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Optimization of Drive Time and Competitiveness in Sports League Design

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OPTIMIZATION OF DRIVE TIME AND COMPETITIVENESS IN SPORTS LEAGUE DESIGN

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OPTIMIZATION OF DRIVE TIME AND COMPETITIVENESS IN SPORTS LEAGUE DESIGN

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Optimization of Drive Time and Competitiveness in Sports League Design

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Club sports, also known as recreational team sports, are prevalent in the metropolitan areas of United States nowadays. However, there is a key concern for organizers, which is how to reduce the time that players spend driving to and from matches while keeping league divisions competitive. We adopt a three-step approach to solve this problem. Initially, we analyze the drive time data between clubs' locations to determine the geographic regions for the league. And then, clubs are assigned to divisions based on their rankings within in the league as well as their home facilities' geographic regions. Finally, divisions are further subdivided to minimize the drive time. Alternatively, we present another two solutions using an integrated model as well as a heuristic. The integrated model focuses on optimizing competitiveness while keeping drive time as a constraint, and the heuristic attempts to improve the drive time while preserving competitiveness. Applying any of the three methods to the game planning to the Tennis Competitors of Dallas, a large and well-established sports league in Texas, USA, we demonstrate that all processes can rearrange the existing divisions in a way that not only shortens the drive time for players, but also maintains an acceptable level of competition.

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INTRODUCTION

1.1 Background

Maureen Connolly was the first woman in history to win the Grand Slam of Tennis, and she founded the Maureen Connolly Brinker Tennis Foundation in 1968 with her friend Nancy Jeffett (The MCB Tennis Foundation, 2022). In 1977, the foundation started the Tennis Competitors of Dallas (TCD) league to promote women’s tennis in the Dallas/Fort Worth area (TCD, 2022). With 563 teams composed of approximately 7,800 players playing at 91 facilities, TCD has grown to become the second largest such league in the United States (Goad, 1999). Teams are placed into divisions, commonly called flights, based on their skill levels. There are typically ten teams in a flight in TCD. Each team plays twice against all the others in the same flight during a season: once at its home facility and another time at the other team's facility. The matches are held on Thursday mornings starting at 9:30 AM, so players on the visiting teams often must drive in rush-hour traffic. Figure 1 shows the locations of all the participating facilities in TCD.

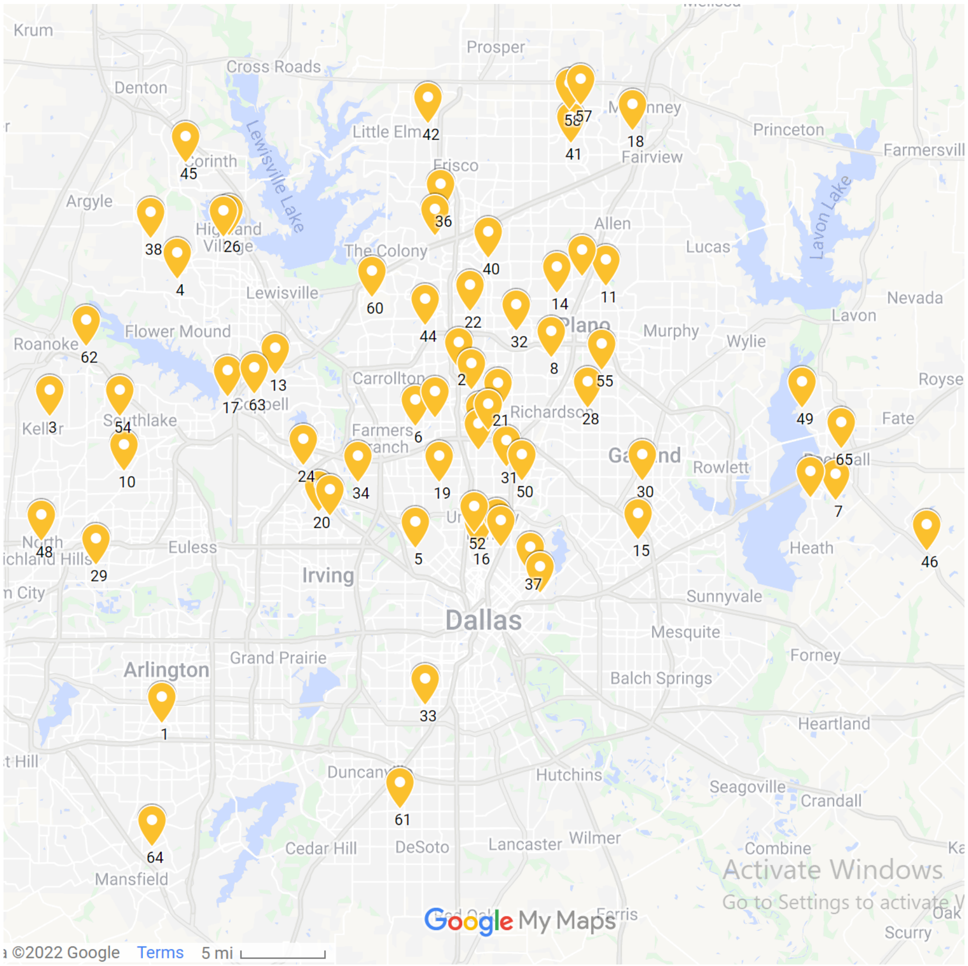

Figure 1 The locations of TCD facilities

Figure 1 shows an area of roughly 400 square miles / 1,036 km^2, which the facilities of TCD are spread across. Unsurprisingly, TCD members complain that the drive time to matches is too long. Looking at the drive time data of teams as an example flight in Table 1, players based at the facility labelled 58 must spend 68 minutes traveling to the facility labelled 51 to compete with their opponents. The over-an-hour-long trip causes disadvantage for visiting teams before their games, and inconvenience to return home in time for players who also have school-age children to care for.

Table 1 Drive time between teams’ facilities in example flight

		Home Team's Facility Number on the Map in Figure 1									
		51	**51**	**12**	**28**	**43**	**05**	**49**	**58**	**02**	**28**
Visiting Team's Facility Number	**51**		0	29	34	29	33	31	49	39	34
	51	0		29	34	29	33	31	49	39	34
	12	32	32		21	5	22	41	31	8	21
	28	33	33	18		18	42	27	30	21	0
	43	32	32	5	21		22	41	31	8	21
	05	22	22	26	36	26		42	42	23	36
	49	36	36	34	25	34	64		42	36	25
	58	68	68	46	36	46	59	56		37	36
	02	38	38	9	21	9	30	37	28		21
	28	33	33	18	0	18	42	27	30	21	
Note: There are two teams based at facility 51 in this flight.											

1.2 Geographic Flighting

Seeking a way to reduce the drive time for TCD’s members without sacrificing the competitiveness of flights, their placement organizers approached the department of Operations Research and Engineering Management at Southern Methodist University for help designing a new flighting system based on geography. The concept of geographic flighting is to shorten drive times by forming flights of teams according to the locations of participating facilities in addition to players’ skill levels. Figure 2 illustrates a proposed geographic partition for TCD’s facilities split into three portions, which are referred to as the Western, Central, and Eastern regions. Facilities in the Western region are represented by the red circles with white cross marks on the map, facilities in the Central region are represented by the yellow circles with white stars in the middle, and facilities in the Eastern region are represented by the blue circles with white dots. To minimize drive time for TCD's members, the method of geographic flighting prohibits forming flights with teams from facilities in both the Western and Eastern regions. Ideally, any given flights consist of teams whose facilities are all in the same region, however this may not always be possible or desirable in terms of competition.

Table 2 presents an example of the drive time data for a geographically designed flight containing six of the teams whose home facilities are listed in Table 1. In Table 1 The longest drive time is 68 minutes for the two teams based at facility 51 drive to facility 58. Whereas in Table 2, the longest drive time for these two teams is reduced to 39 minutes, which is a significant improvement of 43%. However, the drawback of the geographically designed flight is that it is not as competitive as the flight shown in Table 1. A flight with teams ranked consecutively is ideal for competition. The ten teams in the flight are ranked 101 through 110, whereas the teams in the geographically designed flight shown in Table 2 are ranked 101, 102, 103, 105, 106, 109, 112, 116, 117, and 118. The four lowest ranked teams in the flight are at a competitive disadvantage. Thus, TCD must consider a tradeoff between drive time and competition while planning games using geographic flighting.

