Preprint, 11th IFAC Symposium on Dynamics and Control of Process Systems,
including Biosystems
June 6-8, 2016. NTNU, Trondheim, Norway

Robust PID Auto-tuning for the Quadruple Tank System

Clara M. Ionescu, Anca Maxim, Cosmin Copot, Robin De Keyser [*]

[*] *Ghent University, Department of Electrical energy, Systems and Automation, Technologiepark 914 2nd floor, 9052 Gent, Belgium (e-mail: claramihaela.ionescu@ugent.be).*

Abstract: In multi-modular process architectures with independent but interacting subsystems, identification may not be the first choice at hand for closed loop control. A robust relay-based PID autotuning strategy is presented and validated on a quadruple tank system with non-minimum phase dynamics. The controller ensures a specified closed loop robustness, which is of great benefit to the overall performance. The experimental results suggest that the proposed method fulfils the robustness requirement and performs well in various operating conditions of the testbench.

Keywords: Autotuners, Robustness, Non-minimum phase systems, Multivariable control systems

1. INTRODUCTION

Industrial applications of inter-connected systems are manifold and present difficult control issues, such as time delays, multivariable interaction, non-minimum phase dynamics, etc (Bequette (2003); Smith (2010)). Process identification is usually very challenging and time consuming. To simplify this task and to reduce the time required for it, many PID regulators nowadays include autotuning capabilities, i.e. they are equipped with a mechanism capable of computing the 'correct' parameters automatically when the regulator is connected to the plant. This approach copes very well with the dynamic context in factory automation and can be easily implemented in standardized automation software and PLCs.

A specific class of autotuners use relay feedback in order to obtain some information on the process frequency response (Yu (2006); De Keyser and Ionescu (2010)). For multiple input multiple output (MIMO) systems, the autotuning procedure needs to take into account the cross-coupling dynamics in order to converge to stable closed loop controllers (Wang et al. (2008)). Using a relay controller, the outputs will oscillate in the form of limit cycles (after an initial transient). The controller parameters are then iteratively obtained such that these output oscillations converge to the critical frequency of the entire coupled system (Wang et al. (2008)). The number of iterations is usually related to the number of input-output pairings (Johnson and Moradi (2005)).

Classical PID autotuning approaches such as the Åström-Hägglund (AH), Ziegler-Nichols and recent revisited algorithms based on phase margin specifications (Åström and Hägglund (2006); Hang et al. (1991); Ionescu and De Keyser (2012)), identify the critical point on the process frequency response using relay feedback. Their advantage is that they are very simple to apply, i.e. few choices are left for the user (which is indeed an advantage if the industrial engineer lacks theoretical control engineering insight). In (Husek (2014)) has been proposed a model-based method for tuning decentralized PI controllers on the quadruple tank using phase margin specifications. However, care should be taken when single-point specification tuning is performed, since it may lead to unstable results for certain class of processes, as discussed in (Ionescu and De Keyser (2012)).

A characteristic of the relay feedback PID autotuner proposed in our paper is that it guarantees a specified robustness, by means of the modulus margin index. The outline of this paper is as follows. The proposed methodology is given in Section 2, followed by a brief description of the auto-tuning algorithm in section 3. The closed loop control performance and robustness is evaluated and results reported in section 4. A conclusion section summarizes the main outcome of this work.

2. PROPOSED METHODOLOGY

Traditional relay-based autotuning methods such as AH (Åström and Hägglund (2006)) identify one point on the Nyquist curve of the process P; namely, the intersection of the process beeline with the negative real axis as depicted in Figure 1. Using an appropriate PID controller, denoted by C, this initial point is then moved to a specific point in the complex Nyquist plane; e.g. for the original AH-tuner, the beeline of $C * P$ goes through the specific point $-0.6 - 0.28j$ (distance to the point -1 is then 0.5). The theoretical insights from (Ionescu and De Keyser (2012)) claim that single-point specification in the Nyquist plane might be sufficient for some type of processes, but might as well result in poor (low) modulus margin (MM) for other types of processes.

[*] CM Ionescu is a post-doctoral fellow of Flanders Research Foundation, grant nr 12B3415N. All authors are members in the EEDT group, partner of Flanders Make consortium: www.eedt.ugent.be.

