# EXCITED SUPERDEFORMED BANDS $\operatorname{IN}{ }^{191} \mathbf{H g}$ 

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#### Abstract

Two weakly populated rotational bands have been observed in ${ }^{191} \mathrm{Hg}$ with properties (energy spacings, moments of inertia and lifetimes) very similar to those of the previously reported superdeformed band. Based on cranked Woods-Saxon calculations, these structures are interpreted as the first excited bands in the superdeformed minimum of ${ }^{191} \mathrm{Hg}$. Comparisons between the data and the calculations highlight the role of specific orbitals at large deformations.


Following the recent discovery [1] in ${ }^{191} \mathrm{Hg}$ of a band of 12 transitions corresponding to the rotation of a nucleus with a superdeformed (SD) prolate shape (axis ratio $1.65: 1$ ), further investigations were initiated at several laboratories on both the experimental and theoretical questions generated by this finding. SD bands have now been found in ${ }^{190.192 .194} \mathrm{Hg}$ [2-6]. First detailed cranking calculations of singleparticle and quasiparticle energies at large quadrupole deformations ( $\beta \sim 0.5$ ) have also been per-

[^0]formed using Woods-Saxon and modified Nilsson potentials [ $5,7,8$ ]. In the case of ${ }^{194} \mathrm{Hg}$, three bands [5] have been observed in the SD well and the results were explained successfully in the cranked calculations mentioned above. In particular, the rise of the dynamic moment of inertia $\mathcal{f}^{(2)}$ for all three bands as well as the strongly coupled character of the two weakest bands were accounted for satisfactorily [5], and the role of the $j_{15 / 2}$ neutron and $i_{13 / 2}$ proton intruder orbitals was recognized. The same calculations also predict the presence of excited SD bands in ${ }^{191} \mathrm{Hg}$ with dynamic moments of inertia very similar to the one seen in ${ }^{192} \mathrm{Hg}$. Definite predictions on the configurations of these excited bands were given. Hence, their experimental discovery represents a good
test of the quasiparticle spectrum at large deformation and gives insight into the microscopic origin of the shell corrections which are responsible for superdeformation.

Here we report on the discovery of two new band structures in ${ }^{191} \mathrm{Hg}$ which are proposed to correspond to the two lowest excited SD bands in this nucleus. Two level sequences of 12 transitions each were observed. The moments of inertia $\mathcal{J}^{(2)}$ derived from the transition energies are as large as those observed in the previously reported SD band (referred to hereafter as band 1). Furthermore, for both bands $\mathscr{g}^{(2)}$ is observed to increase by $\sim 40 \%$ over the frequency range $\hbar \omega=0.15-0.35 \mathrm{MeV}$, and this increase is comparable to that noted for all other SD bands in this region. It is shown that the data can be interpreted satisfactorily in terms of the gradual alignment of pairs of high- $j$ intruder orbitals within the framework of the cranked Woods-Saxon calculations mentioned above. Specific configurations are proposed for the new level structures.

The results presented below were derived from data sets discussed in previous publications [1,9] where most of the relevant experimental details can be found. Three sets of data were available: two were obtained with thin ${ }^{160} \mathrm{Gd}$ targets consisting of two isotopically enriched $500 \mu \mathrm{~g} / \mathrm{cm}^{2}$ self-supporting foils stacked together while the third used a $1 \mathrm{mg} / \mathrm{cm}^{2}$ ${ }^{160} \mathrm{Gd}$ target on which $14 \mathrm{mg} / \mathrm{cm}^{2} \mathrm{Au}$ was evaporated in order to stop the recoiling evaporation residues. In all measurements the ( ${ }^{36} \mathrm{~S}, 5 \mathrm{n}$ ) reaction was used. The runs with two thin targets were performed at respective beam energies of 172 and 167 MeV while the thick target measurement was carried out at 172 MeV only. The experiments were performed at the ATLAS accelerator with the Argonne-Notre Dame BGO $\gamma$ ray facility which consists of 50 hexagonal BGO elements surrounded by 12 Compton suppressed Ge spectrometers. Data were accumulated in an event-by-event mode with a requirement that at least four (eight in the case of the thick target measurement) detectors of the inner BGO array fire in coincidence with two suppressed Ge detectors.

