

# TRIANGULAR MODELS FOR STUDYING AND MEMORISING TEMPORAL KNOWLEDGE

Katrin Asmussen<sup>1</sup>, Yi Qiang<sup>1</sup>, Philippe De Maeyer<sup>1</sup>, Nico Van de Weghe<sup>1</sup>

<sup>1</sup> CartoGIS Cluster, Dept. of Geography  
Ghent University, Belgium

*E-mail: {Yi.Qiang, Katrin.Asmussen, Philippe.DeMaeyer, Nico.VandeWeghe}@UGent.be*

## Abstract

The study of a lot of disciplines (e.g. history, geology, archaeology and politics) is often connected to temporal information. The nature of temporal information can be very complex. To help students to understand and memorise time-related knowledge, visualisation is a common approach. Currently, time intervals are usually visualised as linear segments. But, if a lot of information with a complex distribution or even incomplete data is displayed, this visualisation is quickly getting overcharged and confusing. The Triangular Model (TM) and its extension the Rough Triangular Model (RTM) are constituting alternatives to the limited linear model. Instead of linear segments, TM and RTM display time intervals as points, polygons or lines. The positions of them determine as well the beginning, the end and the vagueness of an interval. Additionally TM and RTM are also displaying temporal relations and topology of time intervals. Given the spread of the intervals points, polygons or lines *within* the TM, an easy overview about the complex distribution of time intervals can be read as a map which is helpful for students in learning and memorising temporal knowledge. Both TM and RTM deliver a compact visualisation of time-related knowledge where a huge amount of (rough) time intervals can be clearly displayed.

**Keywords** - Time Intervals, Temporal Relations, Triangular Model, Rough Triangular Model

## 1 INTRODUCTION

The study of a lot of disciplines (e.g. history, politics, geology, archaeology) is often connected to temporal information. To help students to study time related knowledge, a visualisation is common approach. For time intervals, the most widely adopted representation is the linear model. This conventional model is often used to visualise temporal information like the sequence of historical events. It represents time intervals (e.g. lifetime of a king) as finite linear segments in a one-dimensional space. In this classical concept, a temporal interval  $I$  is visualised by a segment that is bounded by a begin point  $I^-$  and end point  $I^+$ . But, if a lot of time intervals with a complex distribution or even incomplete interval are displayed, it is quickly getting overcharged and confusing.

The Triangular Model (TM) constitutes an alternative representation to the conventional linear model [8]. It is based on the W-diagram introduced by Kulpa [3]. The basic concept of TM is the construction of two lines through the extremes of a linear time interval. The intersection of  $L_1$  and  $L_2$  is called the Interval Point (IP). The position of the IP in the two-dimensional space

completely determines both, the beginning and end point of the interval. Considering the three possible relations, i.e. smaller than ( $<$ ), equal ( $=$ ) and greater than ( $>$ ), between the beginnings ( $I$ ) and the ends ( $I^+$ ) of intervals, then, according to Allen [1] thirteen possible relations between intervals can be defined. Within TM, these relations are visualised. Each relation thereby corresponds to a specific relation zone (Fig.: 4), which displays, the topological relations between time intervals. Therefore the TM displays temporal locations and topologies of time intervals as a map of spatial geometries.

Additionally, as a lot of disciplines are faced by the problem of handling imprecise time information. Therefore TM has been extended to the Rough Triangular Model (RTM) in order to represent rough time intervals (RTI) which have uncertain beginnings and ends. RTM follows the same principle as the TM. However, RTM represents RTIs as Interval Polygons (IPO) or Interval Lines (IL) instead of using IP. These IPO and IL indicate the zone within the exact IP is located. Size, position and shape of the IPO or IL are providing information about the impreciseness, duration and temporal relation of the displayed rough interval. RTM has been successfully applied to an archaeological case study [7].

This paper begins with introducing the basics of TM in Chapter 2 and the representation of Allen's temporal relations in Chapter 3. That is followed by the description of the RTM (Chapter 4) which is an extension of the TM and can be used for describing RTIs. Chapter 5 is dedicated to illustrate the use of TM and RTM for memorising temporal knowledge including an explanation of the general interpretation of TM and RTM and the illustration of two concrete case studies. The paper ends with conclusions and an future work in Chapter 6.

