Identification of the Poincaré-Steklov Operator in Hybrid FE-BIE Formulations for the Analysis of Internal Resonances

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Abstract

The interest in combining the Finite Element (FE) method with the Boundary Integral Equation (BIE) method led to various hybrid FE-BIE formulations in literature. However, some formulations suffer from breakdown frequencies at which the solution is not uniquely defined, introducing errors due to internal resonances. The occurrence of these errors is investigated by identifying the Poincaré-Steklov operator, which relates the tangential electric field to the equivalent electric current on the boundary of a domain, in both the FE and BIE method. This identification provides new insight in internal resonances for both conformal and non-conformal formulations.

Key words: hybrid FE-BIE, internal resonances, Poincaré-Steklov operator

1 Introduction

The exact formulation for combining the Finite Element (FE) method with the Boundary Integral Equation (BIE) method appears to be very important in order to avoid so-called spurious solutions. Previous contributions demonstrated that formulations applying the Electric Field Integral Equation (EFIE) or the Magnetic Field Integral Equation (MFIE) as BIE method in combination with an FE method contain certain *forbidden frequencies* if the background medium is lossless [1]. At these frequencies, the EFIE and MFIE are not uniquely defined and the sourceless hybrid system contains non-trivial solutions that introduce errors to the result.

Here, these spurious solutions are investigated on an operator level thanks to the concept of the Poincaré-Steklov (PS) operator, which describes the relationship between the tangential electric field and the equivalent electric current on the boundary of a domain. In this way, different properties regarding internal resonances are easily derived for both conformal and non-conformal hybrid FE-BIE formulations [2].

2 Identifying the Poincaré-Steklov operator

In the FE domains, all interior unknowns are eliminated by reducing the system matrix to its Schur complement. This already provides a relationship between the tangential electric field and the equivalent electric current on the boundary of the FE domain: $\mathcal{S}\mathbf{E}^t = jk_0\eta_0\mathbf{J}$. Here, $\mathbf{E}^t$ and $\mathbf{J}$ are the tangential electric field and the equivalent electric current, respectively, $\mathcal{S}$ is the Schur complement, j is the imaginary unit, k_0 is the free space wave number and η_0 is the free space impedance. Hence, the PS operator is easily obtained by scaling this Schur complement with $jk_0\eta_0$.

In the BIE method, the PS operator is obtained from the Calderón projector. This reveals several interesting properties of the PS operator, such as its relation with the BIE integral operators $\mathcal{T}$ and $\mathcal{K}$ [2].

3 Internal resonances

The internal resonance problem originates from the non-uniqueness of the EFIE and the MFIE at certain frequencies. However, the spurious solutions of the sourceless EFIE and MFIE are different from each other. Hence no internal resonances occur when both the EFIE and the MFIE are satisfied. Therefore, the occurrence of internal resonances is investigated by verifying if a hybrid formulation satisfies both the FE wave equation, the EFIE and the MFIE at all frequencies. This verification can be done for both conformal hybrid formulations, where adjacent domains share the same discretisation, and non-conformal hybrid formulations, where all domains are allowed to have independent discretisations.

Thanks to the obtained properties of the PS operator, the breakdown frequencies of several hybrid formulations can be predicted for simple configurations. It can also be demonstrated that only induced resonance currents (or fields) radiate.

4 Conclusion

By identifying the PS operator in both the FE wave equation, the EFIE and the MFIE, it becomes possible to investigate the stability of several hybrid FE-BIE formulations on an operator level. Moreover, the internal resonances can be predicted for simple configurations.

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