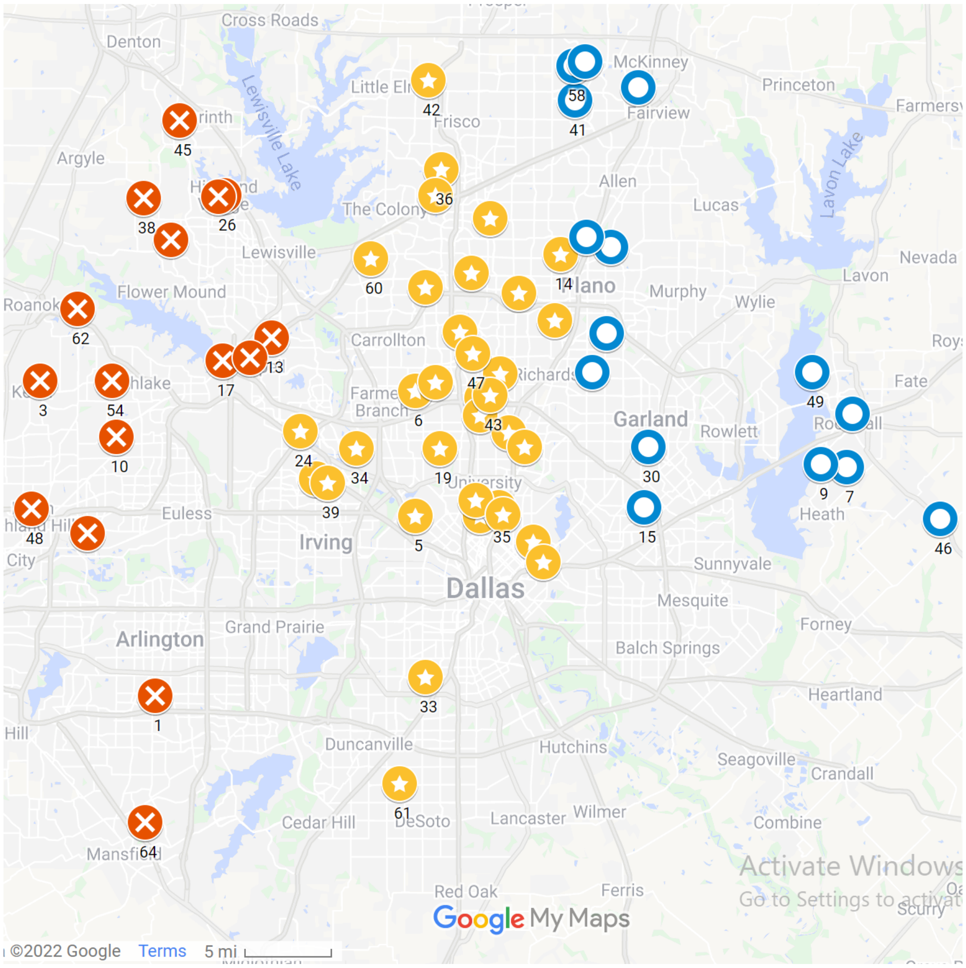

Figure 2 A proposed geographic partition of TCD facilities

Table 2 Drive time for facilities in the geographically designed flight

		Home Team's Facility Number on Map in Figure 2									
		51	51	12	43	05	02	23	51	06	37
Visiting Team's Facility Number	51		0	29	29	33	39	39	0	39	5
	51	0		29	29	33	39	39	0	39	5
	12	32	32		5	22	8	12	32	12	32
	43	32	32	5		22	8	12	32	12	32
	05	22	22	26	26		23	24	22	24	22
	02	38	38	9	9	30		14	38	14	38
	23	36	36	11	11	21	14		36	5	36
	51	0	0	29	29	33	39	39		39	5
	06	36	36	11	11	21	14	5	36		36
	37	5	5	29	29	33	39	39	5	39	
Note: There are three teams based at facility 51 in this flight.											

1.3 An Optimization-based Approach to Geographic Flighting

Rather than thinking intuitively to set the boundaries for the regions as illustrated in Figure 2, we propose a three-step process shown in Figure 3 for geographic flighting in which each step solves a mixed integer programming (MIP) model. The first step is to assign each TCD facility to one of the three regions. Even though we named the three regions as Western, Central, and Eastern regions, they are not necessarily determined by North-South boundary lines as in Figure 2. Instead, the regions are determined by solving a MIP that we refer to as

Model 1. The objective of Model 1 is to minimize the maximum drive time that could possibly result from the geographic flighting. Furthermore, the second step of the process is to assign teams to divisions called double flights. A double flight is a collection of teams whose size is as twice large as a standard flight's. The teams' home facilities in a double flight must be geographically balanced, so that the teams can be split into two standard size flights in a way that no team whose home facility is in the Western region is required to drive to a match in the Eastern region, and vice versa. The assignment placing teams into double flights is done by solving another MIP called Model 2. The objective of Model 2 is to maximize the competitiveness of the double flights. Finally, the last step involves solving a third MIP called Model 3. It splits a given double flight into two standard flights in a way that minimizes the longest drive time for any visiting teams.

The three-step model presented in this praxis builds on a Senior Design project by Cooney, Price, and Snyder (2019). Initially the Senior Design team updated the geographic boundary encompassing the facilities with teams participating in TCD using the latest data, conducted a survey on members' attitudes towards the tradeoff between driving time and competition, collected the travel times, and proposed a geographic flighting strategy. In this praxis, we propose refinements to the approach proposed by Cooney, Price and Snyder (2019) to reduce computation time and make the models more intuitive. An important finding in the survey is that most of the players in the top flights said that competition is much more important to them than drive time. So, those flights are not considered in this study and were not considered by the Senior Design team. The flights that are considered have names like 7A, 7B, 8A, 8B, etc. Flight 7A is the top flight, flight 7B is the second best flight, etc. The idea behind using double flights in Steps 2 and 3 in the three-step process is to replace flights 7A and 7B with flights 7

East and 7 West where flights 7 East and 7 West are mostly composed of teams from the current 7A and 7B flights.

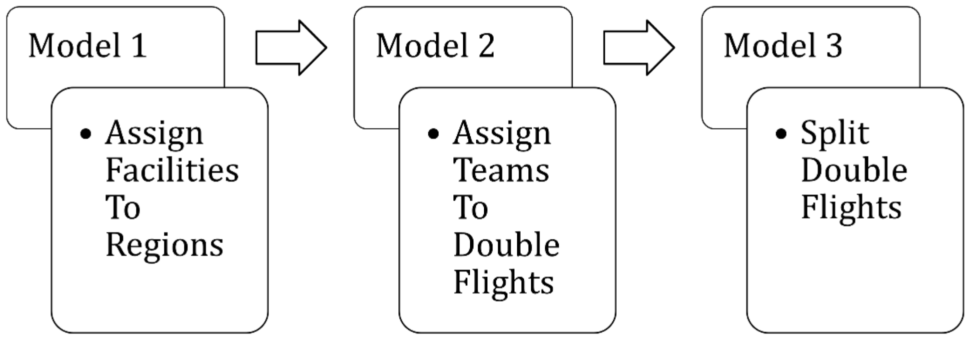

Figure 3 The three-step process

1.4 Literature Review

To the best of our knowledge, the problem posed to us by TCD has not been addressed in the literature. As described in this section, certain parts of the problem have been studied in isolation. By integrating travel time and competition into the decision making, this praxis appears to fill a gap in the literature.

Van Bulck et al. (2020) give a recent survey of the extensive literature on optimizing the schedule for round-robin sports tournaments. These papers typically propose an integer programming model to minimize the total travel time or cost for the teams in the tournament with constraints to ensure that the schedule is fair in various ways. A common measure of fairness is the number of so-called breaks each team has in its schedule. A home break, two consecutive matches played at a team's home facility, is considered an advantage. An away break, two consecutive matches played at opponents' home facilities, is considered a disadvantage. Duran et al. (2021) describe an optimization model used by Argentinean youth soccer leagues to balance the total travel distance for each team while ensuring that the schedule meets requirements on the number and timing of home and away breaks. Riberio and Urruita (2012) describe a similar model for scheduling the top professional soccer league in Brazil that also considers requests from TV networks to have attractive matches played on weekends. Other examples include scheduling a professional league in Chile and the South American qualification round for the 2018 World Cup (Duran, Guajardo, & Wolf-Yadlin, 2012) (Alarcon, et al., 2017) (Duran, Guajardo, & Sauré, 2017). TCD could use this type of model to schedule matches, but they would use it after they have determined which teams are in which flights.

