Classification techniques in complex spatial databases. On the assessment of a network of world cities.

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ABSTRACT

In linking the power centers of the world-economy, a network of world cities provides the spatial outline for the reproduction of society as a capitalist world-system. An exploratory analysis of this global urban system is necessary to attain insight in its functioning, but specifications and analyses based on the use of classic data analysis techniques are hampered by the fact that they cannot assess the various sources of vagueness in this complex network of world cities. It is argued that by replacing the premises of the classic two-valued framework of conventional mathematics by a fuzzy set-theoretical approach where degrees of membership are computed rather than a mere assessment of crisp memberships in clusters, the inherent vagueness of possible classifications of world cities can be taken into account. This assertion is tested by comparing the results of some mainstream data analysis techniques (principal component analysis, crisp c-means clustering) to the results of a classification based on the premises of fuzzy set theory (fuzzy c-means clustering).

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Introduction: fuzzy set theory and its applications

The theory of fuzzy sets was formally introduced by Zadeh (1965), and addressed problems in which the absence of sharply defined criteria is involved. In particular, fuzzy sets aim at mathematically representing the vagueness and lack of preciseness, which are intrinsic in linguistic terms and approximate reasoning. As such, through the use of the fuzzy set theory, ill-defined and imprecise knowledge and concepts can be treated in an exact mathematical way (Tzafestas, 1994). However, this fact does not imply that fuzziness is mere ambiguity or stems from total or partial ignorance. Rather, fuzziness deals with the natural imprecision associated with everyday events (Cox, 1994). To illustrate the problem of imprecision in formalising linguistic terms, take, for instance, a simple statement like “John is tall”. Interpreting this statement in the classical two-valued logical framework of conventional mathematics, this would imply that we would have to design a criterion that unambiguously describes a person as either “tall” or “not tall”. However, in reality, such a statement is abundant with vague and imprecise concepts that are difficult to translate in more precise language without losing some of its semantic value. For example, the statement “John’s height is 178 cm.” does not explicitly state whether he is tall, and if we would state that 180 cm. is tall, this does generally not imply that 178 cm. is not to be considered tall. Furthermore, a person can be considered both tall and not tall depending on one’s perspective. Any crisp analysis resulting in disjoint groups fails to grasp this semantic vagueness (Lakoff, 1972; Zadeh, 1972). Fuzzy set theory aims to provide the mathematical underpinnings for the specification of this inherent vagueness. More formally, Zadeh (1965, p. 338) defined a fuzzy set as “a class of objects with a continuum of grades of membership”. Fuzzy sets are characterized by a membership function which assigns to each object of the set a grade of membership ranging from zero (non-membership of the set) to one (full-membership of the set). Apart from the apparent fuzziness in standard linguistic terminology and everyday events, vagueness is also a problem in classification schemes framed upon the unravelling of patterns in large data sets (Bezdek, 1981; Pal & Dutta Majumder, 1986; Bezdek & Sankar, 1992; Pal and Mitra, 1999). This simple and straightforward example, then, is merely a first step to possible broader applications in the field of mathematical assessments of vagueness drawing on the premises of fuzzy set theory.

The purpose of this article is to provide evidence about the assertion that it is possible to account for different sources of vagueness in large geographical databases by using a fuzzy classification technique. The assertion that a fuzzy set algorithm should be able to offer a more sensitive classification than conventional, crisp methods will be empirically tested by comparing results of more classic data analysis techniques (principal component analysis, crisp clustering algorithm) with results obtained by a clustering algorithm based on the premises of fuzzy set theory. The argument will proceed as follows. First, focusing on possible applications in geography, a brief overview of the premises of both types of classifications is provided in order to distinguish clearly between crisp and fuzzy classifications. Second, a description of the database on the network of world cities, as constructed by the Globalization and World Cities Study Group and Network (GAWC), will be provided. Special attention will be given to theoretical and practical sources of vagueness related to classification analyses in this database. This database on relations between world cities is useful for our analysis for three reasons:

1. Any classification scheme based on the database on world cities should take into account the fact that patterns will never be clear-cut, since the network of world cities is characterized by complexity rather than by a simple hierarchy (Taylor *et al*. 2001a; Sassen, 2000)

2. A great deal of information in this database rests on sparse data, yielding vagueness in *any* classification (Beaverstock *et al*., 1999; Taylor *et al*., 2001b).
3. Some "classical" data analysis techniques (principal component analysis) have been applied on this database (Taylor *et al*., 2001b), providing us the opportunity to assess possible advantages and disadvantages of the use of a fuzzy set-algorithm.

The outset and results of the fuzzy clustering algorithm will be preceded by the outset and results of the associated crisp clustering algorithm. This enables us to show the methodological differences between both approaches, as well as providing additional results that can be compared.

Crisp and fuzzy classifications in geography

The main purpose of unsupervised classification (clustering) of a set of objects is to detect subgroups (clusters) based on similarity or dissimilarity between objects. There are many different approaches to clustering depending on the definitions and interpretation of these subgroups, and each of them may give a different grouping of a dataset. The choice of a particular method will depend on the type of output desired, the known performance of method with particular types of data, and the size of the dataset. For instance, clustering methods may be divided into two categories based on the cluster structure they produce. Non-hierarchical methods divide a dataset into disjoint clusters, whereas hierarchical methods produce a set of nested clusters in which each pair of objects or clusters is progressively nested in a larger cluster until only one cluster remains. The choice of either of these two techniques in this instance, then, depends primarily on the form of the desired output (Kaufman & Rousseeuw, 1990; Everitt *et al*., 2001).

Although hierarchical and non-hierarchical algorithms are two distinct approaches towards the classification of objects, they both share one essential feature: any partition of a set of n objects results in mutually exclusive clusters. In the case of non-hierarchical clustering, the state of clustering is expressed by an n x C matrix $\mathbf{U}=(u_{ic})$, where $u_{ic}=1$ if object i belongs to the cluster c, otherwise $u_{ic}=0$. To ensure that the clusters are disjoint and non-empty, u_{ic} must then satisfy the following conditions (Sato *et al*., 1996):

$$\sum_{c=1}^{C} u_{ic} = 1 \qquad [1]$$

$$u_{ic} \in \{0,1\} \qquad [2]$$

for

$$i = 1,...,n$$
$$c = 1,...,C$$

This classification scheme has certain distinct advantages. For one thing, results are clear-cut, and possible cumbersome interpretations of in-between values are expelled from any analysis since there is no overlap in cluster membership. When applied to the classification of regions or countries based on certain criteria, this fact implies that the only admissible spatial boundaries are unambiguous ones (MacMillan, 1995; e.g. Dezzani, 2001; Arrighi & Drangel, 1986; Van Rossem, 1996). Any location is either entirely situated in a region or a country, or it is not. As a consequence, interpretation of the clustering results is straightforward.

