present in natural language into a binary logic deduction system by introducing the linguistic modifier rather in an axiomatic way. However we feel that, since binary logic can only deal with two kinds of truth values, it can never model the vagueness of linguistic terms to the fullest. More truth values are needed to model the gradual transition.

In the sixties a new kind of many-valued logic, called fuzzy logic, was introduced [25]. Taking truth values in the real unit interval $[0, 1]$ instead of in $\{0, 1\}$, it turned out to be very suitable for the representation of (linguistic) concepts. The introduction of fuzzy set theory (FST) has given rise to an important evolution in the field of computer science; the importance of computing with words has increased tremendously over the last years [28, 29] and it will grow even more in the years to come [27]. Nowadays the fuzzy set theoretical representations of linguistic terms are already applied succesfully in an impressive amount of systems, ranging from control and approximate reasoning systems over preference and decision making systems to database systems. We refer to [1, 21, 24] for some examples in general, and to [4, 17] for some examples in the field of computational linguistics in particular.

The key idea is to model linguistic terms such as "cheap" or "more or less cheap" by a $X \to [0, 1]$ mapping for X a suitable universe (e.g. the universe of prices). When developing an application, the design of such a $X \to [0, 1]$ mapping for each term involved is therefore a fundamental task. Quite early in the history of FST, researchers [16, 26] started studying the possibility to partially automate this not so trivial task in the following way: given a $X \to [0, 1]$ mapping for a term (e.g. "cheap"), how can we automatically generate a suitable $X \to [0, 1]$ mapping for the modified term (e.g. "more or less cheap")? All their efforts resulted in an extensive collection of technical operators transforming one $X \to [0, 1]$ mapping into another (see [14] for an overview).

Although already useful, these so-called hedges have important shortcomings, which are in our opinion due to the fact that they are designed simply to perform a technical transformation, but have no further meaning of their own. In our paper we will present two new approaches with a clear inherent semantics, and we will show that they overcome the shortcomings of the hedges of the first generation. However we will start with a short discussion on the representation of (modified) linguistic terms in general, focussing on the linguistic modifiers "very" and "more or less", and a brief overview of the hedges of the first generation and their shortcomings.

2 Representing linguistic terms

In fuzzy systems, each linguistic term is represented by a fuzzy set on a suitable universe X. A fuzzy set A on X is a $X \rightarrow [0,1]$ mapping, also called the membership function of A. For all x in X, $A(x)$ is called the membership degree of x in A. The class of all fuzzy sets on X is denoted $\mathcal{F}(X)$. Furthermore for A and B in $\mathcal{F}(X)$ we say that $A \subseteq B$ if and only if $A(x) \leq B(x)$, for all x in X. If all membership values of A belong to $\{0,1\}$ then A is called a crisp set. Analogously, traditional set theory is sometimes also called crisp set theory. When representing a linguistic term by a fuzzy set, the most important question is of course: which membership function should we choose?

2.1 Two interpretations

The first fuzzy-set-theoretical representations assumed that very tall $\subseteq$ tall i.e. that very tall is a subset of tall, or to put it more fuzzy, that the degree to which somebody is very tall does not exceed the degree to which he is tall ([16], [26]). This inclusion is called semantic entailment. Psycholinguistic research however showed that the relation between tall and very tall is more complicated ([12], [23]). In an experiment two groups of students were questioned independently [23]. Both groups received a list with the heights of 15 men, each labeled with a character. The first group got the written instruction to indicate all tall men. After this, they were asked to indicate the very tall men. The second group on the other hand was asked from the start to indicate all tall ánd all very tall men. All men that were considered to be very tall by the first group were also considered to be tall by this group. The second group however made a distinction between both categories. While all students of the first group considered the men B (2m25) and M (2m07) to be tall, not one student of the second group placed them in this categorie.

