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A shell-model study of the light Cd isotopes

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Abstract. In an attempt to describe the excitation spectra and transitions strengths of light even-even Cd-isotopes ($A = 98 - 108$), large-scale shell-model calculations in the proton ($2p_{1/2}, 1g_{9/2}$) and the neutron ($2d_{5/2}, 1g_{7/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2}$) model space were performed. Preliminary results are presented and discussed.

1 Introduction

Nuclei in which one type of nucleons (protons or neutrons) constitute a closed shell, are characterized by pairing-dominated energy spectra, in which seniority can be used as an approximate quantum number. This turns out to be the case for, e.g., the Sn ($Z = 50$) and Pb ($Z = 82$) isotopes and the $N = 82$ isotones. The typical energy scale is fixed by the pair-breaking energy (energy gap 2Δ) resulting in a quickly increasing level-density at an energy of ~ 1.5 - 2 MeV. Moreover, the energy of the first excited 2^+ state stays remarkably constant with changing neutron (in the $Z = 50$ and $Z = 82$ isotopes) or proton number (in the $N = 82$ isotones). These results are consistent with the BCS model, or, if the model space allows, with large-scale shell-model calculations [1, 2].

Moving away from the closed shell, adding (or removing) a few valence protons or neutrons, both the excitation energy of the low-lying $2_1^+, 4_1^+, 6_1^+$, .. excited states as well as corresponding $B(E2)$ values, when known, indicate an onset of quadrupole collectivity.

It is an interesting issue to explore in detail how nuclei with just two protons outside the closed shell (or missing) behave, see, e.g., the Cd nuclei ($Z = 48$), Te ($Z = 52$), the Hg ($Z = 80$) and the Po ($Z = 84$) isotopes, as well as isotones with $N = 80$ and $N = 84$.

For the Cd nuclei, an extensive set of experimental data has been obtained over the years, covering essentially the whole $N = 50-82$ neutron major shell, and even going beyond the $N = 82$ closed shell, both on low- and high-spin states, $B(E2)$ values, g-factors, as well as the systematics for those data (see refs.[3, 4] and the references therein). See also ref.[5] for a recent review on the structure of ^{100}Sn and neighbouring nuclei including the light Cd isotopes.

2 Shell-Model Calculations

It is our aim, using the complete ($N = 50 - 82$) neutron model space, i.e. valence neutrons in the $2d_{5/2}$, $1g_{7/2}$, $2d_{3/2}$, $3s_{1/2}$ and $1h_{11/2}$ orbitals, and having ten valence protons in the $2p_{1/2}$ and $1g_{9/2}$ orbitals ($Z = 48$) in the ($Z = 38 - 50$) proton model space, to perform a large-scale shell-model study (LSSM) to calculate and explore a multitude of nuclear structure properties, in particular, the changing structure with increasing number of valence neutrons moving outside the $N = 50$ closed shell. The nucleon-nucleon (NN) interaction that we use is an effective realistic force which was modified to exhibit the correct monopole characteristics for the propagation of the single-neutron energies over the $N = 50 - 82$ shell [6], i.e. starting with experimental single particle energies of ^{89}Sr at $N = 51$ and moving towards the end of the neutron shell, where the shell model reproduces well the neutron hole states in ^{131}Sn.

An important point is the use of the proton and neutron effective charges throughout the full set of Cd nuclei. The procedure used is to fix the proton effective charge e_π fitting the theoretical $B(E2)$ value for the $8_1^+ \rightarrow 6_1^+$ transition to the known experimental value in ^{98}Cd. Having fixed this value, the neutron effective charge e_ν was fixed by comparing the experimentally known and theoretically calculated $B(E2)$ values for the $2_1^+ \rightarrow 0_1^+$ transitions in ^{102,104}Cd [7]. The effective charges used in the current work are $e_\pi = 1.7e$ and $e_\nu = 1.1e$, i.e. the same as in the former shell-model calculations with this interaction [6, 7].

