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NONLINEAR TRANSIENT ISOGEOMETRIC ANALYSIS OF LAMINATED COMPOSITE PLATES BASED ON HIGHER ORDER PLATE THEORY

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Abstract: Geometrically nonlinear transient analysis of laminated composite plates is studied using isogeometric analysis (IGA). Herein, higher-order shear deformation theory (HSDT) is applied in displacement field to ensure by itself the realistic shear strain energy part without shear correction factors. IGA utilizing higher-order B-splines basis functions enables to satisfy easily the stringent continuity requirement of the HSDT model without any additional variables. The nonlinearity of the plates based on the von-Karman strain assumptions is solved by the Newmark time integration associated with the Picard method. Two numerical examples of square composite plates are provided to demonstrate the effectiveness of the proposed method.

Keywords: Laminated composite plate, Isogeometric analysis, Higher-order Shear Deformation Theory, Nonlinear analysis, Newmark integration.

1. INTRODUCTION

Laminated composite plates were widely studied by various scientists with a numerous models including 3D elasticity model [1], layer-wise model [2] or equivalent single layer (ESL) theory. The 3D elasticity solution and layer-wise (LW) model can be recommended to improve the accuracy of transverse shear stresses. However, they have a numerous unknown variables producing much computational cost. Thus, reduction a 3D problem to a 2D problem based on the equivalent single layer theory is considered.

Among the ESL plate theories, classical laminate plate theory (CLPT) relied on the Kirchoff-Love assumptions just provides acceptable results for thin plate. First order shear deformation theory (FSDT) based on Reissner-Mindlin hypothesis, which considers the shear deformation effects, was therefore developed for thin and thick plates. Furthermore, higher order shear deformable theories (HSDT), which include higher-order terms in the approximation of the displacement field has then been devised. It is worth mentioning that the HSDT models ensure non-linear distributions of the shear strains/stresses with traction-free boundary condition at the plate surfaces and provide better results and yield more accurate and stable solutions than the FSDT ones. However, the HSDT requires the C^1-continuity of generalized displacement field which is easily satisfied by the approximated functions from isogeometric analysis (IGA).

IGA [3] firstly proposed by Thomas Hughes fulfils a seamless bridge link between computer aided design (CAD) and finite element analysis (FEA). The basis idea of this approach is using the same B-Spline functions in describing the exact geometry of problem and constructing finite approximation for analysis. Being thankful to higher order continuity of B-Spline functions, IGA naturally verifies the C^1-continuity of plates based on the HSDT assumptions. IGA has been widely applied to the plate structures with various plate models such as CLPT [4], FSDT[5], HSDT[6], four unknown variables refined plate theory (RPT) [7], layerwise [8], etc. The literatures mentioned above, however, did not take into account geometric nonlinearity, except two recent papers [9, 10] based on the FSDT. Therefore, our goal in this paper is firstly extended the HSDT model in study transient analysis of the laminated composite plates. Based on the von-Karman strain which considers small strain and moderate rotation assumptions, the nonlinearity of the plates is formulated using total Lagrange approach and solved by the Newmark time integration associated with the Picard methods. Two numerical examples are given to show the effectiveness of the present formulation.

2. ISOGEOMETRIC COMPOSITE PLATE FORMULATION FOR NONLINEAR ANALYSIS

2.1. The higher-order shear deformation plate theory

The displacement of an arbitrary point in plate can be expressed as:

$$\mathbf{u}=\begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix}=\begin{Bmatrix} u_0 \\ v_0 \\ w \end{Bmatrix}-z\begin{Bmatrix} w_{,x} \\ w_{,y} \\ 0 \end{Bmatrix}+\left(z-\frac{4}{3h^2}z^3\right)\begin{Bmatrix} \beta_x \\ \beta_y \\ 0 \end{Bmatrix} \tag{1}$$

Using the von-Karman assumptions, the nonlinear strain – displacement relation adopts here by neglecting second-order terms of u_0 and v_0 displacements

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\frac{1}{2}\frac{\partial w}{\partial x_i}\frac{\partial w}{\partial x_j} \tag{2}$$

The Cauchy stress tensor $\boldsymbol{\sigma}$ is obtained from the constitutive relation based on Hooke's law

$$\boldsymbol{\sigma}=\mathbf{D}\boldsymbol{\varepsilon} \tag{3}$$

where $\mathbf{D}$ - a square matrix of 5×5 is the elastic constant matrix [11].