In [23] this behaviour is explained by distinguishing the semantical from the pragmatical meaning. Semantical meaning is the literal meaning present in words. For instance it is semantically correct to call somebody of 2m20 tall. Pragmatical meaning on the other hand is added meaning, meaning which we add to the literal meaning, based on world knowledge and some maxims of conversation. According to Grice [10], when one can choose between an utterance A and a more informative utterance B, one has to choose the latter. Therefore it is pragmatically incorrect to call someone of 2m20 tall when one can call him very tall. Pragmatical meaning can coincide with semantical meaning (cfr. the first group of students) but this does not have to be the case (cfr. the second group).

It might be argued that the semantical meaning is precisely the meaning we can model by fuzzy sets, assigning degrees of membership to all objects of the universe X. Pragmatical meaning on the other hand is more concerned with degree of appropriateness. However both can be modelled by $X \rightarrow [0,1]$ mappings, and taken into account that both of them can occur in everyday language, both are worth the study. Following [23] we distinguish two interpretations of linguistic terms.

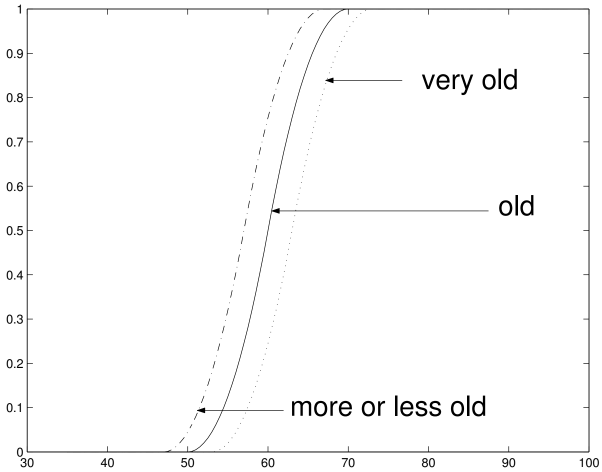

Figure 1: Modifiers based on fuzzy relations: inclusive interpretation

Inclusive interpretation. In the inclusive interpretation, which reflects semantical meaning, it is assumed that semantic entailment [16] holds : for A in $\mathcal{F}(X)$ and x in X

$$x \text{ is } \mathsf{very}\ A \Rightarrow x \text{ is } A \Rightarrow x \text{ is } \mathsf{more\ or\ less}\ A$$

These kind of assumptions are often made in the literature (see e.g. [13, 20, 26]). Representing linguistic terms by fuzzy sets, they correspond to:

$$\mathsf{very}\ A \subseteq A \subseteq \mathsf{more\ or\ less}\ A \tag{1}$$

The underlying semantics is that every object that is very A is also A, and that every object that is A is also more or less A. In this interpretation the membership degree of x in A corresponds to the degree to which x satisfies the term modelled by A: indeed the degree to which an object x is more or less A is always greater than or equal to the degree to which the same object x is A. The same holds for A w.r.t. very A. Figure 1 depicts possible membership functions for the linguistic terms old, more or less old, and very old in the universe of ages expressed in years.

Note that such kind of membership functions are dependent on context and observer and may vary likewise depending on the application and the applicant. However since this is not the central issue of our paper, we will not go into further detail about it and assume that the given membership functions are acceptable to a particular observer in a given context.

Non-inclusive interpretation. In the non-inclusive interpretation, which arises when pragmatical meaning differs from semantical meaning, a term modified by "more or less" or "very" doesn't denote a subset neither a superset of the original term. The terms denote

