Incomplete-Fusion Reactions in the $^{14}N + ^{159}Tb$ System and a "Sum-Rule Model" for Fusion and Incomplete-Fusion Reactions

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Absolute cross sections for a number of binary processes in the $^{14}N + ^{159}Tb$ reaction have been determined by use of γ-particle coincidence techniques. The data are used to test a "sum-rule model" of complete- and incomplete-fusion reactions. The model assumes the reaction probabilities to be scaled by the available phase space and to be constrained additionally by the entrance-channel angular momentum limitations.

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An investigation of the energy dependence of the cross sections for two main incomplete-fusion channels in the $^{12}C + ^{160}Gd$ reaction[1,2] led us to an interpretation of these reactions in terms of a generalized concept of critical angular momentum.[1] According to this concept incomplete-fusion reactions are localized in successive "l windows" above the critical angular momentum for a complete fusion, in a sequence beginning with the capture of the heaviest fragment of a projectile, followed by the capture of lighter fragments at higher angular momenta. Predictions of this model have been confirmed by Geoffroy *et al.*,[3] who measured γ-multiplicity distributions in coincidence with the products of incomplete-fusion reactions.

In this Letter we report on the measurements of the absolute cross sections for a variety of *binary* reactions in the $^{14}N + ^{159}Tb$ collisions at 140 MeV bombarding energy. It is shown that all those binary reactions (with the exception of simple direct transfer) can be understood in terms of the same mechanism that governs the incomplete-fusion processes. Moreover, it seems that this common reaction mechanism comprises also complete fusion. A sum-rule model that enables one to predict absolute cross sections for complete fusion and all incomplete-fusion channels, as well as the distribution of these reactions in l space, is proposed.

The $^{14}N + ^{159}Tb$ system was chosen with the aim to observe a large number of reaction channels [contrary to the $^{12}C + ^{160}Gd$ system studied previously[1,2] for which the important reaction channels were ($^{12}C, \alpha$), ($^{12}C, 2\alpha$), and ($^{12}C, 3\alpha$) only]. Experiments were performed using a 140-MeV ^{14}N beam from the variable-energy cyclotron of the Kernfysisch Versneller Institute bombarding a 1.4-mg/cm^2-thick target of metallic ^{159}Tb. Selection of binary reactions was possible by measuring coincidences between γ rays (used to identify the target-residue nuclei) and charged reaction products detected and identified with a standard ΔE-E telescope consisting of a 50-μm transmission detector and a 5-mm E detector. Particle identification (Z and A) was achieved for all reaction products over the range $2 \leq Z \leq 8$. Details of the experimental techniques used in our experiments are contained in Ref. 2.

Absolute cross sections for different binary reaction channels were obtained by summing the cross sections for all subchannels with appreciable yield:

$$\sigma[^{159}\mathrm{Tb}(^{14}\mathrm{N}, {}^{A}Z)] = \sum_x \sigma(^{14}\mathrm{N}, {}^{A}Zxn)^{173-A-x}(72-Z). \qquad (1)$$

The absolute cross sections for a number of bi-

TABLE I. Cross sections for various reactions in the $^{14}N + ^{159}Tb$ system at 140 MeV determined from particle-γ coincidence data, σ_{expt}, and the calculated cross sections, σ_{calc} ($T = 3$ MeV, $\Delta_l = 2\hbar$, $q_C = 0.06$ fm^{-1}).