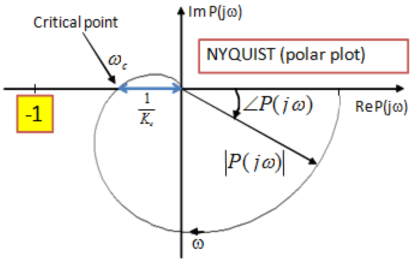

Fig. 1. The Nyquist plot of the process and its intersection with the negative real axis

The goal of this paper is to present and validate a robust relay-based PID auto-tuning method for MIMO processes. The following assumptions about the system under test are made:

- the MIMO processes are open loop stable;
- the MIMO processes are capable of sustaining steady state oscillations when placed under relay feedback control.

For a 2 × 2 process, the proposed algorithm can be summarized in the following steps below. *Step 1*: Close one loop with a relay feedback and leave open the other loop. Suppose one closes loop #1 and leaves open loop #2 as in Figure 2. This initial relay experiment allows us to extract information about the interaction between loops and the critical gain of the loop (recall figure 1). Consequently, we can use the AH methodology to determine a proportional controller (Åström and Hägglund (2006)):

$$K_c = \frac{4d}{\pi A}, \quad P_{c1} = 0.5K_c \qquad (1)$$

where K_c is the critical gain, d is the relay amplitude and A denotes the oscillation amplitude of the first output. Taking a look at the amplitude of the oscillations in the second output, one may decide upon the amount of interaction and whether the pairing is correct. From hereafter, we will consider that the initial pairing u_1 to y_1 is correct.

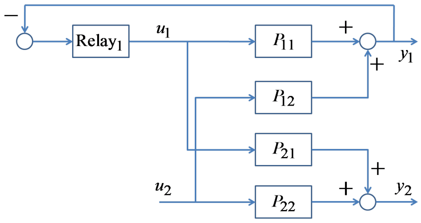

Fig. 2. Proposed MIMO auto-tuning algorithm - Step 1.

Step 2: In the first loop, replace the relay with the proportional controller P_{c1} as depicted in Figure 3. Close the second loop with a relay feedback. Take a note of the amplitude A and period T_c of the oscillations ($\omega_c = 2\pi/T_c$) in the second output.

Step 3: Add a delay (see Appendix for rationale) after the relay block in the closed loop of the second output (i.e. Figure 4), while maintaining the proportional controller P_{c1} on the other loop. Take a note of the amplitude and period of the oscillations for the relay+delay loop.

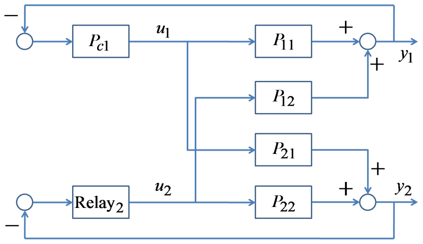

Fig. 3. Proposed MIMO auto-tuning algorithm - Step 2

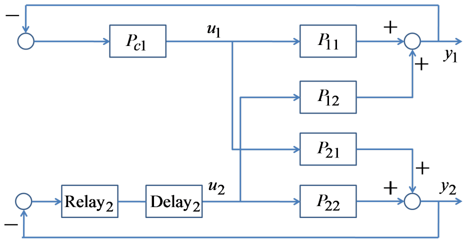

Fig. 4. Proposed MIMO auto-tuning algorithm - Step 3

Step 4: Compute a PID controller for the second loop based on the MM algorithm from (De Keyser et al. (2012)), this will be denoted by PID_2.

Step 5: Replace the relay+delay from the loop selected at step 3 with the PID_2 from Step 4. Close the other loop with a relay feedback like in Figure 5. Take again a note of the *relay loop* the magnitude and phase of the oscillations in the first output.

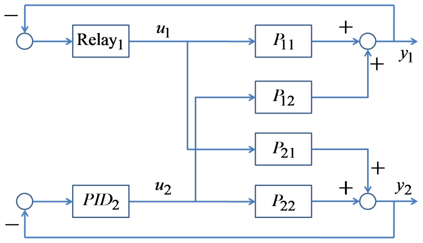

Fig. 5. Proposed MIMO auto-tuning algorithm - Step 5

Step 6: Add a delay after the relay block in the closed loop of the first output (i.e. Figure 6), while maintaining the PID_2 controller on the second loop (as in Step 5). Take a note of the amplitude and period of the oscillations for the relay+delay loop.