Knust (2010) designs an integer linear programming model to aid the Table-Tennis Federation of Lower Saxony (TTVN) in improving its schedule. Since TTVN is an amateur league, Knust's model relaxes some of the strict requirements of professional leagues. For example, breaks are not as much of a concern to TTVN (Knust, 2010). Although they are both amateur leagues, TTVN's scheduling problem is quite different than TCD's. For example ,TTVN teams may play on different days of the week and at different times. So, driving in rush hour traffic is not necessarily a major concern.

Mitchell (2003), and Macdonald and Pulleyblank (2014) present models for changing divisions in the major professional North American leagues when teams move to new cities or when new teams join the league. These papers are related to this praxis since they are concerned with reassigning teams to divisions. However, these models aim to minimize travel time and do not consider competitive balance.

Studies of player-ranking systems for tennis such as (Reid & Morris, 2013) and (Irons, Buckley, & Paulden, 2014) could be of interest to TCD, but are not directly related to this praxis. Additionally, there are papers that study and/or propose systems for team ranking such as (Stefani & Pollard, 2007), (Motegi & Masuda, 2012), and (Criado, Garcıa, Pedroche, & Romance, 2013). This stream of literature would be helpful to TCD if they decide to change the way they rank their teams in the future. In this praxis the team rankings are taken as given input from TCD.

As mentioned previously, TCD assigns teams to flights based on rankings to promote competitive balance. Competitive balance in professional sports leagues is an active research area. In an evaluation of the competitive balance of the Confederation of North, Central America and Caribbean Association Football (CONCACAF) qualifying region for World Cup soccer, Rocke describes three types of balance: "(1) match uncertainty which describes the uncertainty about the result of a special match between two teams; (2) season uncertainty which describes the uncertainty about matches in a particular season; (3) championship uncertainty which describes the dominance of a limited number of teams over the league in a consecutive season as commonly seen in football (Szymanski, 2003)" (Rocke, 2019). In other words, a league, division, or flight is competitively balanced if it is difficult to predict at the beginning of the season which team will win which matches and which team will win the championship. Schelles, Francois, and Dermit-Richards (2022) and Ramchandani et al. (2018) provide recent studies of competitive balance in the major professional European soccer leagues and surveys of related literature, and Plumley et al. (2020) provide a similar study of professional Asian soccer leagues. The literature on competitive balance is mainly descriptive rather than prescriptive. So, it is not directly applicable to this praxis. However, it could be helpful to TDC for analyzing the actual versus predicted change in competitiveness after a switching to a new flighting process.

MODELING

2.1 Facility Assignment

Model 1 assigns facilities to geographic regions in a way that minimizes the maximum potential drive for any teams. Geographic flighting is based on the idea that teams that are too far away from each other will not be placed in the same flight. Therefore, the model only considers drive time between pairs of facilities that are in the same region or in the adjacent regions. All the inputs to the model and decision variables are listed below.

<u>Sets and Parameters</u>

$\mathcal{F}$ – Set of facilities

$\mathcal{R}$ – Set of regions

$\mathcal{A}$ – Set of pairs of regions that might contribute to drive time; $(k, l) \in \mathcal{A}$ if teams based at facilities in region k may be assigned to flights that also contain teams based in region l.

T_i – Number of teams based at facility i

M_i – Maximal number of facilities in region i

N_i – Minimal number of facilities in region i

D_{ij} – Drive time from facility i to facility j (in minutes)

Decision Variables

x_{kilj} – Binary variable equal to 1, if facility i is assigned to region k, and facility j is assigned to region l, and zero otherwise

y_{ki} – Binary variable equal to 1, if facility i is assigned to region k, and zero otherwise

z – Maximum potential drive time resulting from the geographic flighting

The objective function (1) minimizes the maximum potential drive time for any team, z. If facility i is assigned to region k, and facility j is assigned to region l, then it is possible that teams from facility i will be assigned to flights with teams from facility j. Constraint (2) forces z to be at least the drive time from facility i to facility j. Constraint (3) is the logical connection between x and y variables. Constraint (4) assigns each facility to exactly one region. Constraints (5) and (6) ensure that each region has at least the minimum and at most the maximum number of teams. And constraints (7) and (8) set the decisions variables x and y as binary.

Objective Function

$$\text{Minimize} \quad z \quad (1)$$

Constraints

$$z \geq D_{ij} x_{kilj} \qquad \forall i, j \neq i \in \mathcal{F}, \forall (k,l) \in \mathcal{A} \qquad (2)$$

$$y_{ki} + y_{lj} \leq x_{kilj} + 1 \qquad \forall i, j \neq i \in \mathcal{F}, \forall (k,l) \in \mathcal{A} \qquad (3)$$

$$\sum_{k \in \mathcal{R}} y_{ki} = 1 \qquad \forall i \in \mathcal{F} \qquad (4)$$

$$\sum_{i \in \mathcal{F}} T_i y_{ki} \leq M_k \qquad \forall k \in \mathcal{R} \qquad (5)$$

$$\sum_{i \in \mathcal{F}} T_i y_{ki} \geq N_k \qquad \forall k \in \mathcal{R} \qquad (6)$$

$$x_{kilj} \in \{0,1\} \qquad \forall i, j \neq i \in \mathcal{F}, \forall (k,l) \in \mathcal{A} \qquad (7)$$

$$y_{ki} \in \{0,1\} \qquad \forall\, k \in \mathcal{R}, \forall\, i \in \mathcal{F} \qquad (8)$$

2.2 Double Flight Assignment

Model 2 assigns teams to double flights in a way that minimizes a penalty function. Some of the original flights may have to be changed for geographic flighting. Model 2 finds preliminary flight reassignments that minimize a penalty sum for making the geographic flights less competitive than the original flights.

The penalty function is a measure of overall competitiveness. A team will face more competitive opponents if it is moved to higher flights, and less competitive opponents in lower flights. For instance, a team in flight 9 that is moved up to flight 7 to play with stronger competitors gets a penalty of 2, and it would get a penalty of 3 if it is moved down from flight

9 to flight 12 to compete with weaker teams. The penalty function is the sum of the teams' penalties. An ideal penalty function value is zero.

The double flights produced by Model 2 are composed of 20 teams that are split into two flights of 10 teams for the east and west regions by Model 3. Model 2 ensures that there are enough teams from the east and west regions to allow the split in Model 3. For example, a group of 20 teams with 16 from the west region, two from the central region, and two from the east region cannot be split into two 10-team flights under the geographic flighting rules.

Inputs and decision variable for Model 2 are listed below.

Sets and Parameters

$\mathcal{T}$ – Set of teams

$\hat{\mathcal{F}}$ – Set of double flights

$W \subset \mathcal{T}$ – Set of teams assigned to the West region by Model 1

$C \subset \mathcal{T}$ – Set of teams assigned to the Central region by Model 1

$E \subset \mathcal{T}$ – Set of teams assigned to the East region by Model 1

P_{ij}– Penalty for team i is assigned to double flight j

$\bar{M}_j$– Maximum number of teams that may be assigned to double flight j

$\bar{R}_i$– Region of team i based on the facility assignment from Model 1. The region code parameter can be -1, 0 or 1 indicating West, Central (either or East-Central, West-Central), and East regions, respectively.

