New Features of Superdeformed Bands in ¹⁹⁴Hg

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Two new features of superdeformed (SD) bands in the $A \approx 190$ region emerge from a study of ¹⁹⁴Hg with the Gammasphere detector array. A decrease of the dynamic moment of inertia is observed for rotational frequencies $\hbar\omega \geq 0.4$ MeV, confirming long standing expectations based on mean field calculations with pairing. Evidence for a small staggering in the SD transition energies is also observed, suggesting the presence of terms with fourfold symmetry in the SD Hamiltonian.

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In addition to the fission isomers in actinide nuclei, superdeformed (SD) heavy nuclei are now known to exist in regions of the nuclear chart with mass $A \approx 130$, 150, and 190 [1–3]. A striking difference between the SD nuclei near $A = 190$ and those in the other regions is the behavior of the dynamic moment of inertia $\Im^{(2)}$ with the rotational frequency $\hbar\omega$. While the $\Im^{(2)}$ patterns of the SD bands near $A = 130$ and $A = 150$ show pronounced variations, the majority of the SD bands near $A = 190$ display the same large, smooth increase of $\Im^{(2)}$ with $\hbar\omega$ in the frequency range $0.15 \leq \hbar\omega \leq 0.40$ MeV. Current interpretations of this rise of $\Im^{(2)}$ within mean field theories [4,5] invoke the gradual alignment of quasiparticles occupying high-N intruder orbitals (originating from the $i_{13/2}$ proton and $j_{15/2}$ neutron shells) in the presence of pair correlations. In this picture, Pauli blocking of high-N intruder orbits is expected to induce a flattening of the $\Im^{(2)}$ moments of inertia. Such effects are indeed observed, most clearly in the odd-odd Tl isotopes [6,7]. It is a direct consequence of these interpretations that, after the quasiparticle alignments have taken place, $\Im^{(2)}$ will exhibit a downturn with increasing $\hbar\omega$ toward the rigid-body value. Up to now, no downturn in $\Im^{(2)}$ for the SD bands in the $A \approx 190$ mass region has been observed [8], raising some doubt as to our understanding of pair correlations and alignment effects at these large deformations. In the present work, the three SD bands of ¹⁹⁴Hg observed in earlier work [5,9] have been extended to higher rotational frequencies, and the first experimental evidence for the expected downturn in $\Im^{(2)}$ was obtained.

A second important result of this work relates to the possible presence of a higher-order symmetry in SD nuclei. The appearance of a regular staggering pattern in a SD band in ¹⁴⁹Gd [10] has recently been presented as possible evidence for a new symmetry term in the Hamiltonian which is invariant under a rotation by $\frac{\pi}{2}$. It was pointed out that a sequence of four-particle-four-hole (4p-4h) states in ¹⁶O [11], and oscillations in the $K^\pi = 0^-$ bands in the actinide nuclei ²³⁶,²³⁸U [12], might be other cases where this symmetry is realized to different degrees. The increased resolving power obtained from high-fold coincidence data allowed us to determine the SD transition energies in ¹⁹⁴Hg with greater accuracy than in previous work and evidence for a staggering of the γ-ray energies, similar to that observed for the SD band in ¹⁴⁹Gd, is observed in all three bands.

