Processing of logical-physical rules in the control of the autonomous vehicle

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Abstract

Recent advances in intelligent vehicles imply more sophisticated control laws. The standard concept of objective function and models of vehicle and driver represented by differential equations, are not anymore sufficient tools in a future scenario. The capability of reasoning of the machines imposes the use of logic as a fundamental tool to describe requirements of the behavior of the vehicle, and to characterize their response. However, logical statements exhibit a difficulty of integration with the differential physic laws to which the vehicle obeys. There is a clear heterogeneity between mathematics and logic, especially when they must fuse into a single model. The paper proposes an integrated model in which the physics and the logic fuse into a common model, able to generate a meaningful objective function to optimize the behavior through a physical-logic model of the vehicle in the context of control of hybrid dynamical systems. Not negligibly, a logic-statement design helps the autonomous driving to be more acceptable and comprehensible in an insurance and court law context.

1 Introduction

The present paper deals with the problem of autonomous vehicle control in the presence of logical constraints that drive the decision making of the on-board intelligence. Engineering control problems take form, in optimal control theory, in terms of an objective function that provides a measure of the performances and control cost [1]. Moreover, they use a mathematical model of the process to be controlled described by differential equations taking account of the physics laws to which the process obeys. In the case of an autonomous vehicle, the laws of mechanics (or mechatronics) represent the constraint between the vehicle state and its control [2-6]. However, in several circumstances, and not only for technical reasons, such for example because of algorithmic transparency [7] that becomes a crucial element for social and by law acceptability of autonomous vehicles, or of other forms of autonomous robotics [8], the introduction of logical rules to govern the robotic behaviour seems to be natural and preferable. The present paper uses propositional logic as a tool for translating natural language requirements into algebraic constraints [9-11]. The logical statements to prescribe the behaviour of the car are particularly significant and understandable, since they put behavioural rules expressed in terms of natural language. In the present paper, the use of logical statements is included into the decision process through a suitable combination of the physical and logical constraints leading to a hybrid dynamical system [12-18]. A simple example shows how one can operate in practice using logical statements in automated driving.

2 Decision making approach: logical statements and modelling of the physical world

In this paper, automatic decisions of an intelligent route planner develop under the light of formal logic.

The use of sentences coming from natural languages to condition the behaviour of robots has many advantages but at the same time exhibits strong difficulties.

On one side, it requires a tremendous effort to translate the natural language into an artificial one with its own rules. Many problems arise in this context, since human language is a very evolved and complex tool of communications the interpretation and reduction of which into a set of rigid rules is practically impossible. The human way to intend part of the discourse terms and of the possible meaning of its recurring elements is a subject that largely goes beyond the limits of many branches of engineering and mathematics. Although formal logic dates back to the Aristoteles conception, and one of the most famous examples is the syllogistic reasoning scheme, the entire historical evolution of this branch of mathematics (that involves philosophy and linguistics) shows enormous difficulties, and an attempt of giving a mathematical structure to the human language is a hard task and a thorny adventure. Nevertheless, formal logic is the best historical experiment in this direction showing good abilities and success in performing such operation. The investigation of the relationship between human language and its logical translation is not a subject of competence of the authors and is outside the range and scopes of the present paper.

However, the chance of using natural language prescriptions to constraint the operational behaviour of robots has a fundamental advantage with respect to the social acceptability of autonomous machines that will populate the future world. In fact, we can prescribe limits and rules for their behaviour closer to human understanding, even and especially when non engineering specialized people have to take decisions about their use, or a court has to judge their behaviour, or evaluate responsibilities in case of damages or injuries produced by the robot. The civil and criminal codes and the entire structure of the legal system are the most advanced and refined way a human society has given to rule its own life, and they express in terms of natural language. If human beings obey these rules, it is natural to ask machines to obey the same rules. Much less natural is to program machines following only mathematical schemes the translation of which in terms of natural language statements is frequently impossible. Robotic engineering must indeed take care of this problem.

This subject is of inspiration for the present paper that makes an attempt of introducing, into the context of autonomous driving decision, a set of natural language statements, leading to a hybrid dynamical system. The technical problems emerging from this experiment are interesting. We use, as a first step, simple propositional logic. However, since the description of the car behaviour implies the need to introduce a mathematical model of the reality in which the car operates, i.e. of mathematical models of the physical world, the problem is the mixing of models of different nature: logical variables, and real variables, governed by logical rules and differential equations, respectively. They are part of a large family of hybrid control problems.

Our investigation proposes practical examples to introduce natural language prescriptions into the behaviour of an autonomous car, discussing strategies and problems of this operation.