Decision Variable

$\tilde{x}_{ij}$ – Binary variable equal to 1, if team i is assigned to double flight j, and zero otherwise

Objective Function

$$\text{Minimize} \quad \sum_{i \in \mathcal{T}, j \in \mathcal{F}} P_{ij} \tilde{x}_{ij} \quad (9)$$

Constraints

$$\sum_{j \in \hat{\mathcal{F}}} \tilde{x}_{ij} = 1 \qquad \forall i \in \mathcal{T} \qquad (10)$$

$$\sum_{i \in \mathcal{T}} \tilde{x}_{ij} = \bar{M}_j \qquad \forall j \in \hat{\mathcal{F}} \qquad (11)$$

$$\sum_{i \in \mathcal{W}} \tilde{x}_{ij} \leq \frac{\bar{M}_j}{2} \qquad \forall j \in \hat{\mathcal{F}} \qquad (12)$$

$$\sum_{i \in \mathcal{E}} \tilde{x}_{ij} \leq \frac{\bar{M}_j}{2} \qquad \forall j \in \hat{\mathcal{F}} \qquad (13)$$

$$\sum_{i \in \mathcal{E} \cup \mathcal{C}} \tilde{x}_{ij} \geq \frac{\bar{M}_j}{2} \qquad \forall j \in \hat{\mathcal{F}} \qquad (14)$$

$$\sum_{i \in \mathcal{W} \cup \mathcal{C}} \tilde{x}_{ij} \geq \frac{\bar{M}_j}{2} \qquad \forall j \in \hat{\mathcal{F}} \qquad (15)$$

$$\tilde{x}_{ij} \in \{0,1\} \qquad \forall i \in \mathcal{T},\ \forall j \in \hat{\mathcal{F}} \qquad (16)$$

The objective function (9) minimizes the penalty function. Constraint (10) assigns each team to exactly one double flight. Constraint (11) limits the number of teams in each double flight. Constraints (12) and (13) set the upper and lower limits on number of teams in regions. Constraints (14) and (15) balance the number of teams in the east and west regions in each double flight as described above. And constraint (16) sets the decisions variable $\tilde{x}$ as binary.

2.3 Flight Assignment

Model 3 divides teams in a given double flight, f, into standard size flights in a way that minimizes the maximum drive time for all teams. For a given double flight, one of the standard size flights is considered the East flight, flight fE, and the other is the West flight, fW. Inputs and decision variable for Model 3 are listed below.

<u>Sets and Parameters</u>

$\hat{\mathcal{T}}_f$ – Set of teams in double flight f

$\widehat{\mathcal{W}}_\mathrm{f} \subset \hat{\mathcal{T}}_\mathrm{f}$ – Set of teams assigned to double flight f by Model 2 whose home facility is assigned to the West region by Model 1

$\hat{\mathrm{E}}_\mathrm{f} \subset \hat{\mathcal{T}}_\mathrm{f}$ – Set of teams assigned to double flight f by Model 2 whose home facility is assigned to the East region by Model 1

$\overline{D}_{ij}$ – Driving time from the home facility of team i to the home facility of team j (in minutes)

$\bar{R}_i$– Region of team i

$\overline{M}_j$– Maximal number of teams in flight j

<u>Decision Variables</u>

$\hat{x}_{ij}$ – 1, if team j is assigned to new region i, and zero otherwise

$\hat{y}_j$ – Maximum driving time for team j

$\hat{z}$ – Maximum driving time for all teams

<u>Objective Function</u>

$$\text{Minimize} \quad \hat{z} \quad (17)$$

Constraints

$$\hat{y}_j \geq \bar{D}_{ij}\,\hat{x}_{1i}\hat{x}_{1j} \qquad \forall i,j \in \hat{\mathcal{T}}_f \qquad (18)$$

$$\hat{y}_j \geq \bar{D}_{ij}\,\hat{x}_{2i}\hat{x}_{2j} \qquad \forall i,j \in \hat{\mathcal{T}}_f \qquad (19)$$

$$\hat{z} \geq \hat{y}_j \qquad \forall j \in \hat{\mathcal{T}}_f \qquad (20)$$

$$\hat{x}_{1i} + \hat{x}_{2j} = 1 \qquad \forall j \in \hat{\mathcal{T}}_f \qquad (21)$$

$$\hat{x}_{1i} = 1 \qquad \forall i \in \hat{\mathcal{E}}_f \qquad (22)$$

$$\hat{x}_{2i} = 1 \qquad \forall i \in \widehat{\mathcal{W}}_f \qquad (23)$$

$$\sum_{j \in \hat{\mathcal{T}}_f} \hat{x}_{ij} \geq \left\lfloor \frac{|\hat{\mathcal{T}}_f|}{2} \right\rfloor \qquad \forall i \in \{1,2\} \qquad (24)$$

$$\sum_{j \in \hat{\mathcal{T}}_f} \hat{x}_{ij} \leq \left\lceil \frac{|\hat{\mathcal{T}}_f|}{2} \right\rceil \qquad \forall i \in \{1,2\} \qquad (25)$$

$$\hat{x}_{ij} \in \{0,1\} \qquad \forall i \in \{1,2\},\ \forall j \in \hat{\mathcal{T}}_f \qquad (26)$$

$$\hat{y}_j \in \{0,1\} \qquad \forall j \in \hat{\mathcal{T}}_f \qquad (27)$$

The objective function (17) minimizes the maximum drive time for all teams in the double flight. Constraints (18), (19), and (20) ensure that the objective function value is at least as large as the longest drive time in the resulting split of the double flight into two standard size flights. Constraint (21) ensures each team is assigned to the East region of the double flight (i.e., flight fE) or the West region (i.e., flight fW), but not both. Constraints (22) and (23) preassign the teams whose home facilities are in the East and West regions to flights fE and fW, respectively. The last double flight in the TCD example has an odd number of teams. So, constraints (24) and (25) allow an exception so that the difference in the number of teams of the last two standard size flights is one. And constraints (26) and (27) sets the decisions variable $\tilde{x}$ and $\tilde{y}$ as binary.

RESULTS

3.1 General

We used AMPL version 20210731 to generate the MIPs, which were then solved with Gurobi version 9.1.1 on a HP DL380 computer with Dual 16 Core Intel Xeon@2.9GHz processors and 380GB of RAM. Model 1 had 1 continuous and 58,500 binary variables and 116,441 constraints and took 5 minutes of real time to solve. Model 2 had 1,575 binary variables and 220 constraints and took less than 60 seconds of real time to solve. It took a total of 540 seconds of real time to run the nine instances of Model 3. The instances of Model 3 had up to 21 continuous and 18 binary variables and 220 quadratic constraints.

After applying the three-step model to TCD's data, geographic result produced by Model 1 is shown as a graph in figure 4 to present a visualization as well as an overall summary comparing to the original plan in table 3. In figure 4, the three regions are marked by dots, stars, and crosses respectively. Table 3 gives the mean drive time for teams in the given (original) TCD flights compared to the flights determined by our three-step process. There is not a significant change in the minimum drive times; the minimum value is about 16 minutes in each case. The three-step process reduces the averages of the mean and median drive times by approximately two minutes each. The most significant result in this example is that geographic flighting reduces the average of the maximum driving times from over an hour (66.22 minutes) to less than an hour (54.78). Additionally, geographic flighting improves equity by reducing the variation (standard deviation) in driving times.

Table 3 Comparison of drive time statistics between original TCD flighting and the geographic solution for the Fall 2021 Season

	Average over all TCD Teams		
Statistic (per team)	Original Flighting	Geographic Flighting	Change
Minimum	16.89	16.11	-4.62%
Average	31.73	29.14	-8.16%
Median	29.89	27.00	-9.67%
Maximum	66.22	54.78	-17.28%
Standard Deviation	8.76	7.37	-15.87%

We also produced a “visual” solution was by manually assigning facilities to regions to create a more visually intuitive geographical partition (see Figure 2) than one produced by the complete three-step process (see Figure 4). The minimum, average, median, and maximum drive times in the visual solution were 12 minutes, 27.67 minutes, 26.50 minutes, and 62.25 minutes, respectively.