In the analysis, $\gamma-\gamma$ coincidence matrices were obtained where high multiplicity events were selected by requiring that at least ten (eight) detectors of the array fired in prompt coincidence with the Ge detectors at $E_{\text {beam }}=172$ (167) MeV. The final matrices
obtained $95 \times 10^{6}$ and $54 \times 10^{6}$ events for the two thintargets runs at 172 and 167 MeV respectively; the corresponding number for the thick-target measurement is $69 \times 10^{6}$. In all cases the multiplicity condition given above ensures that at least $60 \%$ of the events correspond to the reaction channel of interest (more details can be found in ref. [1]).
The detailed analysis of the coincidence matrices revealed the presence of two bands with an average energy difference of 36 and 37 keV respectively. These values are very similar to that measured in ref. [1] for the first SD band (band 1 ) in ${ }^{191} \mathrm{Hg}$ and are consistent with the spacing expected for a SD shape [8]. Fig. 1 presents spectra for the two new bands which


Fig. 1. $\gamma$-ray spectra in ${ }^{191} \mathrm{Hg}$ obtained by summing the cleanest coincidence gates ( 333,373 and 450 keV for band 2, 352, 429 and 503 keV for band 3) in the coincidence matrix obtained at $E_{\text {beam }}=172 \mathrm{MeV}$. The members of the SD bands and the known yrast transitions are indicated. As discussed in the text, several identified contaminants are also present in the spectra.
are the sum of spectra in coincidence with the cleanest gates. The two bands presented here are rather weak and most of the transitions are contaminated by other lines of stronger intensity. Accordingly, the spectra of fig. 1 contain identified contaminants. Nevertheless the coincidence relationships between the various transitions in the two bands were verified from the individual gates as well as from the observation of a regular grid pattern expected in the twodimensional $\gamma-\gamma$ matrix using the code BANDAID described in ref. [10]. We note that one of the two bands (denoted band 3 in fig. 1) had been mentioned in ref. [1], but no conclusive evidence for its placement was available at that time. The placement of the two bands in ${ }^{191} \mathrm{Hg}$ is based on the following considerations: (1) the transitions in the bands are in coincidence with the $\frac{17}{2}-\frac{13}{2}$ ( 391 keV ) and $\frac{21}{2}-\frac{17}{2}$ ( 629 keV ) transitions in ${ }^{19} \mathrm{Hg}$; (2) the relative variation of the $\gamma$-ray intensity in the two bands between the two beam energies is the same as that observed for band 1 [11] (the two bands each correspond to $0.8 \%$ of the events in ${ }^{19} \mathrm{Hg}$ at 172 MeV and only $0.35 \%$ of the events at 167 MeV ; band 1 has corresponding intensities of $2 \%$ and $0.9 \%$, respectively); (3) the two bands are not observed at lower ${ }^{36} \mathrm{~S}$ beam energies (where only bands in ${ }^{192} \mathrm{Hg}$ are observed [3]) nor are they seen in the ${ }^{160} \mathrm{Gd}\left({ }^{34} \mathrm{~S}, 4 \mathrm{n}\right)$ reaction in which the SD band in ${ }^{190} \mathrm{Hg}$ was observed by Drigert et al. [2]; (4) the fold and sum-energy distributions measured in coincidence with the transitions of interest were found to peak at values similar to those observed for band 1 .
Fig. 2a presents the intensity patterns in the two new bands derived from the analysis of the coincidence gates and compares the data with the corresponding pattern for band 1 [1]. It can be seen that the three bands have essentially the same intensity pattern, i.e. the intensity remains essentially constant in the frequency range $0.175<\hbar \omega<0.24 \mathrm{MeV}$ and decreases gradually at higher rotational frequencies. The decay out of the two bands is seen to occur at lower frequency than in band 1 with the $\gamma$-ray intensity decreasing gradually below $\hbar \omega=0.175 \mathrm{MeV}$. Because of the weak population of the two bands, information on the decay towards the known yrast states is rather fragmentary and judged insufficient to attempt spin assignments to the new states. Indications on the lifetimes of the states was obtained for a few