3 Application examples of autonomous driving

Let us formulate in natural language an elemental logical statement that could be the base of an autonomous driving behaviour to control a micro-fleet of vehicles.

The elemental initial example applies to the vehicles V1 and V2, driving in the same direction on two parallel lanes

Assume we desire to respect the rule expressed in natural language:

If V1 and V2 are too close and they are on the same lane, then one of them must change lane (1)

Moreover, we have the vehicles dynamics physical rule, described as:

$$s'_1 = s_1 + v_1, \quad s'_2 = s_2 + v_2 \tag{2}$$

These statements are particularly simple, but they are useful to clarify the way we operate.

Equation (1) prescribes a logical rule of driving, equation (2) a mathematical law to describe the kinematics of V1 and V2. Symbols $s'_i, s_{i,} v_i$ stand for position along the *i-th* lane at the next time, position along the *i-th* lane at the actual time, speed of the *i-th* vehicle, respectively. The time increment is for simplicity taken as unit, since $s'_1 = s_1 + v_1\Delta t$, and v_1, v_2 can be in general variable with time. Therefore, in the following, the prime stands for *"at the next time"*.

Let us introduce the following variables.

Control logical state variables L_1, L_2

$$V(L_1) = 0 \quad means\ V1\ is\ in\ the\ right\ lane$$

$$V(L_1) = 1 \quad means\ V1\ is\ in\ the\ left\ lane$$

$$V(L_2) = 0 \quad means\ V2\ is\ in\ the\ right\ lane$$

$$V(L_2) = 1 \quad means\ V2\ is\ in\ the\ left\ lane$$

Note $l = V(L)$ is the value l of the logical variable L, where l can be 1 or 0; s_1, s_2 are instead usual real valued state variables. Therefore, the state of the system remains with four state variables of mixed nature, logical and real.

Let us interpret a logical statement as a diadic function that associates to the pair of logical variables p and q (that can assume the values 0-false or 1-true) a logical value, 0 or 1.

Propositional logic introduces some elemental operations among logical variables, such as the operators *disjunction* (symbolic notation $\vee, OR$), *exclusive disjunction* (symbolic notation $\dot{\vee}$, XOR) *conjunction* (symbolic notation $\wedge$, *AND*), *implication* (symbolic notation $\rightarrow, IF-THEN$), *double implication* (symbolic notation $\leftrightarrow$, or also $\equiv, IF\ AND\ ONLY\ IF$), *negation* (symbolic notation $\sim$, *NOT*). Such logical operations establish the correspondence between the two logical variables p and q and the final logical result, following the rules:

$$p \vee q = 1\ if\ at\ least\ one\ of\ the\ two\ logical\ variables\ is\ 1, otherwise\ is\ 0$$

$$p\ \dot{\vee}\ q = 1\ if\ one\ the\ two\ logical\ variables\ is\ 1, but\ not\ both, \qquad otherwise\ is\ 0$$

$$p \wedge q = 1\ if\ both\ the\ two\ logical\ variables\ are\ 1, otherwise\ is\ 0$$

$$p \rightarrow q = 1\ if\ p = 0\ whatever\ q, or\ if\ p = 1\ and\ q = 1, otherwise\ is\ 0$$

$$p \leftrightarrow q\ is\ equivalent\ to\ (p \rightarrow q) \wedge (q \rightarrow p)$$

The true tables condense these results explicitly:

Table 1: **OR**-operator truth table

p	q	$p \lor q$
1	1	1
1	0	1
0	1	1
0	0	0

Table 2: **XOR**-operator truth table

p	q	$p \dot{\lor} q$
1	1	0
1	0	1
0	1	1
0	0	0

Table 3: **AND-**operator truth table

p	q	$p \land q$
1	1	1
1	0	0
0	1	0
0	0	0

Table 4: **IF THEN-**operator truth table

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

From these tables it is possible to formalize a complex logical statement involving logical variables. An equivalent form of these tables can reproduce, by using the Boolean expressions, the logic operators result in an algebraic fashion. Direct verification shows the following correspondence:

Table 5: Boolean Expressions of Previous Truth Tables

LOGICAL OPERATION	BOOLEAN OPERATION
$\boldsymbol{p} \vee \boldsymbol{q}, \quad \boldsymbol{p} \dot{\vee} \boldsymbol{q}$	*p+q-pq,* *p+q-2pq*
$\boldsymbol{p} \wedge \boldsymbol{q}$	*pq*
$\boldsymbol{p} \rightarrow \boldsymbol{q}$	*1-p+pq*
$\boldsymbol{p} \leftrightarrow \boldsymbol{q};\ \boldsymbol{p} \equiv \boldsymbol{q}$	*2pq-p-q+1*
$\sim\boldsymbol{p}$	*1-p*