## 2 REPRESENTING TIME INTERVALS IN TRIANGULAR MODEL

In the classic concept, an interval  $I$  is represented as a pair  $[I, I^+]$  of real numbers with  $I < I^+$ , denoting the left and right end points of the interval, respectively. This time concept is usually visualised as a linear segment in a one-dimensional space, which starts at a point  $I$  and end at a point  $I^+$  (Fig. 1). In the linear model, the vertical dimension is only used to differentiate multiple overlapping intervals. If just a few intervals are displayed in the linear model all information is easy to capture, but as soon the data structure is getting more complex or the time intervals are even incomplete, then this visualisation is quickly getting unstructured and overloaded.

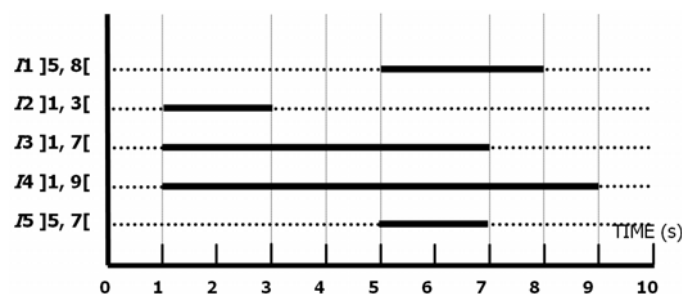


Figure 1: Linear representation of time intervals.

In contrast to that the TM is able to visualise a big amount of time intervals without getting confusing. The basic concept of TM is the construction of two lines through the extremes of a linear time interval (Fig. 2). For each time interval  $I$ , two straight lines ( $L_1$  and  $L_2$ ) are

constructed, with  $L_1$  through  $I^-$ ;  $L_2$  through  $I^+$ , and  $\alpha_1 = \alpha_2$ . The intersection of  $L_1$  and  $L_2$  is called the interval point IP. The position of IP in the two-dimensional space completely determines both, the beginning and end point of the interval. In that way a time interval is represented by a point which is located on a fixed position in a two-dimensional space.

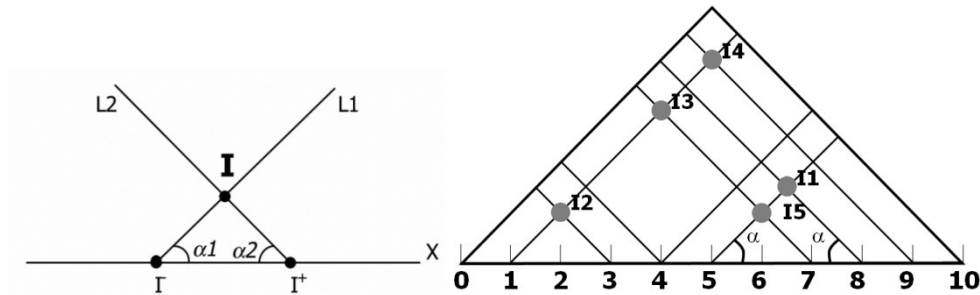


Figure 2: Left: Construction of a simple interval point. Right: Time intervals in TM.

### 3 REPRESENTING TEMPORAL RELATIONS IN TRIANGULAR MODEL

Based on three possible relations between the beginning  $I^-$  and end  $I^+$  of two intervals (smaller than ( $<$ ), equal ( $=$ ) and larger than ( $>$ )), thirteen relations between two intervals are possible (Table 1). These thirteen relations are called Allen's relations [1].

Table 1: Allen's relations of time intervals

$I_1$ equal $I_2$	if $I_1^- = I_2^-$	$\wedge$ $I_1^+ = I_2^+$
$I_1$ starts $I_2$	if $I_1^- = I_2^-$	$\wedge$ $I_1^+ < I_2^+$
$I_1$ started-by $I_2$	if $I_1^- = I_2^-$	$\wedge$ $I_2^+ < I_1^+$
$I_1$ finishes $I_2$	if $I_1^+ = I_2^+$	$\wedge$ $I_1^- > I_2^-$
$I_1$ finished-by $I_2$	if $I_1^+ = I_2^+$	$\wedge$ $I_2^- > I_1^-$
$I_1$ meets $I_2$	if $I_1^+ = I_2^-$	
$I_1$ met-by $I_2$	if $I_2^+ = I_1^-$	
$I_1$ overlaps $I_2$	if $I_2^- > I_1^-$	$\wedge$ $I_1^+ < I_2^+$ $\wedge$ $I_1^+ > I_2^-$
$I_1$ overlapped-by $I_2$	if $I_1^- > I_2^-$	$\wedge$ $I_1^- < I_2^+$ $\wedge$ $I_2^+ < I_1^+$
$I_1$ during $I_2$	if $I_1^- > I_2^-$	$\wedge$ $I_1^+ < I_2^+$
$I_1$ contains $I_2$	if $I_2^- > I_1^-$	$\wedge$ $I_2^+ < I_1^+$
$I_1$ before $I_2$	if $I_1^+ < I_2^-$	
$I_1$ after $I_2$	if $I_2^+ < I_1^-$	

