ROMA TRE
UNIVERSITÀ DEGLI STUDI

COLLANA DEL
DIPARTIMENTO DI ECONOMIA

UNDESIRED MONETARY POLICY EFFECTS IN A BUBBLY ECONOMY

Giuseppe Ciccarone, Francesco Giuli, Enrico Marchetti, Valeria Patella and Massimiliano Tancioni

Working Paper n° 270, 2022

I Working Papers del Dipartimento di Economia svolgono la funzione di divulgare tempestivamente, in forma definitiva o provvisoria, i risultati di ricerche scientifiche originali. La loro pubblicazione è soggetta all'approvazione del Comitato Scientifico.

Per ciascuna pubblicazione vengono soddisfatti gli obblighi previsti dall'art. l del D.L.L. 31.8.1945, n. 660 e successive modifiche.

Copie della presente pubblicazione possono essere richieste alla Redazione.

Esemplare fuori commercio ai sensi della legge 14 aprile 2004 n.106

DIPARTIMENTO DI ECONOMIA

UNDESIRED MONETARY POLICY EFFECTS IN A BUBBLY ECONOMY

Giuseppe Ciccarone, Francesco Giuli, Enrico Marchetti, Valeria Patella and Massimiliano Tancioni

Undesired monetary policy effects in a bubbly economy[*]

Giuseppe Ciccarone

Sapienza University Rome

Francesco Giuli

Roma Tre University

Enrico Marchetti

Parthenope University of Naples

Valeria Patella

Sapienza University Rome

Massimiliano Tancioni

Sapienza University Rome

Abstract

Monetary policy can be responsible for asset price bubble episodes under specific monetary-financial conditions. We evaluate the effects of monetary policy shocks on asset price bubbles by estimating a Markov-switching Bayesian Vector Autoregression on US 1960-2019 data, where states for the interaction of asset prices and monetary outcomes affect the realization of bubbles. We rationalize the evidence with a Markov-switching Overlapping Generations model, generating a bubbly and a no-bubbly economy with a regime-specific monetary policy. By matching the empirical impulse responses, we find that the monetary-financial states of the economy can generate amplified instability under high equity premia and asset price bubble. In a bubbly economy, a monetary tightening is ineffective in reducing stock prices, increasing real rates and inflating bubbles. Expectations to switch to a no bubbly scenario produce stabilizing effects.

Keywords: monetary policy, asset price bubble, Markov-switching, monetary-financial interaction, policy credibility

JEL Codes: C320, D500, E420, E520, E650, G100

1 Introduction

This work contributes to address the long-standing question whether monetary policy should control and is actually able to control financial stability risks. For a long time, the prevailing view was that central banks should abstain from intervening in the presence of stock market bubbles (see Galì (2014)), due to possible unintended consequences of this policy, and to the difficulty to detect actual bubble episodes (Bernanke and Gertler (1999, 2001), Greenspan (2002)). Price and financial stability were perceived as complementary objectives and monetary policy should have hence remained focused on inflation control (Taylor, 2008), and intervene only eventually, with policies aimed at "cleaning up the mess" left by the bubble burst. This view changed after the 2008 financial crisis, making room for a new conventional view on the correct monetary policy stance, described as "leaning against the wind" (LAW). According to this view, central banks should actively act to curb the emergence of bubbles by raising interest rates (see, among others, Miao, Shen and Wang (2019), Allen, Barlevy and Gale (2018)).

Notwithstanding the wide and fierce debate over the last decades,[1] the literature on bubbles has proved unable to: *i)* agree on whether and how monetary policy should respond to perceived deviations of asset prices from fundamentals; *ii)* assess the relevance of the underlying macrofinancial conditions for assessing policy effectiveness on bubbles; *iii)* rationalize the limited existing evidence on the effects of monetary policy on asset price bubbles (Galì and Gambetti, 2015).[2]

In the face of these unsettled issues, the aim of this paper is twofold. We first provide new empirical evidence on the effects of monetary policy shocks on asset prices and their bubble component. We do so by considering the joint nonlinear behavior of asset price and monetary cycles, and find that monetary policy tightening increases or not stock prices depending on the monetary-financial state of the economy. Then, with an OLG model of asset price bubbles, Markov-switching in the monetary policy and in the bubble size, we are able to match salient features of empirical facts and study its theoretical channels. In this type of model agents form expectations over transitioning across monetary policy regimes, with and without asset price bubbles.

As summarized in Figure 1, most asset price fluctuations come from its nonfundamental/bubbly

---

[1] Allen, Barlevy and Gale (2018), Dong, Miao and Wang (2020), Galì (2014), Bernanke and Gertler (1999, 2001), and Miao, Shen and Wang (2019).

[2] These issues relate to: *i)* monetary policy ineffectiveness in long-run models (Martin and Ventura (2016), Ikeda and Phan (2016)); *ii)* the difficulty of including productive capital in "perpetual youth" frameworks (Galì, 2021); *iii)* the preference to be given to the backward-looking sunspot solution around a stable bubbly steady state vs. the forward-looking minimal state variable solution around an unstable bubbly steady state; *iv)* the differences produced by the adoption of an infinite-horizon framework vs. an overlapping generations (OLG) model (Miao, Shen and Wang, 2019).

Figure 1: Data evidence: Monetary and financial facts
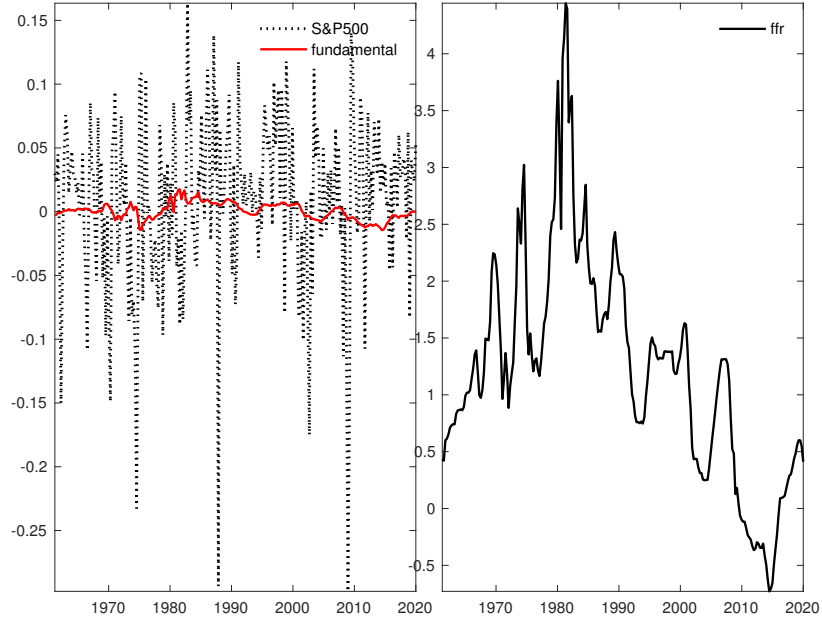
The figure shows the time series for the S&P500 stock price, its decomposition in a fundamental and bubbly component (Galì and Gambetti, 2015), and the federal funds rate over the 1960-2019 US sample.

component (dotted black line, as indentified by Galì and Gambetti (2015)). The sudden drops in asset evaluation after 2000 are almost entirely explained by bubble crashes (2001, 2008, 2012). To address empirically the investigation on the low-frequency relationship between monetary policy, bubbles and asset prices (red solid line), we use a nonlinear framework to capture states in the monetary and financial environment.[3] This results to be responsible for changes in the effectiveness of monetary policy on asset prices. We estimate a Markov-switching structural vector autoregressive (MS-SVAR) model using the most recent data (1960Q1-2019Q4) for the following set of variables: real output, real dividends, output inflation, commodity inflation, the federal funds rate, and the real stock price index. The monetary policy shock is identified recursively (Galì and Gambetti, 2015). By selecting a parsimonious regime-switching specification from a set of nonlinear candidate models, two sources of nonlinearity affect model dynamics. Controlling for time-variation in the variance of structural disturbances, we extract a source of instability driving the interaction of monetary and financial dynamics, namely the interest rate and the stock price equations. This is indicative of the nature of the interrelations of monetary policy behavior and bubble realizations.

---

[3]Nonlinear methods prove particularly apt to detect bubbly episodes, as evidenced in Phillips, Shi and Yu (2015) and Michaelides, Tsionasb and Konstantakis (2016).

3

The main results reached in this first part of the paper can be summarized as follows. First, the recurrent states well align to known historical events, monetary policy changes and financial deepening. On this basis, we label the states in the stochastic component as *variance states*, generating high, medium and low shocks' variance, and those in the systematic component as *monetary-financial states*, generating states of high and low finance. We interpret the former as periods where asset prices, equity returns and real interest rates are higher. Higher premia generate and inflate bubbles, whose component is in turn conditionally higher under the high finance state. Second, from state-dependent impulse responses we find that a monetary tightening may be ineffective in reducing stock prices, inflating their bubble component even further. This happens under states when bubbles exist and are large (i.e., the high finance regime), showing that conditional stock price dynamics depend on the relative size of the bubble. This increases with the real interest rate.[4] Under the high finance regime, monetary policy tightening generates persistent increases in both nominal and real interest rates, thus more sharply affecting output and real dividends. Deeper recession/deflation combines with a positive response of the stock price bubble component which, when dominant over the fundamental one, is responsible for increased asset prices in the long run.

On the theoretical side, we adopt a Markov-switching version of the model for bubbles presented in Ciccarone, Giuli and Marchetti (2019). This is an OLG scheme in line with Galì (2014), Martin and Ventura (2016), Ikeda and Phan (2016)). The model includes borrowers and lenders in the credit market, physical capital accumulation, and financial frictions along the lines of Martin and Ventura (2015, 2016) and Carvalho, Martin and Ventura (2012). These credit market imperfections constrain agents looking for funds to be used for productive investments as the amount of credit that borrowers can obtain is provided by lenders on the basis of the amount of collateral investors can pledge. Nominal frictions in the formation of final goods' prices are also included and provide a role for monetary policy in fixing the nominal interest rate. Furthermore, bequests from old borrowers to young borrowers make it possible to adopt a realistic numerical version of the model.

Markov-switching nonlinearities (see Farmer, Waggoner and Zha (2009)) affect the bubble size and the monetary policy rule, in line with the VAR model selection evidence.[5] Calibration is based on matching some key empirical moments from the VAR, namely impulse responses of output, inflation interest rates and the bubbly component of asset prices. It produces a regime, corresponding

---

[4]This result specifies the conditions under which Galì and Gambetti (2015)'s findings hold.

[5]See Shorfeide (2005), Davig and Leeper (2007), Bianchi (2013), and Faia (2019) for applications to forward-looking Markov switching rational expectations models.

to the high-finance VAR state, where agents invest in bubbles, monetary policy is more persistent and less reactive to inflation, but more reactive to the bubble. Under this regime, the level of the bubble is able to weaken the effect of financial frictions, allowing borrowers to demand more funds.[6]. The matching also generates a regime, corresponding to the low-finance VAR state, with no bubbles, a stronger responsiveness to inflation and lower interest rate persistence.

In line with the empirical evidence, the model predicts that a monetary tightening proves ineffective in reducing stock prices, increasing real rates and inflating bubbles. This happens because the increase in the real interest rate, in addition to the standard demand effect favoring future consumption, generates two additional effects: *i)* a reduction in the demand for credit via a price channel (increased cost of borrowed funds); *ii)* an increase of the share of aggregate savings allocated to non-productive assets displacing investments from productive capital via an allocation channel. These recessive channels more than compensate the expansionary effect of the increase in the bubble size via the collateral channel. Agents expectations play a key role in this mechanism: if investors expect to switch to a no bubbly scenario the impact of the monetary policy shock on real and financial dynamics is dampened.