Using the previously introduced operators and variables, we can translate the natural language sentence (1) into a formal logical expression:

$$If\ B(d) \wedge (L_1 \equiv L_2)$$
$$then\ \{[(L'_1 \equiv\sim L_1) \wedge (L'_2 \equiv L_2)] \vee [(L'_1 \equiv L_1) \wedge (L'_2 \equiv\sim L_2)]\}$$

or in a more compact form:

$$[B(d) \wedge (L_1 \equiv L_2)] \rightarrow \{[(L'_1 \equiv\sim L_1) \wedge (L'_2 \equiv L_2)] \vee [(L'_1 \equiv L_1) \wedge (L'_2 \equiv\sim L_2)]\} \qquad (3)$$

Where $B(d)$ is the boolean variable associated to the cars distance $d = |s_1 - s_2|$. Namely:

$$B(d) = 1, if\ d < d_0$$
$$B(d) = 0, if\ d \geq d_0$$

i.e.:

$$B(d) = \frac{1}{2}\left(1 - \frac{d-d_0}{|d-d_0|}\right) \qquad (4)$$

Then, following Boolean transformations, the elemental relationships hold:

Table 6: Lanes Boolean Expressions

Logical Expression	Algebraic Translation
$\boldsymbol{L_1 \equiv L_2}$	$2l_1l_2 - l_1 - l_2 + 1$
$\boldsymbol{L'_1 \equiv \sim L_1}$	$l'_1 + l_1 - 2l'_1l_1$
$\boldsymbol{L'_2 \equiv L_2}$	$2l_2l'_2 - l_2 - l'_2 + 1$
$\boldsymbol{L'_1 \equiv L_1}$	$2l_1l'_1 - l_1 - l'_1 + 1$
$\boldsymbol{L'_2 \equiv \sim L_2}$	$l'_2 + l_2 - 2l'_2l_2$

To complete the algebraic transformation of the sentence (3), we have (looking at the Table 5):

Table 7: Sentence Boolean Expressions

Logical Expression	**Algebraic Translation**
$[\boldsymbol{B} \wedge (\boldsymbol{L_1} \equiv \boldsymbol{L_2})] \equiv \boldsymbol{Hp}$	$B(2l_1l_2 - l_1 - l_2 + 1) = hp$
$(\boldsymbol{L'_1} \equiv\sim \boldsymbol{L_1}) \wedge (\boldsymbol{L'_2} \equiv \boldsymbol{L_2})$	$(l'_1 + l_1 - 2l'_1l_1)(2l_2l'_2 - l_2 - l'_2 + 1) = e_1e_2$
$(\boldsymbol{L'_1} \equiv \boldsymbol{L_1}) \wedge (\boldsymbol{L'_2} \equiv\sim \boldsymbol{L_2})$	$(2l_1l'_1 - l_1 - l'_1 + 1)(l'_2 + l_2 - 2l'_2l_2) = r_1r_2$
$\{[(\boldsymbol{L'_1} \equiv\sim \boldsymbol{L_1}) \wedge (\boldsymbol{L'_2} \equiv \boldsymbol{L_2})] \vee [(\boldsymbol{L'_1} \equiv \boldsymbol{L_1}) \wedge (\boldsymbol{L'_2} \equiv\sim \boldsymbol{L_2})]\} \equiv \boldsymbol{Th}$	$e_1e_2 + r_1r_2 - e_1e_2r_1r_2 = th$

The final logical expression we desire to implement is:

$$V(Hp \rightarrow Th) = 1$$

This means, whatever the variables values, the sentence is true, meaning the rule of driving (3) holds. Algebraically it transforms as:

$$1 - hp + hp \cdot th = 1$$

i.e.:

$$hp \cdot (th - 1) = 0 \qquad (5)$$

and

$$B(d)(2l_1l_2 - l_1 - l_2 + 1)(e_1e_2 + r_1r_2 - e_1e_2r_1r_2 - 1) = 0$$

i.e.:

$$B(d)(2l_1l_2 - l_1 - l_2 + 1)(e_1e_2 - 1)(1 - r_1r_2) = 0 \qquad (6)$$

This is the product of four factors. It is the algebraic representation of the considered logical constraint by using a Boolean reduction.