Within TM each relation is represented by a zone, these zones are called Fine Relation Zone (FRZ). Given a study period beginning at 0 and ending at 100, all intervals are located within the isosceles triangular of  $I \in ]0, 100[$ . To obtain the best visualisation, the reference interval  $I_2$  ]33,66[ is chosen to be located in the centre of TM. As shown in Fig. 3b several intervals ( $I_{1a}$ ,  $I_{1b}$ ,  $I_{1c}$ ,  $I_2$ ) may exist before interval  $I_2$  ]33,66[. All possible intervals for which  $I_1^+ < I_2^-$  applies are generalised into the relational zone 'before', displayed by the black triangle in Fig. 3c. Note that as Allen worked with open intervals, also the interval zone corresponding to  $I \in ]0, 33[$  is open. The right boundary of the *before* zone represents all intervals which applies  $I_1^+ = I_2^-$ . Therefore, the intervals have their end point at 33, resulting in the *meets* relationship.

Comparing the TM with the linear model, both of them visualise the same fine interval relations, as displayed in Fig. 3a and Fig. 3b. The benefit of TM in visualising time relations

gets spontaneously obvious. Time intervals in Fig.3b are easier for people to capture than the one in Fig. 3a. All the thirteen temporal relation represented in TM are given in Fig. 4.

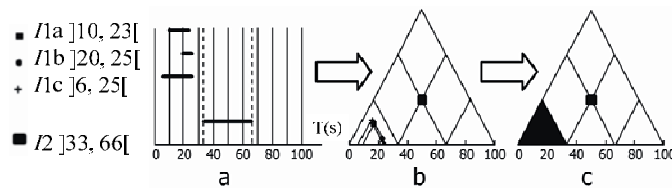


Figure 3: (a) the linear model. and (b)TM. (c)FRZ *before* within TM.

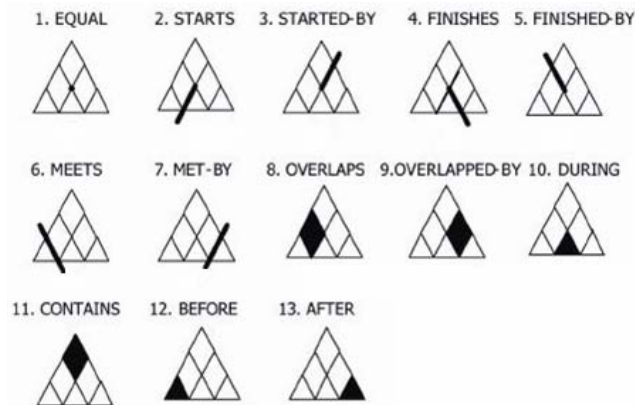


Figure 4: Allen's relations in TM.

#### 4 ROUGH TRIANGULAR MODEL

Students are often faced with problems of learning imprecise temporal knowledge. In these cases, it is not possible to locate the precise location of an interval, but only possible to state that an interval started within a range (e.g. between 1999 und 2004) and ended within a range (e.g. between 1999 und 2004). Rough set theory is one of different approaches in dealing with such imprecise information [5, 4]. Time intervals modelled with rough set theory are called RTIs. The time ranges which are indicating the uncertain area whether a time stamp is within an interval or not is called boundary regions. Boundary regions are situated at the beginning and end of an interval. Due to the nature of data acquisition, RTIs are useful in many time-related disciplines. In archaeology, for instance, it is always difficult to decide the exact beginning and end of a dynasty. The beginning and end of a dynasty is often decided by the age of evacuated artefacts. The age gap between the last evacuated artefact of the former dynasty and the first evacuated artefact of the later dynasty is often creating an uncertain division of two dynasties. The time intervals of these two dynasties can be set as RTIs. Besides history and archaeology, this idea applies to numerous other fields, such as, soil core sampling, socio-economical census and surveys, and opinion polls.

