Test of $\Delta I=2$ staggering in the superdeformed bands of ^{194}Hg

R. Krücken,[1] G. Hackman,[2] M. A. Deleplanque,[1] R. V. F. Janssens,[2] I. Y. Lee,[1] D. Ackermann,[2] I. Ahmad,[2] H. Amro,[2] S. Asztalos,[1] D. J. Blumenthal,[2] M. P. Carpenter,[2] R. M. Clark,[1] R. M. Diamond,[1] P. Fallon,[1] S. M. Fischer,[2] B. Herskind,[3] T. L. Khoo,[2] T. Lauritsen,[2] A. O. Macchiavelli,[1] R. W. MacLeod,[1] D. Nisius,[2] G. J. Schmid,[1] F. S. Stephens,[1] and K. Vetter[1]

[1]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720*

[2]*Argonne National Laboratory, Argonne, Illinois 60439*

[3]*The Niels Bohr Institute, University of Copenhagen, Denmark*

(Received 5 June 1996)

The presence of $\Delta I=2$ staggering in the three known superdeformed (SD) bands of ^{194}Hg has been reexamined in a new experiment with Gammasphere. A relative precision of better than 60 eV was achieved for most transition energies. Staggering plots were extracted and their statistical significance was analyzed. No clear evidence was found for an extended regular $\Delta I=2$ staggering in the three SD bands of ^{194}Hg. However, statistically significant deviations from a smooth reference were observed in the two excited SD bands. Different scenarios are discussed but no firm conclusion about the origin of the observed deviations can be drawn. [S0556-2813(96)50311-0]

PACS number(s): 21.10.Re, 23.20.Lv, 27.80.+w

Superdeformed (SD) nuclei are some of the best quantum rotors known. Their characteristic long sequences of equally spaced transition energies provide a unique opportunity to search for unexpected effects on an energy scale rarely achieved elsewhere in nuclear physics. In this context the recent observation of a regular staggering pattern of the transition energies in the yrast SD band in ^{149}Gd [1], where states differing by four units of angular momentum show a similar energy shift of about 60 eV relative to a (smooth) rotational sequence, is particularly intriguing. Evidence for similar effects has been reported in ^{194}Hg [2], ^{148}Gd [3], ^{192}Tl [4], and in some Ce nuclei [5]. These observations have triggered an intense theoretical effort to understand this phenomenon [6–11]. Some discussions connect this effect with the presence of a C_4 symmetry of the nuclear Hamiltonian [6–8]. Other studies [9–11] argue that the measured energy differences could be related to band crossings. Since the observed energy shifts are only of the order of 100 eV or less, which is at the limit achievable with modern γ-ray arrays for SD transitions, it is essential to confirm the reported effects by new measurements with higher statistics utilizing the still increasing efficiency and resolving power of the currently available detector arrays, such as Gammasphere [12] and Eurogam II [13]. In this Rapid Communication we report results from a new experiment on the known SD bands in ^{194}Hg [14,15] investigating the previously reported [2] staggering in those bands. The very high statistics obtained in this experiment has made possible the determination of the relative transition energies for these bands with a precision of 60 eV or better for most transitions. No extended regular $\Delta I=2$ staggering was found in the three SD bands of ^{194}Hg and the new results do not confirm those previously reported in Ref. [2]. However, deviations of the γ-ray energies from a smooth reference have been established for the two excited SD bands in ^{194}Hg.

Superdeformed states in ^{194}Hg were populated in the reaction ^{150}Nd(^{48}Ca,4n) using a 201 MeV ^{48}Ca beam provided by the 88-inch cyclotron of the Lawrence Berkeley National Laboratory. The emitted γ rays were detected by the Gammasphere array, which at the time of the experiment consisted of 70 Compton-suppressed Ge detectors. A stack of two 500 μg/cm^2 thick Nd targets was used with both sides of each target foil covered with a thin layer of gold (450 μg/cm^2 facing the beam and 220 μg/cm^2 on the other side). A total of 1.4×10^9 coincidence events with fold $\geqslant4$ were recorded on magnetic tape which led, after filtering out random coincidences, to 4.6×10^8, 3.9×10^8, 1.1×10^8, and 3×10^7 triple-, quadruple-, quintuple-, and sextuple-events, respectively, with γ-ray energies below 2 MeV. A gain of 0.125 keV per ADC channel was chosen in order to achieve a high resolution.