In some cases, however, it is not expected that classifications will be clear-cut. As Leung (1987, p. 125) points out, "regions are fundamental analytical units on which most spatial analyses are based. Conventionally, a region is treated as a spatial construct which can be precisely identified and delimited”. However, “…regions may not be precisely identified and boundaries generally exist as zones rather than lines”. In addition to this inherent vagueness in classifications, the clustering of objects based on sparse data is another source of vagueness with respect to the classification of locations.

A possible solution for this problem lies in the use of a fuzzy set-theoretical approach to clustering; that approach discards the unambiguous mapping of the data to classes and clusters, and instead computes degrees of membership specifying to what extent objects belong to clusters. If $u_{ic} \in \{0,1\}$ in [2] is replaced by

$$m_{i_{ic}} \in [0,1] \qquad [3]$$

then the clustering result is more sensitive to vagueness in classifications (Sato *et al*., 1996). In using a crisp clustering algorithm, minor shifts in the data may yield a completely different outcome although the basic pattern in the data may in fact remain pretty much the same. In a fuzzy framework, all places may have a membership in *any* region. In classifying regions where it is more natural to treat them as transient regions between any two areas as fuzzy domains in which the degree of fairness, the cases having almost the same profile or pattern and the gradual change between sample spaces are in fact the expression of fuzziness (Leung, 1987; Rolland-May, 1986; Harris *et al*., 1993).

Since its original outset, fuzzy set theory has been employed in many areas to simulate and manage vague information (Höppner *et al*., 1999). Obviously, these vagueness problems also apply to large geographical databases. MacMillan (1995) has pointed out that fuzzy thinking has been around in geography for as far back as the 1970s. MacMillan himself (1978) and Gale (1972a, 1972b) applied fuzzy set theory with respect to locational decision-making and behavioural geography. However, “at that stage, it did not become fashionable in geographical circles (…)” (MacMillan, 1995, p. 404). More recent examples of applications of the use of fuzzy sets in geography can be found in the domains of spatial analysis (e.g. Leung, 1987; 1988), site selection (e.g. Witlox, 1998), and land-use planning (e.g. Smith, 1992; Xiang *et al*., 1992). Although there are, then, quite a few examples of the use of fuzzy set theory, research topics and methodology issues relying on the use of fuzzy set theory are as yet not a part of mainstream geography. Furthermore, the outset of the basic premises of fuzzy set theory itself was merely the start for myriad studies leading to an explosive growth of both the original core ideas and possible extensions, such as research of expert knowledge systems and neural networks. Possible applications for geographers, then, are not limited to the application of the basic ideas. A whole range of new methods and applications are available now.

One of the major advantages of the use of a fuzzy set-theoretical approach lies in the fact that it is possible to capture various aspects of vagueness (Everitt *et al*., 2001). For instance, fuzzy sets can at the same time capture vagueness due to the sparsity of data and vagueness due to the lack of theoretically defined pre-existent categories. Hence, a minor shift in the data does not necessarily result in a major shift of the classification of in-between values. Rather, a minor shift in the dataset will be reflected by minor changes in membership degree, allowing for a more sensitive approach of the classification scheme. In general, four of the main useful features of fuzzy set methodologies are (Höppner *et al*., 1999; Chi *et al*., 1996):

(i) Fuzzy set theory provides a systematic basis for quantifying vagueness due to incompleteness of information;
(ii) Classes with unsharp boundaries can be easily modelled using fuzzy sets;
(iii) Fuzzy reasoning is a formalism that allows the use of expert knowledge, and is able to process this expertise in a structured and consistent way;

(iv) There is no broad assumption of complete independence of the evidence to be combined using fuzzy logic, as required for probabilistic approaches.

Features and specification of the network of world cities

Urban geographers have long sought to unravel and describe the systematic nature of the spatial arrangement of urban centers. The original outline of Christaller's central place-theory (1933) and Lösch's extensions of this central place-theory (1954) are but two classic examples of such an endeavour. Most of the studies oriented towards the description of the spatial arrangement of such an urban system inherit their physical boundaries from their definition as an integrated economy. Since the beginning of the twentieth century, the world-economy is truly global (Wallerstein, 1983), and hence all cities can be thought of as participating in a single urban system in a Christallerian sense. This global urban network should then theoretically be characterised by functional specialisations as predicted by the spatial optimization processes described by central place-theory.

Although the original outline of central place-theory may still do a reasonably good job in describing the spatial pattern of urbanization on a regional scale or in assessing the location of some service and retail industries at a regional scale, it is not suited to explain patterns of global urbanization. At the most basic level, there are at least four (heavily intertwined) alterations that should be taken into account with respect to the assessment of a global urban network:

(i) The original hierarchy needs to be supplemented by some additional levels (Hall, 2001);

(ii) The combined effect of an ever-increasing globalization and a shift from capitalist production primarily based on manufacturing to a capitalist system focused on knowledge production, suggests that there are new and previously unassessed central place functions in place. This holds especially true with reference to the additional global levels of urbanization (Sassen, 2000);

(iii) Under contemporary globalization, cities are increasingly defined by mutual relations in spaces-of-flows, rather than by relations to their immediate hinterland (Castells, 1996).

(iv) The presumed equivalence between hierarchical position in the urban system and central place functions seems to be altered due to functional specializations among cities (Sassen, 2000).

This extremely brief overview of the most salient features of a global network of world cities has a profound impact on the assessment of this urban system. Clearly, an analysis of this network should concentrate on flows between cities (Smith and Timberlake, 1995; Castells, 1996). Moreover, the flows generated by the spatial strategies of advanced producer services are crucial determinants in this overall space of flows where world cities act as nodes in a complex network (Sassen, 2000). However, irrespective of these theoretical underpinnings on the importance of both (i) relational data and (ii) the role of advanced producer services in these relations, a more precise and practical specification of this network of world cities is obvious. For without such a specification, there can be no detailed study of its nodes, the connections, and how these connections and nodes constitute an integrated whole (Taylor *et al*., 2001b).