Reaction	σ_{expt} (mb)	σ_{calc} (mb)
Fusion	...	1030
(^{14}N, ^{4}He)	84 ± 16	160
(^{14}N, ^{6}He)	< 1.6	0.9
(^{14}N, ^{6}Li)	< 3.5	2.5
(^{14}N, ^{7}Li)	< 6	1.4
(^{14}N, ^{7}Be)	< 1.5	1.3
(^{14}N, ^{8}Be)	36 ± 8	42.7
(^{14}N, ^{9}Be)	5.6 ± 1.1	6.9
(^{14}N, ^{10}Be)	5.8 ± 1.3	6.6
(^{14}N, ^{10}B)	5.3 ± 2.2	2.7
(^{14}N, ^{11}B)	11 ± 4	7.7
(^{14}N, ^{12}B)	3.5 ± 1.2	2.4
(^{14}N, ^{11}C)	2.4 ± 0.7	1.5
(^{14}N, ^{12}C)	36 ± 5	48.9
(^{14}N, ^{13}C)	22 ± 2	29.6

nary reaction channels deduced from particle-γ coincidence data are listed in Table I. In a forthcoming paper we will discuss the results in detail, together with the data on inclusive cross sections, particle-particle coincidence data, and γ-multiplicity measurements. In this Letter we concentrate on a new interpretation of the incomplete-fusion reactions using as a test of the proposed model the cross sections listed in Table I, and the previous data on the $^{12}C + ^{160}Gd$ system.

Our results on the $^{14}N + ^{159}Tb$ system show that the incomplete-fusion reactions characterized by the capture of a massive fragment of a projectile and emission of "fast" light ejectiles, e.g., α particles,[1-5] protons, deuterons, or tritons,[6] belong to a wide class of *binary* reactions in which ejectiles have the average kinetic energies close to those required by the l-matching condition. On the other hand, it is known from inclusive experiments that for this class of reactions the cross sections depend exponentially on the ground-state Q value, Q_{gg}.[7] This striking relation has been interpreted by Bondorf *et al.*[8] in terms of a partial statistical equilibrium of a strongly interacting dinuclear system. Following such phase-space arguments we postulate that the reaction probabilities for all reactions i proceeding via the partially equilibrated system are proportional to the exponential factor as proposed by Bondorf *et al.*[8]:

$$p(i) \sim \exp\{[Q_{gg}(i) - Q_C(i)]/T\}, \qquad (2)$$

where T is an effective temperature, and Q_C is the change of the Coulomb interaction energy due to transfer of charge. Q_C can be parametrized as

$$Q_C \approx q_C(Z_1^f Z_2^f - Z_1^{in} Z_2^{in})e^2, \qquad (3)$$

where Z_1^{in}, Z_2^{in} and Z_1^f, Z_2^f are the atomic numbers of the constituents of the dinuclear system before and after the transfer of charge, respectively, and q_C is a parameter. It should be pointed out that we apply postulate (2) not only to all binary divisions of the dinuclear system, but also to *complete fusion* of the system.

Apart from the phase space, also the *entrance-channel angular momentum limitations constrain the reaction probabilities.* We use the same recipe as that proposed in Ref. 1: For each reaction channel i, the limiting angular momentum is

$$l_{lim}(i) = \frac{\text{mass of projectile}}{\text{mass of captured fragment}} \times l_{cr}(\text{target} + \text{captured fragment}), \qquad (4)$$

where l_{cr} is calculated from the balance of forces[9]:

$$(l_{cr} + \tfrac{1}{2})^2 = \frac{\mu(C_1 + C_2)^3}{\hbar^2}\left[4\pi\gamma\frac{C_1 C_2}{C_1 + C_2} - \frac{Z_1 Z_2 e^2}{(C_1 + C_2)^2}\right]. \qquad (5)$$

Here C_1 and C_2 are the half-density radii, and γ is the surface-tension coefficient. To make our model more realistic we also assume a smooth cutoff in the distribution of the "transmission coefficients" T_l:

$$T_l(i) = \left[1 + \exp\left(\frac{l - l_{lim}(i)}{\Delta_l}\right)\right]^{-1}. \qquad (6)$$

Combining Eqs. (2) and (6), one can write the following sum rule:

$$N_l \sum_i T_l(i) \exp\left(\frac{Q_{gg}(i) - Q_C(i)}{T}\right) = 1, \qquad (7)$$

provided all the reaction channels *(including complete fusion)* are taken explicitely into account. With the normalization factor N_l calculated from Eq. (7) one obtains absolute cross sections for each reaction channel:

$$\sigma(i) = \pi\lambda\!\!\!^{-2} \sum_{l=0}^{l_{max}} (2l+1) N_l T_l(i) \exp\left[\frac{Q_{gg}(i) - Q_C(i)}{T}\right]. \qquad (8)$$