Since B depends on $d = |s_1 - s_2|$, and e_1, e_2, r_1, r_2 are each functions of l_1, l_2, l'_1, l'_2, we can write the previous logical constraint as:

$$E(l_1, l_2, l'_1, l'_2, s_1, s_2) = 0 \qquad (7)$$

Note that:

-It appears that the logical decision variables are not uniquely determined in order to satisfy the logical constraint. In other words, different possibilities exist to satisfy it. In fact, we have four state variables, s_1, s_2, l_1, l_2 constrained by only three equations:

$$\begin{cases} B(d)(2l_1l_2 - l_1 - l_2 + 1)(e_1e_2 - 1)(1 - r_1r_2) = 0 \\ s'_1 - s_1 - v_1 = 0 \\ s'_2 - s_2 - v_2 = 0 \end{cases}$$

that makes it impossible to find a unique solution.

This is a direct consequence of the fact of introducing logical rules. In fact, we meet frequently problems in which they do not define scenarios with a unique solution, but they leave open different possible alternatives. If these alternatives can be equivalent under the point of view of satisfaction of the equation $E=0$, they can be instead different under some other respects, suggesting some decision can be better than others.

-The previous remark suggests an additional criterion, aimed at solving the problem of the alternatives, should appear indicating the most convenient choice among those possible to respect the same logical constraint.

-The additional criterion can have different form and inspiration. We analyse in the following two types of integration requirements. The first consists in introducing an additional logical constraint. The second, more closely resembling the optimal control theory, instead minimizes a penalty function.

3.1 Alternatives disambiguation: Additional logical constraints or logical rules modifications

Let us examine the first chance to eliminate the disambiguation problem.

For example, we can introduce the criterion to make the lane change operating on the slowest vehicle, a reasonable criterion inspired to safety, since a slower vehicle can operate a safer steering. This is a logical criterion, which can take the form:

If a change of lane is needed,, then is the slowest car to change lane

Let us put this sentence into an additional logical statement form. Introducing the Boolean variable

$$B(\Delta v) = \frac{1}{2}\left(1 - \frac{v_1 - v_2}{|v_1 - v_2|}\right) \tag{8}$$

that implies $B(\Delta v) = 1 \; if \; v_1 < v_2, B(\Delta v) = 0 \; if \; v_2 < v_1$, we have:

$$IF$$

$$Hp = B(d)$$

$$THEN$$

$$Th = \left[(IF\left(B(\Delta v)\right) THEN\; ((L'_1 \equiv\sim L_1)\right] \dot{\vee} \left[(IF\left(\sim B(\Delta v)\right) THEN\; ((L'_2 \equiv\sim L_2)\right]$$

That, following the same considerations previously illustrated, becomes:

$$V(Hp \rightarrow Th) = 1$$

$$V(Hp)V(Th) - V(Hp) = 0$$

i.e.:

$$V(Hp)V(Th) - V(Hp) = 0$$

Then:

$$B(d)V(Th) = B(d) \tag{9}$$

It is:

$$\begin{aligned} V(Th) &= [1 - B(\Delta v) + B(\Delta v)(l'_1 + l_1 - 2l'_1 l_1)] \\ &+ [1 - (1 - B(\Delta v)) + (1 - B(\Delta v))(l'_2 + l_2 - 2l'_2 l_2)] \\ &- 2[1 - B(\Delta v) + B(\Delta v)(l'_1 + l_1 - 2l'_1 l_1)][1 - \big(1 - B(\Delta v)\big) \\ &+ \big(1 - B(\Delta v)\big)(l'_2 + l_2 - 2l'_2 l_2)] \\ &= [1 + B(\Delta v)(e_1 - 1)] + [r_2 + B(\Delta v)(1 - r_2)] \\ &- 2[1 + B(\Delta v)(e_1 - 1)][r_2 + B(\Delta v)(1 - r_2)] \\ &= [1 + B(\Delta v)(e_1 - 1)] + [r_2 + B(\Delta v)(1 - r_2)] \\ &- 2[1 + B(\Delta v)(e_1 - r_2) + B(\Delta v)(1 - r_2)(e_1 - 1)] \\ &= [1 - r_2 + B(\Delta v)(-e_1 + 3e_1 r_2 - 2r_2)] \end{aligned}$$

Therefore, the final logical expression is:

$$B(d)[1 - r_2 + B(\Delta v)(-e_1 + 3e_1 r_2 - 2r_2)] = B(d) \tag{10}$$

This permits to complete and summarize our model as:

$$\begin{cases} B(d)(2l_1 l_2 - l_1 - l_2 + 1)(e_1 e_2 - 1)(1 - r_1 r_2) = 0 \\ B(d)[1 - r_2 + B(\Delta v)(-e_1 + 3e_1 r_2 - 2r_2)] = B(d) \\ s'_1 - s_1 - v_1 = 0 \\ s'_2 - s_2 - v_2 = 0 \end{cases}$$

The first two represents two logical statements, the second two physical kinematic rules. The technique of solution implies the first two should be solved with respect to l'_1, l'_2, updating at each time step the two Boolean variables $B(\Delta v), B(d)$ depending on the last two kinematic equations.