The effects we were looking for are only of the order of 100 eV and consequently the analysis plays an important role. Therefore we give hereafter a brief description of the analysis performed to extract the transition energies of the SD bands in ^{194}Hg. An off-line correction of drifts in the ADC gains was performed for each individual detector and they were gain matched by using γ rays from a ^{152}Eu calibration source. Gated coincidence spectra were created for each of the three known SD bands in ^{194}Hg using in each case several sets of gating transitions, all leading to very clean spectra of the SD bands. The sorting procedure was such that for each energy in a coincidence event the remaining energies were checked for their occurrence in the gates. The initial energy was then incremented in a one-dimensional spectrum corresponding to the maximum number of gates satisfied. In this way triple-gated spectra were created as well as spectra with at least four gates satisfied, which, hereafter, we will call quadruple-gated spectra. Figure 1 shows triple-gated spectra for the three SD bands in ^{194}Hg. Using this sorting method [16] leads to statistically independent spectra for the different gate folds, since a given energy was incremented only once. This procedure also avoids the overweighting of single channels (so-called *spikes*) in the spectra due to the unfolding of high-fold events. Avoiding such spikes is crucial for the correct deter-

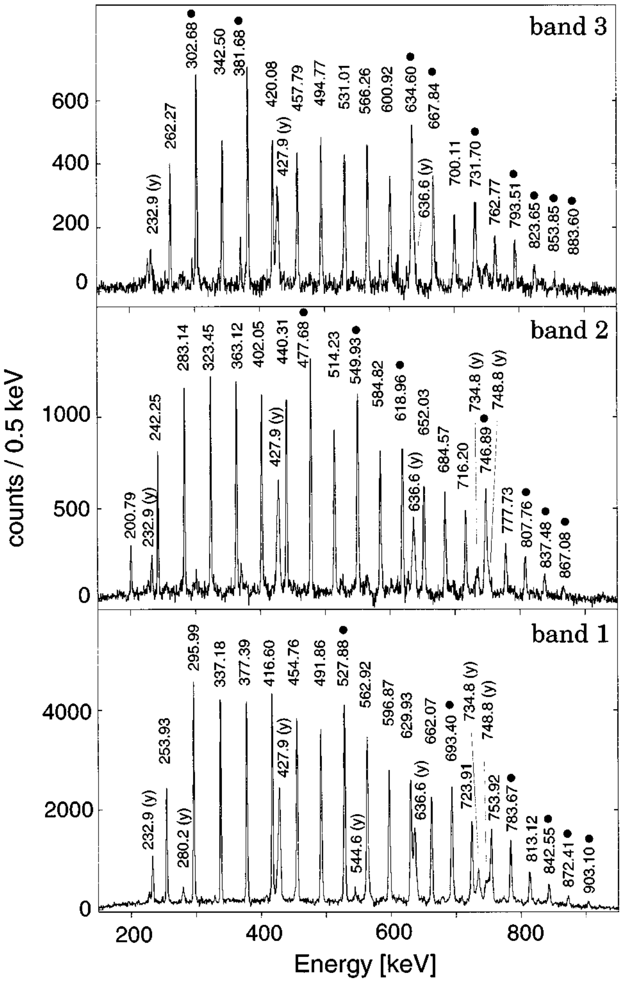

FIG. 1. Triple-gated coincidence spectra for the three superdeformed bands. The transitions marked with filled circles were not used as gates. Strong yrast transitions in ^{194}Hg [19] are labeled with their energy and the symbol ''y.''

TABLE I. Transition energies E_γ and relative intensities I for all transitions of the three superdeformed bands in ^{194}Hg as determined in this work. The intensities are corrected for detector efficiency and internal conversion.