We further postulate that the l_{max} limiting the sum in Eq. (8) is the maximum l for which the

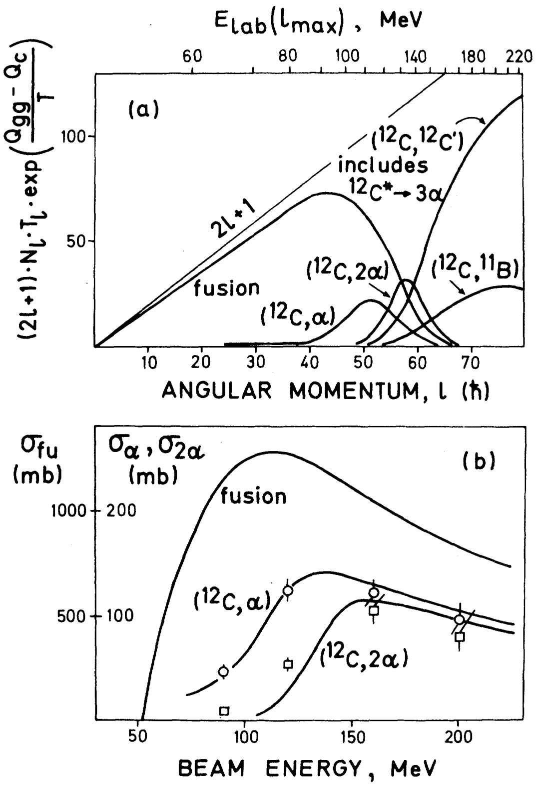

FIG. 1. (a) Reaction probabilities for the main reaction channels in the $^{12}C + ^{160}Gd$ system (T = 3 MeV, $\Delta_l = 2\hbar$, $q_C = 0.06\ \mathrm{fm}^{-1}$). (b) Excitation functions for complete fusion and two main incomplete-fusion channels in $^{12}C + ^{160}Gd$ collisions calculated with the same set of parameters. Experimental data are taken from Ref. 1.

colliding system gets into the region where the total nucleus-nucleus potential is attractive and/or the distance of closest approach is smaller than the sum of the half-density radii. Note that this criterion is identical with that proposed by Glass and Mosel[10] to determine the range of partial waves leading to complete fusion. Now we make the assumption that *this range of partial waves confines complete-fusion and incomplete-fusion reactions together.*

To calculate l_{max} one has to use a certain nucleus-nucleus potential. Results of the numerical calculations presented in this Letter were obtained assuming the point-charge Coulomb interaction potential and the nuclear potential as proposed in Ref. 11, based on the boundary conditions following the liquid-drop model (adiabatic limit for small relative distances, and sudden approximation for proximity configurations). The half-density radii [used, e.g., in Eq. (5) and in calculation of l_{max}] were parametrized as proposed by Myers.[12] Only for the lightest fragments ($A \leq 4$) for which the approximation $C \approx R - b^2/R$ fails (R is the equivalent sharp radius, $b = 1$ fm) the half-density radii were scaled proportionally to $A^{1/3}$ with the constant that matches the Myers parametrization at $A = 4$.

The only free parameters of the model outlined above are the effective temperature T, the diffuseness of the T_l distributions Δ_l (assumed to be the same for all reaction channels), and the Coulomb energy parameter q_C. Contrary to standard statistical-model calculations, the parameter T has no clear interpretation.[8] Therefore it should be regarded rather as an empirical parameter, similarly as Δ_l and q_C.

