The Inventory-Routing problem with uncertain travel times

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Abstract

The central problem studied in this work is the Inventory-Routing Problem (IRP), a combined inventory management and routing problem. One of the main assumptions of this model is that the travel times are predictable and fixed. In this work this assumption is dropped and the IRP is studied subject to uncertain travel times. The only available information is that these travel times are independent and symmetric random variables which can take some value from their support interval.

Keywords: inventory routing problem, uncertainty, robust optimization

For the development of our models we start from a basic version of the IRP described in Coelho et al.(2013) [1]. This basic version minimizes holding and transportation costs while providing a feasible distribution schedule. To avoid that the holding costs would dominate the objective function, we extend the model with a penalty that has to be paid if the tours violate a certain deadline. We propose four different models to face this slightly adapted version of the problem.

The first model consists of two phases. In the first phase we explore the solution space of the problem without taking uncertainty into account. We start from the solution of the nominal problem. Subsequently we add constraints to the problem to enforce lower tours. The holding and transportation costs of the new solution will be higher than the holding and transportation costs of the solution of the nominal problem. However, the probability of paying a penalty is lower which can result in a lower overall cost. We can keep enforcing lower tours until the problem becomes infeasible. The final result is a front of optimal solutions. In the second phase we will apply Monte Carlo simulationto determine the best solution among the solutions of the first phase.

In the second model we use a similar strategy. The big difference is the identification of the most interesting solutions happens not just by enforcing lower tours but by adjusting the degree of robustness. For this purpose we reformulated the problem using a robustness approach developed by Bersimas and Sim(2004) [2]. By starting from the nominal problem and then increasing the level of conservatism we obtain a similar front of solutions as in the previous approach. Subsequently we apply again Monte Carlo simulation to determine the best solution.

In the third model the problem is solved several times for an increasing set of scenarios. To determine which scenarios are included in the scenario set, we use two stages. In the first stage the problem is solved based on the current set of scenarios. We use the solution of this problem in the second stage to find the scenario that is the furthest situated from our solution. This scenario is added to the scenario set after which we return to the first stage. When the scenario set includes enough scenarios, the method converges towards the optimal solution. Note that for this method the variability on the travel times was discretized. The fourth model is similar to the third model except it drops the second phase and just adds random scenarios to the scenario set.

To enhance the performance of our models we propose a number of improvements. The main focus will be on the execution time. Since the original problem is NP-complete, the IRP with uncertain travel times is also. We are confronted with an exponential increase in time both during the exploration of the solution space of the first two methods and during the first stage of the latter two methods. For the first two models we present a proper stopping criterion. For the last two models we present several measures to cope with the increasing complexity of the problem. We design rules for the early fixation of variables, the removal of scenarios out of the scenario set, the use of variance reduction techniques such as antithetic variates to accelerate the convergence of the fourth model, ...

To validate the four models and compare their performance, we set up an experiment using a data set adapted from literature. is on execution time and accuracy. For small instances results indicate that the first model is the best method both in terms of accuracy and execution time. For larger instances the first, second and fourth model are suited. If accuracy is the most important criterion, then the fourth model outperforms the other methods.

References

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