In the classical linear model, RTIs are represented as linear segments with an uncertain beginning range and an uncertain end range (Fig. 5). If there are a huge amount of rough intervals, it is quite difficult for humans to abstract information from this representation. In RTM, a RTI is represented by a geometry which can be an IPO or an IL. For an IPO four lines are constructed respectively from the earliest/latest beginning and the earliest/latest end of an interval. An IL is constructed following the same idea, but instead of having both an uncertain

beginning and end. One of them is known (Fig. 6). IPO and IL are indicating zones in which the uncertain interval is located. We call this representation the Rough Triangular Model (RTM).

In some circumstances, the beginning and end range have intersections. For  $I_5$  in Fig. 6 for example, we only know that it starts between 3 and 5, and ends between 3 and 5. In this case, the interval is not represented as IPO or IL but as a triangle on the X-axis. The shape and size of the geometries representing the intervals are helpful in recognising the characteristics of an interval. Therefore RTM provides a more straightforward and concise visualisation than the classic linear model.

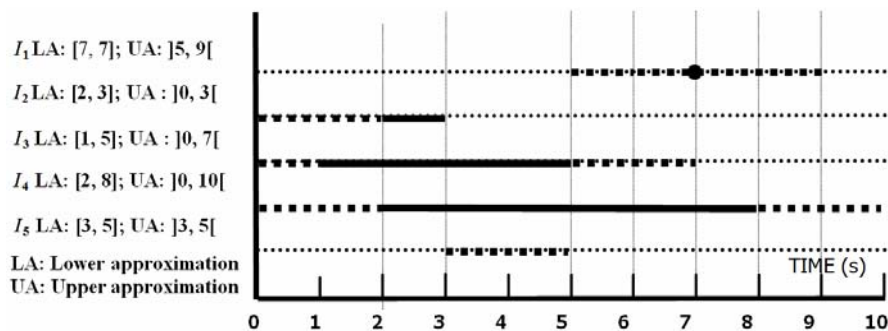


Figure 5: Linear representation of rough time intervals

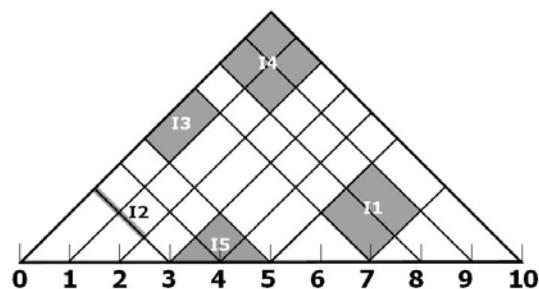


Figure 6: Visualising rough time intervals in RTM.

## 5 APPLICATIONS IN EDUCATION

### 5.1 Interpretation guideline for TM and RTM

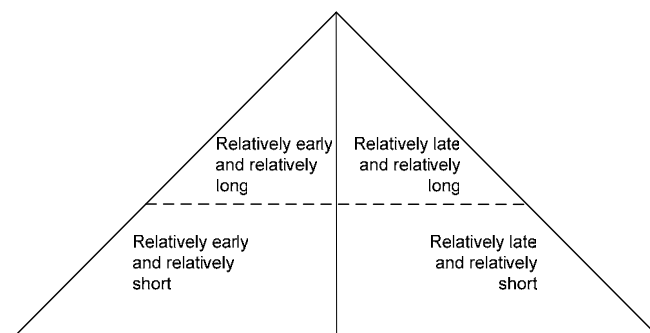


Figure 7: Interpretation guideline for TM and RTM.

TM and the RTM are readable like a map, depending on the position of the IP, IPO and IL it can be observed whether an interval is relatively long or short, relatively early or late, with

respect to the study period. Furthermore it can be observed whether an interval is before or after a specific time event or another interval and whether it is longer or shorter than a specific reference interval. Additionally, all the other time relations of Allen [1] are visualised. The general interpretation guideline is as follows, if an IP, IPO or IL is located relatively low within the TM or RTM then the respective interval is relatively short in relation to the study period. If IP, IPO or IL are relatively high within the model, then the respective interval is relatively long in relation to the study period. If they are situated relatively left or right within the model, then the related interval is more in the beginning and vice versa. Following these principles, intervals in Fig.7 positioned in the left bottom corner are relatively short and relatively early, if the interval is located in the bottom right corner, the represented interval is relatively short and relatively late within the study period. Following the same idea, an IP, IPO or IL situated in the top left part of the model represents an interval which is relatively long and relatively early within the study period. If IP, IPO or IL is situated on the bottom of the triangle the corresponding intervals are extremely short. On the top of the model, they have the longest extent within the study period.