This need for the construction of geographical databases focusing on relations between world cities has been recognized from the very beginning of world city-research (Smith and Timberlake, 1995), but the construction itself has been hampered by methodological problems. This is due to the fact that the bulk of information on cities is attributional data (Short *et al*., 1996). Hence, although all of the definitions and specifications of a network of world cities should be premised upon the existence of worldwide transactions, most of recent research efforts on cities have been centred on studying the internal structures of individual cities and comparative analyses of these cities (Taylor, 1999). Moreover, some earlier attempts towards a specification of the relational character of the network of world cities have remained “ambition rather than reality” (Taylor *et al*., 2001), resulting in *ad hoc* classifications (Friedmann, 1986; Knox, 1995; Sassen, 2000), often limited to the highest ranks in the hierarchy (e.g. Sassen, 1991). The overall aim of the Globalization and World Cities Study Group and Network (GaWC) has been to provide data and research on the relational character of world cities. Arguably one of the most important accomplishments of GaWC was Taylor’s (2001) specification of the world city network, by outlining the construction of connectivity matrices based on data on the presence of advanced producer firms in world cities (Beaverstock *et al*., 1999; Table 1). Connectivities as measure of flows between world cities were derived for each pair of world cities by applying a specific kind of network analysis. Using a specific kind of network analysis was deemed necessary because the nodes in this network (the world cities) are in fact connected by constituent subcomponents (global service firms). That is, although world cities are the formal nodes in this network, they are by themselves at best modest actors in the flows in this network. World cities are only perceived as nodes in that they harbor advanced producer firms that are connected in a complex web of flows. The network of world cities as an interlocking network characterized by boundary penetration relations is defined at two levels: a system-level where the network operates (the network of world cities), and a unit-level consisting of the nodes as actors whose behaviour define the relations (global service firms). Drawing on the formal outline by Knoke and Kuklinski (1982), connectivity measures were derived by computing the sum of the cross products of all of the firms for any pair of cities. These sums reflect the similarity between the cities in terms of global services, and can hence be thought off as a surrogate for particular flows of information and knowledge between the cities when two assumptions are made. First, offices generate more flows within a firm’s network than to other firms in the sector. Although not formally empirically tested, this assumption is plausible, for flows of information and knowledge are indispensable for a seamless service. Second, the larger the office, the more flows will be generated, which will have a multiplicative effect on inter-city relations (Taylor, 2001; Taylor *et al*., 2001c).

To summarize, data on the presence of global service firms in cities (55 cities x 46 firms, Table 1) has been used to derive measures of inter-locking connectivity between cities, resulting in indices of network connectivity, where positions of cities within the world city network can be assessed (55 cities x 55 cities, Table 2). The resulting matrices with connectivities between cities then give way “to various forms of analysis available to simpler types of network. This means the wide repertoire of network techniques from elementary derivation of indices to scaling, ordinating, factoring, clustering and blocking” (Taylor, 2001, p. 192). It is the purpose of this article to complement the specification of this unusual network by a non-classic approach to data analysis.

Exploratory analyses of the network of world cities using 'standard' classification techniques

A Hierarchical classification

GaWC-researchers themselves have undertaken efforts to apply some 'standard' techniques to their data. First, in search for a roster of world cities, Beaverstock *et al*. (1999) identified three hierarchical levels of world cities. Based upon the scores in four global service centers (advertising, banking, accountancy and legal services), 10 Alpha world cities, 10 Beta world cities, and 35 Gamma world cities were identified (Table 3). The initial database consisted of 123 cities, but only 55 cities were classified as world cities. A city was designated as a world city if it served as a global service center for at least two sectors, where at least one of those sectors could be designated as a major service provider. The remaining cities, then, were merely showing evidence of world city formation processes, but this evidence was not strong enough to really call them world cities.

A classification based on principal component analysis

Another classification was provided by Taylor *et al*. (2001b), in applying an exploratory research design using principal component analysis. Principal components analysis (PCA) is a member of the factor-analytic family of multivariate techniques, commonly used to define patterns of independent sources of variation in a data matrix. As such, they are a popular means of producing parsimonious descriptions of large and complex sets of data. It is important to note that the application of this PCA-analysis on world cities was used as an exploratory rather than a confirmatory research design. This choice for an exploratory research design stemmed from the fact that there are 'uncertainties' in the application of the factor analytic family of techniques, and the fact that the world city-network seems to be a complex network rather than a simple hierarchy (Taylor, 2000; Friedmann, 1986). This exploratory research design, then, resulted from a positive approach towards vagueness: the creation of alternative results provides a means for exploring a set of data. Instead of searching for some sort of ideal classification, a multiple-number design allowed for the comparison of results over a range of levels of data reduction (Yates, 1987).

Factor allocation for two components resulted in the identification of two groups of world cities ("Inner Wannabes" versus "Outer Wannabes", Table 4). The generic names of these clusters of cities were derived from the fact that these cities invariably have policies helping them strive for world city status (Short *et al*., 2000). The labelling of these two "wannabe" categories was quite straightforward. Cities with high loadings on the first component were situated in what used to be called the 'third world', plus eastern European cities and some more peripherally located cities in Western Europe, notably in the far south (Mediterranean and Iberian cities) and far north (Scandinavian cities), hence the designation as "Outer Wannabes". Cities with high loadings on the second component were termed "Inner Wannabes", since they are primarily relatively minor US cities plus the 'second cities' in western European countries (Manchester, Birmingham, Barcelona, Lyon, Rome and Rotterdam), and second cities in selected associated countries (Montreal, Melbourne, Cape Town, Rio de Janeiro and Abu Dhabi). Unallocated cities in this analysis cover all parts of the world, but they share one notable feature: they are the major world cities (in the previous allocation termed as Alpha en Beta world cities). Next to this dichotomization of the data, a PCA with 5 and 10 components was applied, yielding new classifications in 'outer cities', 'US cities, 'Pacific-Asian cities', 'Euro-German cities' and 'Old Commonwealth Cities' (Table 5). To summarize, whereas Beaverstock *et al*. (1999) provide a hierarchical classification, Taylor *et al*. (2001b) were able to discern a classification based on a spatial pattern reflecting functional specializations in the network of world cities.

A classification based on a crisp clustering algorithm

Cluster analysis is a rather loose collection of multivariate statistical methods that seek to organize information on variables so that relatively homogenous groups can be formed. All members belonging to the same group or cluster have certain properties in common. Hence, the resultant classification may provide some insight into the data. The classification has the effect of reducing the dimensionality of a data table by reducing the number of rows (cases). The aim of a classical crisp cluster analysis is thus to partition a given set of data or objects into clusters (subsets, groups, classes), with the following properties (Everitt *et al*., 2000):

- Homogeneity within the clusters: data belonging to the same cluster should be as similar as possible.
- Heterogeneity between clusters: data belonging to different clusters should be as different as possible.

The classification of the data is based upon a measure of dissimilarity between the different data points in the matrix. The Euclidean distance is the most simple and common measure of dissimilarity. However, one should consider the fact that (i) different variables as constituent components of the classification analysis may be of different relevance for the classification, and (ii) the range of values should be suitably scaled in order to obtain reasonable distance values (Kaufman & Rousseeuw, 1990). Generally, the second problem can be accounted for by using standardized data (z-scores), for this yields a "unit free" measure. However, since we use connectivity measures that were derived using a singular method and based on real-valued vectors bearing the same meaning (Table 1), this is of no concern here.

Apart from the overall general method (i.e. cluster analysis), one has to choose a particular clustering algorithm. This choice depends both on the type of data available and on the particular purpose (Chi *et al*., 1996). The clustering algorithm that will be used here is a c-means clustering algorithm. A formal specification of this method will be outlined in order to highlight the differences with its fuzzy counterpart. This c-means partitioning method constructs clusters that satisfy the standard requirements of a crisp partition:

- Each group must contain at least one object (no empty clusters).
- Each object must belong to exactly one group (exclusivity of the assignment to a cluster).