Figure 1(a) shows the distributions of the reaction probabilities for the strongest channels in $^{12}C + ^{160}Gd$ close-contact collisions. Reaction channels involving complete fusion and reactions with emission of n, ^{1-3}H, $^{3,4,6}He$, $^{6,7}Li$, ^{7-10}Be, $^{10-12}B$, and $^{11,12}C$ as ejectiles were taken explicitly into the sum rule (7). Because of the largest value of $Q_{gg} - Q_C$, the complete-fusion channel takes almost the whole geometrical cross section for low l values. Only above the critical angular momentum ($l_{cr} \approx 42\hbar$) the probability for complete fusion diminishes and thus makes room for other competing reaction channels. Other important channels next to complete fusion are (^{12}C, α) and (^{12}C, 2α)—again because of the largest $Q_{gg} - Q_C$ values. The l_{lim} values for these two channels differ significantly ($\sim 47\hbar$ and $\sim 61\hbar$, respectively). This is a reason why the reaction probability for (^{12}C, α) decreases first while the probability for the other reaction still increases until its own angular-momentum limitation is reached. As it is seen from Fig. 1(a) the l windows for incomplete-fusion channels postulated in Ref. 1 appear automatically in the model proposed here. However, with parameters that fit the experimental data, the windows are not sharply separated (as assumed in the model of Ref. 1) and the cross sections for particular incomplete-fusion channels are significantly smaller than the geometrical cross sections within the l windows.

Figure 1(b) shows a comparison of the model predictions with the excitation functions for the $^{160}Gd(^{12}C, \alpha)$ and $^{160}Gd(^{12}C, 2\alpha)$ reactions.[1] For simplicity, the parameter T was assumed to be energy independent. (With increasing bombarding energy the thermalized part of kinetic energy is probably shared between more and more degrees of freedom. Therefore the effective temperature of a *partially* equilibrated system may not tend to increase much.) As it is seen from Fig. 1(b), the calculated excitation functions reproduce well the energy thresholds and absolute cross sections for both (^{12}C, α) and (^{12}C, 2α) incomplete-fusion reactions.

The proposed model can be checked more extensively using our new data on the ^{14}N+^{159}Tb system. The absolute cross sections for a number of capture reactions (see Table I) are compared with the model predictions if we assume the same set of parameters as that used for the ^{12}C+^{160}Gd system. The calculated cross sections are given also for other channels for which we could not determine the cross sections. [For instance, we were not able to measure the compound-residue cross section since the (^{14}N, xn) channels in the ^{14}N+^{159}Tb reaction at 140 MeV lead to unknown isotopes of Hf.] There is a close correlation between the calculated and measured cross sections. This result strongly indicates that both the Q_{gg} effects and the entrance-channel limitations built into the model influence the cross sections.

The analysis of the experimental data on the ^{12}C+^{160}Gd and ^{14}N+^{159}Tb systems gives strong arguments in support of the proposed model. An intriguing question is whether the mass and charge distributions of the products of strongly damped reactions in collisions of very heavy systems can also be described by this model. The proposed model couples strongly the complete-fusion reactions with other reaction modes. [Actually, the sum rule (7) simulates in a sense a complete coupled-channels calculation in the no-imaginary-potential limit.] Such an approach offers, therefore, a possible explanation of the nuclear structure effects which show up in magnitudes of the fusion cross sections especially for light systems.[13]

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Spectra of Very Highly Charged Cu- and Zn-Like Ions

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The $4s$-$4p$, $4p$-$4d$, and $4d$-$4f$ transitions of ten copperlike and zinclike ions from Ba^{26+} to W^{45+} have been observed by means of a laser-produced plasma and a 2.2-m grazing-incidence spectrograph. The spectra are accompanied by a prominent continuum lying just below the $4p\,^2P_{1/2}$-$4d\,^2D_{3/2}$ transitions in the copperlike ions. The results support the identification of the resonance lines of Xe^{24+} and Xe^{25+} in the Princeton University ST tokamak by Hinnov.

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The spectra of ions having simple atomic structures are currently of great interest because of their use in the diagnosis of hot plasmas found in controlled-fusion devices. Of special importance are ions in the copper and zinc isoelectronic sequences. These ions contain one and two elec-