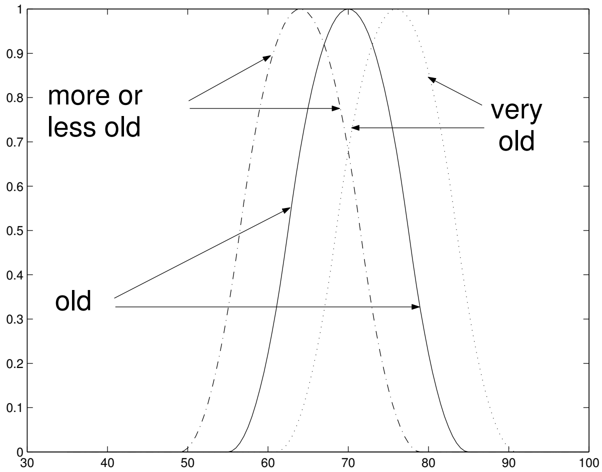

Figure 2: Modifiers based on fuzzy relations: non-inclusive interpretation

different (possibly overlapping) categories. In this case the membership degree of x in A corresponds rather to the degree to which x is representative for the term modelled by A. Figure 2 depicts possible membership functions for old, more or less old, and very old in the non-inclusive interpretation. This interpretation is often used in fuzzy control applications (e.g. [1]).

2.2 Fuzzy modifiers of the first generation

As already indicated in the introduction, the design of a suitable membership function for every term involved in an application is a fundamental task. Furthermore saving all the membership functions in memory can be quite memory consuming. For both these reasons it is desirable to generate the membership function for a modified linguistic term automatically from the membership function of the original term. In practice this is done by representing every linguistic modifier by a fuzzy modifier, i.e. a $\mathcal{F}(X) \rightarrow \mathcal{F}(X)$ mapping [22]. In FST linguistic/fuzzy modifiers are commonly called linguistic/fuzzy hedges. Most of the fuzzy modifiers proposed in the literature (we call them "hedges of the first generation") are (r, t)-decomposable. For r a $[0, 1] \rightarrow [0, 1]$ mapping and t a $X \rightarrow X$ mapping, a fuzzy modifier m on X is called (r, t)-decomposable if and only if for all A in $\mathcal{F}(X)$, for all x in X: $m(A)(x) = r(A(t(x)))$ [18]. t and r are called the pre- and the postmodifier of m respectively. Often either r is the identical mapping $\mathbb{I}_{[0,1]}$ on $[0, 1]$ (i.e. $r(x) = x$, for all x in $[0, 1]$) or t is the identical mapping $\mathbb{I}_X$ on X (i.e. $t(x) = x$, for all x in X). In these cases we talk about *pure premodification* and *pure postmodification* respectively.

Pure postmodification By far the most popular postmodifiers are the mappings $.^\alpha$, for α in $[0, +\infty[$. The associated $(.^\alpha, \mathbb{I}_X)$-decomposable fuzzy modifiers — also called powering modifiers — are originally introduced by Zadeh [26]. When m is a $(r, \mathbb{I}_X)$-decomposable fuzzy modifier then either $(\forall x \in X)(A(x) = 1 \Rightarrow m(A)(x) \neq 1)$ or $(\forall x \in X)(A(x) = 1 \Rightarrow m(A)(x) = 1)$. Suppose we use m(tall) to model very tall in the universe of heights of men, then in the first case no height can be very tall to degree 1, while in the second all heights that are tall to degree 1, are also very tall to degree 1. According to our intuition however a height of 2m00 is clearly very tall to degree 1, while a height of 1m80 is tall to degree 1 but very tall only to a lower degree. This example show that a representation of "very" by a $(r, \mathbb{I}_X)$-decomposable fuzzy modifier leads to counter-intuitive results. Similar remarks can be made for "more or less." Furthermore it's not possible to use a $(r, \mathbb{I}_X)$-decomposable fuzzy modifier for the non-inclusive interpretation.