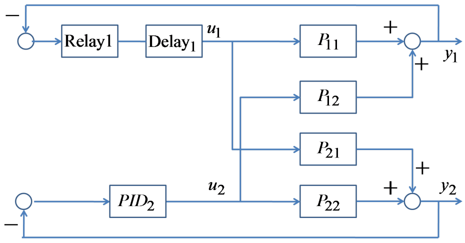

Fig. 6. Proposed MIMO auto-tuning algorithm - Step 6

Step 7: Compute a PID_1 controller for the first loop using the MM algorithm.

Step 8: Repeat steps 5-7. The convergence of the algorithm is established when the output magnitude and phase values in the relay+delay test vary less than 5% from those obtained in the relay test, i.e. this will result in similar PID parameters as in the previous iteration of the corresponding loop.

Notice that the entire procedure can be easily automated. However, a condition for convergence is that minimum one of the loops in the system has a characteristic locus with at least 180 degrees phase lag (Wang et al. (2008)). This condition is not a problem in process industry where high order dynamics are present.

3. CONTROLLER DESIGN

The development of the proposed autotuning algorithm is based on imposing a user-specified robustness, as in (De Keyser et al. (2012)). The robustness specification can be translated using Nyquist plots as a circle of specified radius (r) around the point -1 as drawn in Figure 7 (r=the Modulus Margin, $0 < r < 1$).

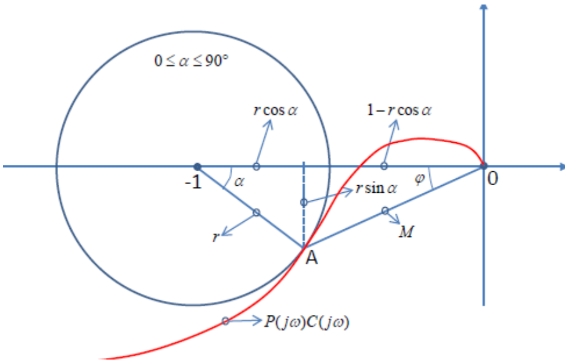

Fig. 7. Supporting figure for the MM algorithm development.

The algorithm searches for the angle α under which the Nyquist curve of the process and controller $P(j\omega)C(j\omega)$ touches the circle in A. The parameters for the PID controller:

$$K_p = \frac{M}{M_P}\cos\varphi, \quad T_d = \frac{1+\sin\varphi}{2\omega_c\cos\varphi}, \quad T_i = 4T_d \tag{2}$$

with M_P defined in Appendix. The values for M and φ are obtained from Pythagorean Theorem applied in Figure 7:

$$M = \sqrt{r^2 - 2r\cos\alpha + 1} \tag{3}$$

$$\varphi = \arctan\frac{r\sin\alpha}{1 - r\cos\alpha} \tag{4}$$

In (Mantz and Tacconi (1989)), it has been shown that optimization of PID free controller parameters leads to complex conjugated zeros, which may induce oscillations in the closed loop performance. To reduce the number of tuning parameters and to have two real zeros in the PID at same location, we impose the rule $T_i = 4T_d$. Also, we do not tackle here the influence of noise and the possibility to use the derivative filter in the design (Segovia et al. (2014)). Although in (Kristiansson and Lennartsson (2006)) it has been shown that imposing $T_i = 4T_d$ does not lead to optimal trade off between robustness and low frequency load disturbance rejection, it will be illustrated in this paper that the obtained performance is satisfactory for a challenging process.

The entire procedure of the algorithm can be summarized as: 1) Find ω_c and $M_P(\omega_c)$ via relay test $[\varphi_c = -180°]$

2) Find ω_c', $M_P(\omega_c')$ and $\varphi_P(\omega_c')$ via a relay+delay test

3) Calculate $\frac{\Delta M_P}{\Delta\omega}$ and $\frac{\Delta\varphi_P}{\Delta\omega}$

4) For $\alpha = 0...90°$, calculate $M(\alpha)$, $\varphi(\alpha)$ and $\delta(\alpha) = \left| \left.\frac{dM}{d\varphi}\right|_{\omega_C} - \left.\frac{dM}{d\varphi}\right|_A \right|$

5) Calculate $M(\alpha^*)$ and $\varphi(\alpha^*)$ with $\alpha^* = \arg min_\alpha \delta(\alpha)$

6) Calculate $\{K_p, T_i, T_d\}$.