For IPO and IL also their shape and size are crucial. If the four sides of the IPO are having the same length (Fig.:6, I4), then the boundary regions of the intervals at the beginning and at the end are having the same length, if two oppositely sides are longer or shorter than the remaining ones, then the boundary regions at the beginning or respectively at the end of the interval is longer or shorter (Fig. 6, I3) In general, as bigger the IPO, as bigger the boundary regions and vice versa. For IL the size is crucial, as longer the line as bigger the regarding boundary region.

## 5.2 Visualising the geological time scale

Earth scientists divide the history of the earth into many time spans, i.e. the geological time scale. The most prevalent representation of the geological time scale is a chronological chart where the time spans are represented as linear segments. Fig. 8 is a typical example where the segments at the vertical direction represent geological time spans, while the nominal titles at the horizontal direction represent the hierarchy. Each time span (e.g. eon, era, period and epoch) has a beginning and an end. But due to the controversy in the earth sciences, the beginnings and ends may have positive/negative deviation. In Fig. 8, the lengths of segments are not consistent with the real lengths of spans, because real lengths of spans vary from millions of years to billions of years. Although this representation gives all the information of the geological time scale it is confusing in recognising the length and the hierarchy of time spans.

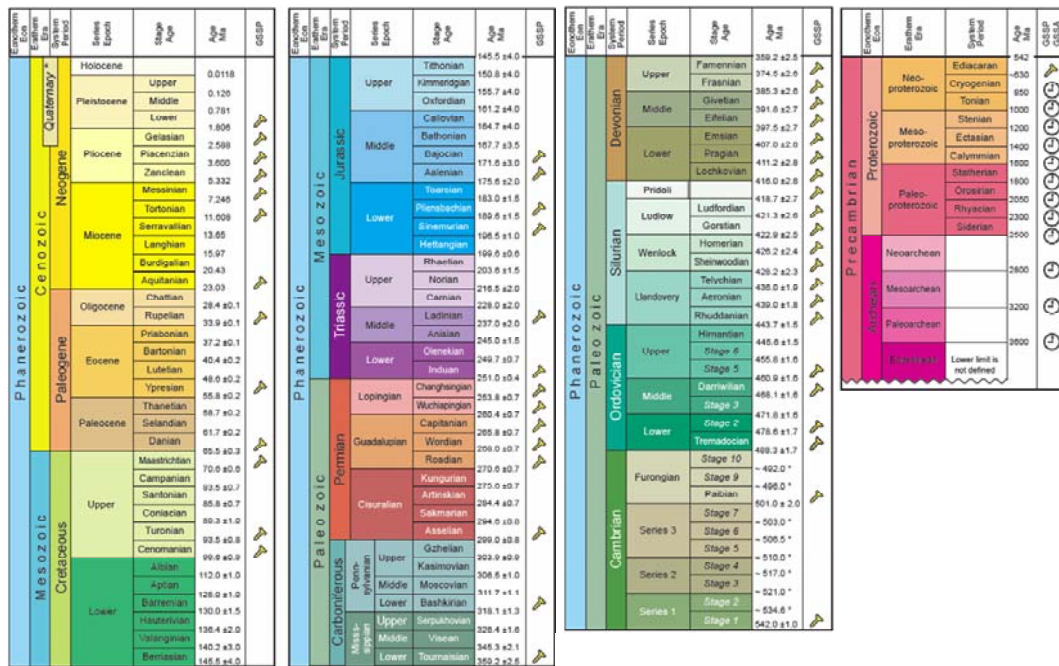


Figure 8: The geological time scale proposed by International Commission of Stratigraphy [7]

Fig. 9 illustrates the triangular representation of the geological time scale. Each time span is represented as a geometry (IP; IPO, IL), depending on the precision of its beginning and end. The time axis points from the past to the present. Colour-coded lines connect the geometries with their beginnings and ends. Blue lines denote eons, green lines denote eras and red lines denote periods. The top figure in Fig. 9 displays the entire geological time scale where all the time spans are visualised as points (with circles around). In this figure, the previous two eons (i.e. Proterozoic and Archean) have longer durations, while the last eon (i.e. Phanerozoic) is relatively shorter and more difficult to read its inner structure. If people zoom in the Phanerozoic eon, the eras and periods in the Phanerozoic eon are clearly displayed (Right-bottom in Fig. 9). If people zoom further into the Paleozoic eon, the periods are actually represented as polygons because they have controversial beginnings and ends (Left-bottom in Fig. 9). The TM visualisation is more interesting when implemented as computer-aided tool where people can zoom in/out or pan the map of interval points.