Pure premodification Premodifiers act on the objects of the universe X. They are mainly studied for X a subset of $\mathbb{R}$, the set of real numbers. The most popular premodifiers are the mappings T_α defined by $T_\alpha(x) = x + \alpha$, for all x in $\mathbb{R}$, α in $\mathbb{R}$. The resulting $(\mathbb{I}_{[0,1]}, T_\alpha)$-decomposable modifiers — also called shifting modifiers — are informally suggested by Lakoff [16] and more formally developed later on [2, 11, 13]. Since they shift the membership function α units to the left or the right they are suitable to model "more or less" and "very" in the non-inclusive interpretation. In the inclusive interpretation shifting modifiers can be used to represent more or less A and very A provided that A is increasing or decreasing, and that X is a subset of $\mathbb{R}$ of course. If A is partially increasing and partially decreasing however (e.g. about 8 p.m.) a shifted version can never be a superset of A and can therefore not be used to model more or less A. In this case an artificial and quite complicated solution to the problem can be found by dividing the membership function into its increasing and decreasing parts and applying a different shift to each part.

A suggestion to use both a non-trivial pre- and postmodifier at once for the inclusive interpretation was made by Novák [18]. Although it is an improvement on the solutions discussed in the previous paragraphs, it can only be applied for a special kind of membership functions and it also involves a process of division of the membership function in increasing and decreasing parts. For a detailed overview of the hedges of the first generation and their shortcomings, we refer to [5, 14].

3 The next generation

The fuzzy modifiers of the first generation are merely artificial operators that transform the membership function of a term A into a membership function that is kind of acceptable (to the designer) for very A or more or less A, but they do not have an inherent semantics. Hence it is no surprise that they are afflicted with the above mentioned shortcomings. We feel that fuzzy modifiers should be endowed with a clear inherent semantics (see also [6]).

3.1 Fuzzy modifiers based on fuzzy relations

Inclusive interpretation We suggest to do this by taking the context, i.e. mutual relationships in the universe into account. More specifically to compute the degree to which an object y is more or less A or very A, we will take a look at the objects that resemble to y. Indeed: according to our intuition a person can be called more or less old if he resembles to someone who is old. Furthermore he is very old if everybody whom he resembles to is old.

Resemblance can be modelled by means of a fuzzy relation R on X, i.e. a fuzzy set on $X \times X$. It is clear that such a resemblance relation should be reflexive and symmetrical. For a more detailed study on how it should look like, we refer to [7]. In Figure 1 for instance we used the fuzzy relation R defined by $R(x, y) = min(1, max(2.5 - 0.5|x - y|), 0))$, with $R(x, y)$ being the degree to which x and y are approximately equal, for all x and y in X. The so-called R-foreset of y is the fuzzy set on X denoted by Ry and defined by $(Ry)(x) = R(x, y)$. If R models resemblance, then Ry is the fuzzy set of objects resembling y, in other words the context of y that we wish to consider.

Furthermore we want to express that y is more or less A if y resembles to an object that is A, in other words if the intersection of A and Ry is not empty. Likewise y is very A if all the objects that resemble to y are A, i.e. if Ry is included in A. Therefore we need to be able to express the degree of overlap between A and Ry as well as the degree to which Ry is included in A. In crisp set theory, intersection and inclusion are defined by means of operators of boolean logic. I.e. if P and Q are crisp subsets of X then

$$P \cap Q \neq \emptyset \text{ iff } (\exists x \in X)(x \in P \wedge x \in Q) \tag{2}$$

$$P \subseteq Q \text{ iff } (\forall x \in X)(x \in P \Rightarrow x \in Q) \tag{3}$$