The model, when coupled with the empirical analysis carried out with the MS-SVAR, can provide an explanation of the magnified recessionary effects (on real rates and on asset bubbles) of a monetary tightening when the economy is in a high finance regime, a state characterized by a relatively great bubble component in asset prices. The outcome is hence a recession/deflation and an increase of the real rate (and hence of the bubble component), that are magnified under the high finance regime. The reason is that in this regime, following the exogenous increase in the policy rate, the reduction in the real rate which is called for by the fall in inflation is partially offset by the Central Bank's reaction to the increased bubble component (which would of course not occur in a non bubbly economy, when interest rates respond only to inflation). This produces a stronger persistence in the nominal interest rate response, higher real rates and a more severe recession/deflation.

The paper is structured as follows. In Section 2, we present the empirical strategy. We then discuss the emerging regimes and results from stochastic simulations. In Section 3, we briefly describe the theoretical model and the dynamics it produces when a shock to the nominal interest rate hits the economy. Section 4 concludes.

---

[6]This approach to link bubbles' dynamics to the monetary policy reactiveness to inflation is also seen in Jarrow and Lamichhane (2022).

2 Empirical evidence

Our empirical analysis is based on a nonlinear multivariate time series model, specified as a Markov-switching structural vector autoregression (MS-SVAR) (Sims and Zha (2006), Sims, Waggoner and Zha (2008)). This modeling structure allows studying the linkages between monetary and financial dynamics, detecting different sources of regime-specific regularities and the conditions under which they occur.

The choice of variables and the identification strategy for the monetary policy shock follow Galì and Gambetti (2015). We extend their analysis by exploiting the properties of the Markov-switching model to stochastically characterize the historical conditions under which their result - that is a monetary tightening leading to a persistent drop in the fundamental component of real stock prices and to an increase in the bubble component - holds. Specifically, we extract regimes affecting the interaction between monetary and financial dynamics. They generate states where the dependence of the stock price on the relative size of the bubbly component matters. Regimes of high stock prices and equity returns are those in which a monetary policy tightening leads to higher fluctuations in real macroeconomic variables and inflation, together with an increase in the bubbly component of stock price. That more than offsets the decline in the fundamental component in the medium to long term.

Our choice of this modelling strategy is based on two main considerations: *i)* the observed dynamics of financial variables generally display abrupt time variation, consistently with discrete changes in market beliefs; *ii)* a regime-switching specification may provide a clearer picture of the sources of time-variation as it allows to distinguish between coefficient-switching and variance-switching dynamics. The former signals changes in structural regularities, that is, shifts in feedback and amplification effects across the variables in the model, whereas the latter account for heteroskedasticity. Based on the estimated MS-SVAR, whose switching structure is selected among a set of model competitors displaying several types of nonlinear structures, the empirical analysis employs smoothed probabilities, state-dependent moments, and impulse response functions (IRFs) to characterize the uneven transmission dynamics over time triggered by an exogenous tightening of monetary policy.

2.1 The MS-SVAR

The MS-SVAR is estimated with Bayesian methods considering quarterly US data spanning the period 1960:Q1-2019:Q4. Six variables are included in the model: the real GDP level y_t, real

dividends d_t, the GDP inflation rate π_t^y, the inflation rate for non-energy commodities π_t^c, the federal funds rate r_t, and the real S&P500 index q_t. All variables enter in logs, with the exception of inflation rates and the policy rate. For the latter, we use the shadow interest rate by Wu and Xia (2016) over the 2009-2016 time interval.[7]

Regime-switching dynamics are introduced as state-dependency in both the systematic (structural coefficients) and the stochastic model components. A Markov chain driving discrete coefficient states, ξ_t^c, controls the former; whereas an independent Markov chain, ξ_t^v, controls the latter, capturing discrete states for shocks' heteroskedasticity. The two chains are collected under a composite process $\xi_t = \{\xi_t^c, \xi_t^v\}$, which is a discrete time-varying wedge affecting shocks and model relations.

The MS-SVAR model is as follows:

$$\mathbf{y}_t' \mathbf{A}_0(\xi_t^c) = \mathbf{c}'(\xi_t^c) + \sum_{i=1}^{\rho} \mathbf{y}_{t-i}' \mathbf{A}_i(\xi_t^c) + \epsilon_t' \mathbf{\Sigma}^{-1}(\xi_t^v) \tag{1}$$

where $\mathbf{y}_t' = \begin{bmatrix} y_t & d_t & \pi_t^y & \pi_t^c & r_t & q_t \end{bmatrix}$, $\mathbf{c}(\xi_t^c)$ is the vector of constants, $\mathbf{A}_0(\xi_t^c)$ is the invertible contemporaneous correlations matrix, $\mathbf{A}_i(\xi_t^c)$ denotes the dynamic cross-correlation matrices for each lag term ρ, and $\mathbf{\Sigma}$ is a diagonal matrix capturing structural shocks' sizes. Following the standard practice for quarterly observations, we fix $\rho = 5$ and adopt Litterman's (1986) random walk prior, consistently with the stochastic properties of the variables. The calibration of prior's hyperparameters follows Sims, Waggoner and Zha (2008), which provide a benchmark for quarterly data MS-SVARs.[8] A multivariate normal distribution for the orthogonal structural shocks ϵ_t is assumed:

$$P(\epsilon_t | \mathbf{Y}^{t-1}, \mathbf{\Xi}_t, \theta, q) = \mathcal{N}(\epsilon_t | \mathbf{0}_n, I_n) \tag{2}$$

where the structural shocks' standard deviations are given by the diagonal elements of $\mathbf{\Sigma}^{-1}(s_t^v)$, θ denotes the vector of the model's structural parameters, $\mathbf{\Xi}_t$ and $\mathbf{Y}^{t-1}$ collect past information on the latent processes and data, respectively. The transition probabilities from state i to state j, $q_{i,j}$, are collected in the composite transition matrix $Q = (q_{i,j})_{(i,j) \in (H \times H)} \in \Re^{h^2}$, where $H = \{1...h\}$ is the set of possible regimes for ξ_t, and $Q = Q^c \otimes Q^v$.[9]

---

[7] A detailed description of variables' definitions and data sources is presented in Appendix A.

[8] Specifically, we adopt the following hyperparameter' structure: $\mu_1 = 1$; $\mu_2 = 1$; $\mu_3 = 0.1$; $\mu_4 = 1.2$; $\mu_5 = 1$; $\mu_6 = 1$. The results are however robust to reasonable changes in these values. The estimation results are generated with one million Gibbs sampling replications. The first 100,000 draws are discarded as burn-in, and then one in every ten draws is retained.

[9] The prior for the transition probabilities is a Dirichlet with parameters implying a symmetric prior average duration of regimes of six quarters. The results are robust to significant variations in the prior regime's duration.

The identification of monetary policy shocks is achieved by imposing the recursive strategy adopted by Galì and Gambetti (2015), implying that only stock prices respond contemporaneously to the federal funds rate shock.

We then test whether there is evidence of regime switches, and evaluate if such changes come outsized shocks (variance-switching, thus heteroskedasticity), or whether they reflect deep structural variations, by computing the (log) marginal data density (MDD) over differently specified competing models. As shown in Appendix B, model selection estimates nonlinearities in both the stochastic (three-state Markov chain) and the systematic (two-state Markov chain) model components. The latter describes a story of state-dependent monetary policy and financial interaction jointly affecting the asset price and interest rate equations. The selected model is labelled $3\xi^v 2\xi^c_{r,q}$. We name the states driving the time-varying covariance matrix and capturing shocks' heteroskedasticity as *variance states*; while the second chain jointly affecting the monetary policy and the asset price equations defines the *monetary-financial states*, $\xi^c = \xi^{mf}$.

2.2 Monetary-financial Regimes

Monetary-financial states reflect nonlinear regularities of structural behaviours and shocks' transmission describing the interaction of monetary and financial phenomena. Their occurrence defines two states: the "high finance state" and the "low finance state". As displayed in Table 1, summarizing some descriptive statistics conditional on states,[10] the former characterizes periods of sustained and stable stock prices and equity returns, higher output, inflation and higher interest rates; the latter, the low finance state, defines periods of lower stock prices and crushes in equity returns. By decomposing the S&P500 stock price index in a fundamental and a bubbly component, as in Galì and Gambetti (2015), stock prices in the high finance state contain an high bubble component, which fully explains the higher stock price levels under this regime. Stock returns are also explained (in levels and standard deviations) by their bubble component, which grows at higher and more stable returns. The price-dividend ratio is substantially higher in the high finance regime.

As for the US historical evolution of regimes, Figure 2 reports the smoothed probabilities evaluated at the posterior mode for the Markov chains emerging under the selected model in the 1960-2019 quarterly sample. The top panel displays states' probabilities for the variance, capturing high (dark gray area), medium (light gray area) and low (white area) shocks' sizes. The bottom panel displays probabilities for the monetary-financial states, where the gray areas identify the high

---

[10]The states' descriptive statistics correspond to the values to which selected moments are expected to converge once a state is in place for an extended period.

Table 1: Descriptive statistics of states: Moments

| | **Monetary-financial states** | | | | | |
| | means | | | standard deviations | | |
Variables	Data	H-fin	L-fin	Data	H-fin	L-fin
y_t	9.086	9.286	8.993	0.516	0.573	0.356
d_t	2.928	3.554	3.735	0.318	0.524	0.500
π_t^y	0.982	0.754	0.709	0.631	0.828	0.354
π_t^c	0.961	1.436	1.091	0.525	3.432	2.873
r_t	1.209	2.092	0.711	0.972	1.609	0.762
q_t	6.535	9.206	3.974	0.706	1.899	1.626
Equity return	0.883	2.714	-2.466	6.589	4.509	8.594
Real ffr	0.407	1.337	0.002	0.708	1.773	0.764
Price/dividend	3.611	5.653	0.239	0.394	1.592	1.918
q_t (bubble)	6.061	6.644	-0.606	0.605	0.870	2.953
Equity R (bubble)	0.797	1.128	-4.369	6.665	4.781	8.597

The table reports the means and standard deviations of model variables and some selected derivations, conditional on staying in each state. The equity return is computed as the growth rate of S&P500 stock prices; the real policy rate is the deviation of the federal funds rate from inflation (based on the GDP deflator); 'q_t (bubble)' is the bubble component of S&P500 stock prices, as decomposed by Galì and Gambetti (2015), and 'Equity return (bubble)' is the return rate of the bubble component of S&P500 stock prices.

finance state and the white areas the low finance state. To enhance regimes' interpretation, the red line displays the evolution of the monetary policy rate over time.

Variance and coefficient regime-switches have a story to be told in terms of known events. Regimes on shocks' variances (low, medium and high) detect high volatility during the 70s, the early 80s and the 2008-09 crisis, reflecting the two oil price turmoils (as in Bianchi and Ilut (2017)), the high volatility of the federal funds rate during the reserve targeting period of the early 80s and the first phases of the global financial crisis. The medium variance regime materializes around the second half of the 80s, including the stock market crash event in 1987 and the recession due to the Gulf War I of the early 90s. The remaining periods, thus the pre-70s, the financial deepening era taking place in the first part of the Nineties and enduring until the 2008 financial crisis, and the post-2009 are characterized by a low variance state, reflecting conditions of relative stability and low exogenous variations in shocks.

Monetary-financial states emerge as result of the interaction of monetary and financial phenomena.[11] Switches towards low-finance states are centered around stock price turning points and

---

[11]They are highly recurrent states, thus do not cover prolonged time spans, as it is implicit in the literature on monetary switches (Sims and Zha (2006) for variance regimes, Bianchi (2013) and Hubrich and Tetlow (2015) for systematic regimes).