Fig. 2. (a) Relative intensities of the $\gamma$-rays in the two new bands as obtained from the coincidence spectra gated on the 429 keV transition (band 3) and on the 333, 373 and 450 keV transitions (band 2). The published [1] intensities for band 1 are also given. The intensities have been normalized to I separately for each band. At a beam energy of 172 MeV , the absolute intensities are $2 \%$ for band 1 and $0.8 \%$ for bands 2 and 3 (see text for details). (b) Comparison between the measured fraction of full Doppler shift for some of the new transitions and the published data of ref. [1]. The solid curve corresponds to the best fit achieved for band 1 ( $Q_{0}=18 \mathrm{eb}$ ), see ref. [1] for details.
of the new transitions from the thick target experiment with the Doppler-shift attenuation method (see ref. [1] for details). Fig. 2b presents the measured fraction of full Doppler shift $F(\tau)$ for some transitions in band 2 (no analysis was possible for band 3 ) and compares the values of $F(\tau)$ with those presented in ref. [1] for band 1. Even though the error bars are rather large (because of the weak population of the band and the presence of contaminants), it is clear that the lifetimes of the states in band 2 are of the same order as the lifetimes of the SD states of band 1. On the basis of these results we conclude that two new SD bands have been observed in ${ }^{191} \mathrm{Hg}$. In view of their respective population, it is proposed that the
two bands correspond to transitions within excited states in the SD minimum and band 1 is then regarded as the SD yrast band.

Fig. 3a presents the dynamic moments of inertia $\mathscr{J}^{(2)}$ for the three bands as a function of the rotational frequency $\hbar \omega$. In all cases, $\mathscr{J}^{(2)}$ is seen to increase with $\hbar \omega$. Furthermore, the average values of $\mathscr{J}^{(2)}$ for bands 2 and 3 ( 110 and $113 \hbar^{2} \mathrm{MeV}^{-1}$, respectively) are somewhat larger than the corresponding value for band $1\left(108 \hbar^{2} \mathrm{MeV}^{-1}\right)$ and are also close to the value reported for ${ }^{192} \mathrm{Hg}\left(113 \hbar^{2} \mathrm{MeV}^{-1}\right)$ [3]. For $\gamma$-ray energies below 400 keV , the transition energies of band 3 are almost exactly intermediate to the energies of band 2: the situation is similar to the one reported for ${ }^{194} \mathrm{Hg}$ [5] where the two excited bands exhibit the same feature over a larger energy range. In analogy with ref. [5], we propose that bands 2 and 3 are "signature partners". The degree of signature splitting can then be inferred from fig. $3 b$ where the experimental single-particle routhians $e^{\prime}$ for bands 2 and 3 are presented as a function of $\hbar \omega$. The computation of these routhians requires knowledge of the
excitation energies and the spins involved in the bands as well as a phenomenological representation of the energy associated with the rotating nuclear core. Under the assumption that the bands are strongly coupled at low $\hbar \omega$, the excitation energies were arbitrarily taken as 4.5 MeV and 4.64 MeV for bands 2 and 3 respectively, and the spins were estimated with the method proposed in ref. [4]. (We note that the spins derived in this way, i.e. $\frac{25}{2}$ and $\frac{27}{2}$ for the bandheads of bands 2 and 3 respectively, differ by one unit and are consistent with the strong coupling picture presented here). The energy of the core was approximated by a Harris parametrization from fits to the low levels of the known yrast SD bands in ${ }^{192} \mathrm{Hg}$ and ${ }^{194} \mathrm{Hg}$ [3-5] as prescribed in ref. [12]. It can be seen from fig. 3b that, with the assumptions outlined above, the two bands may be proposed as signature partners for $\hbar \omega<0.2 \mathrm{MeV}$ (i.e. they follow the same smooth trajectory) and exhibit increasing signature splitting for $\hbar \omega>0.2 \mathrm{MeV}$. This behavior differs from the one observed in ${ }^{194} \mathrm{Hg}$ [5]: in this case the two excited bands show very little signature splitting over