Neglecting the damping effect, the equation of motion obtained from Lagrange's equation using Hamilton's variation principle can be briefly expressed as [12]

$$\int_\Omega \delta\boldsymbol{\varepsilon}^T\boldsymbol{\sigma}\mathrm{d}\Omega+\int_\Omega \delta\mathbf{u}^T\rho\ddot{\mathbf{u}}\mathrm{d}\Omega=\int_\Omega \delta\mathbf{u}^T\mathbf{f}_s\mathrm{d}\Omega \tag{4}$$

where ρ and $\mathbf{f}_s$ are the density and the surface loads, respectively.

2.2. Brief of B-spline functions

A knot vector $\Xi=\{\xi_1,\xi_2,...,\xi_{n+p+1}\}$ is a non-decreasing sequence of parameter values ξ_i , $i=1,...n+p$, where $\xi_i\in R$ called i^{th} knot, p is the order of the B-spline and n is number of the basis functions. Using Cox-de Boor algorithm, the univariate B-spline basis functions $N_{i,p}(\xi)$ are defined recursively

$$\begin{aligned} N_{i,p}(\xi)&=\frac{\xi-\xi_i}{\xi_{i+p}-\xi_i}N_{i,p-1}(\xi)+\frac{\xi_{i+p+1}-\xi}{\xi_{i+p+1}-\xi_{i+1}}N_{i+1,p-1}(\xi) \\ \text{as } p=0 \quad N_{i,0}(\xi)&=\begin{Bmatrix} 1 & if \quad \xi_i\le\xi<\xi_{i+1} \\ 0 & \text{otherwise} \end{Bmatrix} \end{aligned} \tag{5}$$

The multivariate B-spline basis functions are generated by tensor product of the univariate B-splines

$$N_A(\boldsymbol{\xi})=\prod_{\alpha=1}^{d}N_{i_\alpha,p_\alpha}(\xi^\alpha) \tag{6}$$

where $d=1,\ 2,\ 3$ is the dimensional space. Fig. 1 illustrates an example of bivariate B-spline basis function from tensor product of two univariate B-splines $\boldsymbol{\psi}=\{0,0,0,0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1,1,1\}$ and $\Xi=\{0,0,0,\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{3}{5},\frac{4}{5},1,1,1\}$ in ξ and η direction, respectively.

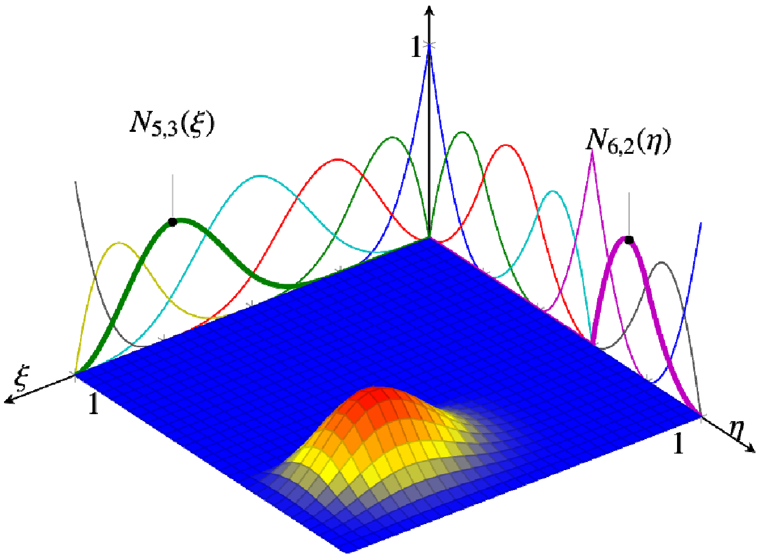

Fig. 1 B-splines basic functions

2.3. Discretization

Using higher-order B-splines basis functions, the displacement field is approximated as:

$$\mathbf{u}^h(\xi,\eta) = \sum_{A}^{m\times n} N_A(\xi,\eta)\mathbf{q}_A \tag{7}$$

where $\mathbf{q}_A$ is the vector of nodal degrees of freedom associated with the control point *A*.

