Probing pairing correlations in Sn isotopes using Richardson-Gaudin integrability

2014 J. Phys.: Conf. Ser. 533 012058

(http://iopscience.iop.org/1742-6596/533/1/012058)

Probing pairing correlations in Sn isotopes using Richardson-Gaudin integrability

S De Baerdemacker[1,2], V Hellemans[3], R van den Berg[4], J-S Caux[4], K Heyde[2], M Van Raemdonck[1,2], D Van Neck[1,2], P A Johnson[5]

[1] Ghent University, Center for Molecular Modeling, Technologiepark 903, 9052 Ghent, Belgium
[2] Ghent University, Department of Physics and Astronomy, Proeftuinstraat 86, 9000 Ghent, Belgium
[3] Université Libre de Bruxelles, PNTPM, CP229, 1050 Brussels, Belgium
[4] Institute for Theoretical Physics, University of Amsterdam, Science Park 904, Postbus 94485, 1090 GL Amsterdam, The Netherlands
[5]Department of Chemistry and Chemical Biology, McMaster University, Hamilton, Ontario, Canada

E-mail: `stijn.debaerdemacker@ugent.be`

Abstract. Pairing correlations in the even-even $A = 102 - 130$ Sn isotopes are discussed, based on the Richardson-Gaudin variables in an exact Woods-Saxon plus reduced BCS pairing framework. The integrability of the model sheds light on the pairing correlations, in particular on the previously reported sub-shell structure.

1. Introduction

Pairing is an important component of the correlations in atomic nuclei at low-excitation energy [1, 2, 3]. The Sn isotopes provide a unique laboratory to probe the neutron-neutron pairing correlations, because the large proton shell gap at $Z = 50$ ensures that the low-lying nuclear structure is largely unaffected by proton particle-hole excitations across the shell gap. Moreover, experimental data of the Sn isotopes in three major shells have become available in recent years thanks to intensive experimental activity with radio-active beam facilities. There exist several theoretical approaches to investigate pairing correlations in atomic nuclei, ranging from fundamental ab initio calculations to studies based on a more phenomenological footing [2]. In the present contribution, we will employ a Woods-Saxon [4] plus level-independent Bardeen-Cooper-Schrieffer (BCS) pairing Hamiltonian [5, 6] as a global probe for pairing correlations in the ground state of Sn. The level-independent, or reduced, BCS Hamiltonian has a complete basis of Bethe Ansatz eigenstates [7, 8], and belongs to the class of Richardson-Gaudin (RG) integrable models [9, 10]. Integrability offers unique opportunities to investigate pairing correlations. On the one hand, the RG variables in the pair-product structure allow for a transparent graphical representation, as well as a clear-cut connection with bosonization approximations [11] via a pseudo-deformation of the quasi spin algebra [12]. On the other hand, physical observables related to particle removal and addition properties [13] can be obtained conveniently using Slavnov's theorem for the RG model [14].

2. Richardson-Gaudin integrability for Sn isotopes

The reduced BCS Hamiltonian is given by [1]

$$\hat{H} = \sum_{i=1}^{m} \varepsilon_i \hat{n}_i + g \sum_{i,k=1}^{m} \hat{S}_i^\dagger \hat{S}_k, \tag{1}$$

with $\hat{S}_i^\dagger = \sum_{m_i>0} (-)^{j_i-m_i} a^\dagger_{j_i m_i} a^\dagger_{j_i -mi}$ the nucleon-pair creation operator in a single-particle level ε_i with (spherical) quantum numbers $(i \equiv n_i, l_i, j_i)$ and of degeneracy $\Omega_i = 2j_i + 1$. This Hamiltonian supports a complete set of Bethe Ansatz eigenstates parametrised by the set of RG variables $\{x\}$ that are a solution of the RG equations [7, 8]. The associated eigenstate energy is then given as $E = \sum_{\alpha=1}^{N_p} x_\alpha + \sum_{i=1}^{m} \varepsilon_i v_i$, with v_i the seniority [15], and N_p the number of pairs.