Both conditions imply that the maximum number of clusters (*C*) cannot be greater than the number of objects to classify (*n*), hence *C* ¡Ü*n*. The second condition also implies that two different clusters cannot have any objects in common and that the *C* clusters together add up to the full data set. Defined more formally, the outset of the crisp clustering problem can be stated as follows (Chi *et al*., 1996).
Let:

$$X = \{x_1, x_2, ..., x_n\} \qquad [4]$$

be a set of samples to be clustered into *C* classes. The clustering process can be considered as an iterative optimization procedure. Suppose that the samples have already been partitioned into *c* classes, be it by random assigning the data points to clusters or through theoretical considerations on potential clusters. The task at hand, then, is to adjust the partition so that the similarity measure (based on the Euclidean distance) is optimized. The criterion function for this optimization procedure is equal to:

$$J(V) = \sum_{k=1}^{n} \sum_{x_k \in C_i} |x_k - v_i| \quad [5]$$

where v_i is the center of the samples in cluster *i*, and

$$V = \{v_1, v_2, ..., v_c\} \quad [6]$$

In order to improve the similarity of the samples in each cluster, we can minimize this criterion function so that all samples are more compactly distributed around their cluster centers. Setting the derivative of **J(V)** with respect to v_i to zero, we obtain

$$\frac{\partial J(V)}{\partial v_i} = \sum_{k=1}^{n} \sum_{x_k \in C_i} (x_k - v_i) = 0 \quad [7]$$

Thus, the optimal cluster center of cluster center v_i is

$$v_i = \frac{1}{n_i} \sum_{x_k \in C_i} x_k \quad [8]$$

where n_i is the number of samples in class *i* and C_i contains all samples in class *i*.

Starting with the initial clusters and their center positions (be it randomly chosen or initially assigned), the samples can now iteratively be regrouped so that the criterion function **J(V)** is minimized. Once the samples have been regrouped, the cluster centers need to be recomputed to minimize **J(V)**. This process then continues for the new cluster centers: the samples are regrouped in order to reduce **J(V)** yielding a new classification with associated cluster centers, and so forth. This iterative process can be repeated until **J(V)** cannot be further reduced or drops below a pre-defined small number ǎ. Obviously, the criterion function is minimized if each sample is associated with its closest cluster center. This means that x_k will be reassigned to cluster i so that $(x_k-v_j)^2$ is minimum when $j=i$. Up to this point, each sample x_k appears only once, that is, it is associated with only one cluster center.

Note that we subscribe to an exploratory rather than a confirmatory research design: we are not looking for a 'best result', rather, the fact that very different results can be found in using a different number of clusters is perceived as the most fruitful approach towards uncertainty in the resulting classifications (Yates, 1987). Here, we shall describe the clustering results for 2, 4 and 8 clusters. In the case with two clusters (*c*=2, Table 6), we note that there is a strong dichotomy between the cities with a high connectivity versus cities with a lower connectivity. Hong Kong, London, Los Angeles, New York, Paris, Singapore and Tokyo are all assigned to the first cluster, all of the other world cities are assigned to the second cluster. All world cities belonging to the first cluster are identified by Beaverstock *et al.* (1999) as Alpha world cities. Only Milan, Frankfurt and Chicago are Alpha world cities that are not classified in the first cluster. However, this is not a surprise when compared to the results of Beaverstock *et al*. (1999), since these cities are found amongst the lower ranked Alpha world cities.

The clustering result for four clusters (*c*=4, Table 7) reveals two clusters containing a subset of the most important Alpha world cities, and two clusters containing the rest of the world cities. The rather odd appearance of a cluster only consisting of Los Angeles and Washington DC may be traced back to the concentration of law firms in Los Angeles and Washington DC, whereas the other cluster containing Alpha world cities is characterised by a concentration in banking and finance services. This corresponds to Sassen's (2000) expectations on functional specializations among American world cities. Taylor *et al*. (2001b; 2001c) were able to define a spatial pattern in their 5-component cut, but the crisp cluster analysis fails to do so. Both clusters 3 and 4 include European cities, cities from the semi-periphery of the world-economy, and a number of American cities, hence no apparent spatial pattern can be discerned.

Arguably the most interesting results were found with the application of the algorithm for eight clusters (*c*=8, Table 8). It shows both the possibilities and the restrictions of the crisp clustering algorithm when applied to the network of world cities. An apparent spatial pattern in the connectivities can be observed. North American cities, German cities and European cities around the old European core form a cluster, Latin American cities and cities in the old European core are assigned to another cluster. The Pacific-Asian world cities have similar connectivities and are, hence, assigned to another cluster (due to their similar relative strength in banking services). However, as in the classification provided by Taylor *et al*. (2001b; 2001c), the classification of some cities (e.g. Johannesburg, Osaka, Toronto, Warsaw) remains open to interpretation.

Summary: classifications based on classical two-valued logic (figure 1)

Exploratory research resting on the application of principal component analysis and cluster analysis clearly reveals some basic patterns in the large and complex data matrices on world cities. However, the use of these standard techniques, although often revealing and promising, still leaves way for additional analysis, i.e.:

(i) The classification of some cities rests on the fact that they are not allocated to any of the components (Taylor, 2001b). As such, the only similarity they bear is the fact that the retrieved factors cannot explain the observed variance in the observed patterns for these unallocated cities.

(ii) The first GaWC classification (Beaverstock *et al*., 1999) assesses a hierarchical classification, whereas the second GaWC classification (Taylor *et al*., 2001b; 2001c) and the crisp clustering algorithm primarily assess spatial patterns. A classification that assesses both functional and hierarchical tendencies, however, would provide some major advantages.

(iii) The original intention of Beaverstock *et al*. (1999) was to account for the bottom end of the scale of the roster of world cities, where uncertainty in classifications reigned.

Classification of this “grey area” under the clearly discernible higher rungs of the global urban hierarchy, however, merely resulted in the conceptualisation of world cities in the “dark grey area”. Cities were dropped from the analysis (316 to 123 (sometimes even to 55)) because of the sparsity of the data. Although the classification of these cities is a huge step forward as compared to the previous *ad hoc* classifications (Friedmann, 1986; Friedmann & Wolff, 1982) and the focus on the top end of the hierarchy (Sassen, 1991), it is still far from complete. The uncertainty due to the sparsity of data, however, tends to prevent the classification of cities only showing weak signs of world city formation.

Fuzzy c-means clustering algorithm

Methodology

In the classical crisp clustering process, each city is assigned to only one cluster and all clusters are regarded as disjoint gatherings of the data set. However, previously, it was argued that the network of world cities constitutes a distinctively non-hierarchical urban structure (Taylor, 2001, p. 192). In other words, the global urban hierarchy of world cities is a complex network system rather than a simple hierarchy. Although the first two ranks stand out (London and New York), this urban system is not a so-called “double-primate” city pattern.