Substituting Eq. (7) into Eq.(2), the generalized strains can be rewritten as:

$$\boldsymbol{\varepsilon} = \sum_{A}^{m\times n}\left(\mathbf{B}_A^L + \frac{1}{2}\mathbf{B}_A^{NL}\right)\mathbf{q}_A \tag{8}$$

where $\mathbf{B}^L$ is the linear part of strain matrix [6], while the nonlinear strain matrix is given as:

$$\mathbf{B}_A^{NL}(\mathbf{q}) = \begin{bmatrix} w_{,x} & 0 & w_{,y} \\ 0 & w_{,y} & w_{,x} \end{bmatrix}^T \begin{bmatrix} 0 & 0 & R_{A,x} & 0 & 0 \\ 0 & 0 & R_{A,y} & 0 & 0 \end{bmatrix} \tag{9}$$

Substituting Eq. (8) into Eq. (4), the equation of motion is written in the following matrix form:

$$\mathbf{Kq} + \mathbf{M}\ddot{\mathbf{q}} = \mathbf{F}^{ext} \tag{10}$$

where $\mathbf{K},\mathbf{M}$ and $\mathbf{F}^{ext}$ are the global stiffness and mass matrices and force vector [13], respectively.

2.4. Solution scheme

From Eq. (10), it is observed that the dynamic equation is dependent upon both time domain and the displacements. To discretize this problem, the Newmark's integration scheme association with the direct iteration method named the Picard method is employed. This solution scheme is detailed in Fig. 2. As noted that the Newmark's constants β and γ are chose as equal to 0.25 and 0.5, respectively [14] and the displacement, velocity and acceleration are set to zero at the initial time.

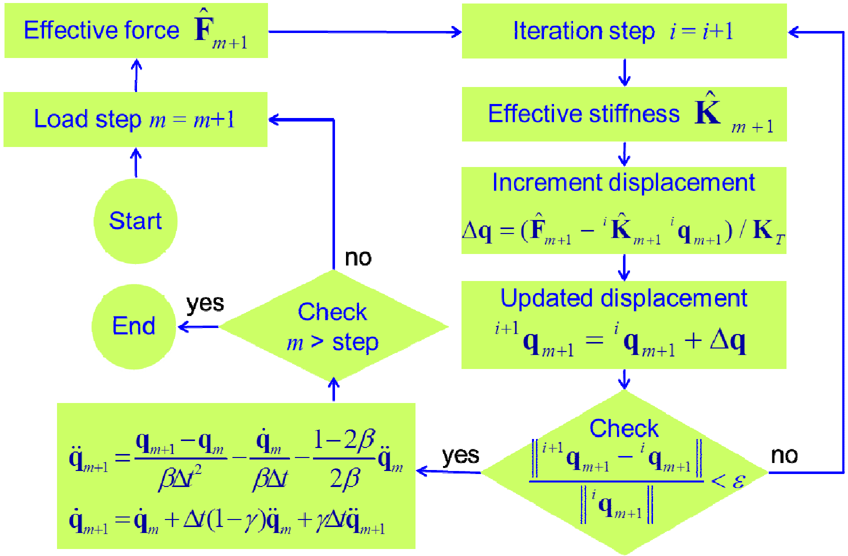

Fig. 2 Flow chart of Picard method.

3. NUMERICAL RESULTS

Firstly, an orthotropic plate [15] with dimensions as; length *L* = 250 mm, thickness *h* = 5 mm is studied for validation. For this problem, the fully simply supported plate is subjected to a uniform step loading of 1 MPa. Its time history of transverse displacement $\bar{w} = w/h$ under both linear and nonlinear analysis is shown in Fig. 3. It is observed that present method predicts the very close deflection response as compared with finite strip method (FSM) [15]. It also clearly exhibits that the magnitude and wavelength of the non-linear response are lower than that of linear behaviour with the same loading intensity.