This method of PID autotuning has been successfully validated on numerous examples depicting various types of processes for single input-single output (SISO) systems (De Keyser et al. (2012); Joita and De Keyser (2012)).The novelty of the proposed method is taking the modulus margin algorithm developed and tested on SISO processes and using it iteratively for tuning the PID controllers in MIMO systems. A robustness curve may be calculated, but it requires the knowledge of the process model ((Arbogast et al., 2008)). In our methodology this is not possible since we do not perform identification of the process.

4. METHOD USED FOR COMPARISON

From the extensive comparison presented in (Menani and Koivo (2001)), it was suggested that the following algorithm could give the best results in terms of relay-based feedback autotuners for MIMO systems. The tuning procedure consists of four steps. In the first step, the critical point of each possible loop is identified using the conventional relay feedback test. In this phase, a set of critical frequencies is formed. In the second step, a design frequency is chosen from the set based on the interaction percent (i.e. least interaction). In the third step, the process steady-state and frequency response matrices are estimated with sufficient accuracy. In the fourth step, the information about the process is used to tune a MIMO PID controller of the form:

$$K_c(s) = K_p + K_I\frac{1}{s} + K_D s \tag{5}$$

with

$$K_I = \rho\det[G(0)]G^{-1}(0), \quad K_D = K_I/4 \tag{6}$$

and

$$K_p = \rho\det||G(j\omega_b)||||G^{-1}(j\omega_b)|| \tag{7}$$

where G denotes the process transfer matrix. The parameter ρ is used to fine tune the closed loop performance, but there are no specific rules for it. Stability is not guaranteed, but it is suggested to use a ω_b at which the interaction is minimal, therefore diminishing the risk of closed loop instability. The advantage of the method over the one proposed in our paper is that it takes interactions into account, hence delivers a controller matrix.

5. RESULTS

For testing the proposed algorithm, we make use of the quadruple water tank benchmark from Quanser. Figure 8 presents the schematic overview of the setup. The control objective is to regulate the level of the water in the lower tanks L_2 and L_4 (expressed in cm) by manipulating the water flows of the two pumps, i.e. the voltages of the two pumps $V_{p1}(t)$ and $V_{p2}(t)$ (expressed in Volts). There is a strong coupling effect between the inputs and the outputs and the current setup exhibits non-minimum phase dynamics; a benchmark that has been investigated by several groups (Johansson (2000); Gatzke et al. (2000); Vadigepalli et al. (2001)).

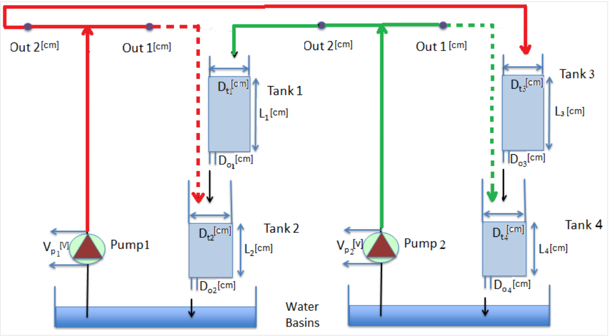

Fig. 8. Schematic diagram of the quadruple tank process from Quanser.

Autotuning of the PID controller via the MM algorithm described in section 2 has been performed using a relay test on the real plant for each loop, giving a robustness specification of 0.7 (i.e. 70%). The final controller parameters were obtained after six iterations and are reported in Table 1. On the real process, the time for one experiment took 5 minutes.

Table 1. PID controller parameters. MM - controller designed via the autotuning method. Subscript denotes the water tank where controlled level was achieved.