There may or may not be hierarchical patterns within the spatial organisation of individual firms at the global scale (depending on their particular strategies), but when aggregated the result is a world city network. It is therefore unlikely that classical, disjoint clusters resulting in clear-cut patterns will be able to provide the most salient results. From both a methodological and a theoretical point of view, it is hardly acceptable that a crisp classification process cannot cater for such a situation. Therefore, we propose to replace the separation of the clusters by a fuzzy notion, in order to represent the real data structures more accurately. The criterion function for the crisp clustering algorithm in [5] is replaced by a fuzzy notion (Chi *et al*., 1996; Höppner *et al*., 1999; all drawing on the seminal work by Bezdek, 1981), based on the iterative minimization of

$$J(U,V) = \sum_{i=1}^{c}\sum_{k=1}^{n} u_{ik}{}^{m} \left| x_k - v_i \right|^2 \qquad [9]$$

where

- $x_1, x_2, \ldots, x_n$ are n data sample vectors
- $\mathbf{V}=\{v_1, v_2, \ldots, v_n\}$ are cluster centers
- $\mathbf{U}=[u_{ik}]$ is a Cxn matrix, where u_{ik} is the ith membership value of the kth input sample x_k,

and the membership values satisfy the following conditions

$$0 \le m_{ik} \le 1$$

$$\sum_{i=1}^{C} m_{ik} = 1$$

$$0 < \sum_{i=1}^{C} m_{ik} < n$$

for $i=1,2,\ldots,C$ and $k=1,2,\ldots,n$.

- $m \in]1,\infty[$ is an exponent weight factor. This weight factor m reduces the influence of small membership values. The larger the value of m, the smaller the influence of samples with small membership values in the optimization procedure outlined below.

The altered objective function is the sum of the squared Euclidean distances between each input sample and its corresponding cluster center, with the distances weighted by the fuzzy memberships. The algorithm is iterative and makes use of the following equations:

$$v_i = \frac{1}{\sum_{k=1}^{n} m_{ik}{}^{m}} \sum_{k=1}^{n} m_{ik}{}^{m} x_{ik} \qquad [10]$$

$$m_{ik} = \frac{\left(\frac{1}{\left| x_k - v_i \right|^2} \right)^{1/(m-1)}}{\sum_{j=1}^{c} \left(\frac{1}{\left| x_k - v_j \right|^2} \right)^{1/(m-1)}} \qquad [11]$$

For the calculation of a cluster center, all input samples are considered in accordance with their membership value. For each sample, its membership value in each cluster depends on its distance to the corresponding cluster center. Following Chi *et al*. (1996), the clustering procedure consists of the following steps:

1. Initialize $\mathbf{U}^{(0)}$ randomly or based on an approximation (for instance, the results of the crisp c-means clustering) by initializing $\mathbf{V}^{(0)}$ and calculating $\mathbf{U}^{(0)}$. The iteration counter α is set to 1, and the number of clusters C and the exponent weight m are chosen.

2. Using the criterion function, the cluster centers ($\mathbf{V}^{(\alpha)}$) can be computed based on the values of the membership values ($\mathbf{U}^{(\alpha)}$).

3. The membership values ($\mathbf{U}^{(\alpha)}$) are then updated based on the new cluster centers ($\mathbf{V}^{(\alpha)}$). This iteration is stopped if $\max | u_{ik}^{(\alpha)} - u_{ik}^{(\alpha-1)} | \leq \varepsilon$, else let $\alpha = \alpha + 1$ and go to step 2, where ε is a pre-specified small number representing the smallest acceptable change in $\mathbf{U}^{(\alpha)}$.

Note that the crisp c-means clustering algorithm can be considered as a special case of the fuzzy c-means clustering algorithms. If u_{ik} is 1 for only one class and zero for all other classes in equation [11], then the criterion function **J(U,V)** used in the fuzzy c-means clustering algorithm is the same as the criterion function **J(V)** used in the crisp c-means cluster algorithm. This is the so-called extension-principle.

Classifications of world cities based on the fuzzy c-means clustering algorithm

Again, in our attempt to provide an alternative classification approach based on fuzzy set-theory, we subscribe to an exploratory research design: there is no definitive way as to the number of clusters we are likely to expect in the data matrix. Therefore, any number of clusters can yield a result that has some interesting conclusions. For two clusters (c=2, Table 9), the results are straightforward. When thresholds are placed on a membership degree of greater than 0.75 and in the interval [0.3-0.75] in the first cluster, we get two cuts of ten world cities comparable to the results of Beaverstock *et al*. (1999). Nine of the ten world cities originally described as Alpha world cities have a membership degree exceeding 0.75 in the first cluster. The only difference is Chicago and Sydney changing places. A minor difference, since Chicago was ranked as one of the lower ranked Alpha world cities, whereas Sydney was originally ranked as one of the higher Beta world cities. Apart from the Chicago/Sydney switch, three of the world cities ranked in the [0.3-0.75] interval are not identified as Beta world cities by Beaverstock *et al*. (1999). Three semi-peripheral cities (São Paulo, Mexico City and Seoul) are replaced by two American cities (Miami and Washington DC) and Taipei. Again, the replaced cities were among the lower ranked Beta world cities, whereas the replacing cities (except for Miami) are to be found in the higher ranks of the Gamma world cities. In short, our results are consistent with the results of Beaverstock *et al*. (1999), since only a few cities located at the edge of the initial classification change their position in the classification based on the fuzzy c-means algorithm.

Computing membership degrees for four clusters (c=4), we can distinguish among several groups. World cities with high membership degrees in the second cluster (>0.75) are exclusively world cities situated in the Pacific-Asian part of the semi-periphery of the world-economy: Seoul, Shangai, Bangkok, Bejing, Jakarta, Kuala Lumpur, Manila and Taipei. This fact indicates that all these cities show a remarkable resemblance in their connectivity profiles. This classification resembles the third category (Pacific-Asian cities) provided by Taylor *et al*. (2001b, Table 8). In contrast with the classification provided by Taylor *et al*. (2001b), Tokyo and Hong Kong are not assigned to a cluster of Asian-Pacific cities, since their highest membership degrees are primarily found in a cluster representing the Alpha world cities: *all* cities scoring > 0.7 in the third cluster are identified by Beaverstock *et al*. (1999) as Alpha world cities. In addition, Tokyo also scores 0.27 in the second cluster. This score means that Tokyo's connectivity profile bears both (i) strong resemblance to that of other Alpha world cities and (ii) some significant (though less strong) resemblance to the Asian-Pacific cluster. This observation implies that this classification scheme is more sensitive towards interpretations, since it provides us with the possibility to discern world cities that have some sort of in-between profile. On the other hand, this classification grasps both hierarchical tendencies (third cluster: Alpha world cities) and functional connectivity patterns (second cluster: Asian-Pacific cities).