Fig. 3. (a) Dynamic moments of inertia $\mathscr{f}^{(2)}$ for the three SD bands in ${ }^{191} \mathrm{Hg}(\odot)$ : band $1, \diamond$ : band 2, $\square$ : band 3). (b) Experimental single-particle routhians for bands $2(\diamond)$ and $3(\square)$ obtained under the assumptions described in the text. The recent data [5] for the excited bands in ${ }^{194} \mathrm{Hg}$ are given for comparison ( + and - ). For all cases the Harris parametrization [12] was taken as $E_{\mathrm{g}}^{\prime}(\omega)=$ $-\left(\omega^{2} / 2\right) \mathscr{L}_{0}-\left(\omega^{4} / 4\right) \mathscr{I}_{1}+\hbar^{2} /\left(8 \mathscr{A}_{0}\right)$ where $\mathscr{f}_{0}=90 \hbar^{2} \mathrm{MeV}^{-1}$ and $\mathscr{\mathscr { F }}=80 \hbar^{4} \mathrm{MeV}^{-3}$. (c), (d) Results from cranked Woods-Saxon calculations. The calculated dynamic moments of inertia are denoted with T in (c), the solid line represents the configuration proposed for band 1 , while the dashed and dotted lines represent the configuration proposed for bands 2 and 3 . The individual contributions of the neutron configurations built on the $\nu 7_{3 / 2}$ ( $\nu$, solid line) and $\nu[642] 3 / 2$ ( $\nu$, dashed and dotted lines) routhians are also given. The contribution due to the protons is shown with the solid line denoted $\pi$. The quasiparticle routhians for $N=111$ are given in (d).
the entire frequency range, as can be seen from fig. $3 b$ where the routhians for the ${ }^{194} \mathrm{Hg}$ bands have been included for comparison ( similar assumptions were made for the excitation energies and spins in this case ). It should be reiterated that the discussion above relies on the fact that the energies in bands 2 and 3 are closely related to one another as well as on the analogy with the situation in ${ }^{194} \mathrm{Hg}$. Clearly, the interpretation would be reinforced by a direct measurement of the excitation energies and spins for any of the bands under discussion.
In order to interpret the results presented above, cranked shell model calculations with pairing were performed using a Woods-Saxon potential. The approach is similar to the one used in ref. [5] and is described in detail in ref. [13]. In the discussion below, use is also made of the neutron and proton sin-gle-particle routhian diagrams for ${ }^{191} \mathrm{Hg}$ which are similar to those presented in ref. [5] and are not repeated here. These calculations reveal a pronounced proton gap at $Z=80$ which remains open for all frequencies considered. The resulting proton configuration contains four $N=6$ ( $\mathrm{i}_{13 / 2}$ ) protons and, according to the convention defined in ref. [14], can be labelled as $\pi 6^{4}$. For the neutron system, single-particle gaps exist at $N=112$ and $N=116$, lying between the second and the third $N=7\left(\mathrm{j}_{15 / 2}\right)$ orbitals with $\Omega=\frac{3}{2}$ and $\frac{5}{2}$, respectively (denoted $\nu 7_{3 / 2}$ and $\nu 7_{5 / 2}$ ). The gaps are separated by the two deformationaligned levels [512]5/2 and [624]9/2. Below the $N=112$ gap and close to the $\nu 7_{3 / 2}$ orbital lics the [642]3/2 orbital which carries little aligned angular momentum. Very similar single-particle level diagrams have been obtained in the Woods-Saxon model of ref. [8]. Fig. 3d presents the neutron quasiparticle routhians for ${ }^{191} \mathrm{Hg}$ calculated using a constant value of the pairing gap ( $\Delta_{\mathrm{n}}=0.41 \mathrm{MeV}$ ) and deformation parameters $\beta_{2}=0.47$ and $\beta_{4}=0.07$. In the calculations the neutron pairing was reduced relative to the ground state value in order to account for the similarity in the behavior of $g^{(2)}$ versus $\hbar \omega$ for ${ }^{192} \mathrm{Hg}$ and ${ }^{191} \mathrm{Hg}$ as was originally suggested by Ye et al. [4]. This point is also discussed in ref. [5] in connection with the similarity in the values of $\mathscr{g}^{(2)}$ for the three bands in ${ }^{194} \mathrm{Hg}$. (In contrast, the proton pairing was slightly increased; $A_{\mathrm{p}}=1.2 \mathrm{MeV}$.)