The experiment was carried out at the 88-Inch Cyclotron Facility of the Lawrence Berkeley Laboratory. Excited states in ¹⁹⁴Hg were populated with the reaction ¹⁵⁰Nd(⁴⁸Ca,4n)¹⁹⁴Hg at a beam energy of 206 MeV. The target consisted of a stack of two isotopically enriched, self-supporting ($\approx 500\ \mu\mathrm{g\,cm^{-2}}$) ¹⁵⁰Nd foils. The γ rays emitted in the reaction were detected with the Gammasphere detector array [13]. In this particular experiment, 32 large Compton-suppressed germanium detectors were used. Approximately 5.0×10^8 triple- and higher-fold coincidence events (yielding 3.1×10^8 and 4.2×10^7 expanded fourfold and fivefold events, respectively) were stored on magnetic tape. Spectra obtained for the three SD bands in ¹⁹⁴Hg are shown in Fig. 1. All SD transitions reported earlier [5,9] are clearly observed, together with known ¹⁹⁴Hg yrast transitions associated with the decay out of the SD bands. Compared with previous work the three SD bands have all been extended to higher spins by two or three transitions. These transitions represent on average less than 10% of the maximum intensity in the respective bands. Furthermore, the accuracy in the determination of the transition energies is higher than that
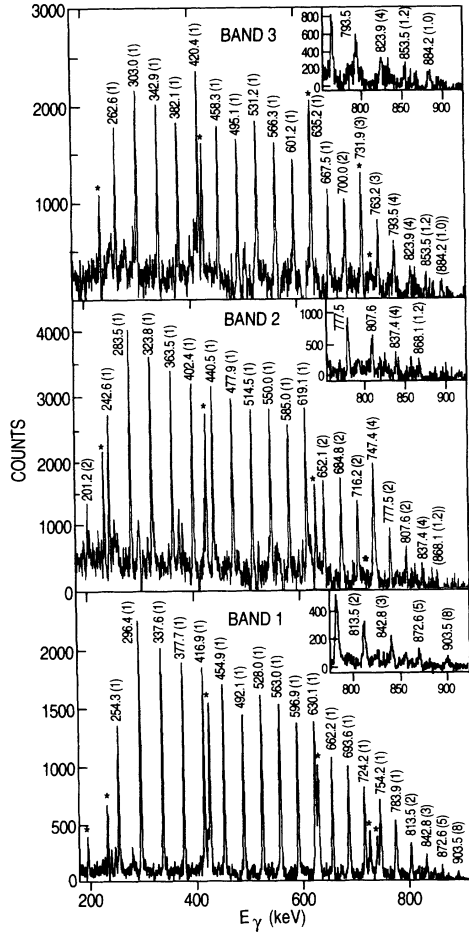
FIG. 1. Coincidence γ-ray spectra for SD bands 1, 2, and 3 in ^{194}Hg. The spectrum for band 1 was obtained from the sum of all possible combinations of threefold coincidence gates on all transitions with $E_\gamma \leq 873$ keV. The spectra for bands 2 and 3 are obtained from sums of all possible combinations of twofold gates, excluding those which bring in contaminating lines (e.g., the 619.4, 747.5, 807.6, and 868.1 keV gates for band 2 and the 303.0, 382.1, 635.2, and 732.2 keV gates for band 3). The insets show high-energy portions of the spectra with expanded scales. Low-lying yrast transitions are marked with a *. In all cases, background contributions have been subtracted.

of the previous work due to the greater resolving power achieved with high-statistics high-fold data. The energies were determined using a conventional peak-fitting routine and the deduced uncertainties in the transition energies take into account the effects of the counting statistics, the dispersion, and variations in the peak positions due to different background subtractions. The last contribution is difficult to derive unambiguously in high-fold data. In this specific case, the adopted errors play an important role in the analysis and we have therefore taken the conservative approach of including the maximal contributions to the uncertainties in the peak centroids for various "reasonable" background subtractions.

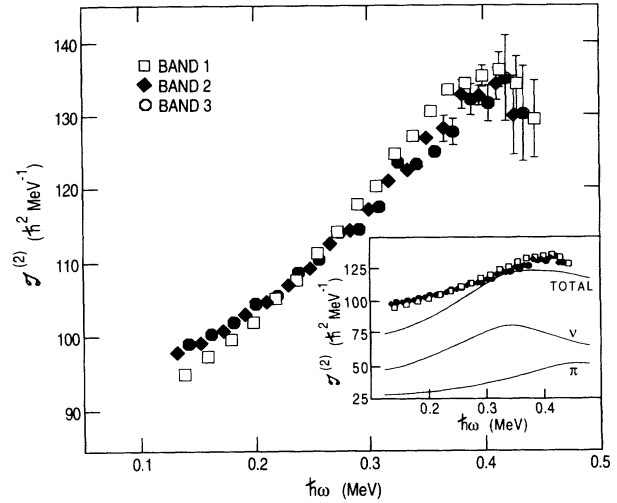With the extension of the three SD bands towards

FIG. 2. Dynamic moments of inertia $\mathfrak{I}^{(2)}$ as a function of $\hbar\omega$ for the SD bands in ^{194}Hg. The last points for bands 2 and 3 are somewhat uncertain. The inset shows results from new, modified TRS calculations for the yrast SD configuration (see text for details). The three curves in the inset represent the calculated total $\mathfrak{I}^{(2)}$ values and their decomposition into neutron and proton contributions.

higher energies, it is now possible to extend the dynamic moments of inertia $\mathfrak{I}^{(2)}$ up to frequencies $\hbar\omega \geq 0.4$ MeV as shown in Fig. 2. For band 1 a change in the slope of $\mathfrak{I}^{(2)}$ with $\hbar\omega$ is noticeable for $\hbar\omega \approx 0.37$ MeV and, for the first time in this mass region, a clear turnover is present around $\hbar\omega \approx 0.43$ MeV. The characteristics of bands 2 and 3 are very similar to band 1 although the uncertainties are larger.