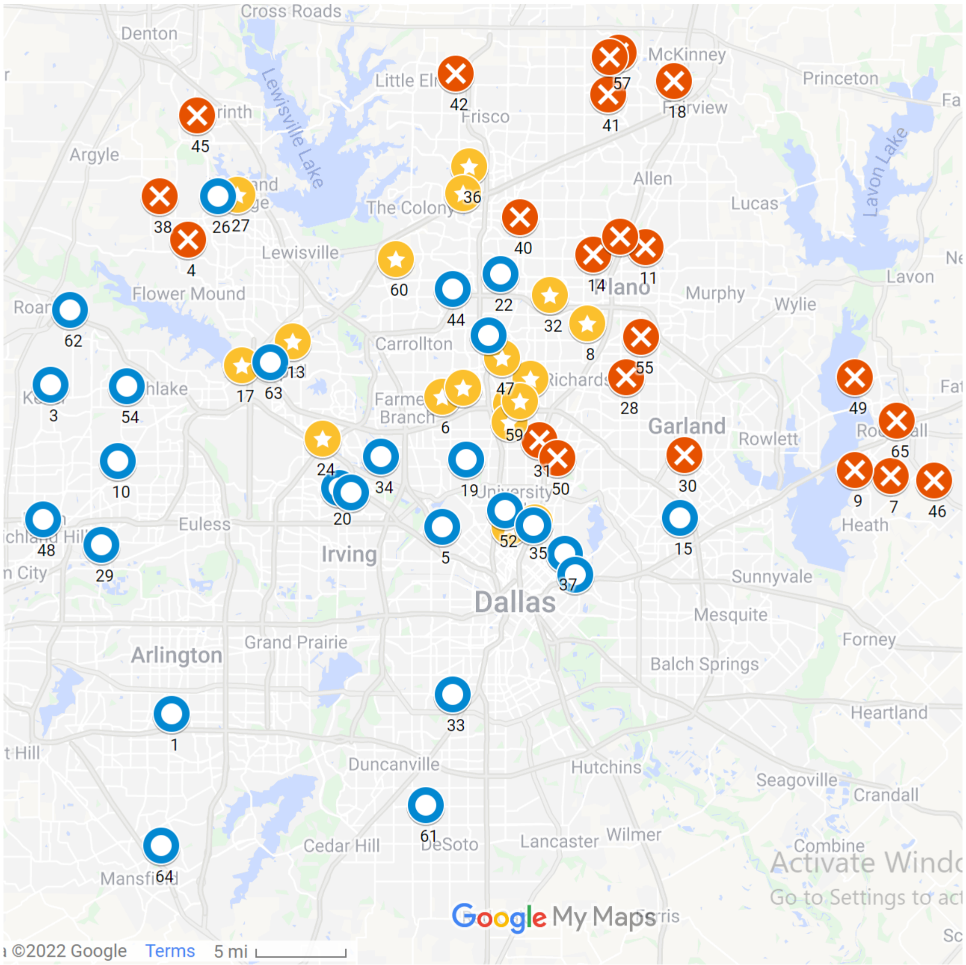

Figure 4 Facility assignment determined by the three-step process for the Fall 2021 Season

3.2 Validation

To check the robustness of the system, a few more sets of data are applied to it, which are from the Fall season of 2019 to Spring 2021. Note that the input data for the five seasons in our experiment have significant differences. The team rankings change each season. Additionally, some teams leave TCD each year and new teams join the league. Tables 4 through 7 give the drive time statistics for geographic fighting produced by the three-step process for the other four TCD seasons. Across the five seasons, the reduction in the average maximum drive time resulting from geographic flighting ranges from a low of 4.55% to a high of 19.96%. The reduction in the average drive time ranges from 3.81% to 11.86%. In all five cases, the maximum final penalty per team is two units.

Table 4 Comparison of drive time statistics between original TCD flighting and the geographic solution for the Fall 2019 Season

	Average over all TCD Teams		
Statistic (per team)	Original Flighting	Geographic Flighting	Change
Minimum	15.00	16.33	8.89%
Average	30.42	27.84	-8.50%
Median	29.44	26.67	-9.43%
Maximum	64.00	52.00	-18.75%
Standard Deviation	9.51	7.86	-17.30%

Table 5 Comparison of drive time statistics between original TCD flighting and the geographic solution for the Spring 2020 Season

	Average over all TCD Teams		
Statistic (per team)	Original Flighting	Geographic Flighting	Change
Minimum	16.67	16.33	-2.00%
Average	30.59	29.43	-3.81%
Median	28.83	28.11	-2.50%
Maximum	61.22	49.00	-19.96%
Standard Deviation	8.91	7.96	-10.71%

Table 6 Comparison of drive time statistics between original TCD flighting and the geographic solution for the Fall 2020 Season

	Average over all TCD Teams		
Statistic (per team)	Original Flighting	Geographic Flighting	Change
Minimum	15.88	14.78	-6.91%
Average	30.98	27.30	-11.86%
Median	29.66	26.06	-12.15%
Maximum	53.43	51.00	-4.55%
Standard Deviation	8.15	7.43	-8.76%

Table 7 Comparison of drive time statistics between original TCD flighting and the geographic solution for the Spring 2021 Season

	Average over all TCD Teams		
Statistic (per team)	Original Flighting	Geographic Flighting	Change
Minimum	18.33	17.78	-3.03%
Average	31.42	29.18	-7.14%
Median	29.78	27.78	-6.72%
Maximum	55.67	52.67	-5.39%
Standard Deviation	8.32	6.88	-17.32%

Table 8 presents summary statistics of the range of penalty values for all teams in the geographic flighting solution across five seasons. The largest penalty is never more than 2 and the minimum is 0 in the three of the five seasons. And the mean penalty is 1 in all five seasons, and mean penalty ranges from 0.88 to 1.05 this shows that the penalties are consistently in narrow range for various data sets.

Table 8 The penalty comparison across five seasons

	Fall 2019	**Spring 2020**	**Fall 2020**	**Spring 2021**	**Fall 2021**
Minimum	1	1	0	0	0
Average	1.05	1.00	0.88	0.94	1.00
Median	1	1	1	1	1
Maximum	2	1	1	1	2

ALTERNATIVES TO GEOGRAPHIC FLIGHTING

4.1 Integrated Model

So far, TCD has not adopted geographic flighting. They reviewed the results summarized in Chapter 3 and decided that many teams would not consider their individual drive time savings to be large enough to convince them to switch from the current flighting system to geographic flighting. The three-step process has some limitations. It alternates between optimizing drive time and optimizing competition. A model that considers both objectives at the same time could produce better results. Geographic flighting places all teams that share a given home facility in the same region, which limits the possibilities for assigning teams to double flights. For these reasons, we developed the integrated model described in this chapter. Instead of using geographic flighting, the integrated model assigns teams to flights in a way that minimizes the maximum penalty per team subject to a user-specified limit on the maximum drive time. All the inputs to the model and decision variables are listed below.

<u>Sets and Parameters</u>

T – Set of teams

F – Set of facilities

$\hat{F}$ – Set of flights

H_i – Home facility of team i

M_i – Number of teams in flight i

D_{ij} – Drive time (in minutes) from facility i to facility j

D – Maximum allowable drive time (in minutes)

P_{ij} – Penalty if team i is assigned to flight j

<u>Decision Variables</u>

p – Maximum penalty

x_{ij} – Binary variable equal to one if team i is assigned to flight j, and zero otherwise

y_{ij} – Binary variable equal to one if team drives from facility i to facility j, and zero otherwise

<u>Objective</u>

$$\text{Minimize} \quad p \quad (29)$$

<u>Constraints</u>

$$\sum_{j\in\hat{F}} P_{ij}x_{ij} \leq p \quad \forall i \in T \quad (30)$$

$$\sum_{j\in\hat{F}} x_{ij} = 1 \quad \forall i \in T \quad (31)$$

$$\sum_{i\in T} x_{ij} = M_j \quad \forall j \in \hat{F} \quad (32)$$

$$D_{ij}\, y_{ij} \leq D \quad \forall i, j \neq i \in F \quad (33)$$

Link X and Y: $$x_{ik} + x_{jk} \leq y_{H_iH_j} + 1 \quad \forall i, j \neq i \in T,\ \forall k \in \hat{F} \quad (34)$$

$$x_{ij} \in \{0,1\} \quad \forall i \in T,\ \forall j \in \hat{F} \quad (35)$$

$$y_{ki} \in \{0,1\} \quad \forall i, j \neq i \in F \quad (36)$$

The objective function (29) minimizes the highest potential penalties for all teams, p. Constraint (30) limits the total penalty for moving teams in and out of a flight. Constraint (31) assigns teams to flights. Constraint (32) sets the size of flights. Constraint (33) ensures the drive time in each facility-to-facility trip not exceed the maximum. Constraint (34) links variables x and y. And constraints (35) and (36) set the decisions variables x and y as binary.