The single-particle levels are provided by a Woods-Saxon potential [4], for which we used a recent global parametrisation [16], and the single-particle energy spectrum for ^{100}Sn is given in Table 1. We followed a global prescription $g = g_0/\sqrt{A}$ for the pairing interaction, in order to reproduce the 3-point pairing gaps $\Delta^{(3)}(A) = (-)^A[BE(A) - 2BE(A-1) + BE(A-2)]$ [4], presented in Figure 1b. The two-neutron separation energies $S_{2n} = [BE(A) - BE(A-2)]$ [4] are

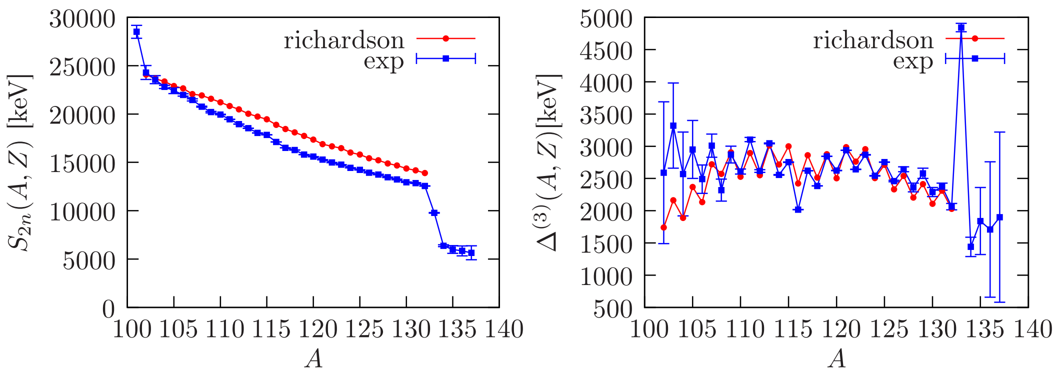

Figure 1. Experimental (squares) and theoretical (circles) two-neutron separation energies S_{2n} (a) and three-point pairing gaps $\Delta^{(3)}$ (b). Experimental data taken from [17].

given in Figure 1a, following a general linear trend, with the exception of a small kink around mid shell, signaling a sub-shell closure. The calculated curve is smoother than the experimental values at this point, consistent with the overestimated pairing gaps $\Delta^{(3)}$ around mid shell. Recent measurements showed a decrease in the $B(E2 : 0_1^+ \to 2_1^+)$ strength around mid shell [18], which was qualitatively attributed [19] to this sub-shell effect in the seniority scheme [15]. Figure 2 depicts the RG variables for the ground state of the even-even $^{102\text{-}130}$Sn isotopes, and sheds more light on the sub-shell structure. Weakly correlated pair states give rise to a clustering of RG variables around the single-particle poles in the complex plane, whereas collective pairing states organise the RG variables along a broad arc in the complex plane [10, 12]. The pairing interaction in the lighter isotopes is strong enough to distribute the RG variables along an arc in the complex plane, however the arc only extends over the $d_{5/2}$ and $g_{7/2}$ sub-shell single particle poles. For the heavier nuclei, the pairs separate into two distinct sets, with seven RG variables clustering around the $d_{5/2}$ and $g_{7/2}$ sub-shell poles and the remaining forming a collective arc around the other poles. For medium-heavy nuclei, there is a gradual transition between both situations. This structure can be quantified using the pseudo-deformation scheme, where all RG variables can be labeled according to their collective behaviour in the Tamm-Dancoff Approximation (TDA) (see Table 1) [12]. From the table, it can be seen that the TDA structure is consistent

Journal of Physics: Conference Series **533** (2014) 012058 doi:10.1088/1742-6596/533/1/012058