Figure 2: US: Variance and monetary-financial states
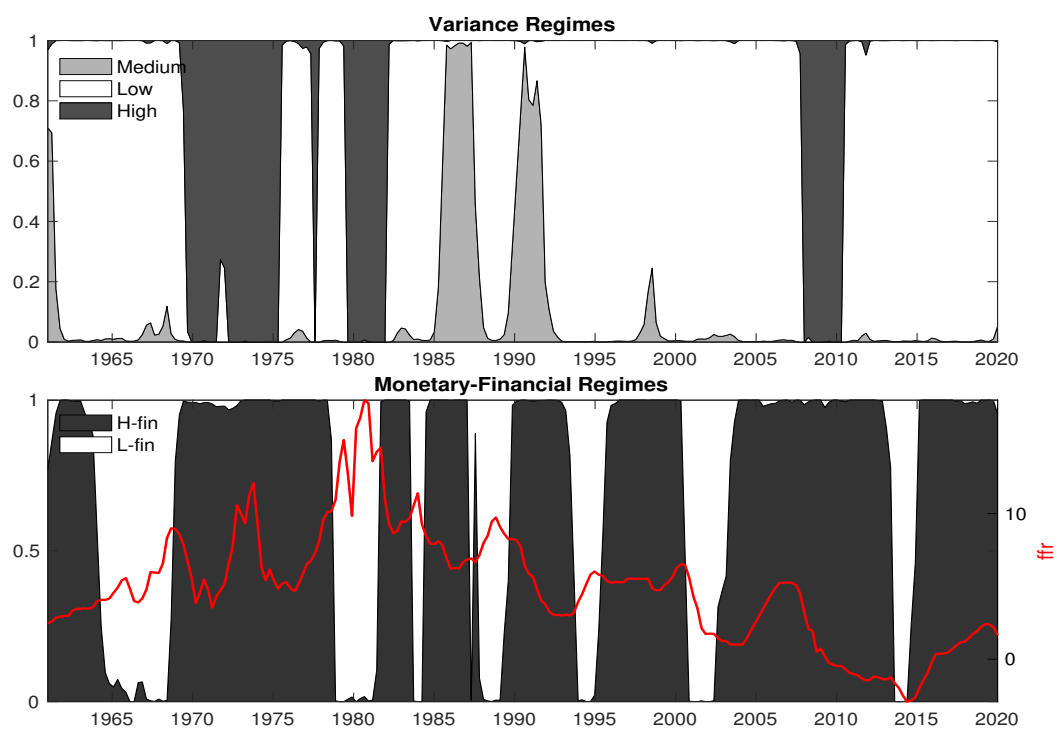
The figure shows the states' smoothed probabilities at the posterior mode from the best-fit benchmark model, $3\xi^{v}2\xi^{c}_{r,q}$. The upper plot displays the smoothed probabilities for the variance states. The bottom plot displays the smoothed probabilities for states emerging on the monetary policy and the asset price equations. The series for the interest rate is shown below the latter.

occur in the late 60s, during the first half of the 80s (Volker era), during the dot-com bubble (2001), and the 2015-16 stock market sell-off, including the Chinese stock market turbulence, the Greek debt default and the effects of the end of quantitative easing in US.

2.3 Impulse response Analysis

We evaluate the regime-specific conditional dynamics generated by an unexpected increase in the federal funds rate considering regime-dependent impulse response functions (IRFs).[12] In order to enhance the interpretation of results with respect to the transmission dynamics across regimes, we condition the IRFs to a 1% contractionary monetary shock in all states.[13]

In Figure 3, impulse responses are state-dependent with respect to the monetary-financial regime in place, and display different channels of monetary transmission, depending on whether the high or low finance state is in place. Specifically, an interest rate shock triggers a drop in real GDP and real dividends, which in turn worsens expectations of future profitability and reduces stock price evaluations, as predicted by the traditional theory.[14] The high finance state, namely the state where the bubbly component of stock prices is higher, features more amplification and less policy effectiveness in stabilizing target variables, leading to stronger and more persistent drops in real activity and dividends, and a further increase in real interest rates. Notwithstanding the drop in expected dividends, the fall in stock price decrease is lower under the high finance (bubbly) state, and vanishes after one year. This is because asset prices do not only co-move with dividends, but they embed an additional premium resulting from nonfundamental sources, otherwise called bubbly component of asset prices, which grows with real rates.

In Figure 4, by decomposing the stock price impulse response as in Galì and Gambetti (2015) in a fundamental and a nonfundamental component,[15] we note that, while the fundamental component decreases with dividends, the bubbly component of asset prices increases. This is more evident under the high finance state, where the bubbly share in asset prices is higher and increases with the real interest rate. This result confirms Galì and Gambetti (2015)'s evidence, but underlines that its relevance is regime-specific and depends on recurrent monetary-financial states of the

---

[12]Forecast error variance decompositions (FEVDs) are in Appendix C.

[13]Note that variance states do not affect the model-generated transmission dynamics, which depend only on the model's coefficients. The relative contribution of the monetary policy shock to each variable's variance is also unaffected by such a re-scaling.

[14]The impulse response of inflation does not display a price puzzle, a well-recognized issue in the empirical literature (Sims, 1992). This result is obtained by applying the Estrella (2015)'s overidentifying restriction. However, the response of inflation to the monetary tightening remains low with respect to theoretical results in the literature.

[15]where the former is the present value of future discounted dividends and the latter the remaining part.

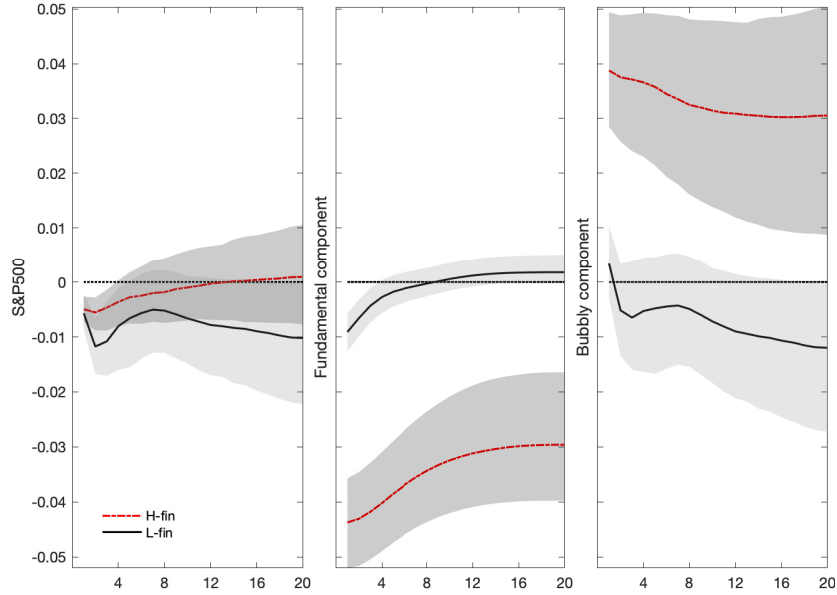Figure 3: Monetary Policy shock. Impulse responses
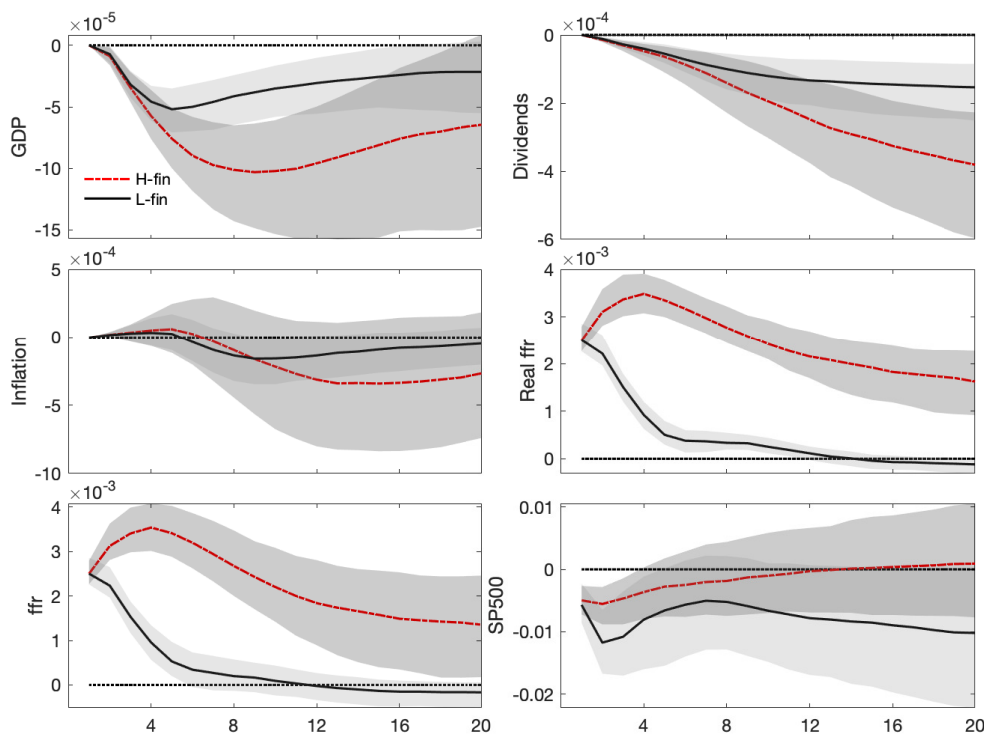
The figure shows the impulse responses to a 1% monetary policy shock. Difference across regimes reflect nonlinearities in the monetary policy and asset price equations only, driven by ξ^{mf}.

Figure 4: Monetary Policy shock. Impulse responses: stock price decomposition

The figure shows the impulse responses of asset prices, and their decomposition in a fundamental and bubbly component, to a 1% monetary policy shock. Differences across regimes reflect nonlinearities in the monetary policy and asset price equations only, driven by ξ^{mf}.

economy. Policy ineffectiveness to dampen the emergence of financial rational bubbles depends on the financial state of the economy and its interaction with the endogenous component of monetary policy. Moreover, it emerges in periods when stock prices are high and less volatile, when the real economy responds more to monetary policy shocks, and stock prices composition features the build-up of a bubbly component.

3 A Markov-switching model for bubbles

The analytical model, synthetically described in Appendix D, introduces Markov switches in Ciccarone, Giuli and Marchetti (2019). The economy is populated by overlapping generations (OLG) of agents living for two periods; within each period, young and old agents coexist in equal and constant proportion. This OLG framework is characterized by three main elements: *i)* frictional financial markets; *ii)* physical capital accumulation; *iii)* sticky prices. Households are grouped into two types: borrowers and lenders in the credit market. Borrowers can invest in physical capital and can trade an additional asset which is modeled as "pure" bubble, analogous to a pyramid scheme. Due to asymmetric information between creditors and debtors, and to the absence of

13

state-contingent securities, the amount of credit that can be obtained by borrowers varies with the amount of collateral that can be pledged, which depends also on the (expected) value of the bubbly asset. Due to price stickiness, monetary policy can affect the real macroeconomic variables.

The economy produces one intermediate good and a continuum of differentiated final goods. By using capital and labor, both traded in competitive markets, a representative firm produces the intermediate good, which is sold under perfect competition to a continuum of monopolistically competitive final producers. Final goods can be consumed or transformed into new physical capital. The nominal prices of the final goods are sticky.

Within each generation, the two classes of agents participate to the final goods market, to the markets for productive inputs and to the financial markets. The savers work when young, consume the final goods when young and when old, and save part of their labor income when young to purchase credit contracts paying nominal interest. The firms producing the final output are owned by the old savers, who pass them on as a bequest at the end of their lives, when young savers enter their old age. Profits and interest payments on credit contracts finance the consumption of old savers. Borrowers, who also consume the final good over their lifetime, invest, when young, in the productive ("fundamental") and in the non-productive ("bubbly") assets, and finance this expenditure borrowing in the financial sector and using the resources left to them as a bequest by the borrowers of the previous generation.