With these ingredients kept constant in the LSSM study of the Cd nuclei, it is the NN interaction acting in the large model space that produces the nuclear structure properties as a function of increasing neutron number. In particular, we concentrate on a study of the interplay of the pairing part of the NN force that prefers broken pair states to be ordered in some particular sequence (dictated by the seniority quantum number) versus the quadrupole component (mainly proton-neutron part if many neutrons are present) of the NN force, that prefers to generate quadrupole collective excitation modes.

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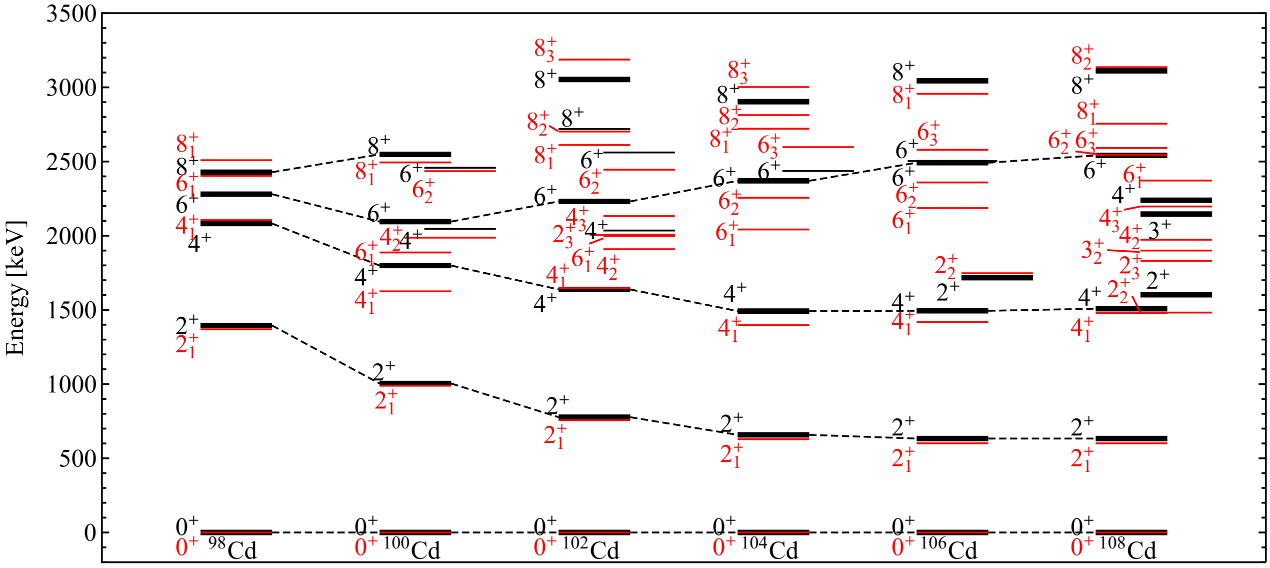

Figure 1. Experimental (black) and LSSM (red) energy spectra of the even-even light Cd isotopes. Experimental data are taken from refs.[3, 4, 6] and the NNDC database [8].

The large scale shell model calculations were performed with the code ANTOINE [1]. The calculations presented here were performed in the full model space without any additional truncations. The m-scheme matrix dimension for m=0 in ^{108}Cd was about 10^8.