	K_p	T_i	T_d
$MM2$	0.27	10.85	2.71
$MM4$	0.27	12.81	3.02

The MIMO PID controller parameters for the autotuner from (Menani and Koivo (2001)) are:

$$K_{pp} = 0.001 * \begin{bmatrix} 0.7719 & 0.5481 \\ 0.5940 & 0.7666 \end{bmatrix} \quad (8)$$

$$K_{dd} = \begin{bmatrix} 0.0022 & -0.0044 \\ -0.0048 & 0.0031 \end{bmatrix} \quad (9)$$

$$K_{ii} = \begin{bmatrix} 0.0090 & -0.0174 \\ -0.0193 & 0.0125 \end{bmatrix} \quad (10)$$

Figure 9 depicts the results for setpoint tracking in water reservoir #2. The reference is changed from 5 cm to 6 cm, while keeping the reference for the water level L_4 constant to 5 cm. Figure 10 depicts the results for disturbance rejection in water reservoir #2. The disturbance consists in eliminating the water from reservoir #1 directly in the water pool, without going trough Tank2. This represents a load step disturbance. As a result, the level L_2 drops and the controller changes the pump voltage to recover the reference value of 6 cm. Due to the coupling between the pumps (recall Figure 8), the level L_4 will also be disturbed; results are not shown since the performance is similar.

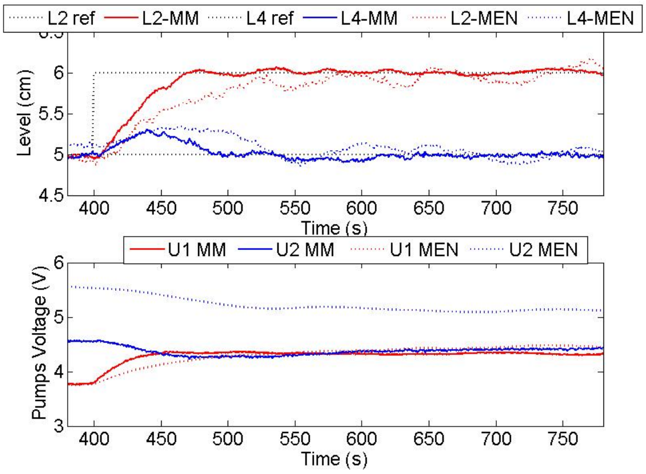

Fig. 9. Setpoint change experiment in nominal operating point.

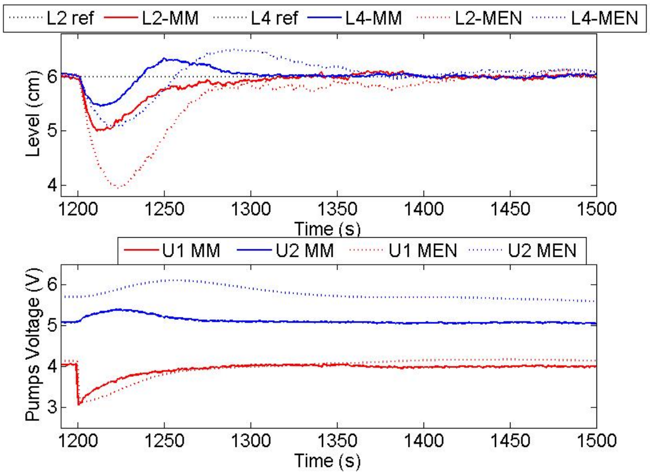

Fig. 10. Disturbance rejection experiment in nominal operating point.

One may conclude that both auto-tuning control loops perform similarly.

A verification that the MM auto-tuner fulfils the robustness specification of 70% is done through the Nyquist plot in Figure 11. It can be observed that this specification is fulfilled.

In order to test the robustness of the two control strategies, we re-make the setpoint and disturbance rejection tests at a significantly different operating point (15 cm). The results are given in Figures 12 and 13, where we could conclude that our method outperforms that from (Menani and Koivo (2001)).

Also, a mean squared error (MSE) index was used to indicate the performance of each autotuner in both experiments (i.e. the nominal case and the robustness test). The results are given in Table 2.

6. CONCLUSIONS

The auto-tuning method proposed in this paper for robust PID controllers is based on relay feedback and can be eas-

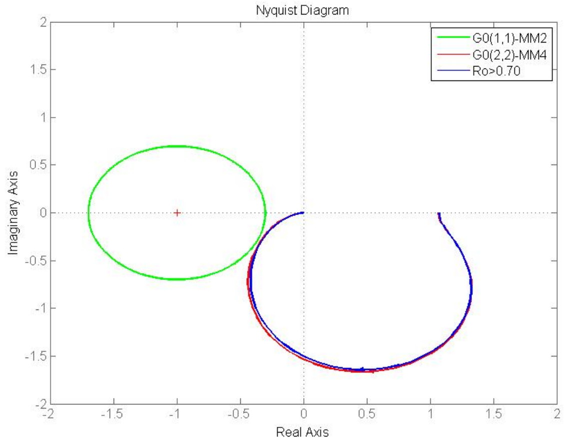

Fig. 11. Nyquist plot of the closed loop for modulus margin specification validation.