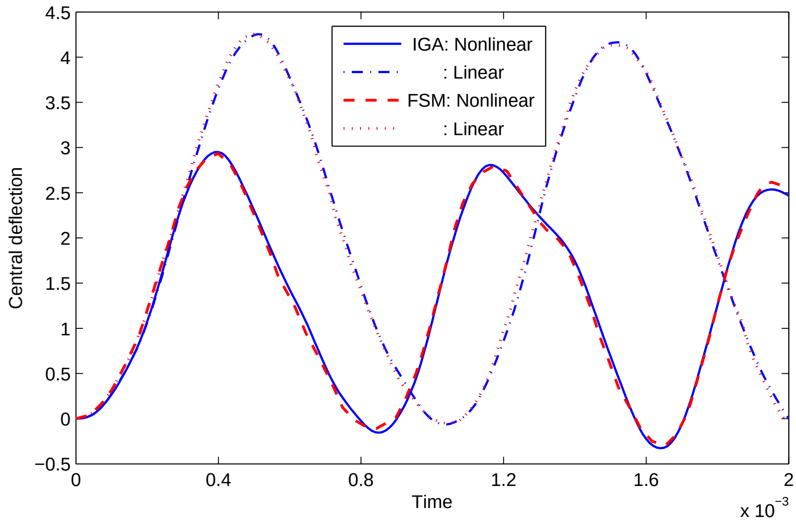

Fig. 3 Time history of the transverse displacement of an orthotropic plate.

Next, the dynamic response of three layer [0/90/0] thick plate [16] is investigated. The plate is square in dimension of *h* = 0.1526m, *L/h* = 5. The transverse load is sinusoidally distributed in spatial domain and is assumed to vary with time as follows

$$f_0(x,y,t) = q_0 \sin(\frac{\pi x}{a})\sin(\frac{\pi y}{b})F_0(t) \tag{11}$$

in which $q_0 = 0.689\ \text{GPa}$ and value of force $F_0(t)$ depicted in Fig. 4 depends on loading types: step, triangular, sinusoidal and explosive blast, respectively. Once again the observation in Fig. 5 is that nonlinear analysis takes the lower central deflection and higher frequency than that of the linear one.

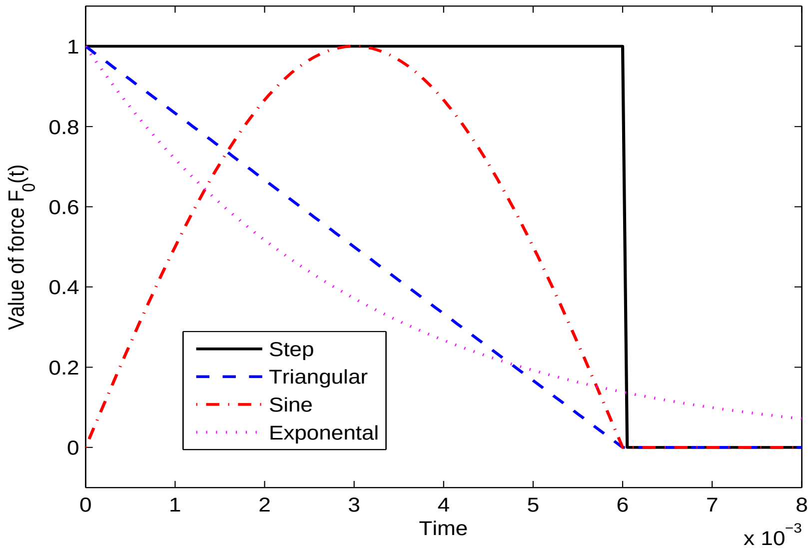

Fig. 4 Time history of load $F_0(t)$.

