Dual Basis for the Fully Linear LL Functions

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The use of Buffa-Christiansen (BC) basis functions in the Calderon preconditioning of the electric field integral equation (EFIE) has been widely acknowledged during the recent years. The BC functions are dual functions of the lowest order (0.5) curl-conforming rotated Rao-Wilton-Glisson functions (RWG). The BC functions are defined as a linear combination of div-conforming RWG functions on a barycentrically refined mesh. Even though the number of elements in the refined mesh is six times larger, the computation time is decreased because the resulting matrix to be inverted is more well conditioned decreasing the number of iterations when solving the matrix equation iteratively.

Another widely accepted use of the BC functions is encountered in the discretization of the magnetic field integral equation (MFIE). It has been shown earlier that if the MFIE is tested in the dual of it's range with the rotated BC functions, the accuracy is superior compared to the Galerkin testing with the RWG functions. However, it's also well known that the accuracy of the MFIE also increases (even with Galerkin testing) when the surface current is represented with fully linear LL functions instead of the lowest order RWG functions. This raises the question, whether the accuracy of the MFIE with the LL functions could be increased even further, if it is also tested in the dual of its range.

A dual basis for the rotated fully linear (LL) div-conforming basis functions is proposed in this work. Closed form formulas are given which can be used to represent the dual basis in terms of div-conforming functions in the barycentrically refined mesh. The Gram matrix linking the dual basis and the rotated LL basis is then analyzed to show that the mapping between the spaces is invertible. The dual basis is then used both in the Calderon preconditioning of the EFIE and as testing functions for the MFIE to analyze the benefits.