The counterparts of $\wedge$ and $\Rightarrow$ in fuzzy logic are called triangular norm and implicator respectively. An increasing $[0, 1]^2 - [0, 1]$ mapping $\mathcal{T}$ is called a triangular norm if it is commutative, associative, and satisfies the boundary condition $\mathcal{T}(x, 1) = x$ for all x in $[0, 1]$. A $[0, 1]^2 - [0, 1]$ mapping $\mathcal{I}$ is called an implicator if for all x in $[0, 1]$ $\mathcal{I}(x, .)$ is increasing, $\mathcal{I}(., x)$ is decreasing and $\mathcal{I}(1, x) = x$. One can verify that for every triangular norm $\mathcal{T}$: $\mathcal{T}(0, 0) = \mathcal{T}(0, 1) = \mathcal{T}(1, 0) = 0$ and $\mathcal{T}(1, 1) = 1$, while for every implicator $\mathcal{I}$: $\mathcal{I}(0, 0) = \mathcal{I}(0, 1) = \mathcal{I}(1, 1) = 1$ and $\mathcal{I}(1, 0) = 0$. This guarantees that triangular norms and implicators are extensions of their classical boolean counterparts. In the examples in this section we use the Łukasiewicz triangular norm $\mathcal{T}_{\text{L}}$ and it so-called residual implicator $\mathcal{I}_{\text{L}}$. They are defined by $\mathcal{T}_{\text{L}}(x, y) = \max(0, x + y - 1)$ and $\mathcal{I}_{\text{L}}(x, y) = \min(1, 1 - x + y)$ for all x and y in $[0, 1]$.

The existential and universal quantifiers in (2) and (3) can be generalized by taking respectively the supremum and the infimum over the universe. Putting everything together,

Term	*Fuzzy set*	*Conditions*
very A	$R^{\heartsuit}(A)$	R is a resemblance relation
more or less A	$R^{\clubsuit}(A)$	R is a resemblance relation

Table 1: Modifiers based on fuzzy relations - inclusive interpretation

we suggest to model more or less A and very A by the fuzzy sets $R^{\clubsuit}(A)$ and $R^{\heartsuit}(A)$ respectively, with for all y in X:

$$R^{\clubsuit}(A)(y) = \sup_{x \in X} \mathcal{T}(A(x), (Ry)(x))$$

$$R^{\heartsuit}(A)(y) = \inf_{x \in X} \mathcal{I}((Ry)(x), A(x))$$

$R^{\clubsuit}(A)$ and $R^{\heartsuit}(A)$ can also be interpreted as the direct and superdirect image of A under R (see [8]), and as the upper and the lower fuzzy rough approximation of A under R (see [9]). The meaning of these formulas becomes very clear when A is a crisp singleton, i.e. $A = \{z\}$ for some z in X or in other words $A(z) = 1$, and $A(x) = 0$ for all other x in X. Then, regardless whether Ry is crisp or not:

$$R^{\clubsuit}(A)(y) = R(z, y)$$

In the chosen representation this corresponds to "y is more or less $\{z\}$ to the degree to which z and y resemble," which is according to our intuition. To show that the presented framework respects the semantic entailment needed in the inclusive interpretation (cfr. Formula (1)), we will proof the following proposition which holds for an arbitrary reflexive fuzzy relation. A fuzzy relation R is called reflexive if and only if $(\forall x \in X)(R(x, x) = 1)$.

Proposition 3.1 *If R is a reflexive fuzzy relation on X then for all A in $\mathcal{F}(X)$:*

$$R^{\heartsuit}(A) \subseteq A \subseteq R^{\clubsuit}(A)$$

Proof. For all y in X:

$$\begin{aligned} R^{\heartsuit}(A)(y) &\leq \mathcal{I}(R(y, y), A(y)) \\ &\leq \mathcal{I}(1, A(y)) \\ &\leq A(y) \\ &\leq \mathcal{T}(A(y), 1) \\ &\leq \mathcal{T}(A(y), R(y, y)) \\ &\leq R^{\clubsuit}(A)(y) \end{aligned}$$

According to the definition of fuzzy set inclusion given at the beginning of Section 2, this proves the proposition. □

	beautiful	average	ugly
snowwhite	1.00	0.00	0.00
witch	0.00	0.30	0.70
wolf	0.00	0.00	1.00
dwarf	0.10	0.70	0.20
prince	0.80	0.20	0.00
red-hood	0.50	0.50	0.00