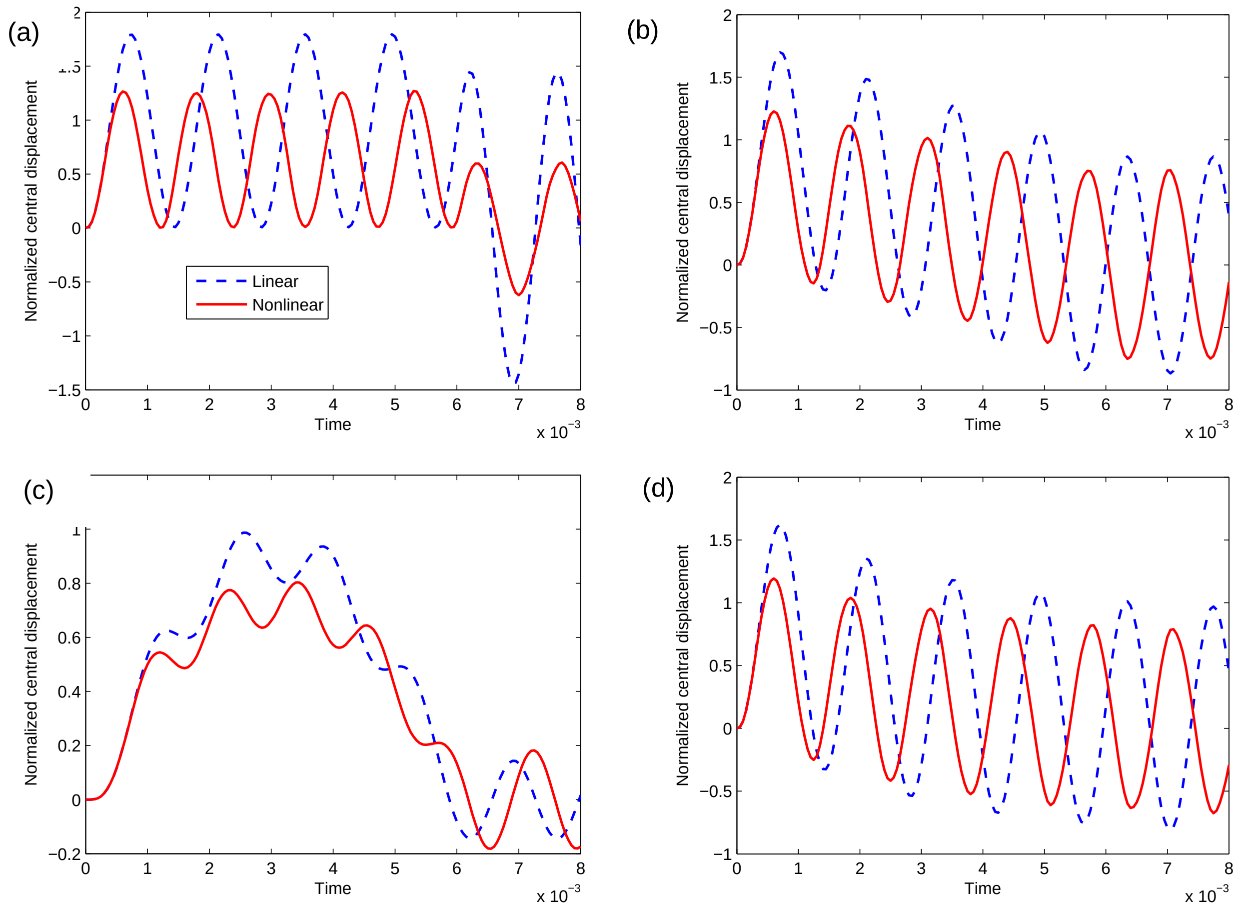

Fig. 5 Effect of different loadings on the deflection respond of the cross-ply [0/90/0] square laminated plate: (a) step; (b) triangular; (c) sine and (d) explosive blast loading.

4. CONCLUSIONS

An effective numerical procedure based on IGA and HSDT has been presented for geometrically nonlinear transient analysis of the laminated composite plates. Herein, using cubic approximation functions, the present method naturally satisfies the C^1 continuity across inter-element boundaries without any additional variables. The nonlinearity of the plates based on the von-Karman strain assumptions is solved by the Newmark time integration associated with the Picard method. The obtained results are in good agreement with available solutions in the literature. It is also concluded that the nonlinear analysis produces lower magnitude and wavelength of the transverse displacement as compare with linear one.

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