Other spatial patterns are found when assessing the membership degrees in the first and the fourth cluster: all Latin American world cities (Buenos Aires, Mexico City, São Paulo, Santiago and Caracas) score >0.8 in the first cluster, whereas most US cities (Atlanta, Boston Dallas, Houston, Minneapolis and Montreal) score high in the fourth cluster. The European cities are scattered mostly over two clusters, with a concentration of German cities in one group, bearing resemblance with the classification provided by Taylor *et al*. (2001b).

Some cities are very hard to classify (e.g. San Francisco's minimum membership degree is 0.1942 and its maximum membership degree is 0.3476), while other cities seem to be 'hanging' in-between two clusters, yielding additional interesting profiles. For instance, Melbourne and Sydney have a very fuzzy profile, yielding memberships of about 0.4 in both the second cluster (Asian-Pacific world cities) and the first cluster (Latin American world cities). Rather than bearing solely resemblance with Asian-Pacific world cities, as would be the case in classifications based on a two-valued logic, Melbourne and Sydney have a connectivity profile in-between that of Latin American world cities and Asian-Pacific cities, yielding almost equal membership degrees in both clusters. Using the fuzzy c-means algorithm, then, vagueness in the connectivity profile of Melbourne and Sydney can be assessed. In other words, a marginal shift in service profiles and hence connectivity structure could lead to a complete (and unwanted) shift in classification in a crisp classification, whereas the use of the fuzzy clustering algorithm adapts its resulting classification in a more sensitive way.

Conclusion

The data provided by the Globalization and World Cities Study Group and Network (GaWC) on the relational character of the network of world cities can be analysed with routine data analysis techniques. However, principal component analysis and a crisp clustering algorithm make it very hard to assess patterns in the relational data, since it is often characterized by different sources of vagueness. Sparse data at the basis of all classifications and theoretical considerations on the presence of a complex pattern rather than a clear-cut hierarchy make that crisp classifications of world cities have a highly uncertain character. Therefore, rather than applying data analysis strategies based on the classical two-valued framework of conventional mathematics, we have applied a clustering algorithm that is based on the premises of fuzzy set theory.

After outlining the results of other attempts towards an exploratory analysis on the network of world cities, we have described the crisp and fuzzy c-means clustering algorithms for unsupervised classification. In both algorithms, the distance of an input sample to the center of the cluster is used as a criterion to measure the cluster compactness. In the hard c-means algorithm, an input sample belongs to one cluster only, while in the fuzzy c-means algorithm the degree to which an input sample belongs to a cluster is represented by a membership value. Preliminary results of the application of a fuzzy set-algorithm on the 55x55-matrix provided by GaWC, point out that it is possible (i) to assess both hierarchical tendencies and connectivity patterns (e.g., the case of Tokyo), and (ii) to reveal previously hidden information, especially with respect to the assessment of world cities exhibiting an 'in-between' connectivity profile (e.g. Melbourne and Sydney). Therefore, the use of membership values provides more flexibility and makes the clustering result more useful in practical applications, especially when (i) the data is hampered by sparsity and (ii) identifying in-between values is the specific aim for the data analysis. Using this technique, then, it might be possible to assess connectivity patterns for cities originally expelled from the analysis due to sparsity of the data.

There are, however, some drawbacks. First, although the use of a fuzzy clustering algorithm may reveal some additional information in exposing more sensitivity in the classification, the classification of some objects is hard to interpret. San Francisco, for instance, has for c=4 significant memberships in *all* clusters, yielding a very fuzzy pattern, and making it impossible to classify it in a convincing way. Moreover, since membership values are computed for all clusters using an intensive optimization procedure, a more sensitive interpretation also implies a larger task at hand in interpretation itself.

Acknowledgement
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Table 1: Extract of the distribution of offices for 46 global advanced producer service firms over 55 world cities (collected by Taylor, P.J. and Walker, D.R.F.).

	KPMG	Coopers & Lybrand	Ernst & Young International	…
Amsterdam	3	3	1	…
Atlanta	3	3	2	…
Bangkok	1	1	1	…
…	…	…	…	…

Table 2: Extract of the inter-city matrix on the symmetrical relations between 55 world cities (collected by Taylor, P.J. and Walker, D.R.F.).

	Amsterdam	Atlanta	Bangkok	…
Amsterdam	0,333333343	0,118421055	0,285087705	…
Atlanta	0,118421055	0,157894731	0,100877196	…
Bangkok	0,285087705	0,100877196	0,342105269	…
…	…	…	…	…

Table 3: A roster of world cities (Beaverstock *et al*. 1999).

Alpha world cities	London, Paris, New York, Tokyo, Chicago, Frankfurt, Hong Kong, Los Angeles, Milan and Singapore
Beta world cities	San Francisco, Sydney, Toronto, Zurich, Brussels, Madrid, Mexico City, Sao Paulo, Moscow and Seoul
Gamma world cities	Amsterdam, Boston, Caracas, Dallas, Dusseldorf, Geneva, Houston, Jakarta, Johannesburg, Melbourne, Osaka, Prague, Santiago, Taipei, Washington, Bangkok, Beijing, Montreal, Rome, Stockholm, Warsaw, Atlanta, Barcelona, Berlin, Buenos Aires, Budapest, Copenhagen, Hamburg, Istanbul, Kuala Lumpur, Manila, Miami, Minneapolis, Munich and Shanghai

Table 4: Cities allocated to two components in a principal component analysis (Taylor *et al.*, 2001b).

	Component I: "Outer Wannabes"	Component II: "Inner Wannabes"
>0.7	Istanbul, Athens, Cairo, Montevideo, Sofia, Beirut, Prague	St Louis, Indianapolis
0.6-0.69	Dubai, Bucharest, Mumbai, Karachi, Tel Aviv, Budapest, Casablanca, Nairobi, Manila, Zagreb, Warsaw, Lisbon, Santiago, Quito, Moscow, Taipei	Charlotte, Kansas City, Atlanta, Seattle, Vancouver, Perth, Pittsburgh, Brisbane, Denver, Manchester, Adelaide
0.5-0.59	Panama City, Kuwait, Calcutta, Jakarta, Bangalore, Chennai, Caracas, Seoul, Kuala Lumpur, Lima, Vienna, Kiev, Johannesburg, Auckland*, Jeddah, Madrid, Amsterdam, Nicosia, Helsinki, Copenhagen, Dublin, Ho Chi Minh City	Portland, Houston, Philadelphia, Boston, Dallas, Minneapolis, Cleveland, Montreal, Melbourne, Birmingham, Cape Town, San Diego, Auckland, Barcelona, Calgary
…	…	…

* indicates second highest loading for a city

Cities unallocated to two components:
Antwerp, Berlin, Chicago, Cologne, Dusseldorf, Frankfurt, Hamilton, London, Luxembourg, Mexico City, Munich, Nassau, New York, Singapore, Stockholm, Sydney, Tokyo, Wellington, Zurich.

Table 5: Cities allocated to five components in a principal component analysis (loadings above 0.4; Taylor *et al*., 2001b).