^{194}Hg-1		^{194}Hg-2		^{194}Hg-3	
E_γ (keV)	I (%)	E_γ (keV)	I (%)	E_γ (keV)	I (%)
		200.79 (6)	36 (5)		
253.93 (4)	58 (3)	242.25 (6)	75 (5)	262.27 (6)	76 (5)
295.99 (3)	97 (3)	283.14 (6)	100 (5)	302.68 (6)	90 (5)
337.18 (3)	95 (3)	323.45 (6)	97 (5)	342.50 (6)	90 (5)
377.39 (3)	97 (3)	363.12 (6)	100 (5)	381.68 (6)	97 (5)
416.60 (3)	100 (3)	402.05 (6)	100 (5)	420.08 (6)	100 (5)
454.76 (3)	99 (3)	440.31 (6)	102 (5)	457.79 (6)	99 (5)
491.86 (5)	101 (3)	477.68 (6)	101 (5)	494.77 (6)	102 (5)
527.88 (3)	100 (3)	514.23 (6)	100 (5)	531.01 (7)	104 (5)
562.92 (3)	94 (3)	549.93 (6)	102 (5)	566.26 (6)	105 (5)
596.87 (5)	89 (3)	584.82 (6)	99 (5)	600.92 (6)	95 (5)
629.93 (3)	87 (3)	618.96 (6)	89 (5)	634.60 (11)	91 (5)
662.07 (4)	82 (3)	652.03 (6)	86 (5)	667.84 (7)	89 (5)
693.40 (4)	76 (3)	684.57 (7)	84 (5)	700.11 (6)	86 (5)
723.91 (6)	68 (3)	716.20 (6)	71 (5)	731.70 (17)	72 (5)
753.92 (6)	56 (3)	746.89 (19)	54 (5)	762.77 (6)	61 (5)
783.67 (8)	48 (3)	777.73 (6)	44 (5)	793.51 (6)	48 (5)
813.12 (3)	33 (3)	807.76 (8)	35 (5)	823.65 (13)	30 (5)
842.55 (6)	19 (3)	837.48 (7)	18 (5)	853.85 (12)	13 (5)
872.41 (13)	10 (3)	867.08 (24)	8 (5)	883.60 (22)	9 (5)
903.10 (18)	5 (3)				

mination of the transition energies, specifically in the quadruple-gated spectra. Different backgrounds were subtracted from the triple-gated spectra in order to test for systematic effects arising from these subtractions. The background-subtraction procedure involved subtracting varying amounts of an $(n-1)$-fold spectrum as background for an n-fold spectrum. For the present analysis no background was subtracted from the quadruple-gated spectra. The Doppler shift for SD transitions above 700 keV was found to vary from the average recoil velocity [$v/c=0.0200\,(5)$] for the lower energy transitions due to the fact that these decays occur in the target foils and their thin gold backing while the recoiling nuclei are still slowing down. The correction method proposed by Cederwall *et al.* [17] was used to take these effects into account and this improved the peak resolution for those high-energy transitions by 10–15%.

The transition energies E_γ were determined by using a conventional fitting routine from the triple- and quadruple-gated spectra corresponding to different gating conditions. In the different spectra analyzed it was found that the E_γ values for some transitions showed fluctuations of the order of $2-3\sigma$, where σ is the statistical uncertainty of E_γ, resulting from the utilized fitting procedure. Such fluctuations are expected even in the case of a purely statistical behavior and thus we have extracted the overall distribution of transition-energy values with respect to their average. We found it to be in good agreement with a statistical distribution: 61% of the data were within 1σ of their average, 31% were in the range 1–2σ and 8% were found to be outside a 2σ range with respect to the corresponding average E_γ. Despite this remarkable statistical behavior (a statistical distribution would give values of 68%, 27%, and 5%), systematic effects due to background subtraction procedures and different gating conditions could not be excluded and have been taken into account in the evaluation of the final uncertainties.

To ensure that the results were not biased by some detail of the data reduction, the same data were analyzed independently in a different manner. In this parallel analysis double- and triple-gated spectra were background corrected with the operator-based subtraction method of Ref. [18]. Each individual double-gated spectrum was inspected for cleanliness before adding it to the summed spectrum. The measured transition energies and their uncertainties were found to be consistent with those reported in Table I.

Table I summarizes the transition energies and relative intensities derived in the present experiment for the three SD bands in ^{194}Hg. It must be emphasized that the observed relative intensities in Table I cannot rule out the presence of small contaminating peaks under the SD transitions of interest at a level of <3% for band 1 and of <5% in bands 2 and 3. The possible influence of such contaminants was investigated and leads to position uncertainties of 30 eV in band 1 and 60 eV in bands 2 and 3. Furthermore, some of the yrast transitions in ^{194}Hg [19] that are in true coincidence with the SD bands lead, in specific cases (i.e., for the 746.89-keV transition in band 2 and the 634.60-, 731.70-keV transitions in band 3), to significant uncertainties in the determination of those SD transition energies. The uncertainties given in Table I are dominated by the uncertainties arising from possible or observed contaminants but also take into account the uncertainties arising from different background subtractions and gating conditions as well as the possible statistical fluctuations. The improvement in the overall precision of the present measurement compared with that in Ref. [2] is more than a factor of two.