The solution of the logical expressions is very simple in the present context, since only four possibilities exist for the logical state variables: $l'_1 = 0, l'_2 = 0$; $l'_1 = 0, l'_2 = 1$; $l'_1 = 1, l'_2 = 0$; $l'_1 = 1, l'_2 = 1$. Therefore, in general, the sequential check of all the possible logical variable combinations is a possible simple option of solution.

Note, in this case we did not introduce any distinction between state variables and control variables.

The use of the implication operator implies, under the logical point of view, that there are further conditions that produce ambiguity, and again not unique mathematical solutions. In fact, the *if-then* table shows the known rule for implication: if the premise is false, then conclusion con be equally false or true. Latin rule to illustrate this fact is *ex falso sequitur quodlibet*, i.e. from a false premise everything can follow (false or true). This implies that, when imposing $V(Hp \rightarrow Th) = 1$, for $V(Hp) = 0$, this condition becomes an identity, whatever the values of the involved logical variables, and in this way, it does not add any useful information to the decision model. In fact if $V(Hp \rightarrow Th) = 1$ then $V(Hp \rightarrow Th) = 1 - p_{Hp} + p_{Hp} q_{Th} = 1$, i.e. $p_{Hp}(q_{Th} - 1) = 0$, that for $p_{Hp} = 0$, vanishes collapsing into a trivial identity. This suggests modifying or correct the implication operator for this application looking at the expression 1+p-q instead of 1-p+pq that is capable to eliminate the ambiguity never collapsing into an identity if the premise is false (i.e. when p=0).

The decision model modifies as:

$$\begin{cases} B(d)(2l_1 l_2 - l_1 - l_2 + 1) - e_1 e_2 - r_1 r_2 + e_1 e_2 r_1 r_2 = 0 \\ B(d)(2l_1 l_2 - l_1 - l_2 + 1) - \big[B(\Delta v)e_1 + \big(1 - B(\Delta v)\big)r_2 - 2\big(B(\Delta v)e_1\big(1 - B(\Delta v)\big)r_2\big)\big] = 0 \\ s'_1 - s_1 - v_1 = 0 \\ s'_2 - s_2 - v_2 = 0 \end{cases}$$

The numerical solution of this system of conditions produces valid results. In Figure 1 the two logical variables are plotted as function of time, considering the logical additional condition:

If a change of lane is needed,, then is the slowest car to change lane

and in with Figure 2 an additional condition:

If a change of lane is needed,, then is the fastest car to change lane

It appears that the logical variables make the jump when the Boolean variable of the distance is 1, i.e. when the cars become closer.

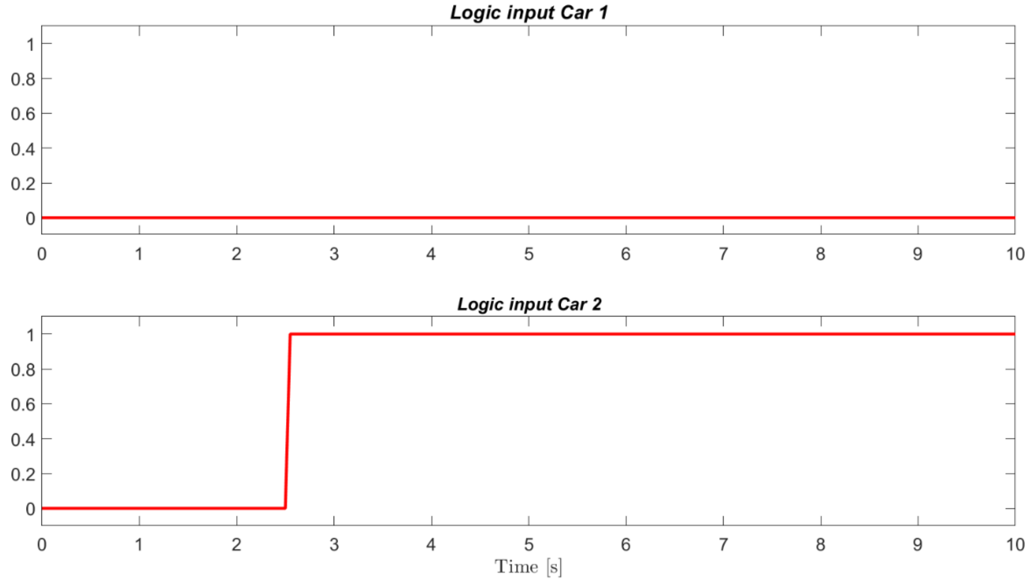

Figure 1: Logical Inputs to the Controller (case of lane change operated by the slowest car)