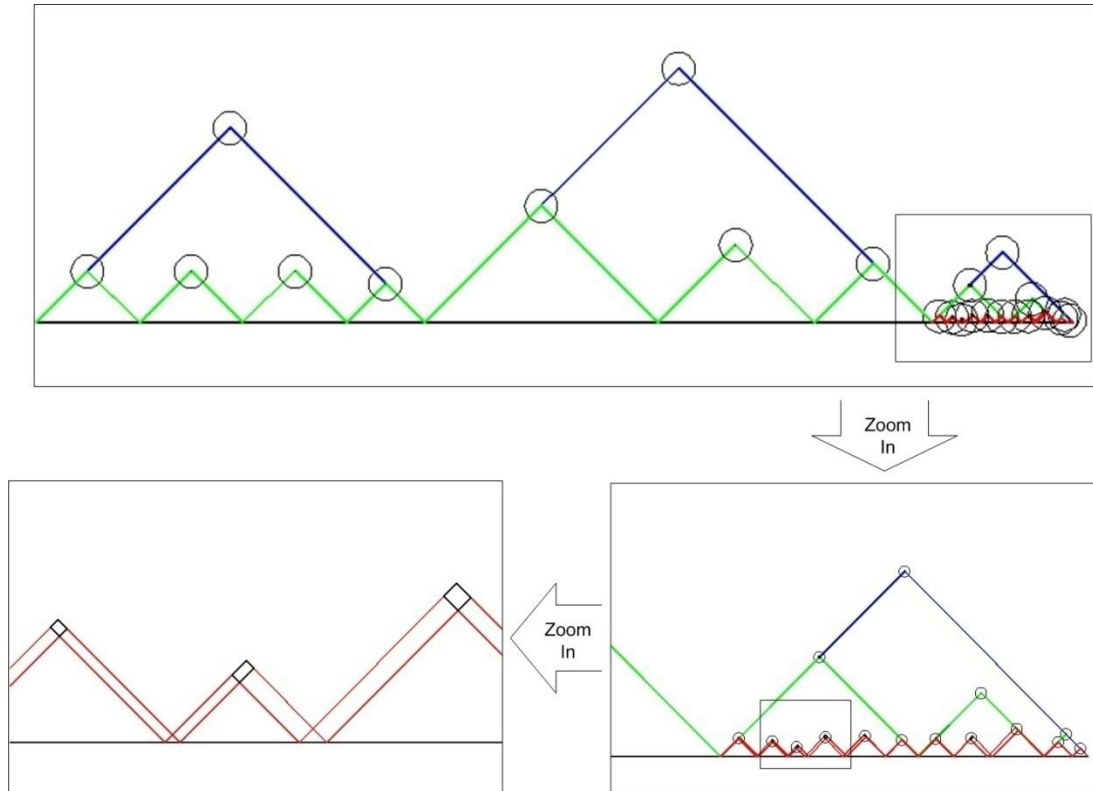


Figure 9: The TM/RTM representation of the geological time scale

### 5.3 Archaeological application of Rough Triangular Model

In order to evaluate RTM in visualising imprecise temporal information; we applied it into an archaeological use case [6]. During World War I, aerial photos covering the Belgian-German front line in West-Flanders (Belgium) have been taken. From these aerial photos, we can observe whether a feature such as a fire trench, a gun position or a barrack, was not yet present, was present or was destroyed. For each feature, several photos with according date of acquisition exist. Hence, if we can find (1) the last photo where the feature is not yet found, (2) the first photo where the feature is found, (3) the last photo where the feature is found, and (4) the first photo where the feature is destroyed, we can derive a RTI representing the feature's uncertain lifetime (Fig. 10).