Productive investments add to the capital stock, which the young borrowers buy from the old ones at the end of the period. The representative intermediate firm rents physical capital from borrowers and hands to them the remuneration of capital, when they become old. The bubbly assets are valued on the expectation of their re-sale value. Each generation of borrowers issues new bubbles with random initial value, which are traded in the market for bubbles, alongside the old bubbles issued by previous generations and sold to the young one.

In the credit market, identical and perfectly competitive banks accept the deposits demanded by savers and use them to supply the loans demanded by borrowers at the nominal loan rate. At the end of each period, loans and deposits (plus interest) are paid back, banks' balance sheets clear and banks shut down, to open again at the beginning of the next period. Savers can hence hold two types of financial assets: money, supplied by the Central Bank, and bank deposits. The Central Bank sets the rate of interest on deposits by following a dynamic rule, as described below.[16]

---

[16]This is equivalent to allowing households to buy government assets and to hold bank deposits, which represent a form of private liquidity (Aksoy, Basso and Coto-Martinez, 2013). As riskless deposits and riskless non-contingent government bonds are perfect substitutes in the savers' portfolios, assuming no arbitrage conditions, the deposit rate always equals the government bond rate (Freixas and Rochet, 1997). The central bank is then also setting the nominal

In the market for bubbles, old borrowers supply the outstanding bubbles issued in the previous period and can issue new bubbles. Both types of bubble are demanded by young borrowers. Young savers, who supply labor inelastically, enter period t without previously accumulated cash holdings and deposits, receive money wage income and deposit at banks. At the end of each period, deposits are repaid, together with interest earnings. A cash-in-advance constraint requires agents to allocate money balances and money wage income for consumption, net of the deposits they make at banks. The old savers receive the aggregate profits obtained from retailer firms and from banks, and are not interested in carrying financial assets to the future (bequest motive).

Credit market imperfections affect the behavior of banks, which may not always obtain the full repayment of the loans provided to the borrowers. To obtain loans, borrowers must then provide credit intermediaries with collateral. They can pledge only a fraction of their future resources, but can create and exchange a bubbly asset that can also be used as collateral. The overall guarantee provided by borrowers eliminates the need for banks to add a risk premium, on top of the riskless rate, to the rate of interest on loans.

The OLG model is Markov-switching in the bubble size and the monetary policy rule, according to a two-state Markov process ξ_t^{mf} capturing monetary-financial regimes. This generates two states, namely the high finance state and the low finance state, in line with the empirical evidence. The Markov process evolves according to a transition matrix H^{mf} collecting the transition probabilities to move across states. In this modelling framework, agents are allowed to form expectations in each period to switch to a different monetary-financial state of the economy.

3.1   The market for bubbles

The equilibrium between demand (by young borrowers) and supply (by old borrowers) of bubble in every period is $B_{t+1} = B_t + B_{t+1}^N$, where $B_{t+1} \geq 0$ is the physical amount of the bubble supplied at $t+1$ and B_{t+1}^N represents the newly issued bubbles. The bubble equilibrium equation can be expressed as:

$$Q_{t+1} = R_{t+1}^B Q_t + Q_{t+1}^N \tag{3}$$

where P_{t+1}^B is the real price of the bubble, $R_{t+1}^B = P_{t+1}^B / P_t^B$ is the real factor of return on the bubble and $Q_t = P_t^B B_t$ is the real value of the bubble. The new bubble creation process is linked to the economy's size through the parameter $\omega > 0$: $q_t^N = \omega y = \omega A k^\alpha$, where $q_t^N = Q_t^N / g^t$ is the trend-less value of the new bubble and y, k are, respectively, the stationary levels of output and

interest rate on public (outside) liquidity (in zero net supply), reflecting in reduced form open market operations.

15

capital;[17] hence ω measures the bubble-to-output ratio. If $\omega = 0$, the economy sets itself into a "no bubble" stationary state in which the level of capital k_{NB} is generally lower than that of a "bubbly" economy ($\omega > 0$).

We assume the bubble share to follow a two-state Markov process ξ_t^{mf}, capturing the financial side of the monetary-financial interaction we evidence in the VAR. The Markov process produces two states, namely the high finance state when the bubble share assumes higher values and the low finance state when it is smaller. The monetary policy counterpart will be discussed in the next section.

Since the bubble-to-output ratio is a steady state parameter, this is computed as the ergodic mean value of the Markov-switching shares, $\omega^{mf} = \bar{\omega}$ (Foerster et al., 2016). See Appendix D.5 for more details.

In order to understand the effects of $\bar{\omega}$ on the stationary value of capital k, and hence on y, take the equilibrium relationship between k and the real interest factor R in the credit market, $Ak^{\alpha-1} = f_B(R)$ from equation (D.26), and solve it with respect to k. Then, compute the following derivative:

$$\frac{dk(R)}{d\bar{\omega}} = \frac{gk(R)^{2-\alpha}}{(1-\alpha)AR}\left[\frac{\phi r^k - \frac{\varepsilon_{R,\bar{\omega}}}{\bar{\omega}}\left(\frac{r^k}{\mu\alpha}(\phi\mu\alpha + \bar{\omega}) + (1-\delta_K)\phi\right)}{\frac{g}{R}(\phi\mu\alpha + \bar{\omega})} + \frac{(1-\phi)\eta}{\eta+\beta}\mu\alpha\right]$$

$$\varepsilon_{R,\bar{\omega}} = \frac{dR/R}{\frac{d\bar{\omega}}{\bar{\omega}}} > 0, \quad r^k = \alpha\mu Ak^{\alpha-1}$$

The sign of $\frac{dk(R)}{d\bar{\omega}}$ defines two different regions for the effect on capital of an increase in the bubble share. We are in *crowding-in (crowding-out)* when an increase in the bubble share ends up stimulating/dampening the accumulation of capital:

$$\bar{\omega}\phi r^k - \left[\left(r^k + 1 - \delta_K\right)\phi + \frac{q^N}{k}\right]\varepsilon_{R,\bar{\omega}} \gtrless 0$$

Therefore, the direction in which bubble share $\bar{\omega}$ in the (locally unique) stationary state affects capital depends on three different channels acting on the three terms of the above inequality:

- *Collateral channel*: an increase in the bubbly asset (higher $\bar{\omega}$) slackens the collateral constraint allowing borrowers to demand more funds and invest more. This yields a positive effect on k

---

[17]The term g is the economy's exogenous growth factor, while A and α are the parameters of the Cobb-Douglas production function.

and y (crowding-in), through the term $\bar{\omega}\phi r^k$;

- *Price channel*: a higher $\bar{\omega}$ increases the cost of borrowed funds, leading borrowers to demand less funds and to invest less. This produces a negative effect on k and y (crowding-out), via the term $\varepsilon_{R,\bar{\omega}}$;

- *Asset allocation channel*: a higher $\bar{\omega}$ increases the quantity of the bubbly asset to be purchased, crowding out productive investment expenditures. This effect increases with the total return on capital, and it yields a negative effect on k and y (crowding-out), through the term $\left(r^k + 1 - \delta_K\right)\phi + \frac{q^N}{k}$.

The final effect of an exogenous increase of $\bar{\omega}$ on k (i.e., whether the economy is in a crowding-in or in a crowding-out regime) is determined by the relative size of these three channels.

3.2 Monetary policy

In order to simulate an exogenous shock to the nominal interest rate (i_t), we assume that the monetary authority sets the policy rate according to the following instrument rule:[18]

$$\widehat{(1+i_t)} = \rho_{i,\xi_t^{mf}}\widehat{(1+i_{t-1})} + \left(1 - \rho_{i,\xi_t^{mf}}\right)\left(\delta_{\pi,\xi_t^{mf}}\pi_t + \delta_{q,\xi_t^{mf}}\hat{q}_t\right) + e_t^i \qquad (4)$$
$$e_t^i \sim i.i.d. \left(0; \sigma_{e^i}^2\right)$$

where $\widehat{(1+i_t)}$ is the deviation of the monetary policy factor from its steady state value, $\delta_\pi > 1$ is the policy reaction parameters to inflation, and $\delta_q \geqslant 0$ denotes the policy reaction to the bubble ($\delta_q > 0$ indicates a LAW policy). This (empirical) rule makes the interest rate react not only to changes in the inflation rate π_t, with ρ_i capturing the persistence of the policy rate.

We set the policy reaction to inflation and the bubble, and the policy persistence as functions of a two-state Markov process, $\xi_t^{mf} \in \{\text{H-fin, L-fin}\}$, which jointly affects the steady state bubble-to-output ratio. This process evolves according to a transition matrix, H^{mf}, whose calibration is given by the VAR estimates.

---

[18]The percentage deviations from the stationary state are indicated with a hat, e.g., for the generic (trendless) variable z_t: $\hat{z}_t = \frac{z_t - z}{z}$.

3.3 Solution method

We solve the model using the efficient perturbation methods applied to Markov-switching models elaborated by Maih (2015), and differently proposed also by Foerster et al. (2016).[19] A detailed description of the solution method is reported in Appendix E. The model's first-order approximated solution can be written in the following form:

$$\Upsilon_t = T_{\xi_t^{mf}}(\Upsilon_{t-1}, \sigma, \epsilon_t) \tag{5}$$

$$T_{\xi_t^{mf}} = T_{\xi_t^{mf}}(z) + DT_{\xi_t^{mf}}(z)(z_t - z)$$

where Υ_t is the vector of model's variables, $T_{\xi_t^{mf}}$ the Taylor first-order expansion, σ defines the perturbation parameter, ϵ_t the vector of structural shocks and DT the matrix of first-order derivatives. The expansion point is $z_{\xi_t^{mf}} = (\Upsilon, 0, 0)$, where $z_t = (\Upsilon_{t-1}, \sigma, \epsilon_t)$ and Υ identifies the variables' steady states.

3.4 Impulse responses

We compute the impulse response functions to the monetary policy shock. We calibrate the model at annual frequency by fixing $\beta = 0.96$, $\alpha = 0.33$, $\gamma_b = 1$, $g_s = 1.02$, $\gamma_s = 0.2$, $\mu = 1 - 1/42$, $\delta_k = 0.1$, $\beta_{beq} = 42$, $\phi = 0.03$, $A_2 = 0.3$, $\sigma_i = 0.01$ (equivalent to Ciccarone, Giuli and Marchetti (2019) but at the annual frequency). The transition probabilities are fixed at the annual counterparts of the VAR estimates, namely we set the probability to move from the high-finance to the low-finance regime to 0.1834, and the transition probability from low-finance to high-finance to 0.4205.

The Markov-switching parameters, such as ω, ρ_i, δ_π and δ_q, are estimated by using impulse response matching with respect to the VAR results. More specifically, we match four regime-dependent VAR impulse responses: output, inflation, interest rate, and the bubble component of asset prices. Table 2 reports the results.

The estimation leads to a regime of high-finance which corresponds to a bubbly economy with a more persistent and less reactive monetary policy rule with respect to inflation. As for the reactiveness to the bubble, this regime reflects a LAW monetary policy rule ($\delta_q > 0$). Under this regime, the stationary level of the bubble is high enough to locate the economy in crowding-in, with

---

[19]They differ in the fixed point around which they perform the Taylor series expansion. Foerster et al. (2016) expand around the steady state associated to the ergodic mean of the parameters. The approach pioneered by Maih (2015), instead, expands around the steady state associated to each regime taken in isolation. In our case, the regimes do not alter the steady state. Therefore, the two approaches would not differ.

Table 2: Model calibration based on impulse response-matching

Parameter	Meaning	Strategy	Value
ω(H-fin)	Bubble share	IRF Matching	0.0143
ω(L-fin)	of GDP	IRF Matching	0
ρ_i(H-fin)	Monetary policy	IRF Matching	0.379
ρ_i(L-fin)	persistence	IRF Matching	0.198
δ_π(H-fin)	Reactiveness of ffr	IRF Matching	0.866
δ_π(L-fin)	to inflation	IRF Matching	1.531
δ_b(H-fin)	Reactiveness of ffr	IRF Matching	2
δ_b(L-fin)	to bubble	IRF Matching	0

The estimated values are generated by matching the regime-dependent impulse responses of output, inflation, interest rate, and the asset price bubble component. They are weighted based on their confidence intervals.

higher stationary levels of real rates, capital and output. Instead, the L-finance regime corresponds to a no-bubbly economy, with a stronger reactiveness to inflation and lower interest rate persistence.