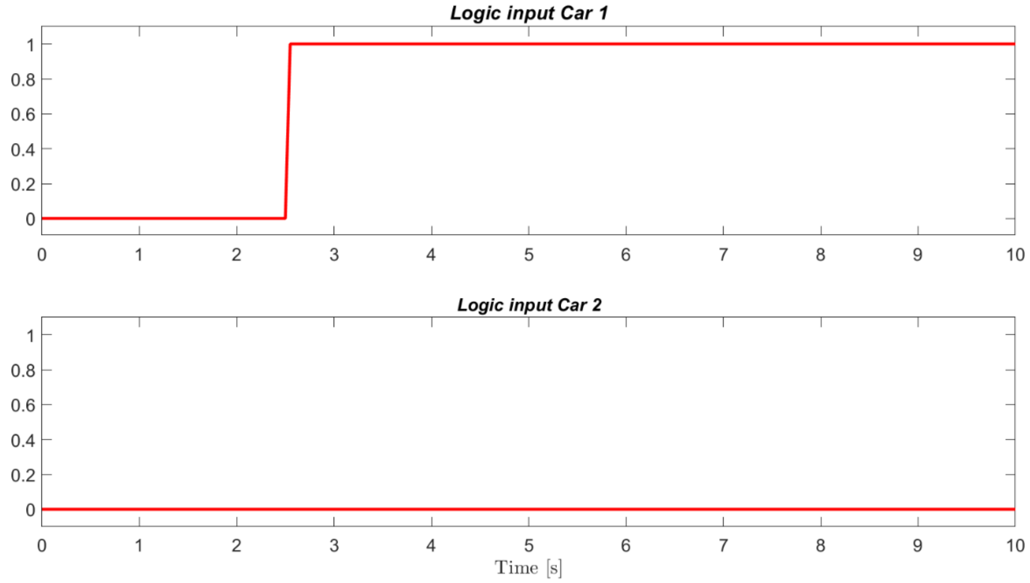

Figure 2: Logical Inputs to the Controller (case of lane change operated by the fastest car)

3.2 Alternatives disambiguation: Cost function minimization

Let us introduce a different approach to solve the disambiguation problem and let us describe the problem with three cars using a more complex model.

The introduction of a minimizing cost function is an effective way to eliminate disambiguation. In fact, when more than one solution exists, if one asks to determine the best solution that minimizes the objective function, a ranking of the different solutions is produced on the basis of the value of the objective function of each of the considered solutions. This allows selection, among the possible alternatives, of the one minimizing the objective function, solving the disambiguation problem.

We start with an analogous logical requirement illustrated at the beginning of the paper, now specified for three vehicles:

P=If V1 and V2 are too close and they are on the same lane, then one of them must change lane;

OR

Q=If V1 and V3 are too close and they are on the same lane, then one of them must change lane;

OR

R=If V2 and V3 are too close and they are on the same lane, then one of them must change lane;

i.e. $V(P \vee Q \vee R) = 1$.

The formal algebraic form of the three previous statements becomes:

$$p = 1 + B(d_{12})(2l_1l_2 - l_1 - l_2 + 1)(e_1e_2 - 1)(1 - r_1r_2)$$
$$q = 1 + B(d_{13})(2l_1l_3 - l_1 - l_3 + 1)(e_1e_3 - 1)(1 - r_1r_3)$$
$$r = 1 + B(d_{23})(2l_2l_3 - l_2 - l_3 + 1)(e_2e_3 - 1)(1 - r_2r_3)$$

Therefore, $V(P \vee Q) = p + q - pq$ and $V(P \vee Q \vee R) = (p + q - pq)(1 - r) + r = 1$.