Although the present data indeed reveal the expected $\mathfrak{I}^{(2)}$ turnover, standard cranked-shell-model (CSM) calculations with pairing [4,5] underpredict the frequency at which the downturn in $\mathfrak{I}^{(2)}$ occurs and are also unable to reproduce the behavior of $\mathfrak{I}^{(2)}$ at lower frequencies. In an attempt to improve the situation, new, modified total-Routhian-surface (TRS) calculations have been carried out. In these calculations, the monopole-pairing interaction was treated self-consistently by means of the Lipkin-Nogami method [14,15] and a revised parametrization was used for the Woods-Saxon potential [16] which yields moments of inertia which no longer require renormalization [17]. In the new TRS calculations both pairing and deformation are varied self-consistently whereas in earlier calculations only one of these was so treated [5,17]. The results of the new TRS calculations for the yrast SD configuration are shown in the inset of Fig. 2. The rise of $\mathfrak{I}^{(2)}$ with $\hbar\omega$ is due mainly to the successive alignments of neutrons and protons occupying high-N intruder orbitals. An additional contribution to the rise in $\mathfrak{I}^{(2)}$ comes from the response of the mean field to the alignment processes. The deformation of the SD minimum is calculated to decrease slightly with increasing rotational frequency [the (β_2, β_4) values decrease from $(0.48, 0.065)$ to $(0.46, 0.045)$ between $\hbar\omega = 0.20$ MeV and 0.50 MeV]

which in turn results in an additional gain in alignment. (A similar, but slightly smaller, decrease in deformation with $\hbar\omega$ is also predicted in recent cranked-Hartree-Fock calculations [18].) While the new TRS calculations reproduce the main trends of $\Im^{(2)}$ as a function of $\hbar\omega$, there clearly remains room for improvement. At the lowest frequencies, the absolute value of $\Im^{(2)}$ is still underpredicted. Better agreement between theory and experiment for the detailed behavior of $\Im^{(2)}$ at the highest frequencies is also desirable. It is possible that the addition of higher-order terms to the pairing interaction will remedy some of these difficulties, as indicated by preliminary studies [19,20].

The observation that the variations of $\Im^{(2)}$ with $\hbar\omega$ in bands 2 and 3 follow closely those of band 1 is an indication that the behavior of $\Im^{(2)}$ is determined mainly by the effects of high-N intruder orbitals. Indeed these bands are interpreted as one of the signature pairs based on the excited $\nu[624]9/2 \otimes [512]5/2$ configuration [5]; i.e., these excited SD bands have the same intruder occupation as the yrast SD configurations in both ^{194}Hg and ^{192}Hg. However, the fact that band 3 has transition energies (as well as $\Im^{(2)}$ values) virtually identical to those of the ^{192}Hg SD band goes far beyond this statement and a full understanding of SD moments of inertia will have to include the remarkable phenomenon of "identical bands" [21].

A close inspection of the $\Im^{(2)}$ moments of inertia (Fig. 2), which are derived from the difference between the energies of consecutive transitions, reveals the presence in each band of a small $\Delta I = 2$ staggering. Thus, it appears that the energy levels in the bands are experiencing small, alternating energy shifts. The staggering in the γ-ray-transition energies is illustrated in Fig. 3, where the staggering parameter ΔE_γ is plotted using a smooth reference, derived from a quadratic interpolation of the transition energies E_γ:

$$\Delta E_\gamma(I) = \tfrac{3}{8}\left\{ E_\gamma(I) - \tfrac{1}{6}[4E_\gamma(I-2) + 4E_\gamma(I+2) \right.$$
$$\left. - E_\gamma(I-4) - E_\gamma(I+4)]\right\}.$$