4.2 Model Result

Table 9 compares TDC's flighting for the fall 2021 season (original) with the results of solving the integrated model for a range of maximum drive time values. By design, the solution to the integrated model has a maximum penalty of p = 0 when the maximum drive time D is set to the largest drive time in the facility-to-facility drive time matrix. Starting with this value, we decreased D by increments of one minute until the problem became infeasible for D = 30 minutes. As indicated by the last column in Table 9, TCD could arrange the flights in a way that limits the maximum drive time to 31 minutes for all teams. This is a significant reduction from the average maximum drive time of 54.78 minutes resulting from geographic flighting (see Table 3). However, the maximum penalty p = 18 is too high for practical use. Setting D = 47 minutes produced a solution with p = 2, which is the same as the maximum penalty for geographic flighting. This shows the advantage of using the integrated model over geographic flighting.

Table 9 Comparison of average drive time between the original and new flighting

D (minutes)	77	57	47	44	38	31
Minimum (minutes)	16.89	18	17.22	13	9.22	3.22
Median (minutes)	29.89	28	29.11	22.89	20.11	16.33
Mean (minutes)	31.74	29.01	29.75	22.87	20.32	16.11
Maximum (minutes)	66.22	50.67	48	36.22	30.78	28.89
Standard Deviation (minutes)	8.76	6.51	6.79	5	4.34	4.96
p	0	1	2	3	8	18
	Change Compared to TCD's Flighting					
Minimum (minutes)	0	6.57%	1.95%	-23.03%	-45.41%	-80.94%
Median (minutes)	0	-6.32%	-2.61%	-23.42%	-32.72%	-45.37%
Mean (minutes)	0	-8.60%	-6.27%	-27.95%	-35.98%	-49.24%
Maximum (minutes)	0	-23.48%	-27.51%	-45.30%	-53.52%	-56.37%
Deviation (minutes)	0	-25.68%	-22.49%	-42.92%	-50.46%	-43.38%

Table 10 shows the distribution of penalties for five different *D* values. When $D = 47$ minutes, most of the teams have a penalty of 0 or 1, and 82 out of 185 (44%) have a penalty of 2. Decreasing D below 47 minutes not only increases the maximum penalty, but also dramatically increases the number of teams with a penalty of 2 or larger.

Table 10 Number of teams for each penalty in Fall 2021

185 Teams		Maximum Drive Time				
		57 Mins	**47 Mins**	**44 Mins**	**39 Mins**	**31 Mins**
Penalty	**1**	72	57	47	25	23
	2		82	52	20	20
	3			54	33	17
	4				21	18
	5				15	11
	6				10	16
	7				35	12
	8				13	10
	9					10
	10					12
	11					5
	12					8
	13					3
	14					4
	15					2
	16					3
	17					3
	18					1

We used the integrated model to make plots for TCD that show the tradeoff between competition and drive time over multiple seasons. In each case we started with the largest D value in the drive time matrix and reduced D by increments of one minute until the problem became infeasible. Each time a decrease in D caused an increase in p, we recorded the combination of D and p to make the plot. For example, Figure 5 shows that for the fall 2021 season, decreasing the maximum drive from 77 minutes to 57 minutes increases the maximum penalty from zero to 1. Decreasing the maximum drive time from 57 minutes to 47 minutes increases the maximum penalty from 1 to 2, and so on. Figures 6 through 9 show the plots for the spring 2021, fall 2020, spring 2020, and fall 2019 seasons. In our conversations with TCD it was determine that penalties larger than 2 are not acceptable. Figures 5 through 9 indicate that acceptable flights can be obtained for maximum drive times in the range of 44 to 54 minutes.