Given the above calibration for the two states, Figure 5 shows the model-based impulse responses in the two regimes, as compared the VAR counterparts. Following a positive shock to the nominal interest rate, the model predicts a recession/deflation, together with an increase in the real rate R_t.[20] More importantly, in a bubble economy (H-fin regime) the direction of the reaction of the bubble q_t is in line with our empirical evidence in Section 2, thus the contractionary monetary policy inflates bubbles' valuation.

The economic mechanism underlying these results is as follows: the shock e_t^i has a direct impact, due to the nominal rigidity, on the real interest rate R_t. In addition to the standard demand channel acting through the Euler equation and favoring future consumption, the increase in the real interest rate generates two additional effects: on the one side, the demand for credit is reduced, due to the working of a price channel (the increased cost of borrowed funds); on the other side, the value of the bubble rises and an allocation channel re-directs more resources towards the purchase of the bubble. These recessive channels more than compensate the effect of the increase in the bubble size q_t on credit demand, due to the slackening of the collateral constraint which allows borrowers to demand more funds (collateral channel); the prevalence of recessionary drivers is related to the

---

[20]The distance between model and VAR-based impulse responses of inflation is due to the weak and delayed reaction in the price dynamics generated by the VAR.

Figure 5: Monetary Policy shock. Regime-dependent impulse responses
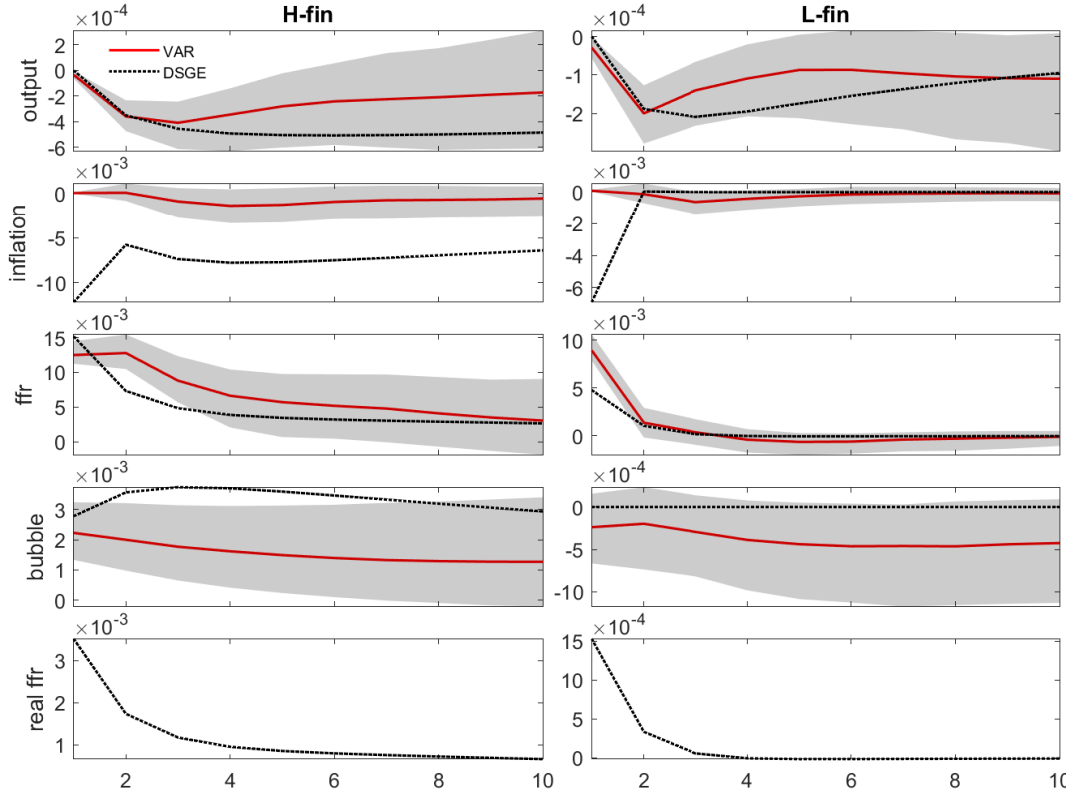
The figure shows the model-implied versus VAR-based impulse responses of selected variables (output, inflation, nominal interest rate, bubble, and the real interest rate) to a 1% monetary policy shock. Regimes affect the steady state parameter ω and the monetary policy rule, namely ρ_i, δ_π and δ_q. The impulse response matching is based on the following grid: $\omega(H-fin) \in [0, 0.018]$, $\omega(L-fin) \in [0, 0.01]$, $\rho_i(H-fin) \in [0.3, 0.6]$, $\rho_i(L-fin) \in [0.1, 0.3]$, $\delta_q(H-fin) \in [0, 0.2]$, $\delta_q(L-fin) \in [0, 2]$, $\delta_\pi(H-fin) \in [0.8, 2]$, $\delta_\pi(L-fin) \in [1.25, 2]$. The corresponding loss function, computed on the distance between VAR and DSGE impulse responses at the estimates, is equal to 0.1438.

fact that the shock is straightforwardly directed to the interest rate and does not directly affect the bubble size. The outcome is hence a recession/deflation coupled with a raise in the bubble value. The policy reaction to the fall in inflation π_t tends to mitigate the recessionary effects of e_t^i, by reducing the amplitude of the downturn of output and inflation.

As for the role played by the other estimated parameters of the Taylor rule (i.e. ρ_i, δ_q), the recession/deflation and the increase in the real rate R_t (and hence of the bubble component) are magnified when $\rho_i > 0$ and/or $\delta_q > 0$, namely under the H-finance regime. The reason is that, following the exogenous increase in the policy rate, the policy reaction is dampened by the presence of the smoothing parameter ρ_i and/or by the presence of the LAW policy reaction ($\delta_q > 0$). Under LAW, the signal sent from the fall of inflation, which calls for a reduction of the real rate R_t, is partially offset by the increase in the bubble component. The response of the Central Bank results in an effort which is too feeble in stabilizing the real interest rate. This explains more persistence in the nominal interest rate response, higher real rates, and more severe recession/deflation, as compared to the L-finance regime. Indeed, in a no bubbly economy interest rates respond to inflation only (and more than in H-fin), dampening the initial effects of the shock.

The IRF analysis of the theoretical model suggests a simple interpretation of the policy regimes singled out by the MS-VAR analysis of Section 2: a Central Bank which is too timid in its reaction to exogenous shocks hitting the interest rate could be at the origin of both recessions and growth in the bubble value.

3.5 Policy scenarios

To assess policy credibility under the two regimes, we generate counterfactual scenarios in Figure 6, where we make each regime at a time more persistent and more likely to realize. We label as 'HF credible' the scenario where we set to 0 the probability to go from HF to LF, and increase by 50% that of transitioning from LF to HF (from 0.1834 to 0.2751). We label as 'LF credible' the scenario where we set to 0.01 the probability to go from LF to HF, and increase by 50% that of transitioning from HF to LF (from 0.4205 to 0.6307).

When agents expect the HF policy ($\rho_i > 0$, $\delta_q > 0$) to last longer (HF credible), the effect of a monetary shock is exacerbated both in terms of deeper recession/deflation and bubble's inflation. In the long run the increase in asset valuation boosts borrowing capacity and favors a faster exit out of the crisis.

If, instead, agents assign higher probability to switch in the LF regime (LF credible), they also

21

Figure 6: Impulse responses to monetary policy shock. Counterfactual scenarios
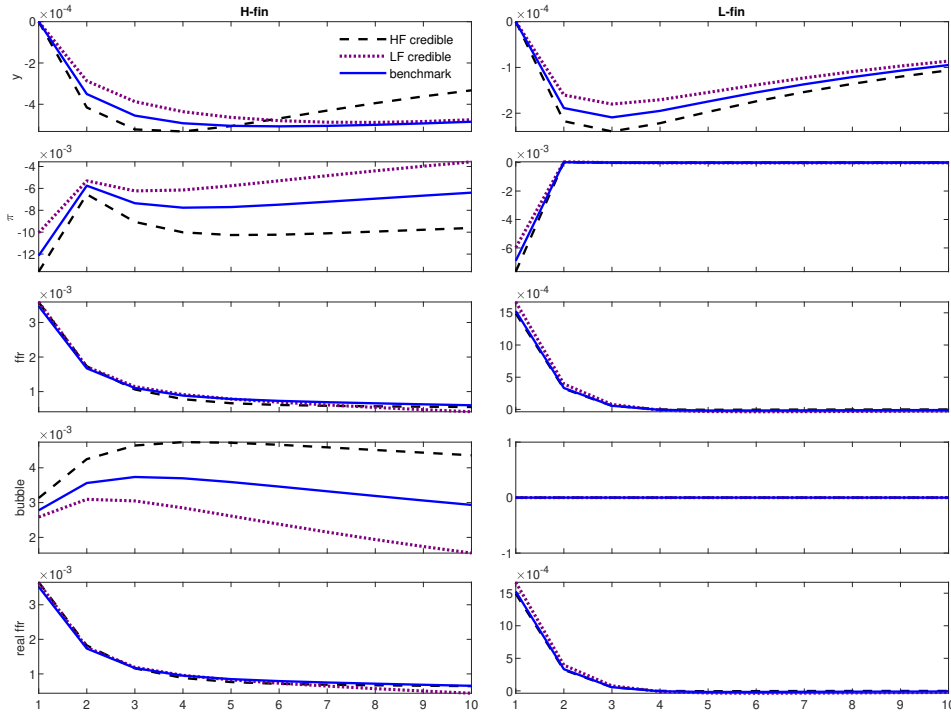
The figure compares the impulse responses extracted from the estimated model to those from counterfactual scenarios defined as follows: a HF credible scenario increases the probability to switch from L-fin to H-fin, and decrease the probability from H-fin to L-fin; a LF credible scenario increases the probability to switch from H-fin to L-fin, and decrease the probability from L-fin to H-fin.

assign higher probability to the belief that the policy rule will be more stabilizing in the future. They hence behave so as to make the deflationary and recessive effects of the shock smoother. In a nutshell, in a bubbly world, the impact of monetary tightening depends on how agents form their beliefs on the credibility of the monetary policy and on the state of the bubble.

4 Conclusions

The empirical aim of this paper is to contribute to the evidence that an unexpected monetary tightening determines an increase in the stock priceâs bubble component. To this aim, we have estimated a Markov-switching structural vector autoregressive (MS-SVAR) model over U.S. data, and we have found that this unpleasant effect of monetary policy mainly occurs under "high finance states", when monetary policy shocks lead to more persistent increases in interest rates and more

sharply affect output and real dividend dynamics. In such regimes, the bubble component of stock prices will positively respond to the interest rate increase if its size dominates over that of the fundamental component, increasing asset prices in the long run. This result is described over identified events describing the history of the US joint determination of monetary and financial facts.

We have then evaluated the ability of a model for rational bubbles with a Markov-switching bubble size and monetary policy to match and rationalize the empirical findings. The model predicts instead that a monetary tightening proves ineffective in reducing stock prices in a bubbly economy, increasing real rates and inflating bubbles' valuation. Under this state (i.e., a high finance regime, as described by the MS-VAR), monetary policy is also less reactive to inflation, favouring more persistent recession/deflation. The model's predictions are hence in line with the empirical findings.