It should be noted that our definition of the staggering parameter ΔE_γ is different from the one used in Ref. [10]. This is done because of the convenience of a quantity which directly measures the deviation of the γ-ray energies from a smooth behavior. Our reference also has the advantage of being more general, in the sense that it accommodates a varying moment of inertia. It is worth pointing out that we have applied various other methods for deriving ΔE_γ, yielding consistent staggering patterns. The oscillations shown in Fig. 3 are associated with large relative uncertainties but a few features may be noticed: (i) This pattern is present even at the lowest frequencies and the oscillation amplitude appears to increase with frequency, at least for bands 2 and 3. (ii) The oscillation appears to be larger in magnitude for bands 2 and 3. (iii) In bands 2 and 3, an inversion in the phase of the oscillations appears to take place at $\hbar\omega \approx 0.3$ MeV.
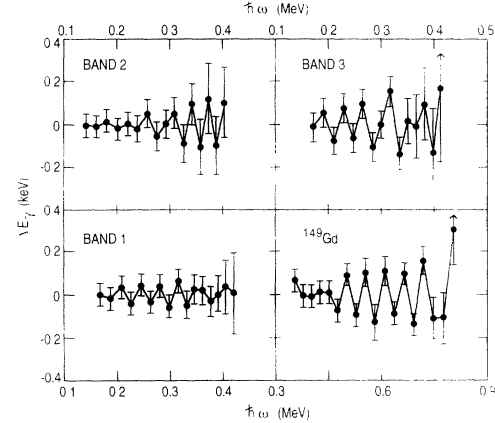
FIG. 3. Staggering ΔE_γ of the transition energies in the SD bands in ^{194}Hg and in the yrast SD band in ^{149}Gd. The formula used to derive the staggering is given in the text.

The oscillations are comparable in magnitude (~ 100 eV for bands 2 and 3, ~ 40 eV for band 1), but occur at lower frequencies than in ^{149}Gd, the only other SD nucleus for which this surprising effect has been reported [10]. The staggering in the transition energies translates into a corresponding staggering of the level energies with half the magnitude, which represents a perturbation of the order of 10^{-4} of the level separations in the bands. Although the observed energy staggering is very small, it is not likely to be an artifact of the data related to nonlinearities in the detection system. The transition energies reported here agree very well with those of Refs. [5,9], and the previous measurements were performed in separate experiments, with different detector arrays and their associated electronics. In fact, the oscillations in $\Im^{(2)}$ are detectable to some extent in the earlier works, but are placed on a firmer ground here because of the improved accuracy of the transition energies. As stated above we have taken a conservative approach in the error analysis of the measured transition energies. This is indeed visible in the staggering plots in Fig. 3 since the adopted errors normally would imply a larger variation in the ΔE_γ points than is actually observed.

We present here the results of one of several possible methods to estimate the significance of the observed staggering ΔE_γ. In this method, the signs of the deduced ΔE_γ values in each band were changed for every other transition, and the new average $\overline{\Delta E_\gamma'}$ and uncertainty σ' of the altered values were compared with the corresponding values $\overline{\Delta E_\gamma}$ and σ of the original data. If the staggering is consistent with a random fluctuation, the distributions of the altered values and the original data will be statistically the same. On the other hand, a statistically significant difference will indicate the existence of a staggering. A confidence level (C.L.) which measures the difference between the two distributions can be defined as

$$\text{C.L.} = \int_{-\infty}^{+\infty} \text{erf}(\Delta E_\gamma, \overline{\Delta E_\gamma}, \sigma) g(\Delta E_\gamma, \overline{\Delta E_\gamma'}, \sigma') d\Delta E_\gamma,$$

TABLE I. Table of statistical confidence levels for observed $\Delta I = 2$ staggering patterns. The superscript r indicates that a certain rotational frequency range was selected: $0.45 \leq \hbar\omega$ (MeV) ≤ 0.75 for the case of the ^{149}Gd band and $0.15 \leq \hbar\omega$ (MeV) ≤ 0.35 for the case of band 1. The superscript i indicates that an inversion of the phase of the oscillation was applied at $\hbar\omega = 0.30$ MeV (bands 2 and 3). See the text for further details.