Thus, we have the logical statement:

$$(p + q - pq)(1 - r) + r - 1 = 0 \quad (11)$$

Assume each car obeys the dynamics rule $v'_i - v_i = F_i$, where the force F_i (braking or accelerating) is now a control parameter. The kinematic rule $s'_i - s_i = v_i$ still holds. Suppose one desires to have a “fast” traffic, i.e. possibly the three speed would be high, so it is desirable the value $\sum_{i=1}^{3} v_i^2(t_k)$ is possibly large. However, we do not want to increase the transportation cost too much, that we assume to be proportional to $\sum_{i=1}^{3} F_i^2(t_k)$ that implies accelerations (negative or positive) are penalized. Assume that also “fast steering” maneuvers are penalized, for example including into the penalty function the term $\sum_{i=1}^{3} v_i(t_k)[l_i(t_k) - l_i(t_{k-1})]^2$, that is a mixed term, including logical and real state variables: a change of lane is more penalized when the speed is high. Therefore, one builds up the objective function:

$$J = \sum_{k=1}^{N}[A\sum_{i=1}^{3} v_i^2(t_k) - B\sum_{i=1}^{3} F_i^2(t_k) - C\sum_{i=1}^{3} v_i(t_k)[l_i(t_k) - l_i(t_{k-1})]^2] \quad (12)$$

where *A*,*B*,*C* are three weighting parameters.

For a simpler example of two cars, analogous to the previous, we desire minimize J subject to the constraints:

$$\begin{cases} B(d)(2l_1l_2 - l_1 - l_2 + 1) - e_1e_2 - r_1r_2 + e_1e_2r_1r_2 = 0 \\ s'_1 - s_1 - v_1 = 0 \\ s'_2 - s_2 - v_2 = 0 \\ v'_1 - v_1 = F_1 \\ v'_2 - v_2 = F_2 \end{cases}$$

Under the mathematical point of view, this problem exhibits the difficulty of involving the two logical variables l_i, which are not real and assumes only the two values 0 and 1. This does not permit the direct application of the Pontryagin approach and leads the problem on the ground of hybrid systems. Specific methods are available in this field (see for example [16-18]).

Let us solve here the problem dealing with the logical variables introducing them in the context of real variables by inserting a continuous form also for them. Let us replace l_1, l_2 by ξ_1, ξ_2 with the definition:

$$l_1 = 1 + \beta\ atan(\alpha\xi_1),\ l_2 = 1 + \beta\ atan(\alpha\xi_2)$$

Where the new variables ξ_1, ξ_2 are now real, and with a suitable choice of α, β, they give back l_1, l_2 that are in the range (0,1). Their values except for a small interval close to zero, produce the desired values 0 and 1.

Therefore, with this variable change our problem is characterized by real variables and solved using the standard procedures.

The numerical solution of the previous minimization problem is implemented. In Figure 3, the real control variables, i.e. the forces applied to the two cars, appear together with the logical variables. The forces start with a high intensity to accelerate the two cars, then their speed (see Figure 4) stabilize at some optimal values. When their distance becomes smaller and reaches the threshold for the lane change, the fastest car changes its lane (see Figure 3). Figure 4 shows the state variables of the system.