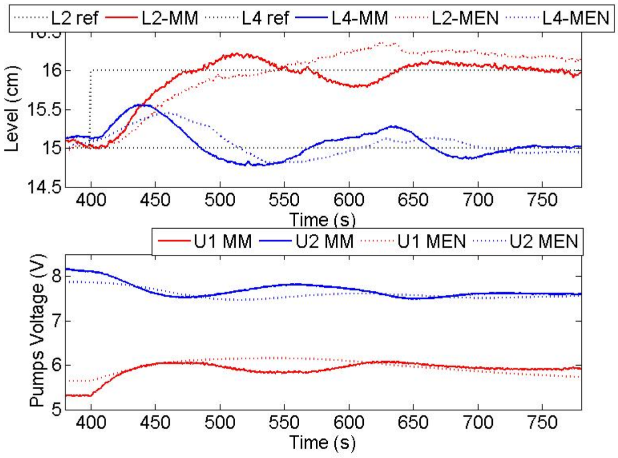

Fig. 12. Robustness test - setpoint tracking at 15cm.

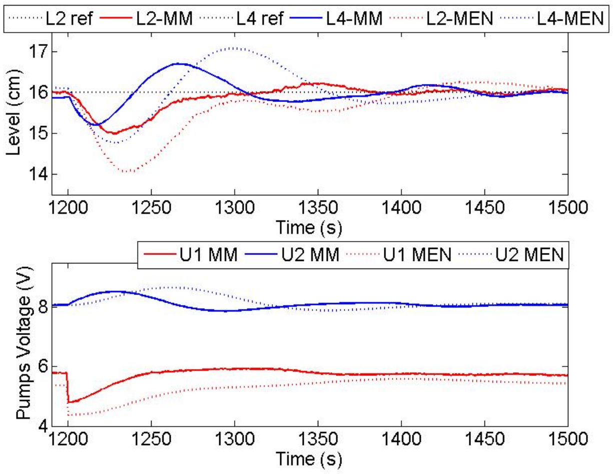

Fig. 13. Robustness test - disturbance rejection at 15cm.

ily implemented in PLCs. The method has been successfully validated on a multivariable, strongly coupled, non-minimum phase system, i.e. the quadruple tank system from Quanser.

Table 2. Mean Squared Error (MSE) index. MMnom, MMrob - MSE for the autotuning method in nominal case and robustness experiment respectively. MENnom, MENrob - MSE for the comparative method in nominal and robustness experiment respectively. L_2, L_4 denotes the water tank where controlled level was achieved.

	L_2	L_4
$MMnom$	0.0430	0.0365
$MMrob$	0.0625	0.0702
$MENnom$	0.1366	0.0583
$MENrob$	0.1979	0.1271

APPENDIX

The algorithm searches for the angle α under which the Nyquist curve of the process and controller $P(j\omega)C(j\omega)$ touches the circle in A. From Figure 7 we get:

$$\begin{aligned} P(j\omega_c)C(j\omega_c) &= M_P e^{j\varphi_P} * M_C e^{j\varphi_C} = \\ &= M_P M_C e^{j(180^\circ+\varphi_C)} = M e^{j(180^\circ+\varphi)} \end{aligned} \tag{11}$$

where:

$$\varphi = \varphi_C \quad \text{and} \quad M = M_P M_C \tag{12}$$

Next, the textbook frequency response for the PID controller is written:

$$C\left(j\omega_c\right) = K_p\left[1 + j\left(T_d\omega_c - \frac{1}{T_i\omega_c}\right)\right] \tag{13}$$

Considering

$$\tan\varphi_C = T_d\omega_c - \frac{1}{T_i\omega_c} \tag{14}$$

and

$$M_C = K_p\sqrt{1+\tan\varphi_C^2} = \frac{K_p}{\cos\varphi_C} \tag{15}$$

and imposing two identical zeros for the controller ($T_i = 4T_d$), we obtain

$$\tan\varphi_C = T_d\omega_c - \frac{1}{4T_d\omega_c} \Rightarrow T_d\omega_c = \frac{1+\sin\varphi}{2\cos\varphi} \tag{16}$$