	C.L. All data points	C.L. Selected range/inversion
^{149}Gd	0.97	0.99^r
Band 1	0.80	0.94^r
Band 2	0.82	0.94^i
Band 3	0.62	0.95^i

where $\operatorname{erf}(\Delta E_\gamma, \overline{\Delta E_\gamma}, \sigma) = \int_{-\infty}^{\Delta E_\gamma} g(x, \overline{\Delta E_\gamma}, \sigma) dx$ and $g(x, \overline{\Delta E_\gamma}, \sigma)$ is a Gaussian function centered at $\overline{\Delta E_\gamma}$ with standard deviation σ. Two identical distributions will give a confidence level of 0.5, whereas distributions separated by 1, 2, or 3 standard deviations give confidence levels of 0.76, 0.92, or 0.98, respectively. The first column of Table I gives the C.L. considering all data points in each band. All bands show a positive effect, and in bands 1 and 2 the effect is greater than 1 standard deviation. In the well known $\Delta I = 1$ staggering in signature split bands both changes in amplitude and signature inversions for different regions of the bands are observed. Therefore we consider that similar effects might be present in the $\Delta I = 2$ staggering and estimates are also included in Table I (i) allowing for the possibility that the staggering is weak or absent in part of the frequency range (band 1 and the ^{149}Gd band) and (ii) allowing for an inversion in the phase of the oscillations (bands 2 and 3). The results indicate a relatively high significance of greater than 2 standard deviations of the observed staggering under these assumptions. It is noteworthy that for band 3 there are yrast lines close in energy to some SD lines. This makes the staggering analysis for this excited band subject to larger systematic errors, which are hard to estimate, and the results may therefore be less reliable.

It has been suggested in Ref. [10] and references therein that this regular perturbation of a rotational pattern over many transitions, which results in $\Delta I = 4$ families of states, might be indicative of the presence of a perturbation term in the Hamiltonian associated with an invariance under a rotation by $\frac{\pi}{2}$ and possible scenarios were briefly discussed. Several specific observations can be drawn from the ^{194}Hg case which may provide additional information on the origin of this effect. In each of the three SD bands of ^{194}Hg, eight high-N intruder orbitals are occupied (four $N = 7$ neutron and four $N = 6$ proton orbitals), and, as discussed above, alignment processes involving these high-N orbitals are taking place in the frequency range where the oscillations are not only present, but increase in magnitude (at least

in bands 2 and 3). The apparent inversion of the oscillations in bands 2 and 3 may indicate some dependence on the intrinsic configurations involved, even when the active intruder orbitals remain the same. These results may point to additional aspects of these oscillations as compared with the suggestions made in Ref. [10], which were related to the occupation of a single $N = 7$ neutron and to the assumption that the alignment of a pair of $N = 6$ protons has been completed at the frequencies where the effect is present.

In summary, a downturn has been observed above $\hbar\omega = 0.4$ MeV in the $\Im^{(2)}$ moments of inertia of the ^{194}Hg SD bands, in agreement with the general expectations of mean field theories with pairing. In addition, evidence for a small staggering in the level energies is observed, indicating that the SD states can be grouped into $\Delta I = 4$ families.

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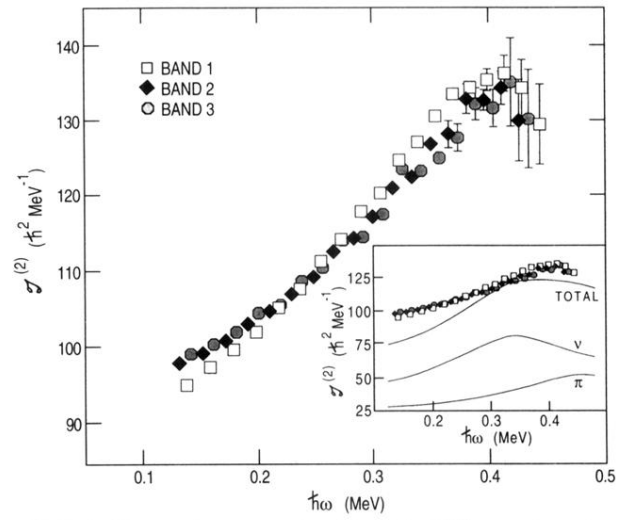[21] F. S. Stephens et al., Phys. Rev. Lett. **64**, 2623 (1990); **65**, 301 (1990).

FIG. 2. Dynamic moments of inertia $\mathfrak{I}^{(2)}$ as a function of $\hbar\omega$ for the SD bands in ^{194}Hg. The last points for bands 2 and 3 are somewhat uncertain. The inset shows results from new, modified TRS calculations for the yrast SD configuration (see text for details). The three curves in the inset represent the calculated total $\mathfrak{I}^{(2)}$ values and their decomposition into neutron and proton contributions.