Figure 1 shows the results of the LSSM compared to the experimental spectra for the even-even $^{98-108}$Cd isotopes. The lowest 2^+ and 4^+ states energies are well reproduced by the shell-model calculations while the higher-spin states show deviations in energy. The calculated ratios of the energy of the first 4^+ and 2^+ states are known as the R(4/2) ratios. As shown in figure 2, the R(4/2) ratio for calculated even Cd isotopes compares well with the experimental values. The R(4/2) ratio is often used as a signature for the collectivity of the nucleus. Values below 2 correspond to single-particle structures, while higher values of R(4/2) correspond to "collective" structures. A value of exactly 2 corresponds to the ratio for a harmonic vibrator, while a value of 3.3 corresponds to the case of a rigid rotor. At $N = 50$, ^{98}Cd shows a seniority-type of spectrum (see figure 1) which also corresponds to a single particle R(4/2) ratio (see figure 2), as expected for a nucleus only two proton-holes away from a doubly-magic nucleus, i.e. ^{100}Sn. By increasing the number of valence neutrons outside the closed $N = 50$ shell, the experimental R(4/2) ratio raises above the pure vibrational value of 2, reaches a maximum of about 2.4 at ^{108}Cd and stabilizes afterwards around a value of 2.3. The increase of collectivity in the light Cd nuclei as demonstrated by the R(4/2) behaviour also shows up in a steady increase of the electric quadrupole transition strength between the low-lying yrast states. Starting from ^{104}Cd a clear collective yrast band, up to the 6^+ state, is present in the calculations with values of $B(E2)$ decently reproducing known experimental values [9]. Therefore we have calculated the evolution of the $B(E2)$ reduced transition probabilities along the yrast line, in particular concentrating on the $2_1^+ \rightarrow 0_1^+$ and $4_1^+ \rightarrow 2_1^+$ transitions. For transitions from higher spin 6^+,8^+ states, it is of importance to locate the strongest $B(E2; 6_i^+ \rightarrow 4_1^+)$ and $B(E2; 8_i^+ \rightarrow 6_j^+)$ transitions in order to identify strongly correlated levels when attempting to join them in a given band structure. Consequently we have studied in some more detail emerging band structures from the shell-model calculations for ^{106,108}Cd and results are in preparation for publication [9].

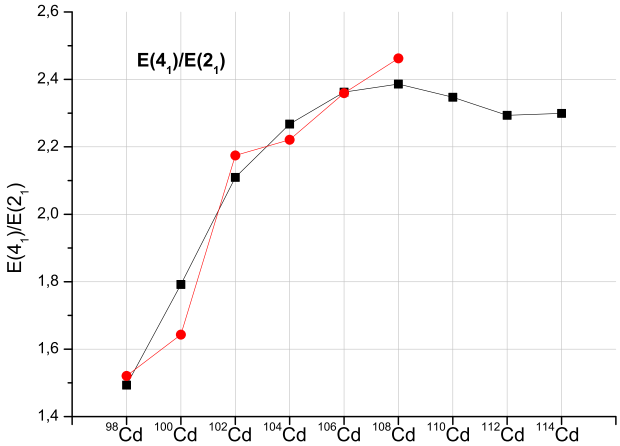

Figure 2. Experimental (black squares) and LSSM (red circles) R(4/2) ratios for the light even Cd isotopes.

The study of the calculated neutron and proton single-particle orbital partial occupations numbers for the ground state, 0_1^+, as a function of the valence neutron number, increasing from 0 to 10 (^{98}Cd to ^{108}Cd), reveals a “simultaneous” filling of the $\nu 2d_{5/2}$ and $\nu 1g_{7/2}$ orbitals with the $\nu 2d_{5/2}$ orbital being the lower one and respectively

Table 1. Occupation numbers of proton and neutron valence orbitals in the wave function of selected calculated states in the even $^{98-108}$Cd nuclei. In the table only the occupation number for the $\pi 2p_{1/2}$ proton orbital is given, while the occupation number for the other proton valence orbital $\pi 1g_{9/2}$ can be easily calculated, i.e. the sum of both is equal to 10. The numbers for the different orbitals are given as a function of the Cd isotope mass number. The occupation numbers for the different states are separated by double horizontal lines.