Table 2: Fuzzy sets in the universe of fairy-tale characters

R	snowwhite	witch	wolf	dwarf	prince	red-hood
snowwhite	1.00	0.00	0.00	0.00	1.00	0.50
witch	0.00	1.00	1.00	0.50	0.00	0.00
wolf	0.00	1.00	1.00	0.00	0.00	0.00
dwarf	0.00	0.50	0.00	1.00	0.00	0.88
prince	1.00	0.00	0.00	0.00	1.00	1.00
red-hood	0.50	0.00	0.00	0.88	1.00	1.00

Table 3: Resemblance relation on the universe of fairy-tale characters

As stated above it is natural to assume reflexivity for a fuzzy relation modelling resemblance; indeed every object resembles to itself to the highest degree. Therefore our framework garantuees semantic entailment. Furthermore it imposes no restrictions on the membership functions and it can be applied to all kinds of universes (numerical as well as non-numerical), as we will illustrate in the example below. The resemblance relation can be defined differently from universe to universe, thereby allowing to express some context-dependency.

Example 3.1 *Table 2 shows the membership degrees of the fuzzy sets* beautiful, average *and* ugly *in the universe X of 6 fairytale characters, while Table 3 defines a resemblance relation R on X. For more details on the construction of R we refer to [7]. The modified fuzzy sets obtained using the representational scheme of Table 1 are given in Table 4. Comparing our technique to the traditional fuzzy modifiers discussed in Section 2, we notice that there is no clear shifting operation on the non-numerical universe of fairy-tale characters, hence no straightforward way to use shifting hedges. The application of powering hedges is technically possible, but can never yield the results obtained in Table 4. In fact the* prince *who is* beautiful *only to degree 0.80, is considered* more or less beautiful *to degree 1 in our representation. Likewise snowwhite who is* average *to degree 0 is promoted a bit to being* more or less average *to degree 0.20. This is due to her resemblance to other fairy-tale characters who are already* average *to some degree greater than 0. Since modifiers with pure postmodification, such as powering modifiers, can not make the difference between objects belonging to degree 1 to the original fuzzy set and belonging to degree 1 to the modified fuzzy set (and analogously for the degree 0) they can not yield such results.*

	more or less beautiful	more or less average	more or less ugly	very beautiful	very average	very ugly
snowwhite	1.00	0.20	0.00	0.80	0.00	0.00
witch	0.00	0.30	1.00	0.00	0.00	0.70
wolf	0.00	0.30	1.00	0.00	0.00	0.70
dwarf	0.38	0.70	0.20	0.10	0.62	0.12
prince	1.00	0.50	0.00	0.50	0.00	0.00
red-hood	0.80	0.58	0.08	0.22	0.20	0.00

Table 4: Modified fuzzy sets

*Similar remarks can be made for "*very*".*

Non-inclusive interpretation For the non-inclusive interpretation suitable fuzzy relations R can easily be chosen such that $R^{\clubsuit}(A)$ imitates the behaviour of shifting as discussed in Section 2.2. Hence for such suitable relations R_1 and R_2 we can use ${R_1}^{\clubsuit}(A)$ to model more or less A and ${R_2}^{\clubsuit}(A)$ to model very A. Furthermore for quite similar fuzzy relations not only a shift but also a change of shape can be obtained, making the representational scheme even more flexible. Note that this allows the use of the same powerful framework for both the inclusive and the non-inclusive interpretation. However since no further inherent semantics is yet assigned to this technique in the non-inclusive interpretation, we omit the details here. An example is given in Figure 2.

3.2 The horizon approach

Another approach to give inherent semantics to modifiers is the horizon idea developed by Novák [19, 20] for the inclusive interpretation. We extend it to the non-inclusive interpretation. For both interpretations we also integrate a mathematical notion of distance, namely a pseudo-metric, so the technique can be used not only in numerical universes but in pseudo-metric spaces in general. We recall that an $\mathcal{M}^2 - [0, +\infty[$ mapping d is called a pseudo-metric on $\mathcal{M}$ if and only if $d(x, x) = 0$, $d(x, y) = d(y, x)$ and $d(x, y) + d(y, z) \geq d(x, z)$, for all x, y, and z in $\mathcal{M}$. $(\mathcal{M}, d)$ is then called a pseudo-metric space.