Replacing (15) and (16) into (12) we get the parameters for the PID controller:

$$K_p = \frac{M}{M_P}\cos\varphi, \quad T_d = \frac{1+\sin\varphi}{2\omega_c\cos\varphi}, \quad T_i = 4T_d \tag{17}$$

where M and φ are obtained from Pythagorean Theorem applied in figure 7:

$$M = \sqrt{r^2 - 2r\cos\alpha + 1} \tag{18}$$

$$\varphi = \arctan\frac{r\sin\alpha}{1 - r\cos\alpha} \tag{19}$$

Now we can express the variation of the modulus with the phase, which describes the tangent to a circle of radius r around the point -1:

$$\left.\frac{dM}{d\varphi}\right|_{\omega_c} = \frac{{}^{\mathrm{d}M}\!/_{\mathrm{d}\omega}}{{}^{\mathrm{d}\varphi}\!/_{\mathrm{d}\omega}} \tag{20}$$

with

$$\frac{dM}{d\omega} = \frac{d(M_P M_C)}{d\omega} = M_P \frac{dM_C}{d\omega} + \frac{M}{M_P}\frac{dM_P}{d\omega} \tag{21}$$

and

$$\frac{d\varphi}{d\omega} = \frac{d(\varphi_P + \varphi_C)}{d\omega} = \frac{d\varphi_P}{d\omega} + \frac{d\varphi_C}{d\omega} \tag{22}$$

In order to compute (20), we need

$$\frac{dM_C}{d\omega} = \frac{dM_C}{d\tan\varphi_C}\frac{d\tan\varphi_C}{d\omega} \tag{23}$$

which is further equal to

$$K_p \sin\varphi_C \frac{1}{\omega\cos\varphi_C} = \frac{M\sin\varphi_C}{M_P\omega} \tag{24}$$

and

$$\frac{d\varphi_C}{d\omega} = \frac{d\tan\varphi_C/_{d\omega}}{d\tan\varphi_C/_{d\varphi_C}} = \frac{1/_{\omega}\cos\varphi_C}{1/_{\cos\varphi_C{}^2}} = \frac{\cos\varphi_C}{\omega} \tag{25}$$

Substituting, we have

$$\left.\frac{d\mathrm{M}}{d\varphi}\right|_{\omega_C} = \mathrm{M}\frac{\frac{\sin\varphi}{\mathrm{\omega_c}} + \frac{1}{\mathrm{M_P}}\left.\frac{dM_P}{d\omega}\right|_{\omega_c}}{\frac{\cos\varphi}{\mathrm{\omega_c}} + \left.\frac{\mathrm{d}\varphi_\mathrm{P}}{\mathrm{d}\omega}\right|_{\omega_c}} \tag{26}$$

Since for autotuning purposes we do not have a model for the process, we will approximate the derivative of the process frequency response in ω_C using differences:

$$\left.\frac{dM_P}{d\omega}\right|_{\omega_c} \approx \left.\frac{\Delta M_P}{\Delta\omega}\right|_{\omega_c} = \frac{M_P(j\omega_c) - M_P(j\omega')}{\omega_c - \omega'} \tag{27}$$

$$\left.\frac{d\varphi_P}{d\omega}\right|_{\omega_c} \approx \left.\frac{\Delta\varphi_P}{\Delta\omega}\right|_{\omega_c} = \frac{\varphi_P(j\omega_c) - \varphi_P(j\omega')}{\omega_c - \omega'} \tag{28}$$

where $M_P(j\omega')$ and $\varphi_P(j\omega')$ are the modulus and phase of the process at a frequency ω' which is close to the critical frequency ω_c. This can be easily obtained using a relay test with time delay, $\tau_d = \frac{\Delta\varphi_P}{\omega_C}$, corresponding to a specified $\Delta\varphi_P$ (De Keyser et al. (2012)). Next, we compute the variation of the modulus with the phase, in the point A, from Figure 7:

$$\left.\frac{dM}{d\varphi}\right|_{\mathrm{A}} = \frac{\mathrm{dM}/_{\mathrm{d}\alpha}}{\mathrm{d}\varphi/_{\mathrm{d}\alpha}} = \mathrm{M}\frac{\sin\alpha}{\cos\alpha - \mathrm{r}} \tag{29}$$

where

$$\frac{dM}{d\alpha} = \frac{r\sin\alpha}{M},\quad \frac{d\varphi}{d\alpha} = \frac{r\,(\cos\alpha - r)}{M^2} \tag{30}$$