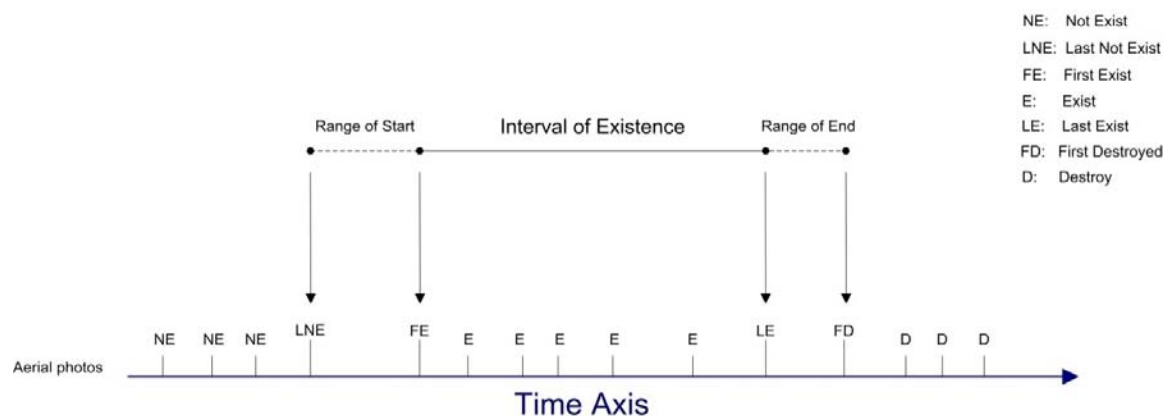


Figure 10: Uncertain existence time of an archaeological feature.



The lifetime of these features are represented as IPO which are denoting zones within the IP are located. Some of the polygons may overlap or partially overlap. More overlapped areas are marked with darker grey, and vice versa. Fig. 11 and Fig. 12 visualise existence time of gun positions and breastworks respectively. Comparing these two visualisations, we can have a direct overview about how the two types of features are temporally distributed. As in the model most polygons of gun positions are distributed right of the breastworks zones, comparing Fig. 11 with Fig. 12, people may observe that gun positions exist relatively later than breastworks. It also can be observed that breastworks generally exist longer than gun positions because most of the breastworks zones are higher on the y-axis than the zones of the gun positions. However, the information may quite difficult to abstract in the classic linear model.

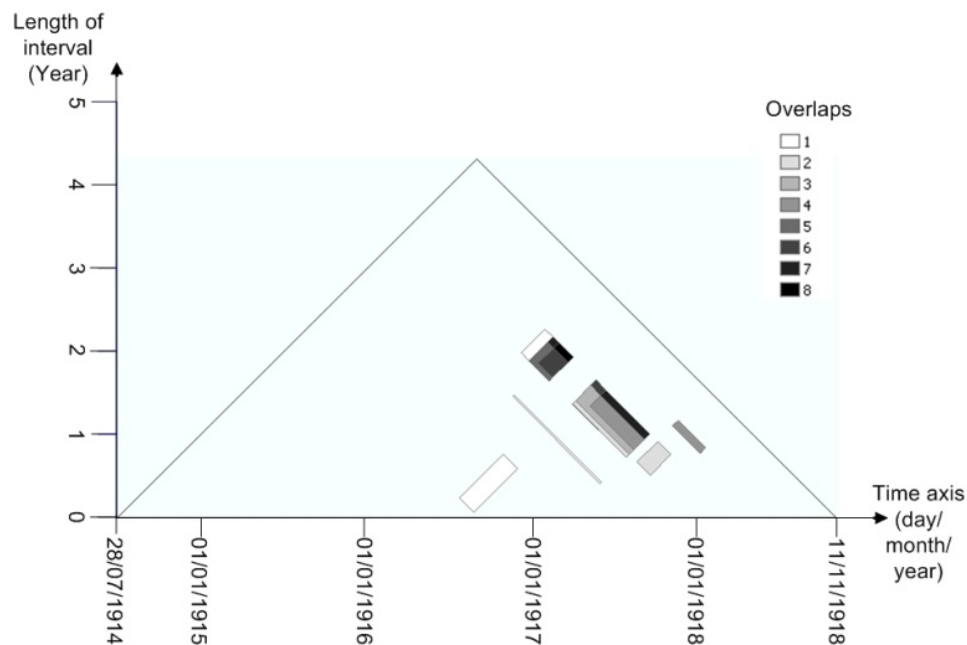


Figure 11: Rough interval zones of gun positions.