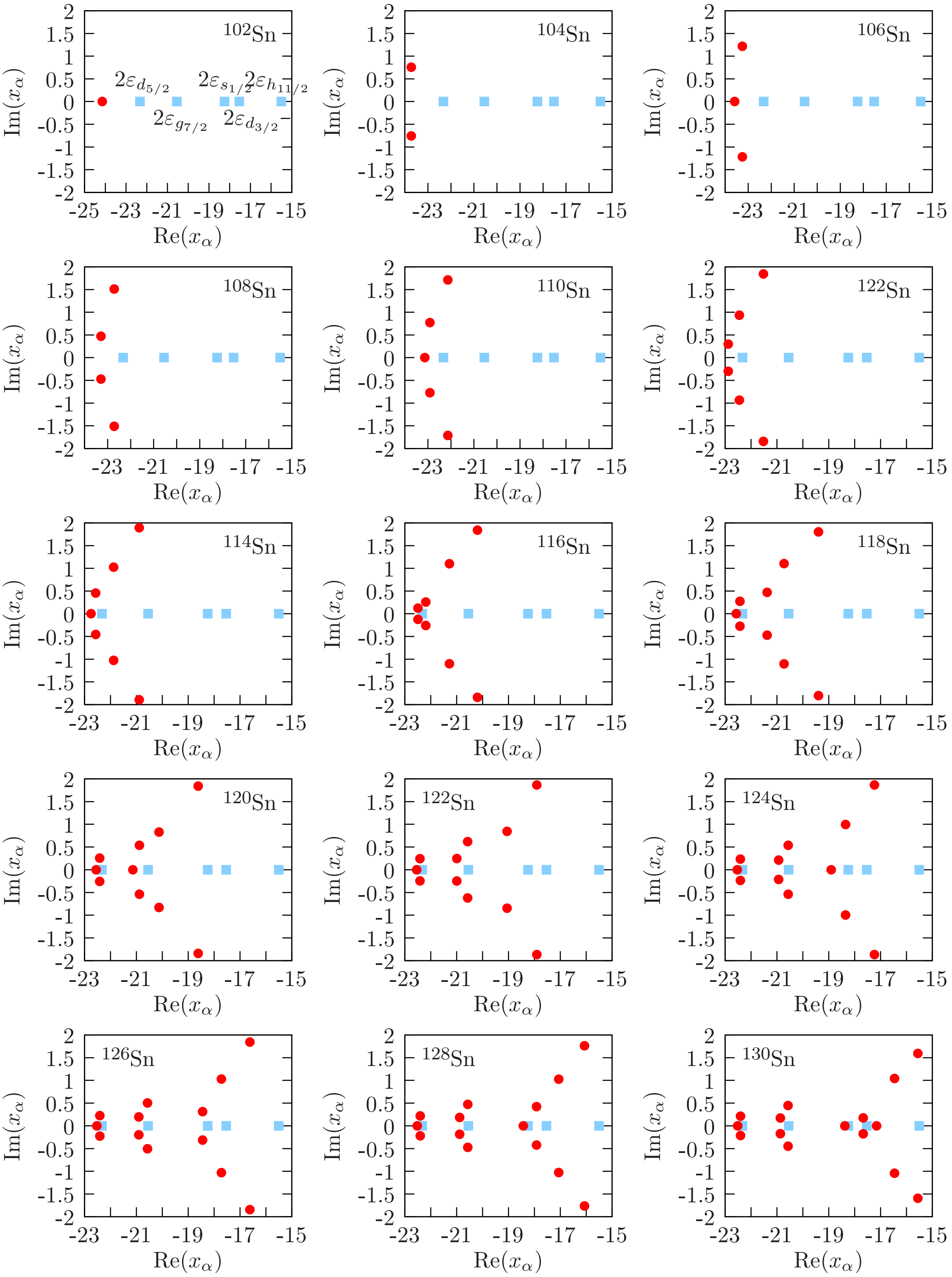

Figure 2. The RG variables (circles) and single-particle poles (squares) of the even-even $^{102\text{-}130}$Sn isotopes.