I	II	III	IV	V
OUTER CITIES	US CITIES	PAC.-ASIAN CITIES	EURO-GERM. CITIES	OLD-COMM. CITIES
784 Tel Aviv	769 St Louis	740 Taipei	782 Berlin	716 Perth
767 Sofia	703 Cleveland	726 Tokyo	768 Munich	715 Adelaide
753 Kuwait		725 Bangkok	703 Hamburg	
730 Helsinki		703 Jakarta		
730 Quito				
724 Beirut				
696 Casablanca	680 Dallas	664 Beijing	697 Cologne	687 Brisbane
681 Athens	664 Kansas City	658 Manila	660 Stuttgart	657 Hamilton
670 Nairobi	650 Pittsburgh	633 Seoul		616 Birmingham
666 Montevideo	634 Portland	630 Kuala Lumpur		
664 Jeddah	633 Atlanta	607 Hong Kong		
660 Bucharest	631 Seattle			
650 Indianapolis	623 Charlotte			
645 Cairo	622 Denver			
642 Lagos	620 Detroit			
629 Panama	607 Philadelphia			
624 Lima				
608 Vienna				
599 Dubai	560 Boston	598 Guangzhou	593 Frankfurt	547 Manchester
595Copenhagn	557 San Diego	593 Shanghai		504 Nassau

595 Oslo 592 Zagreb 590 Karachi 586 Chennai 584 Bangalore 572 Istanbul 570 Lisbon 553 Bratislava 535 Kiev 534 Nicosia 533 Calcutta	524 Washington 524 Minneapolis 502 San Francis 500 Houston	560 Ho Chi Min 516 Istanbul 511 Mumbai 500 Singapore	569 Paris 530 Budapest 530Dusseldorf 519 Warsaw 511 Milan 508 Luxembg	501 Vancouver 501 Nicosia
495 Riyadh 492 Prague 468Auckland 461 Moscow 457 Johannesbg 452 Cape Town 448 Manila 446 Budapest 427 Mumbai 424 Warsaw 421 Port Louis 418 Santiago	499 Melbourne 473 Los Angeles 462 Vancouver 437 Chicago 425 Miami 410 Montreal 409 Toronto	455 Sao Paulo 443 Caracas 416 New Delhi 405 Santiago	482 Antwerp 460 Prague 452Rome 437 Lyons 433 Amsterdam 402 Moscow	457 Abu Dhabi 453 Montreal 442 Auckland 441 Calgary 426 London 423 Dubai 410 Port Louis 408 Dublin 402 Wellington

Table 6: Crisp c-means clustering algorithm for c=2.

Cluster 1: Alpha world cities	Cluster 2
Hong Kong, London, Los Angeles, New York, Paris, Singapore, Tokyo	All other world cities

Table 7: Crisp c-means clustering algorithm for *c*=4.

Subset of Alpha world cities		Other world cities	
Cluster 1	Cluster 2	Cluster 3	Cluster 4
LosAngeles, Washington D.C.	London, Paris, Tokyo, Hong Kong, New York	Amsterdam, Buenos Aires, San Francisco, Singapore,…	Copenhagen, Joahannesburg, Atlanta, Kuala Lumpur,…

Table 8: Crisp c-means clustering algorithm for *c*=8.

Alpha and Beta world cities					Gamma world cities		Pacific-Asian world cities
Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Cluster 6	Cluster 7	Cluster 8
London, New York	Chicago, San Francisco	Paris, Brussels.	Los Angeles	Washington D.C.	Seven North-American cities: Atlanta, Boston, Dallas, Houston, Miami, Minneapolis, Montréal. Four German cities: Hamburg, Düsseldorf, Berlin, Münich. Six cities around the old European: Prague, Budapest, Istanbul, Rome, Stockholm, Copenhagen. Osaka, Johannesburg.	Six European core cities: Barcelona, Frankfurt, Amsterdam, Milan, Madrid, Zürich. Five Latin American cities: São Paulo, Buenos Aires, Santiago, Mexico City, Caracas. Moscow, Toronto, Tokyo, Warsaw.	Bangkok, Bejing, Hong Kong, Jakarta, Kuala Lumpur, Manila, Seoul, Shangai, Singapore, Taipei, Melbourne.

Table 9: Memberships degrees for *c*=2 (*m*=1.2).

Alpha world cities
Beta world cities

	Cluster 1	Cluster 2
Amsterdam	0.0983	0.9017
Atlanta	0.0405	0.9595
Bangkok	0.1841	0.8159
Barcelona	0.0864	0.9136
Bejing	0.0665	0.9335
Berlin	0.0121	0.9879
Boston	0.1037	0.8963
Brussels	0.6399	0.3601
Budapest	0.0429	0.9571
Buenoas Aires	0.0398	0.9602
Caracas	0.0315	0.9685
Chicago	0.4336	0.5664
Copenhagen	0.0186	0.9814
Dallas	0.1202	0.8798
Dusseldorf	0.0724	0.9276
Frankfurt	0.8678	0.1322
Geneva	0.0589	0.9411
Hamburg	0.0254	0.9746
Hong Kong	0.9617	0.0383
Houston	0.0338	0.9662
Istanbul	0.0567	0.9433
Jakarta	0.1904	0.8096
Johannesburg	0.041	0.959
Kuala Lumpur	0.0872	0.9128
London	0.9631	0.0369
Los Angeles	0.7787	0.2213
Madrid	0.7118	0.2882
Manila	0.0254	0.9746
Melbourne	0.1186	0.8814
Mexico City	0.4149	0.5851
Miami	0.2053	0.7947
Milan	0.8017	0.1983
Minneapolis	0.0258	0.9742
Montréal	0.0412	0.9588
Moscow	0.4468	0.5532
Münich	0.0148	0.9852
New York	0.9385	0.0615
Osaka	0.0142	0.9858
Paris	0.9459	0.0541
Prague	0.1187	0.8813
Rome	0.018	0.982
San Francisco	0.733	0.267
São Paulo	0.3059	0.6941
Santiago	0.0469	0.9531

Seoul	0.181	0.819
Shangai	0.0805	0.9195
Singapore	0.9212	0.0788
Stockholm	0.0518	0.9482
Sydney	0.8455	0.1545
Taipei	0.4264	0.5736
Tokyo	0.9629	0.0371
Toronto	0.4132	0.5868
Warsaw	0.1359	0.8641
Washington DC	0.5621	0.4379
Zürich	0.6725	0.3275

Table 10: Membership degrees for c=4 (m=1.2).

