High-Order Calderón Multiplicative Preconditioner for Time Domain Electric Field Integral Equations

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Marching-on-in-time (MOT) time domain electric field integral equation (TDEFIE) solvers are often used to analyze scattering of transient electromagnetic fields from perfect electrically conducting (PEC) surfaces. Unfortunately, just like their frequency domain counterparts, MOT-TDEFIE solvers suffer from dense-mesh breakdown when the linear dimensions of the patches in a surface mesh are small compared to the space-time step and/or the order of the spatial basis functions used to expand the surface currents is high. This breakdown phenomenon manifests itself in the form of ill-conditioned MOT system matrices and slow convergence rates of the MOT iterative solver.

Recently, a Calderón multiplicative preconditioner (CMP) for regularizing MOT-TDEFIE solvers by leveraging Calderón identities was proposed (Kools et al., IEEE Trans. Antennas Propagat., 57, 2009, 2352-2364). The Calderón identities express the fact that the square of the TDEFIE operator is a “compact perturbation of the identity” and their application leads to well-conditioned MOT-TDEFIE system matrices even when the mesh includes sub-wavelength geometric features. As the preconditioner is multiplicative in nature, it is easily integrated into existing codes. The CMP uses two separate discretizations of the EFIE operator, one in terms of standard Rao-Wilton-Glisson (RWG) basis functions, and the other in terms of Buffa-Christiansen (BC) basis functions. Unfortunately, the low order character of these functions limits the accuracy and efficiency of current CMP-MOT-TDEFIE solvers.

This article describes a high-order CMP (HO-CMP) for MOT-TDEFIE solvers. The HO-CMP exchanges the RWG functions for Graglia-Wilton-Peterson ($\mathrm{GWP}(p)$) ones, which comprise of products of scalar polynomials (complete up to order p) and RWG basis functions (Graglia et al., IEEE Trans. Antennas Propagat., 45, 1997, pp. 329-342). Similarly, it exchanges BC functions for a new set of pth-order div- and quasi curl-conforming ($\mathrm{DQCC}(p)$) functions that, when used in conjunction with the $\mathrm{GWP}(p)$ functions, give rise to a well-conditioned Gram matrix that links both function sets while guaranteeing the annihilation of the square of the discretized hypersingular component of the TDEFIE operator (Valdés et al., IEEE Antennas Propagat. Soc. Int. Symp, 2009). The MOT-TDEFIE system matrices are discretized just as in any standard high-order time-domain solver using $\mathrm{GWP}(p)$ basis functions. The preconditioner is built using a localized TDEFIE operator discretized with $\mathrm{DQCC}(p)$ basis functions. The use of a localized instead of a “full” TDEFIE operator allows for a dramatic reduction in the computational complexity and memory requirements of the HO-CMP. Due to the properties of the $\mathrm{GWP}(p)$-$\mathrm{DQCC}(p)$ pair, the preconditioned system is guaranteed to be well-conditioned regardless of the mesh density and the order p of the basis functions used. This fact is corroborated by our numerical results.