with the discussed sub-shell structure. The lightest isotopes are consistent with a collective TDA condensation in the $d_{5/2}$ and $g_{7/2}$ sub shell, whereas the TDA structure of the heavier isotopes points towards a normal filling of the $d_{5/2}$ and $g_{7/2}$ sub shell, with the additional pairs collectively distributed over the $s_{1/2}$, $d_{3/2}$ and $h_{11/2}$ sub shell.

level	ε_i [MeV]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$d_{5/2}$	-11.164	1	2	3	4	5	6	7	8	3	3	3	3	3	3	3
$g_{7/2}$	-10.275	0	0	0	0	0	0	0	0	6	7	4	4	4	4	4
$s_{1/2}$	-9.124	0	0	0	0	0	0	0	0	0	0	4	5	6	1	1
$d_{3/2}$	-8.766	0	0	0	0	0	0	0	0	0	0	0	0	0	6	2
$h_{11/2}$	-7.754	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5

Table 1. The single-particle energies ε_i obtained from a Woods-Saxon potential [16], and the TDA eigenmode decomposition of the 0^+ground state for the even-even isotopes $^{102\text{-}130}$Sn. The number of active pairs N_p in the isotope ^{A}Sn is given in the upper row ($N_p = (A - 50)/2$).

3. Conclusions

We have investigated pairing correlations in the Sn isotopes by inspecting the location of the RG variables with respect to the single-particle poles in the complex plane, generated by a schematic Woods-Saxon plus reduced BCS Hamiltonian. The results point towards a sub-shell structure, consistent with previous studies. We expect this structure to also be reflected in the relevant transition rates; this will be investigated in future publications

Acknowledgments

SDB is an FWO-Vlaanderen post-doctoral fellow and acknowledges an FWO travel grant for a "long stay abroad" at the University of Amsterdam (The Netherlands). VH acknowledges financial support from the FRS-FNRS Belgium. This project is also supported by Belspo IAP Grant no P7/12 (SDB, VH, and KH).

References

[1] Heyde K 1994 *The nuclear shell model* (Berlin: Springer-Verlag)
[2] Dean D J and Hjorth-Jensen M 2003 *Rev. Mod. Phys.* **75** 607
[3] Brink D and Broglia R 2005 *Nuclear superfluidity, pairing in finite systems* (Cambridge: University Press)
[4] Bohr A and Mottelson B 1998 *Nuclear Structure, Vol.2* (Singapore: World Scientific)
[5] Bardeen J, Cooper L N and Schrieffer J 1957 *Phys. Rev.* **108** 1175
[6] Bohr A, Mottelson B R and Pines D 1958 *Phys. Rev.* **110** 936
[7] Richardson R W 1963 *Phys. Lett.* **3** 277
[8] Richardson R W and Sherman N 1964 *Nucl. Phys.* **52** 221
[9] Gaudin M 1976 *J. Phys. (Paris)* **37** 1087
[10] Dukelsky J, Pittel S and Sierra G 2004 *Rev. Mod. Phys.* **76** 643
[11] Ring P and Schuck P 2004 *The Nuclear Many-Body Problem* 3rd ed (Berlin: Springer)
[12] De Baerdemacker S 2012 *Phys. Rev. C* **86** 044332
[13] Grasso M, Lacroix D and Vitturi A 2012 *Phys. Rev. C* **85** 034317
[14] Faribault A, Calabrese P and Caux J S 2008 *Phys. Rev. B* **77** 064503
[15] Talmi I 1993 *Simple models of complex nuclei* (Chur: Harwood academic publishers)
[16] Schwierz N, Wiedenhöver I and Volya A 2007 (*Preprint* `nucl-th/0709.3525`)
[17] Audi G, Wapstra A H and Thibault C 2003 *Nucl. Phys. A* **729** 337
[18] Jungclaus *et. al.* 2011 *Phys. Lett. B* **608** 110
[19] Morales I O, Van Isacker P and Talmi I 2011 *Phys. Lett. B* **703** 606