The economic explanation of these results is that the exogenous shock increases the real interest rate; this, in addition to the recession acting through the intertemporal allocation of consumption, translates into an increase in the rate at which a rational bubble grows, thus re-directing more resources towards the purchase of bubbly assets. These recessionary channels more than compensate the expansionary effect generated by the slackening of the collateral constraint. A Taylor rule which reacts to asset prices is less successful in stabilizing the shock with respect to a strict inflation targeting, since the signal sent from the fall of inflation, which calls for a reduction of the policy rate, is partially offset by the increase in the bubble component. The outcome is a more severe recession/deflation, coupled with a raise in the bubble value.

Our conclusion is that a theoretical model with frictional financial markets and sticky prices can provide a straightforward and convincing interpretation of the policy regimes singled out by the MS-SVAR analysis: recessions and growth in bubble values may be the offspring of a Central Bank which is too timid in its reaction to exogenous interest rate shocks.

References

Aksoy, Y., H. Basso, and J. Coto-Martinez. 2013. "Lending Relationships and Monetary Policy." *Economic Inquiry*, 51: 368–393.

Allen, F., G. Barlevy, and D. Gale. 2018. "Theoretical Model of Leaning Against the Wind." *ederal Reserve Bank of Chicago, Working paper*, 2017–16.

Benati, F, and C. Surico. 2009. "VAR Analysis and the Great Moderation." *American Economic Review*, 99: 1636–1652.

Bernanke, B.S., and M. Gertler. 1999. "Monetary Policy and Asset Price Volatility." *Economic Review, Federal Reserve Bank of Kansas City*, 84: 17–51.

Bernanke, B.S., and M. Gertler. 2001. "Should Central Banks Respond to Movements in Asset Prices?" *American Economic Review*, 91: 253–257.

Bianchi, F. 2013. "Regime Switches, Agents' Beliefs, and Post-World War II U.S. Macroeconomic Dynamics." *Review of Economic Studies*, 80: 463–490.

Bianchi, F., and C. Ilut. 2017. "Monetary/fiscal policy mix and agents' beliefs." *Review of economic Dynamics*, 26: 113–139.

Carvalho, V.M, A. Martin, and J. Ventura. 2012. "Understanding Bubbly Episodes." *American Economic Review*, 102: 95–100.

Chib, S. 1995. "Marginal likelihood from the Gibbs output." *Journal of the American Statistical Association*, 90: 1313–1321.

Ciccarone, G., F. Giuli, and E. Marchetti. 2019. "Should central banks lean against the bubble? The monetary policy conundrum under credit frictions and capital accumulation." *Journal of Macroeconomics*, 59: 195–216.

Davig, T., and E.M. Leeper. 2007. "Generalizing the Taylor Principle." *American Economic Review*, 97: 608–617.

Dong, F., J. Miao, and P. Wang. 2020. "Asset bubbles and monetary policy." *Review of Economic Dynamics*, 37: S68–S98.

Estrella, A. 2015. "The Price Puzzle and VAR identification." *Macroeconomic Dynamics*, 19: 1880–1887.

Faia, E. 2019. "Insolvency-Illiquidity, macro externalities and regulation." *CEPR Discussion Paper*, DP14055: 251–301.

Farmer, R., D. Waggoner, and T. Zha. 2009. "Understanding Regime-switching Rational Expectations Models." *Journal of Economic Theory*, 144: 1849–1867.

Fernández-Villaverde, J., P. Guerron-Quintana, and D.F. Rubio-Ramirez. 2010. "Fortune or Virtue: Time-Variant Volatilities Versus Parameter Drifting in U.S. Data." *NBER Working Paper*, 15928.

Foerster, A., J. Rubio-Ramirez, D. Waggoner, and T. Zha. 2016. "Perturbation methods for Markov-switching DSGE models." *Quantitative Economics*, 7: 637–669.

Freixas, X., and J.C. Rochet. 1997. *Microeconomics of Banking.* MIT Press, Cambridge, Massachusetts.

Galì, J. 2014. "Monetary Policy and Rational Asset Price Bubbles." *American Economic Review*, 104: 721–752.

Galì, J. 2021. "Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations." *American Economic Journal: Macroeconomics*, 13: 121–67.

Galì, J., and L. Gambetti. 2015. "The Effects of Monetary Policy on Stock Market Bubbles: Some Evidence." *American Economic Journal: Macroeconomics*, 7: 233–257.

Greenspan, A. 2002. "Opening Remarks. In: Federal Reserve Bank of Kansas City Economic Symposium Rethinking Stabilization Policy." 1–10.

Hubrich, K., and R.J. Tetlow. 2015. "Financial stress and economic dynamics: The transmission of crises." *Journal of Monetary Economics*, 70: 100–115.

Ikeda, D., and T. Phan. 2016. "Toxic Asset Bubbles." *Economic Theory*, 61: 241–271.

Jarrow, R., and S. Lamichhane. 2022. "Risk premia, asset price bubbles, and monetary policy." *Journal of Financial Stability*, 60.

Liu, Z., D.F. Waggoner, and T. Zha. 2011. "Sources of macroeconomic fluctuations: A regime-switching DSGE approach." *Quantitative Economics*, 2: 251–301.

Maih, J. 2015. "Efficient perturbation methods for solving regime-switching DSGE models." *Norges Bank Working Paper*, 1.

Mankiw, G., and R. Reis. 2002. "Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve." *Quarterly Journal of Economics*, 117: 1295–1328.

Martin, A., and J. Ventura. 2015. "Managing Credit Bubbles." *Journal of the European Economic Association*, 14: 753–789.

Martin, A., and J. Ventura. 2016. "The International Transmission of Credit Bubbles: Theory and Policy." *Journal of Monetary Economics*, 76: 537–556.

Meng, X., and W.H. Wong. 1996. "Simulating ratios of normalizing constants via a sample identity: a theoretical exploration." *Statistica Sinica*, 6: 831–860.

Miao, J., Z. Shen, and P. Wang. 2019. "Monetary Policy and Rational Asset Price Bubbles: Comment." *American Economic Review*, 109: 1969–90.

Michaelides, P.G., E.G. Tsionasb, and K.N. Konstantakis. 2016. "Non-linearities in financial bubbles: Theory and Bayesian evidence from SP500." *Journal of Financial Stability*, 24: 61–70.

Phillips, P.C.B., S. Shi, and J. Yu. 2015. "Testing for multiple bubbles: historical episodes of exuberance and collapse in the SP 500." *international Economic Review*, 56: 1043–1078.

Shorfeide, F. 2005. "Learning and Monetary Policy Shifts." *Review of Economic Studies*, 8: 392–419.

Sims, C. 1992. "Interpreting the macroeconomic time series facts: the effects of monetary policy." *European Economic Review*, 36: 2–16.

Sims, C.A., and T. Zha. 2006. "Where there regime switches in U.S. monetary policy?" *American Economic Review*, 96: 54–81.

Sims, C.A., D.F. Waggoner, and T. Zha. 2008. "Methods for Inference in Large Multiple-equation Markov-switching Models." *Journal of Econometrics*, 146: 255–274.

Taylor, J.B. 2008. "The Costs and Benefits of Deviating from the Systematic Component of Monetary Policy." *Keynote address at the Federal Reserve Bank of San Francisco, Conference on Monetary Policy and Asset Markets.*

Wu, J.C., and F.D. Xia. 2016. "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound." *Journal of Money, Credit, and Banking*, 48: 253–291.

Appendices

A   Data

We use quarterly US data spanning the period 1960:Q1-2019:Q4. The six variables included in the MS-SVAR are the same employed by Galì and Gambetti (2015): log-real GDP y_t, log-real dividends d_t, the GDP deflator inflation π_t^y, the inflation rate for non-energy commodities π_t^c, the federal funds rate r_t and the log-real S&P500 index q_t. For the policy rate, we get rid of the zero-lower bound issue by considering the Wu and Xia (2016) shadow interest rate for the 2009-2016 time interval. Table A1 below summarizes the variable's data sources and their transformations used in the estimates. The federal funds rate, real output and its deflator are all taken from the Federal Reserve Economic database (FRED), dividends and stock prices are taken from the updated Shiller' stock market database, and non energy commodity prices are taken from the World Bank's (WB) The Pink Sheet historical database on commodity prices.

Table A1: Marginal Data Densities for model selection

Table A1.1 Data sources and their transformations

Variable	Definition	Source	Transf.
y_t	Real GDP	FRED	$\log(Y_t)$
d_t	Real SP Comp. dividends	Shiller' Stock Market data	$\log(\frac{D_t}{P_t^y})$
π_t^y	GDP-deflator inflation	FRED	$\Delta\log(P_t^y)$
π_t^c	Inflation on non energy comm.	WB - The Pink Sheet	$\Delta\log(P_t^c)$
r_t	Federal funds rate (shadow rate)	FRED (Wu and Xia, 2020)	—
q_t	Real SP Comp. stock price	Shiller' Stock Market data	$\log(\frac{Q_t}{P_t^y})$

B   MS-SBVAR. Model selection

The selected model is obtained from the combination of specifications featuring an increasing number of parameters governed by the Markov chains in the MS-SVAR. To take model complexity under control and numerical computation feasible, we allow no more than three states for the chain on shock variances and two states for the chain on the structural parameters.[21]

---

[21]Limiting the nonlinear structure to a compact size unavoidably comes at the risk of ignoring potentially relevant additional states. However, keeping the state space to a minimal dimension on the shocks' variance domain allows the identification of meaningful changes in the systematic model component, which may otherwise be hidden under the proliferation of stochastic states. The unobservables would become uninformative for the identification of the

Table B2 compares the MDD measure of fit of the constant-coefficient model (Chib (1995)'s method) to several MS specifications. As obtaining an accurate measure of MDD is extremely challenging for Markov-switching multiple-equation models, we evaluate MDDs over three metrics differing in the weighting function used for their numerical approximation: the new modified harmonic mean method (MHM) suggested by Sims and Zha (2006), Meng and Wong (1996)'s bridge sampling method, and the Müller's method (Liu, Waggoner and Zha, 2011). By comparing differently specified models, it results that data strongly favor a model with a three-state chain on shock's variances and an independent two-state chain on the VAR structural parameters affecting jointly the monetary policy and the asset price equations, $3\xi^v 2\xi^c_{r,q}$. A set of additional information can be extracted. First, all models with time-varying properties outperform the linear model. Second, a model with regime-switching variances with up to three states ($3\xi^v$) improves substantially over the linear case, but is itself dominated by models with regime switches in both the systematic and stochastic components. Third, introducing MS dynamics in the systematic model component featuring the real-sector only worsens model fit when compared to the model with variance regimes only, while introducing nonlinearities in the monetary policy or the asset price equations improves model performance ($3\xi^v 2\xi^c_r, 3\xi^v 2\xi^c_q$). Finally, it is the interaction of monetary and financial structural dynamics ($3\xi^v 2\xi^c_{r,q}$) to best describe our data, outperforming the independent alternative ($3\xi^v 2\xi^c_r 2\xi^c_q$).

C   MS-SBVAR. Forecast error variance decomposition

Table C3 reports the forecast error variance decomposition for each state in the model systematic component, indicating the fraction of variance of model variables explained by the monetary policy shocks on impact, at the one-year and five-year time horizons. The displayed results are for the low variance state only. The FEVDs show that the relevance of the policy shock is higher under the high finance state in the long term. The fraction of GDP variance explained by the monetary shock is negligible in the short term and it is around 3% after 5 years in periods when a monetary shock generates amplified effects and stock prices contain an higher share of stock price bubble component. The same holds for the other model variables. Interest rates amplified persistence under the high finance state is evident also in the short run.

---

actual source of nonlinearity in the SVAR (Benati and Surico (2009), Fernández-Villaverde, Guerron-Quintana and Rubio-Ramirez (2010)).