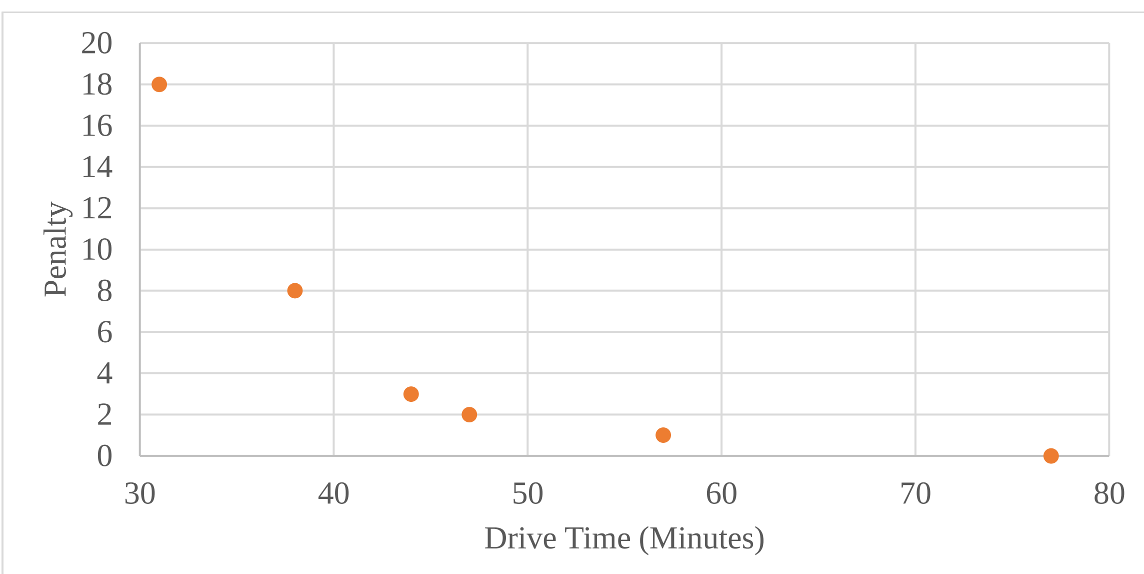

Figure 5 Penalty versus drive time for Fall 2021

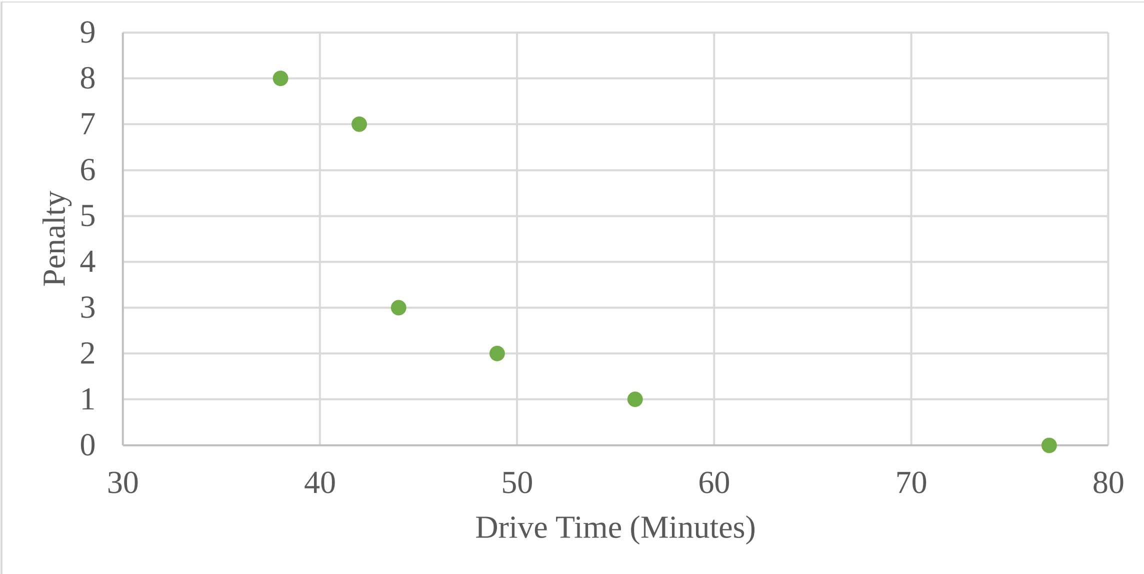

Figure 6 Penalty versus drive time for Spring 2021

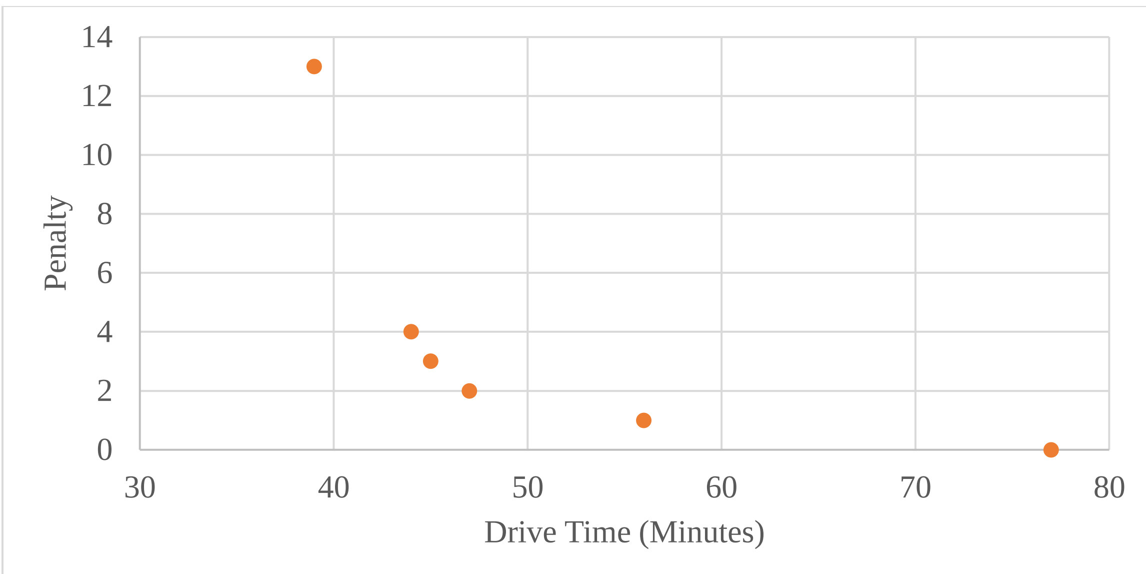

Figure 7 Penalty versus drive time for Fall 2020

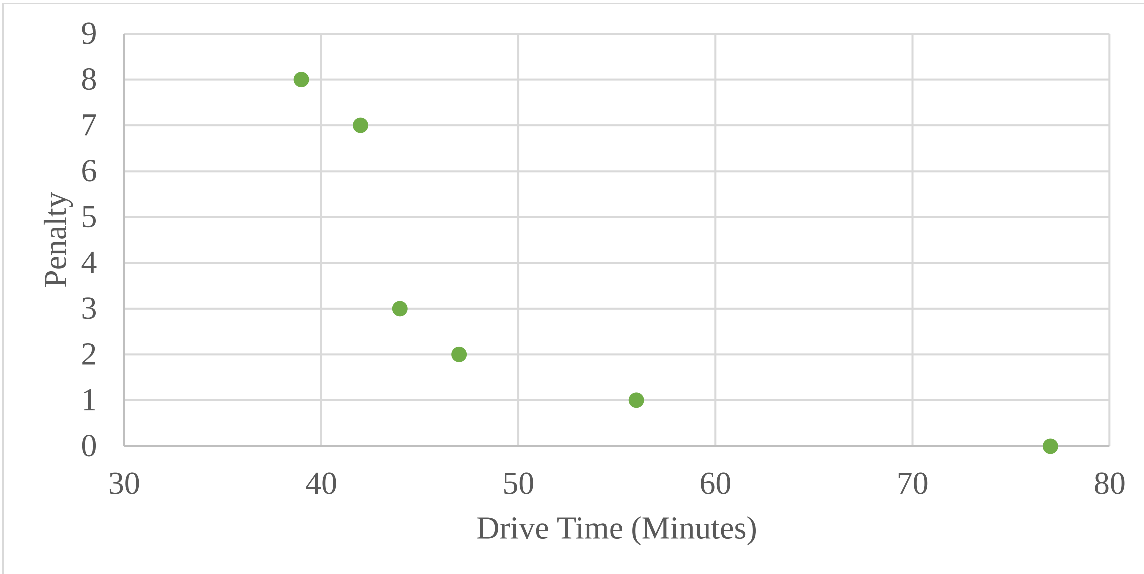

Figure 8 Penalty versus drive time for Spring 2020

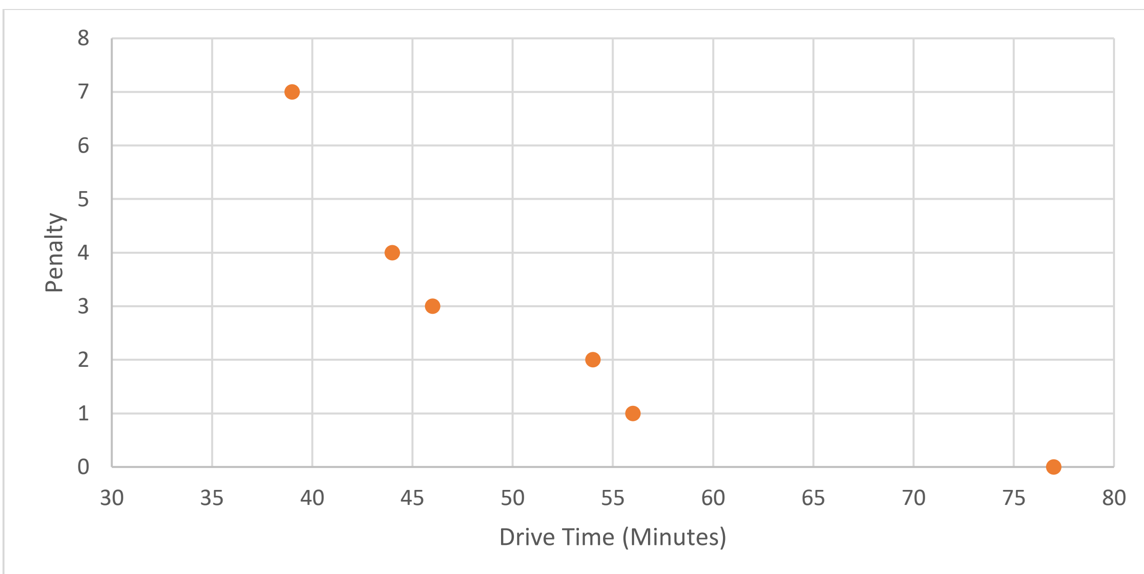

Figure 9 Penalty versus drive time for Fall 2019

Table 11 shows the distribution of penalties for each of the five seasons with D values that produced solutions with a maximum penalty of 1 or 2. Recall that the average penalty using geographic flighting was 1.05, 1.00, 0.88, 0.94, and 1.00 for fall 2019, spring 2020, fall 202, spring 2021, and fall 2021. And the average of the maximum driving times in the geographic solutions was 52, 49, 41, 52.68 and 54.78 minutes for fall 2019, spring 2020, fall 202, spring 2021, and fall 2021, respectively. Since those time are averages, there are some teams in each season with larger maximum driving times in the geography solutions. Thus, we argue that overall, the integrated model produces better solutions than geographic flighting.

Table 11 Number of teams for each penalty for five seasons

Season	**Maximum Drive Time**	**Highest Penalty**	**Number of Teams with of**			**Average Penalty**
			0	1	2	
Fall 2021	57 minutes	1	113	72	/	0.39
	47 minutes	2	51	82	52	1.01
Spring 2021	56 minutes	1	93	76	/	0.45
	49 minutes	2	43	66	60	1.10
Fall 2020	56 minutes	1	96	56	/	0.37
	47 minutes	2	37	70	45	1.05
Spring 2020	56 minutes	1	58	112	/	0.66
	47 minutes	2	34	72	64	1.18
Fall 2019	56 minutes	1	71	107	/	0.60
	54 minutes	2	84	65	29	0.69

4.3 Heuristic

For the ease of TCD's organizers, we introduce a technique that helps solve their problem in drive time and competitiveness without adopting using integer programming. The design relies on using spreadsheets and applying specific rules to the drive time matrices between the home facilities of their tennis teams. Teams are needed to be placed into flights based on their rankings to get it started. The method aims to reduce the longest drive time in each flight by moving teams to either up or down one flight, in such a way that the competitiveness within the flights is sustained.