Finding iteratively the angle $\propto^*$ for which the error $\left|\left.\frac{dM}{d\varphi}\right|_{\omega_C} - \left.\frac{dM}{d\varphi}\right|_A\right|$ is minimum implies the optimal controller parameters for a specified modulus margin r.

REFERENCES

Arbogast, J., Beauregard, B., and Cooper, D. (2008). Intuitive robust stability metric for pid control of self-regulating processes. *ISA Transactions*, 47(4), 420–428.

Åström, K. and Hägglund, T. (2006). *Advanced PID Control.* ISA, Berlin Heidelberg.

Bequette, B. (2003). *Process Control: Modelling, Design and Simulation.* Prentice-Hall, NJ.

De Keyser, R. and Ionescu, C. (2010). A comparative study of three relay-based pid autotuners. In *Proc. of the IASTED Conf. On Modelling, Identification and Control.* ACTA Press, Phuket, Thailand. AsiaMIC 2010.

De Keyser, R., Joita, O., and Ionescu, C. (2012). The next generation of relay-based pid autotuners (part 2): A simple relay based pid autotuner with specified modulus margin. In *IFAC Conf. on Advances in PID Control.* International Federation of Automatic Control, Brescia, Italy. 10-3182/20120328-3-IT-3014.00022.

Gatzke, E., Meadows, E., Wang, C., and Doyle III, F. (2000). Model based control of a four-tank system. *Comp and Chem Eng*, 24(2-7), 1503–1509.

Hang, C., Åström, K., and Ho, W. (1991). Refinements of the ziegler-nichols tuning formula. *Ctrl Th and Appl*, 138(2), 111–118. doi:0143-7054.

Husek, P. (2014). Decentralized pi controller design based on phase margin specifications. *IEEE Trans Ctrl Syst Tech*, 22(1), 346–351.

Ionescu, C. and De Keyser, R. (2012). The next generation of relay-based pid autotuners (part1): Some insights on the performance of simple relay-based pid autotuners. In *IFAC Conf. on Advances in PID Control (PID12).* International Federation of Automatic Control, Brescia , Italy. ISSN: 1474-6670.

Johansson, K.H. (2000). The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Trans Ctrl Syst Tech*, 8(3), 456–465.

Johnson, M. and Moradi, M. (2005). *PID Control (Chapters 5-11).* Springer-Verlag, London.

Joita, O. and De Keyser , R. (2012). Validation of a simple relay based pid autotuner with specified modulus margin. In *IEEE Int. Conf. on Automation, Quality and Testing, Robotics.* IEEE Xplore, Romania.

Kristiansson, B. and Lennartsson, B. (2006). Robust tuning of pi and pid controllers. *IEEE Ctrl Syst Mag*, 26(1), 55–69.

Mantz, R. and Tacconi, E. (1989). Complementary rules to ziegler nichols rules for regulating and tracking controller. *Int J Ctrl*, 49(-), 1465–1471.

Menani, S. and Koivo, H. (2001). A comparative study of recent relay autotuning methods for multivariable systems. *Int J of Systems Science*, 32(4), 443–466.

Segovia, V.R., Hägglund, T., and Åström, K. (2014). Measurement noise filters for common pid tuning rules. *Ctrl Eng Prac*, 32(Nov), 43–64.

Smith, C. (2010). *Advanced Process Control.* Wiley, AIChe, Hoboken, New Jersey.

Vadigepalli, R., Gatzke, E., and F.J. Doyle III (2001). Robust control of a multivariable experimental four-tank system. *J Proc Ctrl*, 40(8), 1916–1927.

Wang, Q.C., Ye, Z., Cai, W.J., and Hang, C.C. (2008). *PID Control for Multivariable Processes.* Springer-Verlag, Berlin Heidelberg.

Yu, C. (2006). *Autotuning of PID controllers - A relay feedback approach.* Springer-Verlag, London.