	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Amsterdam	0.524123	0.306897	0.004424	0.164556
Atlanta	0.02802	0.050987	0.001817	0.919177
Bangkok	0.049657	0.918786	0.004012	0.027546
Barcelona	0.49155	0.352507	0.004947	0.150996
Bejing	0.053499	0.88279	0.001879	0.061832
Berlin	0.018868	0.026729	0.000344	0.954059
Boston	0.097505	0.188287	0.009952	0.704257
Brussels	0.615905	0.189492	0.061804	0.132799
Budapest	0.270859	0.538072	0.003472	0.187597
Buenoas Aires	0.650816	0.284257	0.00114	0.063787
Caracas	0.633092	0.196391	0.001641	0.168876
Chicago	0.329475	0.23966	0.059238	0.371627
Copenhagen	0.076183	0.072819	0.001196	0.849802
Dallas	0.203804	0.171604	0.01073	0.613862
Dusseldorf	0.137256	0.240522	0.006825	0.615397
Frankfurt	0.643882	0.149012	0.13084	0.076266
Geneva	0.221924	0.551812	0.0037	0.222564
Hamburg	0.0401	0.050294	0.00107	0.908535
Hong Kong	0.05245	0.049765	0.886345	0.01144
Houston	0.076289	0.186034	0.002741	0.734937
Istanbul	0.150946	0.487548	0.007376	0.35413
Jakarta	0.171782	0.799264	0.002761	0.026192
Johannesburg	0.162217	0.370754	0.004099	0.46293
Kuala Lumpur	0.064078	0.787284	0.005949	0.142689
London	0.006198	0.003729	0.988516	0.001557
Los Angeles	0.074831	0.106491	0.743886	0.074792
Madrid	0.88104	0.085558	0.012942	0.02046
Manila	0.047634	0.928858	0.000408	0.023101
Melbourne	0.378934	0.412471	0.006964	0.201632
Mexico City	0.855859	0.100751	0.005759	0.03763
Miami	0.341053	0.292682	0.032896	0.33337
Milan	0.808024	0.098918	0.045067	0.047991
Minneapolis	0.030464	0.069439	0.00156	0.898537
Montréal	0.091494	0.080668	0.002173	0.825666
Moscow	0.457956	0.261764	0.075146	0.205134
Münich	0.023434	0.033815	0.000455	0.942296
New York	0.009922	0.006757	0.979927	0.003394
Osaka	0.021008	0.061036	0.000636	0.917321
Paris	0.165979	0.078834	0.724841	0.030346
Prague	0.364467	0.354509	0.014034	0.26699
Rome	0.11081	0.188571	0.001409	0.69921
San Francisco	0.347625	0.194246	0.263695	0.194434
São Paulo	0.95959	0.030323	0.000983	0.009104
Santiago	0.809093	0.120572	0.001107	0.069227
Seoul	0.081293	0.865856	0.004585	0.048267
Shangai	0.057417	0.808512	0.005624	0.128447
Singapore	0.263969	0.296401	0.406517	0.033113

Stockholm	0.372888	0.158211	0.004673	0.464227
Sydney	0.417064	0.417061	0.108132	0.057743
Taipei	0.102919	0.846042	0.01474	0.036299
Tokyo	0.008328	0.271475	0.696712	0.023484
Toronto	0.80892	0.118192	0.007306	0.065582
Warsaw	0.401989	0.292484	0.014729	0.290798
Washington DC	0.185934	0.182946	0.328281	0.302839
Zürich	0.77475	0.13121	0.032399	0.06164

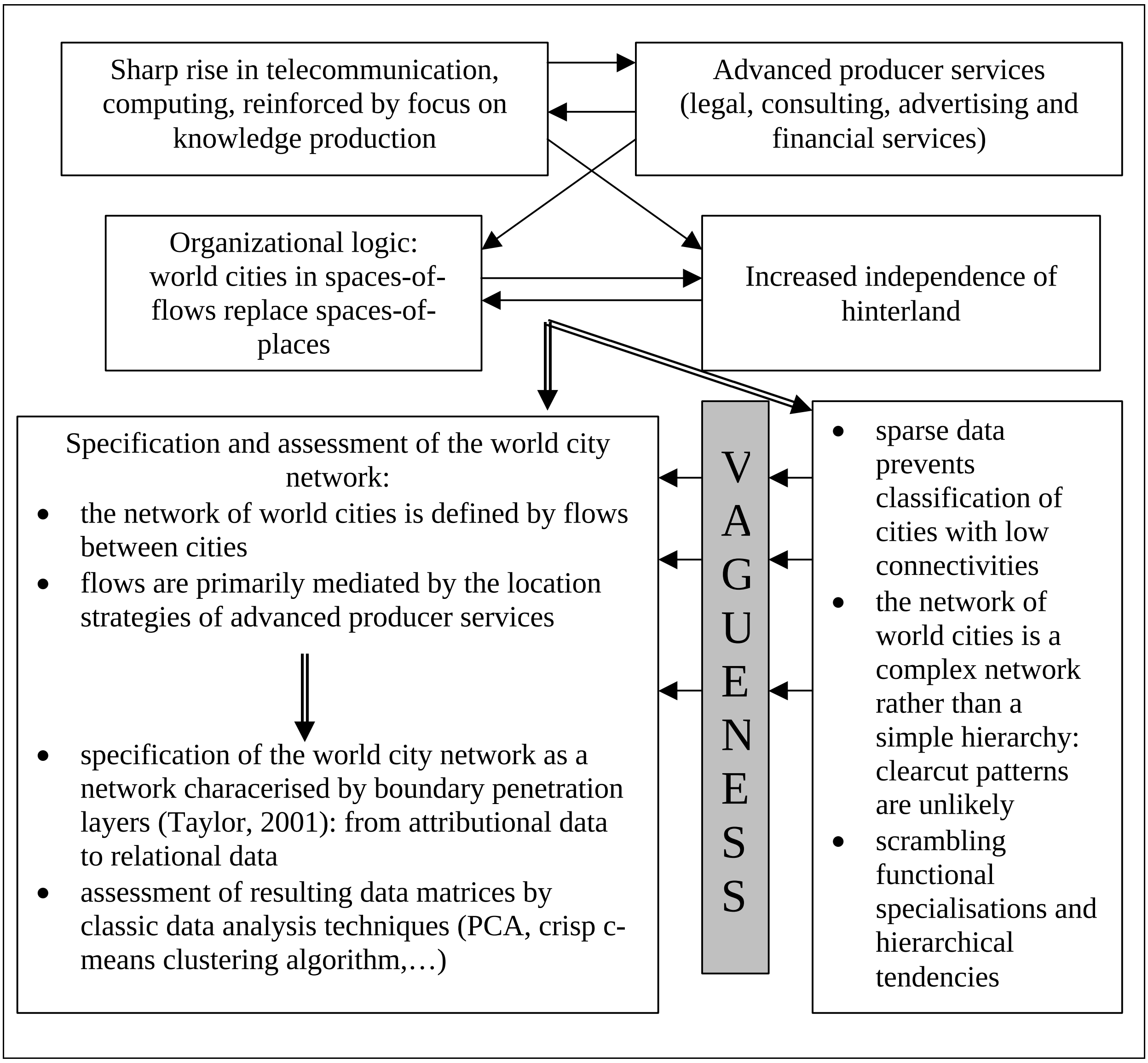

Figure 1: Vagueness in the assessment of a network of world cities.