Table B2: Marginal Data Densities for model selection

Method—Model	Linear	$2\xi^v$	$3\xi^v$		
Chib	2906.6				
SWZ MDD		3104.5	3214.3		
Muller MDD		3104.9	3213.0		
Bridge MDD		3104.6	3211.7		
Method—Model	$3\xi^v2\xi^c_r$	$3\xi^v2\xi^c_r2\xi^c_q$	$3\xi^v2\xi^c_{r,q}$	$3\xi^v2\xi^c_q$	$3\xi^v2\xi^c_{real}$
SWZ MDD	3259.5*	3275.8*	3243.3	3241.7	3230.3
Muller MDD	3219.4	3219.4	3222.4	3216.6	3197.6
Bridge MDD	3212.7	3212.7	3221.2	3215.3	3196.4

The table reports the log MDDs associated with different models under the new modified harmonic mean, the bridge and Müller methods. The Markov-switching models under assessment are defined as follows: a two-state Markov chain on all shock variances $(2\xi^v)$; a three-state chain on all shock variances $(3\xi^v)$; two independent Markov chains, one three-state chain on all shock variances and a two-state chain on the interest rate equation $(3\xi^v, 2\xi^r)$; three independent Markov chains, one three-state chain on all shock variances, a two-state chain on the interest rate equation and one chain governing asset price equation $(3\xi^v2\xi^r2\xi^q)$; two independent Markov chains, one three-state chain on all shock variances and a two-state chain jointly on the interest rate and the asset price equations $(3\xi^v, 2\xi^{r,q})$; two independent Markov chains, one three-state chain on all shock variances and a two-state chain on the asset price equation $(3\xi^v, 2\xi^r)$; two independent Markov chains, one three-state chain on all shock variances and a two-state chain on the equations defining the real-sector variables $(3\xi^v, 2\xi^{real})$.

Table C3: Forecast error variance decomposition - Monetary policy shock. Low-variance state.

	Monetary-financial states					
	L-fin	H-fin	L-fin	H-fin	L-fin	H-fin
	Output		Dividends		Inflation-GDP	
1qr	0.101	0.050	0.091	0.052	0.004	0.002
1yrs	2.064	1.462	0.407	0.267	0.057	0.032
5yrs	0.892	2.625	1.164	3.037	0.251	0.600
	Inflation-Commodity		Federal funds rate		Stock price	
1qr	0.022	0.008	43.992	63.622	0.875	1.019
1yr	0.150	0.081	14.689	27.206	0.873	0.754
5yrs	0.368	0.481	4.139	6.428	0.455	0.845

The table presents the fraction of variances (computed from the posterior median) of model variables, explained by the monetary policy shock at various horizons, under each regime. The results are for the low-variance state.

D A model for bubbles

D.1 Savers/lenders

The preferences of the representative saver are specified by the following utility function:[22]

$$\mathcal{U}_t^s = \frac{\left(C_{1,t}^s\right)^{1-\gamma_s} - 1}{1 - \gamma_s} + \beta \frac{\left(\mathbb{E}_t C_{2,t+1}^s\right)^{1-\gamma_s} - 1}{1 - \gamma_s} \tag{D.6}$$

where $\gamma_s \in [0;1)$ is the inverse of the intertemporal elasticity of substitution, $\beta \in (0;1)$ is the subjective discount factor and $\mathbb{E}_t$ represents the (conditional) expectation operator. $C_{1,2}^s$ is an index of the saver's aggregate consumption of the final goods in the two periods:

$$C_{1,t}^s = \left(\int_0^1 C_{1,t}^s (j)^{\frac{\epsilon-1}{\epsilon}} \, dj\right)^{\frac{\epsilon}{\epsilon-1}}; \qquad C_{2,t+1}^s = \left(\int_0^1 C_{2,t+1}^s (j)^{\frac{\epsilon-1}{\epsilon}} \, dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{D.7}$$

where ϵ is the elasticity of substitution between the $j-$types of final goods.

The budget constraints are:

$$C_{1,t}^s = \frac{W_t}{P_t} - L_t^{S_{real}}; \qquad C_{2,t+1}^s = L_t^{S_{real}} \frac{P_t}{P_{t+1}} \left(1 + i_t\right) + \Pi_{t+1}^R \tag{D.8}$$

where $L_t^{S_{real}}$ is the amount of savings, $\frac{W_t}{P_t}$ is the real wage and Π_{t+1}^R are real profits.

The optimization problem of savers is then:

$$\max_{C_{1,t}^s, C_{2,t+1}^s, L_t^{S_{real}}} \mathcal{U}_t^s \quad \text{s.t.:} \ (\text{D.8})$$

from the first order conditions of this problem the supply of funds (expressed in real terms) can be obtained:

$$L_t^{S_{real}} = \frac{\beta^{\frac{1}{\gamma_s}}}{\beta^{\frac{1}{\gamma_s}} + \left(\mathbb{E}_t R_{t+1}\right)^{1 - \frac{1}{\gamma_s}}} \left(\frac{W_t}{P_t} - \frac{\mathbb{E}_t \left(\Pi_{t+1}^R\right)}{\beta^{\frac{1}{\gamma_s}} \left(\mathbb{E}_t R_{t+1}\right)^{\frac{1}{\gamma_s}}}\right) \tag{D.9}$$

where $R_t = \frac{P_t}{P_{t+1}} \left(1 + i_t\right)$ is the real interest rate.

---

[22]This corresponds to an Epstein-Zin utility function when agents are "risk neutral". See Martin and Ventura (2015).

D.2 Borrowers/investors

We assume that, differently from savers, borrowers want to leave some resources $S_{t+1} \geq 0$ to the next generation:[23]

$$\mathcal{U}_t^b = \frac{\left(C_{1,t}^b\right)^{1-\gamma_b} - 1}{1 - \gamma_b} + \beta \frac{\left(\mathbb{E}_t C_{2,t+1}^b\right)^{1-\gamma_b} - 1}{1 - \gamma_b} + \eta \frac{\left(\mathbb{E}_t S_{t+1}\right)^{1-\gamma_b} - 1}{1 - \gamma_b} \tag{D.10}$$

where the parameter $\gamma_b \in (0;1)$ can be different from γ_s and $\eta > 0$. The young borrower can also use part of his/her resources to buy investment goods whose aggregate index is: $I_t = \left(\int_0^1 I_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$:

The borrowers' budget constraints can be written as:

$$C_{1,t}^b = \frac{L_t^D}{P_t} + S_t - Q_t - I_t - (1 - \delta_K) K_t; \tag{D.11}$$

$$C_{2,t+1}^b = r_{t+1}^k K_{t+1} + (1 - \delta_K) K_{t+1} - S_{t+1} + R_{t+1}^B Q_t + Q_{t+1}^N - \frac{1 + i_t^L}{P_{t+1}} L_t^D \tag{D.12}$$

where L_t^D is the agents' demand for funds; $\frac{1+i_t^L}{P_{t+1}} L_t^D$ is the amount to be repaid when old; Q_t is the amount of the bubble purchased when young and $R_{t+1}^B Q_t + Q_{t+1}^N$ represents the accruals from selling the bubble when old, i.e., the bubble purchased when young augmented with its factor of return, plus the value of the newly created (and sold) bubble. The rate r_{t+1}^k is the rental price of physical capital, so that $r_{t+1}^k K_{t+1}$ is the physical capital income obtained by the old agent. The amount $(1 - \delta_K) K_{t+1}$ represents the value of the remaining capital stock (net of depreciation, at the constant rate $\delta_K \in (0;1)$) that old agents sell to young agents.

Finally, the following capital accumulation constraint holds:

$$K_{t+1} = I_t + (1 - \delta_K) K_t \tag{D.13}$$

Credit market imperfections. Credit market imperfections affect the behavior of banks, which may not always obtain the full repayment of the loans (capital plus interest) provided to the borrowers, $L_t^D \left(1 + i_t^L\right)$, due, e.g., to a risk of bankruptcy leading to default, or forms of misbehavior by the borrowers. As a consequence, borrowers cannot obtain loans without providing credit intermediaries with collaterals given by the sum of a fraction $\phi \in (0;1)$ of their future resources and of the re-sell value of their bubbly asset B_t.

---

[23]The inclusion of the bequest term is needed in order to calibrate the steady state of the model at yearly frequency.

The banks' problem can then be written as:

$$\max_{L_t^D} \Pi_t^{bank} = \left(1 + i_t^L\right) L_t^D - (1 + i_t) D_t; \quad \text{s.t. } D_t = L_t^D$$

and the optimality condition implies: $i_t^L = i_t$.

Being $D_t = L_t^D = L_t^S$, it follows that the borrowing constraint - which we here assume to hold with equality - can be written as:

$$\frac{(1 + i_t)}{P_{t+1}} L_t^D = \phi \left(r_{t+1}^k + 1 - \delta_K\right) K_{t+1} + R_{t+1}^B Q_t + Q_{t+1}^N \tag{D.14}$$

The optimization problem of the borrowers is:

$$\max_{I_t, L_t^{D\,real}, Q_t, Q_{t+1}^N, S_{t+1}} \mathcal{U}^b \quad \text{s.t. } \text{(D.11), (D.12), (D.14)} \tag{D.15}$$

First of all, if the collateral constraint always holds, the demand for credit funds will be represented by equation (D.14), rewritten with the appropriate expectation operators:

$$L_t^{D\,real} = \frac{1}{\mathbb{E}_t R_{t+1}} \left[\phi \mathbb{E}_t \left(r_{t+1}^k K_{t+1}\right) + \phi \left(1 - \delta_K\right) K_{t+1} + \mathbb{E}_t Q_{t+1}\right] \tag{D.16}$$

From the first order conditions for a maximum, we derive the equilibrium condition in asset (and credit) markets:

$$\mathbb{E}_t R_{t+1}^B = \mathbb{E}_t R_{t+1} \tag{D.17}$$

From the first order conditions of (D.15) with respect to I_t, we get:

$$K_{t+1} = \frac{\beta^{\frac{1}{\gamma_b}} \left(L_t^{D\,real} - Q_t + S_t\right) + \left[(1 - \phi) \left(\mathbb{E}_t R_t^K\right)\right]^{-\frac{1}{\gamma_b}} S_{t+1}}{\beta^{\frac{1}{\gamma_b}} + \left[(1 - \phi) \left(\mathbb{E}_t R_t^K\right)\right]^{1 - \frac{1}{\gamma_b}}} \tag{D.18}$$

where the return factor is: $R_t^K = r_t^k + 1 - \delta_K$. The first order conditions of (D.15) with respect to capital determine also the following relation:

$$\mathbb{E}_t R_{t+1}^K > \mathbb{E}_t R_{t+1} = \mathbb{E}_t R_{t+1}^B$$

which must be satisfied in equilibrium.[24]

Finally, the first order condition of (D.15) with respect to S_{t+1} (recall that S_t is predetermined for the young agent) leads to:

$$S_t = (1 - \phi) \left[1 + \left(\frac{\beta}{\eta} \right)^{\frac{1}{\gamma_b}} \right]^{-1} R_t^K K_t$$

that is, the amount of the bequest is proportional to the net resources $R_t^K K_t$ deriving from capital ownership.