$0^+_1 \setminus A =$	98	100	102	104	106	108
$\pi 2p_{1/2}$	1.76	1.88	1.95	1.97	1.98	1.98
$\nu 2d_{5/2}$	-	1.09	2.38	3.22	3.82	4.41
$\nu 1g_{7/2}$	-	0.67	1.11	2.01	3.07	3.94
$\nu 2d_{3/2}$	-	0.14	0.30	0.45	0.62	0.82
$\nu 3s_{1/2}$	-	0.07	0.16	0.24	0.33	0.49
$\nu 1h_{11/2}$	-	0.03	0.05	0.08	0.16	0.34
$0^+_2 \setminus A =$	98	100	102	104	106	108
$\pi 2p_{1/2}$	0.24	1.75	1.93	1.97	1.98	1.98
$\nu 2d_{5/2}$	-	0.70	2.44	3.67	4.51	4.17
$\nu 1g_{7/2}$	-	1.20	1.21	1.63	2.51	3.65
$\nu 2d_{3/2}$	-	0.03	0.17	0.40	0.52	0.70
$\nu 3s_{1/2}$	-	0.03	0.13	0.25	0.36	0.58
$\nu 1h_{11/2}$	-	0.04	0.05	0.05	0.10	0.90
$8^+_1 \setminus A =$	98	100	102	104	106	108
$\pi 2p_{1/2}$	2.00	2.00	2.00	1.99	1.99	1.99
$\nu 2d_{5/2}$	-	1.19	2.35	3.31	3.89	3.40
$\nu 1g_{7/2}$	-	0.57	1.20	2.07	3.26	3.59
$\nu 2d_{3/2}$	-	0.14	0.27	0.33	0.46	0.62
$\nu 3s_{1/2}$	-	0.08	0.14	0.26	0.30	0.35
$\nu 1h_{11/2}$	-	0.02	0.04	0.03	0.09	2.04

exhibiting an occupation number of about one neutron more compared to the close-lying $\nu 1g_{7/2}$ orbital, while the occupation numbers for the other neutron orbitals remain relatively small (see table 1). This implies a strong mixing of configurations with neutrons in the $\nu 2d_{5/2}$ and $\nu 1g_{7/2}$ orbitals in the description of the 0^+_1 state. Similar orbital occupation behaviour is found also for other states. It is worth noticing that at ^{108}Cd the 0^+_2 state suddenly shows an increase of the $\nu 1h_{11/2}$ occupation number raising close to 1 (see table 1), manifesting the onset of the role played by the $1h_{11/2}$ neutron orbital. This effect is even more pronounced for the 8^+_1 state in ^{108}Cd where the trend of an increasing occupation number for $\nu 2d_{5/2}$ is reduced and a value larger than 2 for the $\nu 1h_{11/2}$ orbital occupation number is observed (see table 1). This can be described using the picture of an even number (2,4,...) of neutron "particle-hole" excitations mostly from the $\nu 2d_{5/2}$ to the $\nu 1h_{11/2}$ orbital. An occupation number of more than 2 for the $\nu 1h_{11/2}$ orbital means that the wave function contains mainly configurations with 2 neutrons in $\nu 1h_{11/2}$ orbital and a minor mixing of configurations with more than 2 neutrons in $\nu 1h_{11/2}$ orbital, once again showing the increasing influence of the $\nu 1h_{11/2}$ orbital for the nuclear structure as one approaches the middle of the neutron $N = 50 - 82$ shell.

3 Conclusions and outlook

In conclusion, we have performed LSSM calculations for the light even-even $^{98-108}$Cd isotopes in the proton $(2p_{1/2}, 1g_{9/2})$ and the neutron $(2d_{5/2}, 1g_{7/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2})$ model space. The energy spectra describing the lowest yrast states are well reproduced over the whole calculated range, while the higher-lying yrast and non-yrast states show deviations comparing with the experimental data. By comparing the calculated transition strengths and branching ratios with experimental ones, it is possible to associate a specific shell-model state with a corresponding experimental one, neglecting the order of the specific state within the calculated set of states [9]. The increasing collectivity in the light Cd isotopes, when moving away from the $N = 50$ closed shell, is well reproduced. Shell-model occupation numbers are discussed and the onset of filling of the neutron $\nu 1h_{11/2}$ orbital is noticed for selected states in ^{108}Cd. Since the present approach contains the full model space, as discussed before, both collective modes as well as multiple broken-pair states (in particular the high-spin states 8^+,10^+,12^+,...) can result within a single framework. The results will be presented in a forthcoming paper [9].

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