For each band the deviation of the γ-ray energies from a smooth reference ΔE_γ was determined by calculating the fourth derivative[1] of the γ-ray energies $E_\gamma(I)$ at a given spin I by

$$\Delta E_\gamma(I)=\frac{3}{8}\left[E_\gamma(I)-\frac{1}{6}[4E_\gamma(I-2)+4E_\gamma(I+2) -E_\gamma(I-4)-E_\gamma(I+4)]\right]. \quad (1)$$

This expression was previously used in Ref. [2] and is identical to the expression for $\Delta^4 E_\gamma(I)$ in Ref. [11]. We chose to use the expression above in order to be able to follow higher order changes in the moments of inertia of the SD bands. The effects discussed below are certainly also visible in all lower derivatives. Fig. 2 shows the resulting values of ΔE_γ for the entire frequency range of the three SD bands in ^{194}Hg from the present experiment on the left side and the results from Ref. [2] on the right side. The uncertainties for ΔE_γ given in Fig. 2 are calculated using the standard error propagation method. We are aware that the given uncertainties of the individual ΔE_γ values do not account for the correlations induced in the staggering pattern by the change of individual E_γ values. The effect of these correlations will be discussed later in this paper.

It is an important question how the new results compare to those previously reported [2]. The resulting staggering plots shown in Fig. 2 differ in several aspects from those of Ref. [2] even though for most data points the ΔE_γ values are consistent within the given uncertainties. Let us now briefly consider each band separately.

The previously observed regular staggering pattern of the order of 40 eV in band 1 was not observed. There appears to be a very small oscillation with an amplitude of about 20–25 eV in the range of $\hbar\omega=0.25$–0.35 MeV, but it has little statistical significance (see discussion of confidence level below). It is noteworthy that this pattern is in phase with that seen in the previous report.

The low-frequency range ($\hbar\omega<0.25$ MeV) of the staggering plot for band 2 is very similar to that of the previous work. A significant deviation from a smooth behavior sets in for $\hbar\omega\geqslant 0.3$ MeV, which is somewhat higher in frequency than seen in Ref. [2]. While the oscillation starts with the same phase in both measurements, the inversion at

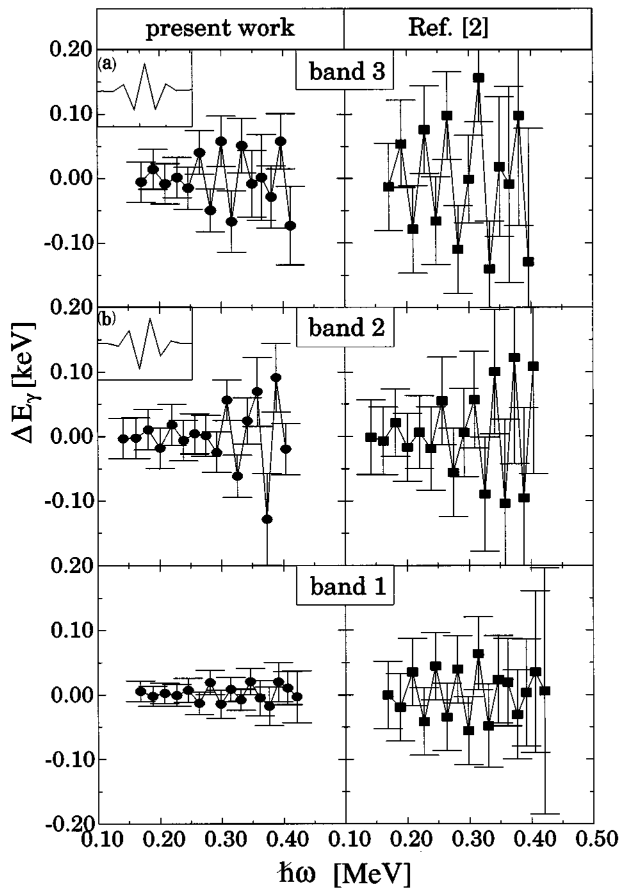

FIG. 2. The left panels show the fourth derivative ΔE_γ (see text for definition) of the γ-ray energies of the three superdeformed bands in ^{194}Hg vs rotational frequency $\hbar\omega$, determined in this work. The right panels show the results from Ref. [2]. The insets show staggering patterns expected from a band crossing scenario with the crossing frequency near a given level (a) and at the midpoint between the two levels (b).

$\hbar\omega=0.35$ MeV was not observed earlier. It is important to realize that the staggering plot of the high-frequency part of band 2 depends critically on the position of the 746.89-keV ($\hbar\omega\approx 0.375$ MeV) transition. The precise determination of this energy is complicated by the presence of the 748.8-keV ($5^-\rightarrow 4^+$) yrast transition [19]. We are, however, confident that this interfering transition has been consistently taken into account, since its centroid and shape have been accurately determined from spectra in coincidence with other yrast transitions.