Let (X, d) be a pseudo-metric space and let Ω be an element of X called the *observer*. In real life situations the objects that are close to Ω (closer than some distance α) are clearly visible to him (visible to degree 1). The visibility of an object x of X to Ω drops with the distance $d(\Omega, x)$ between Ω and that object. Somewhere in the far distance, behind the horizon (say, at distance γ), the objects aren't visible at all anymore (visible to degree 0). α and γ characterize the vision of the observer; therefore we call them the *quality of vision parameters*. It is natural to assume that $\alpha \in \mathbb{R}$, $\gamma \in \mathbb{R}$ and $\alpha < \gamma$. The visibility of objects in X to an observer Ω in X with quality of vision parameters α and γ can now be expressed by a $X \to [0, 1]$ mapping $\mathbb{H}(\Omega, (\alpha, \gamma))$ with the following characteristics:

Term	*Corresponding fuzzy set*	*Conditions*
A	$\mathbb{H}(\Omega, (\alpha, \gamma))$	
very A	$\mathbb{H}(\Omega, (\alpha_1, \gamma_1))$	$\alpha_1 \leq \alpha$, $\gamma_1 \leq \gamma$
more or less A	$\mathbb{H}(\Omega, (\alpha_2, \gamma_2))$	$\alpha \leq \alpha_2$, $\gamma \leq \gamma_2$

Table 5: Horizon approach - inclusive interpretation

(H.1) $d(\Omega, x) \leq \alpha \Rightarrow \mathbb{H}(\Omega, (\alpha, \gamma))(x) = 1$
(H.2) $\gamma \leq d(\Omega, x) \Rightarrow \mathbb{H}(\Omega, (\alpha, \gamma))(x) = 0$
(H.3) $d(\Omega, x) \leq d(\Omega, y) \Rightarrow \mathbb{H}(\Omega, (\alpha, \gamma))(x) \geq \mathbb{H}(\Omega, (\alpha, \gamma))(y)$

Furthermore for two couples of quality of vision parameters (α_1, γ_1) and (α_2, γ_2) such that $\alpha_1 \leq \alpha_2$ and $\gamma_1 \leq \gamma_2$, it should hold that

(H.4) $\mathbb{H}(\Omega, (\alpha_1, \gamma_1))(x) \leq \mathbb{H}(\Omega, (\alpha_2, \gamma_2))(x)$

which expresses that the visibility is better if the quality of vision parameters are higher. An example of such a mapping is $f = \mathbb{H}(\Omega, (\alpha, \gamma))$ defined by, for all x in X,

$$f(x) = \begin{cases} 1 & \text{if } d(\Omega, x) \leq \alpha \\ 1 - \frac{d(\Omega, x) - \alpha}{\gamma - \alpha} & \text{if } \alpha < d(\Omega, x) < \gamma \\ 0 & \text{otherwise} \end{cases}$$

Inclusive interpretation To model a term A in X (e.g. the term "small" in the universe of heights), we place the observer Ω with quality of vision parameters (α, γ) in the element of X that satisfies the term A the best (the smallest height, i.e. 0). The visibility of an object x of X to Ω corresponds to the degree to which x satisfies A. Then we make the observer take off his glasses, which changes the quality of his vision to the worse: it is now characterized by two new quality of vision parameters (α_1, γ_1) such that $\alpha_1 \leq \alpha$, $\gamma_1 \leq \gamma$. The objects he can still see without his glasses must be very close to him, so they must be very A. Finally we give the observer back his glasses and a telescope as well, thereby improving the quality of his vision. Now he can see all objects that are more or less A. This is summarized in Table 5 in which Ω is the element of X that satisfies A the best. Note that the characteristics imposed on $\mathbb{H}$ and the conditions on the quality of vision parameters guarantee that Formula (1) is respected. The membership functions in Figure 3 are constructed according to this scheme, using the mapping f defined above with $d(x, y) = |x - y|$ for all x and y in X (the universe of ages).