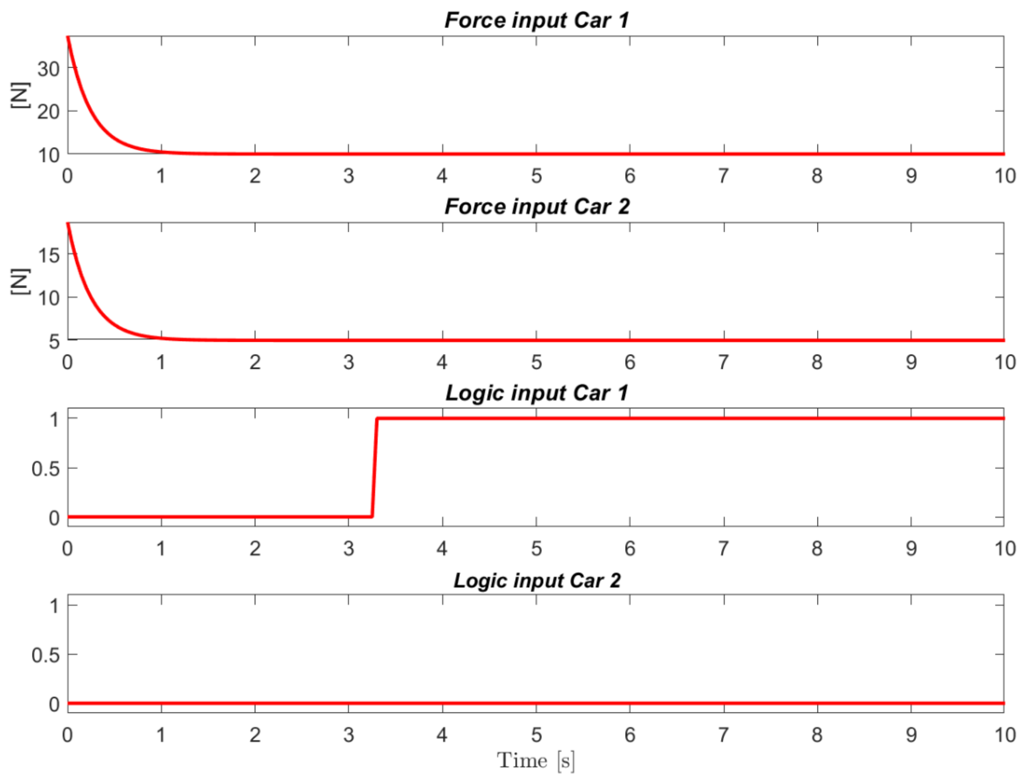

Figure 3: Physical and Logical Inputs to the Controller

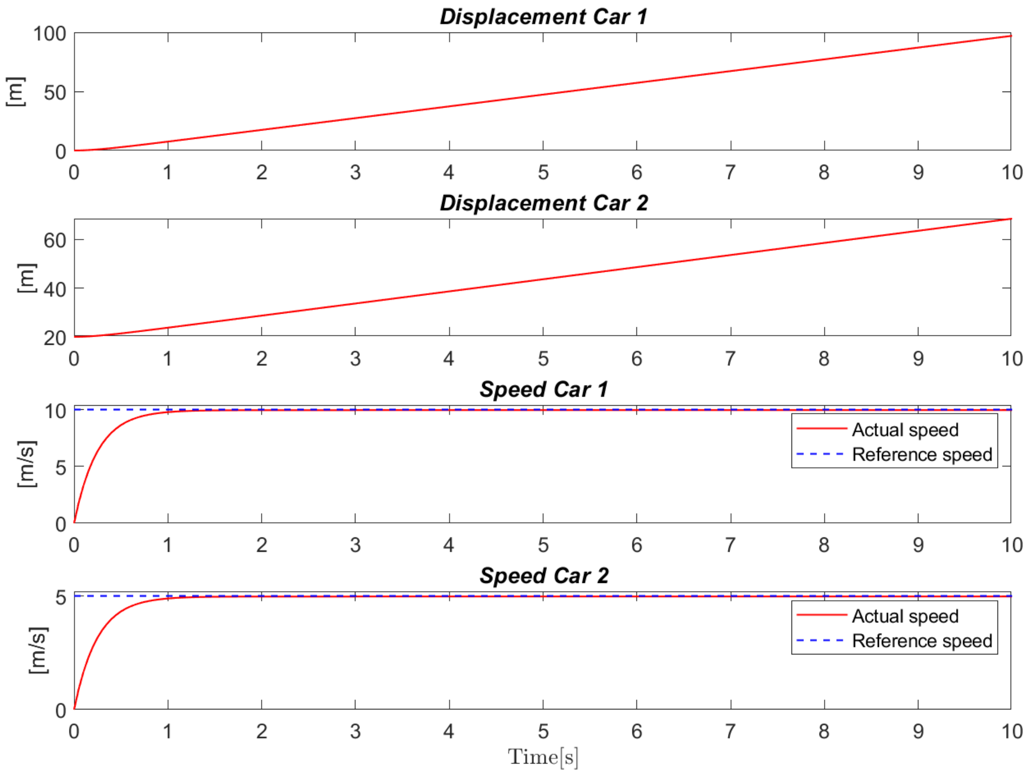

Figure 4: State Variable

4 Conclusions

The present paper attacks an important problem in the context of autonomous driving and more in general of autonomous robotics. In fact, there is an increasing pressure from the society for a better understanding of the behavior rules adopted by autonomous robots not under the human supervision. Some of the classical formulations of control, such as optimal control theory, based only on physical laws constraints do not make explicit the rules at which the robot obeys. Frequently, the translation into natural language of their behavior is impossible and this produces legal and ethical problems in the acceptability of governing robotics laws. The courts will be faced soon with the problem of evaluating the responsibility of damages and injuries produces by autonomous robots, and a clear statement of the rules at which the device obeys becomes a crucial question.

The present paper makes very elemental examples of decision models that use differential equations and propositional logic statements applied to the autonomous driving of a car. The statement of logical rules that translate natural language behavior requirements shows a different nature with respect to differential laws. Using propositional logic to translate the natural language sentences, we show these translate in turn into a Boolean arithmetic representation, and mix with the differential models, describing the dynamics of the car in the physical environment. Numerical results show the process works correctly.

Our team is at work to implement this method on much more complex examples, using the illustrated method to drive the experimental car platform Auto Sapiens, the real autonomous car in our lab.

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