D.3 Intermediate and final firms

Intermediate firm. The firm's production technology is of the Cobb-Douglas type:

$$X_t = F(K_t; N_t) = A K_t^\alpha \left(g^t N_t \right)^{1-\alpha}; \qquad \alpha \in (0; 1) \tag{D.19}$$

where $A > 0$ is a scale factor. We assume that the economy experiences a growth process driven by exogenous (Harrod-neutral) technical progress embodied in the growth rate $g > 1$. The intermediate firm profit, expressed in real terms, is:

$$\Pi_t^X = \frac{P_t^X}{P_t} A K_t^\alpha \left(g^t N_t \right)^{1-\alpha} - \frac{W_t}{P_t} N_t - r_t^k K_t$$

where P_t^X / P_t is the real price of the intermediate good. The demand functions for inputs, stemming from profit maximization (together with $N_t = 1$), are equal to:

$$\frac{W_t}{P_t} = g^t (1 - \alpha) A \left(\frac{K_t}{g^t} \right)^\alpha \frac{P_t^X}{P_t}$$

$$r_t^k = \alpha A \left(\frac{K_t}{g^t} \right)^{\alpha-1} \frac{P_t^X}{P_t}$$

Final goods producers. The production function of the $j - th$ producer is linear in the unique input $X_t(j)$:

$$Y_t(j) = X_t(j) \tag{D.20}$$

---

[24]This condition guarantees that borrowers are willing to borrow funds in the credit market, as the return on the real capital they can buy with them is greater than the factor of return on credit that will be paid to savers.

and the monopolist faces a demand for the $j - th$ good equal to:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}(C_t + I_t) \qquad \text{where } C_t \equiv C_{1,t}^b + C_{2,t}^b + C_{1,t}^s + C_{2,t}^s \tag{D.21}$$

The final producer's real profit is: $\Pi^R(j)_t = \frac{P_t(j)}{P_t}Y_t(j) - \frac{P_t^X}{P_t}X_t(j)$, where $\frac{P_t^X}{P_t}X_t(j)$ is the real cost of production. Hence the firm's marginal cost mc_t is $mc_t = \frac{P_t^X}{P_t}$.

The monopolist sets the price $P_t^o(j)$ so as to solve the problem:

$$\max_{P_t^o(j)} \Pi^R(j)_t \quad \text{s.t. (D.20),(D.21)}$$

and the real price of the individual good j writes:

$$\frac{P_t^o(j)}{P_t} = \left(\frac{\epsilon}{\epsilon - 1}\right)mc_t. \tag{D.22}$$

As we assume nominal rigidities, in every t some of the prices $P_t^o(j)$ can be equal to a value set some period in the past. We may hence assume that, in general, the average price index P_t is a function not only of the current mc_t, but also of the level of prices and marginal cost expected in the past:[25]

$$P_t = P(mc_t; \mathbb{E}_{t-1-s}P_t; \mathbb{E}_{t-1-s}mc_t)_{s=0,1,2...,\infty} \tag{D.23}$$

Under a flexible price regime in which $P_t^o(j) = P_t$, equation (D.22) univocally sets the value of the marginal cost $mc_t = \mu = 1 - \frac{1}{\epsilon} \in (0;1)$, $\forall t$, where $\frac{1}{\mu}$ is the mark-up over production costs.

D.4 Monetary policy

In order to calculate the predicted dynamic responses of the model economy to an exogenous shock to the nominal interest rate (i_t), we assume that the monetary authority sets the policy rate according to equation 4, discussed in the main text.

D.5 Stationary state

The economy converges to one unique steady state, where the endogenous model variables are constant through time, prices are fully flexible, so that $P(j) = P$, $mc = \mu$ and $y(j) = y = \int_0^1 y(j)\,dj = \bar{y} = x$, zero trend inflation is assumed: $\pi = 0$, and the bubble-to-output share is at its

---

[25]We use the Mankiw and Reis (2002) "sticky information" model.

ergodic level, $\omega = \bar{\omega}$.[26] The latter is equal to the weighted average of the two state-specific shares ω^{mf}, where the weights are the ergodic states' probabilities.

The equilibrium system can be reduced to the following set of equations:

$$
\begin{aligned}
gk &= \frac{\left(l^D - q\right)\left(\eta^{\frac{1}{\gamma_b}} + \beta^{\frac{1}{\gamma_b}}\right) + \eta^{\frac{1}{\gamma_b}}\left(1 - \phi\right)\left(\mu\alpha Ak^\alpha + 1 - \delta_K\right)k}{\left(\eta^{\frac{1}{\gamma_b}} + \beta^{\frac{1}{\gamma_b}}\right) + \left[\left(1 - \phi\right)\left(\mu\alpha Ak^\alpha + 1 - \delta_K\right)\right]^{1 - \frac{1}{\gamma_b}}} & \text{(D.24)} \\
l^S &= \frac{\beta^{1/\gamma_s}}{\beta^{1/\gamma_s} + R^{1 - \frac{1}{\gamma_s}}}\left[\mu\left(1 - \alpha\right) - \frac{g\left(1 - \mu\right)}{\epsilon\left(\beta R\right)^{1/\gamma_s}}\right]Ak^\alpha; \\
l^D &= \frac{g}{R}\left(\phi\mu\alpha Ak^\alpha + \phi\left(1 - \delta_K\right)k + q\right); \\
l^D &= l^S; \quad q = \frac{R}{g}q + q^N;
\end{aligned}
$$

By assuming $\bar{\omega} > 0$, which requires $\omega^{mf} \neq 0$ at least in one state, the economy is in a *bubbly stationary equilibrium*, i.e. a vector $\left(l^D, l^S, k, R, q\right)$ solves equations (D.24) under $q^N > 0$, where the bubble market $q = \left(\frac{g}{g-R}\right)q^N$ poses a constraint: given $q^N \geq 0$, the interest rate on the credit (and the bubble) market must be small enough: $g > R = R^B$. This condition is necessary for the young agents to be able to buy the bubble (using their resources and the lent funds).

From the equilibrium $l^D = l^S$ we obtain:

$$
\phi\left(1 - \delta_K\right) = \left\{\frac{\beta^{1/\gamma_s}}{\beta^{1/\gamma_s}/R + R^{-\frac{1}{\gamma_s}}}\left[\frac{\mu\left(1 - \alpha\right)}{g} - \frac{1}{\epsilon\left(\beta R\right)^{1/\gamma_s}}\right] - \phi\mu\alpha - \frac{\bar{\omega}g}{g - R}\right\}Ak^{\alpha - 1} \quad \text{(D.25)}
$$

Furthermore, as we also assume that $\gamma_b = 1$, the borrowers' accumulation equation is:

$$
\begin{aligned}
r^k &= \alpha\mu\left(Ak^{\alpha - 1}\right) = \mu\alpha \cdot f_B\left(R\right) & \text{(D.26)} \\
f_B\left(R\right) &= \frac{g\left(1 + \frac{1}{\beta + \eta}\right)}{\frac{g}{R}\left(\phi\mu\alpha + \bar{\omega}\right) + \left(\frac{(1-\phi)\eta}{\eta+\beta}\right)\mu\alpha} - \frac{\left(1 - \delta_K\right)\left[\left(\frac{(1-\phi)\eta}{\eta+\beta}\right) + \phi\frac{g}{R}\right]}{\frac{g}{R}\left(\phi\mu\alpha + \bar{\omega}\right) + \left(\frac{(1-\phi)\eta}{\eta+\beta}\right)\mu\alpha}
\end{aligned}
$$

E Model Solution Method

The Markov-switching OLG model is solved using the perturbation method of Foerster et al. (2016). They develop an iterative procedure that approximate the model's solution by guessing a set of approximations under each regime; given a guess, each regime's approximation follows from standard

---

[26]The steady state must be independent of the realization of any regime in the discrete Markov process governing parameter changes (Foerster et al., 2016). We follow the literature and define the steady state with the ergodic mean values of the Markov-switching bubble-to-output share.

perturbation techniques, and the iterative algorithm stops when obtained approximations equal the guesses.

In order to technically describe the solution method, it's convenient to stack our variables into a group of exogenous and endogenous predetermined variables, $\mathbf{x}_t \in \mathbb{R}^{n_x}$, and a group of non-predetermined (control) variables, $\mathbf{y}_t \in \mathbb{R}^{n_y}$. Then, we define the vector of structural shocks as $\epsilon_t \in \mathbb{R}^{n_\epsilon}$ and the switching parameters' vector as $\theta(\xi_t^{mf}) \in \mathbb{R}^{n_\theta}$. Given the vector of state variables $(\mathbf{x}_{t-1}, \epsilon_t, \xi_t^{mf})$, the equilibrium conditions for our model have the following general form:

$$\mathbb{E}_t \mathbf{f}\Big(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \mathbf{x}_{t-1}, \chi\epsilon_{t+1}, \epsilon_t, \theta(\xi_{t+1}^{mf}, \chi), \theta(\xi_t^{mf}, \chi)\Big) = 0_{n_y+n_y}$$

where $\mathbf{f}$ is a nonlinear function. Then, the algorithm works as an extension of conventional perturbation methods (Judd, SGU), where not only ϵ_{t+1} is perturbed, but also the switching parameters, $\theta(\xi_{t+1}^{mf})$, $\theta(\xi_t^{mf})$. Since in our model the steady state is affected by the policy regime in place, the perturbation function for $\theta(\xi_t^{mf})$ is: $\theta(k, \chi) = \chi\theta(k) + (1 - \chi)\bar{\theta}$, where $\chi \in \mathbb{R}$ is the perturbation parameter, k indicates a generic regime and $\bar{\theta} = [\theta(1)...\theta(n_s)]\bar{p}$ is the ergodic mean of $\theta(\xi_t^{mf})$.

Stacking the regime-dependent solutions for $\mathbf{y}_t$ and $\mathbf{x}_t$, the algorithm assumes the following functional forms for $\mathbf{Y}_t = y_t(\mathbf{e}_{s_t}^T \otimes \mathbf{I}_{n_y})^{-1}$ and $\mathbf{X}_t = x_t(\mathbf{e}_{s_t}^T \otimes \mathbf{I}_{n_x})^{-1}$:

$$\mathbf{Y}_t = \mathbf{G}(\mathbf{x}_{t-1}, \epsilon_t, \chi) = \begin{bmatrix} g_{\xi_t^{mf}=1}(\mathbf{x}_{t-1}, \epsilon_t, \chi) \\ \vdots \\ g_{\xi_t^{mf}=n_s}(\mathbf{x}_{t-1}, \epsilon_t, \chi) \end{bmatrix}$$

$$\mathbf{X}_t = \mathbf{H}(\mathbf{x}_{t-1}, \epsilon_t, \chi) = \begin{bmatrix} h_{\xi_t^{mf}=1}(\mathbf{x}_{t-1}, \epsilon_t, \chi) \\ \vdots \\ h_{\xi_t^{mf}=n_s}(\mathbf{x}_{t-1}, \epsilon_t, \chi) \end{bmatrix}$$

where $\mathbf{g}_{\xi_t^{mf}} : \mathbb{R}^{n_x+n_\epsilon+1} \to \mathbb{R}^{n_y}$ and $\mathbf{h}_{\xi_t^{mf}} : \mathbb{R}^{n_x+n_\epsilon+1} \to \mathbb{R}^{n_x}$ are continuously differentiable regime-dependent solutions. Second-order perturbation around the point $(\mathbf{x}_{ss}, \mathbf{0}_{ss}, 0)$ is represented by:

$$\mathbf{G}(z_t) \approx \mathbf{Y}_{ss} + D\mathbf{G}(\mathbf{z}_{ss})(\mathbf{z}_t - \mathbf{z}_{ss}) + \frac{1}{2}\sum_{l_1}^{n_z}\sum_{l_2}^{n_z} D_{l_2}D_{l_1}\mathbf{G}(\mathbf{z}_{ss})(z_{t,l_1} - z_{ss,l_1})(z_{t,l_2} - z_{ss,l_2})$$

$$\mathbf{H}(z_t) \approx \mathbf{X}_{ss} + D\mathbf{H}(\mathbf{z}_{ss})(\mathbf{z}_t - \mathbf{z}_{ss}) + \frac{1}{2}\sum_{l_1}^{n_z}\sum_{l_2}^{n_z} D_{l_2}D_{l_1}\mathbf{H}(\mathbf{z}_{ss})(z_{t,l_1} - z_{ss,l_1})(z_{t,l_2} - z_{ss,l_2})$$

where $\mathbf{z}_t = [\mathbf{x}_{t-1}, \epsilon_t, \chi]$, $\mathbf{z}_{ss} = [\mathbf{x}_{ss}, 0_{n_\epsilon}, 0]$, and $z_{t,l}$ and $z_{ss,l}$ are the lth components of $\mathbf{z}_t$ and $\mathbf{z}_{ss}$.