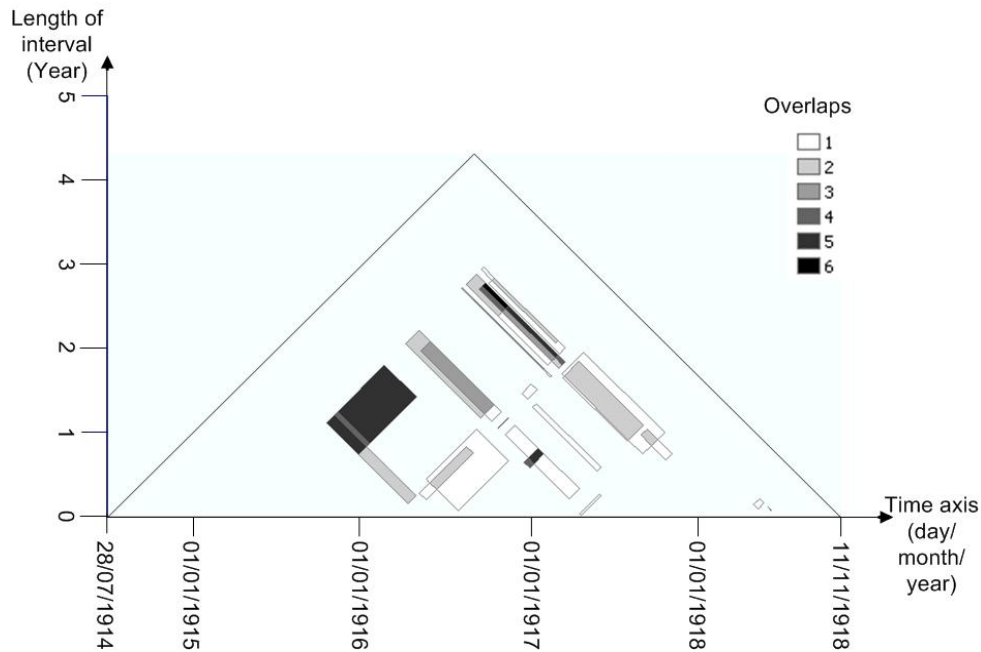


Figure 12: Rough interval zones of breastworks.

## 6 CONCLUSIONS AND FUTURE WORK

This paper presents an alternative visualisation of time intervals which can be used in representing various types of time-related knowledge. Compared with the classic linear model, TM and RTM showing advantages in following aspects. First of all they are providing compact visualisations of time intervals with the aid of points, polygons and lines instead of just line segments. Secondly, each IP, IPO and IL in TM and RTM is having a fix position in a two-dimensional space. Therefore, a set of time intervals is displayed in one single time map which makes it easier for students to study and memorise the distribution and topology of time intervals and the temporal information e.g. the geological time scale.. Thirdly, relations of time intervals are represented as zones, which are easier for people to recognise than comparing extremes of linear intervals. The triangular models establish the possibility for students to gather a mental map of e.g. historical events time lines do not just need to study by hard; with this approach a visual impression of the sequence of events is created. These advantages of TM and RTM are helping student in studying and memorising temporal knowledge in various disciplines.

But, on the practical side, the user friendliness of the TM and the RTM needs to be improved. TM and RTM for the use of studying and memorising temporal data presented in this paper are still having a basic level and show the principles of these new approach. However, the layout and the labelling of TM and RTM needs to be rectified. The example of chapter 5.shows the benefit of the visualisation of TM and RTM in contrast to the conventional line model, but is still lacking a user friendly layout and labelling of the information to the respective IP, IPO or IL.

Future work will have to concentrate on the design of labelling the model to ensure an easy overview not just about the time interval but also over the information's which are linked to them. Further more the models (TM and RTM) should get implemented in teaching material,

as well in print media as well in software. Interactive software which allows zooming is delivering a promising environment for this way of representation of time intervals. With zooming in and out unlimited time periods can be displayed ranging from the beginning of calculation of time till seconds or even smaller time durations.

## References

- [1] Allen, J.F., *Maintaining Knowledge about Temporal Intervals*. Communications of the ACM, 1983. **26**(11): p. 832-843.
- [2] Gradstein, F.M., J.G. Ogg, and A.G. Smith, *A geologic time scale 2004*. 2004: Cambridge University Press. 589
- [3] Kulpa, Z., *Diagrammatic representation for a space of intervals*. Machine Graphics & Vision, 1997. **6**: p. 5-24.
- [4] Pawlak, Z., *Rough Sets: Theoretical Aspects of Reasoning About Data*. 1991, Dordrecht, Netherlands: Kluwer Academic Publishers.
- [5] Pawlak, Z., *Rough Sets*. International Journal of Information and Computer Science, 1982. **11**(5): p. 341-356.
- [6] Qiang, Y. et al., *Visualizing Rough Time Intervals in a Two-Dimensional Space*, Conference proceeding IFSAWorld Conference and EUSFLAT, 2009
- [7] Van de Weghe, N., et al., *The triangular model as an instrument for visualising and analysing residuality*. Journal of Archaeological Science, 2007. **34**: p. 649-655.
- [8] Van de Weghe, N. and P. De Maeyer. *A two-dimensional temporal model: base of a spatio-temporal model and a spatio-temporal data format*. in *the 11 the GIS research UK conference (GISRUK)*. 2003. London, UK.