Teams in all flights are available for swapping in the beginning. An adjacent flight pair is chosen at each iteration, and the heuristic attempts to reduce the maximum drive time in at least one of the two flights by swapping teams causing the maximum drive times in both flights. Division ranking determines which flights are neighbors. For instance, flight 7B is one rank lower than flight 7A, so they are adjacent, and flight 7B is adjacent to flight 8A, etc. An iteration of the heuristic follows the sequence of the steps listed below.

1. Select an available flight that has the largest maximum drive time.
2. Select another available flight that is adjacent to the flight selected in Step 1.

   If there are two choices in this step, **Then**

   > **If** the two flights have different maximum drive times, **Then**
   >
   > Select the flight with the larger maximum drive time
   >
   > **Else**
   >
   > Select the one of the two neighboring flights with the higher rank

3. Let f be the higher ranked flight of the two selected in steps 1 and 2.
4. Let g be the lower ranked flight of the two selected in steps 1 and 2.

5. If it is possible, swap a team in flight f with a team in flight g; this step is described in more detail below.
6. Mark flights f and g as unavailable for swapping.

Step 5 of the heuristic has two objectives. The first one is to reduce the maximum drive time in the selected flights, and the other one is to keep the resulting flights as competitive as possible. For the first objective, Step 5 makes a list, I, of the pairs of teams (row-column entries in the drive time matrix) in flight f that cause the maximum drive time for the flight, and a corresponding list, J, of the team pairs in flight g with maximum drive times. For the second objective, Step 5 makes an ordered list of pairs of teams ($i \in I,\ j \in J$) to be considered for swapping. The pairs of teams are considered in order until an acceptable swap is found, or the list is exhausted (i.e., no swap is possible). Let t_f and t_g be the maximum drive times in flights f and g before the swap and let $t_{\hat{f}}$ and $t_{\hat{g}}$ be the maximum drive times after the swap. A swap is acceptable if one of the two maximum drive times is reduced and the maximum drive time in the other flight is not increased. Mathematically, a swap is acceptable if $\max(t_{\hat{f}}, t_{\hat{g}}) < \max(t_f, t_g),\ t_{\hat{f}} \leq t_f$ and $t_{\hat{g}} \leq t_g$. The next paragraph explains how the list of the swapping team pairs is ordered.

The team numbers used in the notation correspond to the team rankings. For example, team 1 is ranked higher than team 2, team 2 is ranked higher than team 3, etc. A team moved from flight f to flight g in a swap is "demoted" to a lower ranked flight and the team moved in the other direction is "promoted" to a higher ranked flight. Ideally, the teams being selected for the swap from lists I and J are the "closest" in terms of ranking to the flights they are moved into. Therefore, the list order of the team pairs for swapping is determined by taking all pairs ($i \in I,\ j \in J$), sorting the pairs by decreasing order of i (the team from flight f) and then sorting by increasing order of j (the team from flight g). For instance, if $I = \{35, 39\}$ and $J = \{41, 42, 43, 44\}$, then the ordered list of swaps to consider is $1st$ $(39, 41)$, $2nd$ $(39, 42)$, $3rd$ $(39, 43)$, $4th$ $(39, 44)$, $5th$ $(35, 41)$, $6th$ $(35, 42)$, $7th$ $(35, 43)$, and finally $8th$ $(35, 44)$.

Since Step 6 marks both flights unavailable, the heuristic stops after seven or eight iterations. If it is desired, further potential reduction in the maximum drive time for the flights can be executed by repeating the heuristic. Comparing to integer programming approaches, one advantage of the heuristic is that the placement committee can monitor the overall competitiveness of the flights and stop the procedure at any point they like.

Table 11 shows an example of drive time in two adjacent flights. Teams 54, 56, and 57 in flight 9B on the left cause the maximum drive time of 77 minutes, while teams 62 and 69 in the adjacent flight 10A cause the maximum drive time of 71 minutes. Team 56 is the largest number of pairs causing the maximum drive time in its flight, and team 62 in the other flight is closer to flight 9B than team 69. Therefore teams 56 and 62 are chosen to swap to get the new maximum drive times, which are 56 minutes in the new flight 9B and 68 minutes in the new flight 10A.

Table 12 Drive time between teams' home facilities in flights 9B and 10A

Flight 9B	**T. 51**	**T. 52**	**T. 53**	**T. 54**	**T. 55**	**T. 56**	**T. 57**	**T. 58**	**T. 59**	**T. 60**
Team 51		20	25	28	5	54	28	37	20	5
Team 52	21		17	21	21	59	21	44	0	21
Team 53	27	17		28	27	54	28	49	17	27
Team 54	37	25	34		37	77	0	56	25	37
Team 55	5	20	25	28		54	28	37	20	0
Team 56	64	62	59	77	64		77	67	62	64
Team 57	37	25	34	0	37	77		56	25	37
Team 58	36	46	51	42	36	74	42		46	36
Team 59	21	0	17	21	21	59	21	44		21
Team 60	5	20	25	28	0	54	28	37	20	
Flight 10A	T. 61	T. 62	T. 63	T. 64	T. 65	T. 66	T. 67	T. 68	T. 69	T. 70
Team 61		23	42	36	33	14	14	40	56	23
Team 62	33		68	59	52	28	28	52	71	47
Team 63	34	49		33	39	41	41	18	40	23
Team 64	27	42	22		15	28	28	16	39	26
Team 65	27	40	34	16		24	24	33	34	27
Team 66	13	22	49	37	28		0	38	51	29
Team 67	13	22	49	37	28	5		38	51	29
Team 68	28	35	17	16	30	25	25		42	14
Team 69	54	71	44	32	39	51	51	51		59
Team 70	16	29	21	27	27	22	22	18	49	

4.4 Heuristic Result

The heuristic was implemented by hand for each of the five seasons from fall 2019 to fall 2021. Since it was very consuming, the heuristic was stopped after all flights were marked unavailable for swapping; this required 6 to 8 iterations depending on the season. The results are presented in table 13. The heuristic decreased the minimum of the maximum drive time by about 20%; the largest decrease is in the Spring 2021 season. However, there was very little change in the largest maximum drive time values. In practice, we would recommend allowing each flight to be involved in more than one swap before making it unavailable. That would allow more opportunity to reduce the largest drive times. As shown in table 13, the heuristic did decrease the median and mean of the maximum driving times by 1.64% to 13.24% and 3.6% to 12.35%, respectively.

Table 13 Comparison of maximum drive time across five seasons

	Season	Minimum	Median	Mean	Maximum	Iterations
Original	Fall 21	44	68	67.79	77	
	Spring 21	56	68	67.06	77	
	Fall 20	49	61	62.56	77	
	Spring 20	49	61	62.06	77	
	Fall 19	44	67	62.78	77	
Heuristic	Fall 21	39	59	59.42	74	8
	Spring 21	46	64	62.65	77	7
	Fall 20	46	60	60.31	77	6
	Spring 20	46	56	57.38	77	7
	Fall 19	44	62	58.12	74	8
Change	Fall 21	-11.36%	-13.24%	-12.35%	-3.90%	
	Spring 21	-17.86%	-5.88%	-6.58%	0	
	Fall 20	-6.12%	-1.64%	-3.60%	0	
	Spring 20	-6.12%	-8.20%	-7.54%	0	
	Fall 19	0	-7.46%	-7.42%	-3.90%	

CONCLUSION

The three-step process for geographic flighting significantly reduces drive time while maintaining an acceptable level of competition for TCD. Rather than producing a strictly geographic partition of the facilities, our process allows for overlapping regions as shown in Figure 3. A strictly geographic partition like the one illustrated in Figure 2 would be easier for TCD to implement and for its members to understand. However, we found that such a partition is too limiting in terms of competition. The integrated model presented in Chapter 4 gives even more significant improvements in drive time. In particular, it reduces the maximum drive time by over 20% in some cases. However, the user must manually set the limit on drive time and then adjust it for competitiveness if needed. The heuristic is also able to lower the drive time, but it would have to be implemented in code to be practical. It was implemented by hand for a the praxis to valid that concept, which was much more time consuming than running the integer programming models.

There are two areas can be worked on in the future. One direction is to implement the heuristic as an application such as a spreadsheet that uses Visual Basic, or other computer programming languages like Python or Java. Another one is to adapt the models and heuristic to take carbon emissions into account (Demir, Hrušovský, Jammernegg, & Van Woensel, 2019).

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