Non-inclusive interpretation For the inclusive interpretation we have kept the observer Ω fixed on some object of X, and we have changed his quality of vision parameters.

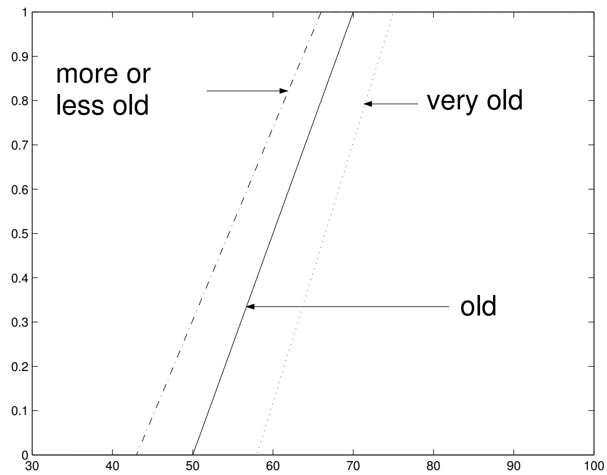

Figure 3: Horizon approach: inclusive interpretation

Term	*Corresponding fuzzy set*
A	$\mathbb{H}(\Omega_1, (\alpha, \gamma))$
very A	$\mathbb{H}(\Omega_2, (\alpha, \gamma))$
more or less A	$\mathbb{H}(\Omega_3, (\alpha, \gamma))$

Table 6: Horizon approach - non-inclusive interpretation

In the non-inclusive interpretation we will do exactly the opposite: we keep the quality of vision fixed, but we make the observer walk through the universe. More in particular, to model a term, we will place the observer on the object of X that is most representative for that term. In Table 6 Ω_1, Ω_2, and Ω_3 are the elements of X most representative for A, very A, and more or less A respectively. See Figure 4 for an example of membership functions generated according to this scheme using the mapping f and d as defined above.

Conclusion and future work

After indicating the shortcomings of traditional fuzzy modifiers, we have presented two new approaches. Unlike the hedges of the first generation, these new kinds of fuzzy modifiers are not merely technical operators, but are endowed with clear inherent semantics. For the fuzzy modifiers based on fuzzy relations this is achieved by taking the context into account (more specifically: mutual relationships between objects). In the horizon approach (an extension of the work initiated by Novák) the modified terms are characterized by a change of quality of vision parameters or a change of the observer, giving them a semantics smoothly integrated in the surrounding horizon idea. Since this new generation of hedges

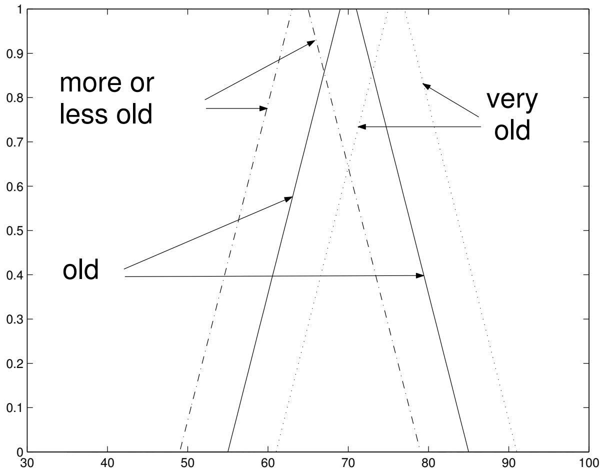

Figure 4: Horizon approach: non-inclusive interpretation

has a strong semantical ground, they lead to membership functions which are closer to human understanding. Furthermore they can be used on all kinds of membership functions with (hardly) no restriction. Our further work will focus on the application of these new kinds of fuzzy modifiers in fuzzy systems.

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