The calculations indicate that the yrast SD configuration is built on the aligned $\nu 7_{3 / 2}$ routhian with
parity and signature quantum numbers ( $\pi=-$, $r=+i$ ). We associate this configuration with band 1 . Bands 2 and 3 can be understood as the signature partners built on the [642] $3 / 2$ orbital with quantum numbers ( $\pi=+, r=+\mathrm{i}$ ) and ( $\pi=+, r=-\mathrm{i}$ ), respectively. The evolution of the calculated $\mathscr{g}^{(2)}$ values for the 3 bands as a function of $\hbar \omega$ is given in fig. 3 c where the individual contributions of the proton and neutron configurations under discussion are also presented. The major contribution to the rise of $\mathscr{g}^{(2)}$ in the three bands is caused by the gradual alignment of the $\pi 6^{4}$ protons. The neutron contributions differ for band 1 and bands 2 and 3 . In band 1 , the $N=7$ neutron crossing is blocked whereas it is present in bands 2 and 3 . As a consequence, the neutron contribution to $\mathscr{g}^{(2)}$ remains almost constant for band 1 while an additional increase is calculated for bands 2 and 3 . The calculations reproduce the data rather well for $\hbar \omega \geqslant 0.2 \mathrm{MeV}$ as can be seen from a comparison between figs. 3a and 3 c . As discussed in ref. [13], the larger deviations between theory and experiment at lower frequencies can be attributed to the renormalization procedure used in the calculations. (We note that the same calculations also reproduces the $\mathscr{g}^{(2)}$ values for the SD band in ${ }^{192} \mathrm{Hg}$ [15].) The signature partner of band $1(\pi=-, r=-i)$ is calculated to lie several hundred keV higher in excitation energy because of the large signature splitting (fig. 3d) and, as a result, the corresponding rotational band should be more difficult to observe experimentally.
The calculated values of $\mathscr{f}^{(2)}$ in bands 2 and 3 are similar to that of the lowest SD in ${ }^{192} \mathrm{Hg}$ since all these bands have the same content in intruder orbitals (four $N=6$ ( $\mathrm{i}_{13 / 2}$ ) protons and four $N=7\left(\mathrm{j}_{15 / 2}\right)$ neutrons). Furthermore, it is seen in fig. 3d that the $\nu[642] 3 / 2$ routhians form a coupled structure below $\hbar \omega \sim 0.15 \mathrm{MeV}$, while signature splitting increases gradually at higher frequencies. In the calculations without pairing this signature splitting is present as well, although slightly reduced [5]. These features are present in the data and can be considered as strong arguments in favor of the assignments presented here. It should, however, be mentioned that another possible assignment for bands 2 and 3 could involve the neutron orbitals [512]5/2 and [624]9/2 mentioned above. This possibility cannot be excluded a priori as the calculations are expected to reproduce the positions of the single-particle states to within a few
hundred keV only. However, these orbitals are calculated to remain strongly coupled over the entire range in $\hbar \omega$ and no significant signature splitting would result. This is contrary to the experimental observations and reinforces our assignment based on the $\nu$ [642]3/2 orbital for this SD nucleus.
In summary, two new SD bands have been reported in ${ }^{191} \mathrm{Hg}$. These bands are characterized by slightly larger moments of inertia than the previously reported SD yrast struct are. In addition, some signature splitting between the two bands is present at $\hbar \omega>0.2 \mathrm{MeV}$. These observations are corroborated by Woods-Saxon cranking calculations which emphasize the role of the $N=6$ ( $\mathrm{i}_{13 / 2}$ ) proton, $N=7$ ( $\mathrm{j}_{15 / 2}$ ) and [642] $3 / 2$ neutron orbitals.

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