The staggering pattern for band 3 observed in this work agrees with the previous result only in the frequency range $\hbar\omega=0.25$–0.325 MeV. The discrepancy for $\hbar\omega\geqslant 0.325$ MeV may be linked to the presence of two interfering γ rays close to the 634.60-keV and 731.70-keV transitions. In both cases, strong yrast transitions (the $4^+\rightarrow 2^+$ and $6^+\rightarrow 4^+$ lines) in coincidence with the SD band interfere with the SD transitions. These two transitions have an important influence on the staggering plot in Fig. 2. The large uncertainty for the 731.70-keV transition is due to its proximity to the 734.8-keV ($6^+\rightarrow 4^+$) transition in ^{194}Hg. Therefore, it remains unclear whether the regular staggering in band 3 continues towards higher frequencies. The previously reported

[1]The expression given is in fact the finite difference approximation to the forth derivative d^4E_γ/dI^4 of the transition energies.

staggering pattern is in phase below $\hbar\omega=0.25$ MeV but the amplitude of only 15 eV is considerably smaller than the previous one of 80 eV.

To evaluate the statistical significance of the staggering pattern an analysis was performed in terms of the confidence level defined in Ref. [2]. In this method the distribution of ΔE_γ values around their average is compared to the distribution obtained when the sign of every other data point is changed. The separation of these distributions in terms of their standard deviation (by definition the standard deviations of both distributions are equal), gives a measure of the significance of the observed effect (including amplitude and regularity). Assuming a regular staggering over the whole frequency range, we have determined the confidence levels for the three SD bands in ^{194}Hg. The confidence level for band 1 is 0.7σ, which cannot be called statistically significant and reflects that all ΔE_γ values are within their uncertainties consistent with $\Delta E_\gamma=0$. For band 2, a confidence level of 1.4 σ was found for a regular staggering over the whole range. We also investigated the significance of those short regular oscillations that differ from the smooth reference outside their uncertainties. In band 2 the two frequency ranges $\hbar\omega=0.29-0.35$ MeV and $\hbar\omega\geqslant0.35$ MeV exhibit larger confidence levels of 2.5σ if analyzed individually. But the opposite phases of the oscillations in these regions lead to a small overall confidence level. However, each of these frequency ranges exhibits a significant deviation from a smooth reference. Band 3 exhibits the most regular pattern, which is represented by a 3.1σ confidence level. However, this large confidence level is mainly due to the oscillations observed in the frequency range $\hbar\omega=0.25$–0.325 MeV. The region below exhibits only very small confidence levels of about 0.5σ, while a value of 1.3σ is obtained for frequencies $\hbar\omega>0.325$ MeV. This leads to the conclusion that there is no clear evidence for a regular oscillation extending outside the frequency range $\hbar\omega=0.25$–0.325 MeV.

An additional statistical analysis was performed in order to investigate the fact that changes of individual E_γ values will have a correlated influence on the ΔE_γ plot, since each γ-ray energy is used in the calculation of five ΔE_γ values [see Eq. (1)]. In this analysis we have determined the probability that the observed staggering plots can be produced by a smooth rotational sequence of γ-ray energies that obey the experimental uncertainties. We find that there is a 47.8% probability that the staggering plot of band 1 is produced by a smooth rotational sequence. For bands 2 and 3 this probability is 10.8% and 0.4%, respectively. One may therefore conclude that the statistical analyses support the presence of significant deviations from a smooth reference in bands 2 and 3. Though not necessarily over the whole band, as discussed above.

The following conclusions can be drawn about the individual bands. (i) Within the quoted uncertainties, band 1 shows no significant deviation from a smooth behavior over the entire frequency range. (ii) A deviation from a smooth behavior of the γ-ray energies is obvious in band 2 for rotational frequencies above 0.3 MeV. However, this deviation does not correspond to a regular oscillation pattern, since a phase inversion is observed at $\hbar\omega=0.35$ MeV. (iii) A short regular staggering pattern of the order of ±50 eV is visible in band 3 in the frequency range $\hbar\omega=0.25$–0.325 MeV. Whether the observed pattern continues or not at higher and/or lower frequencies remains an open question due to the magnitude of the uncertainties involved.

We now want to discuss the present data in relation to some suggested interpretations. The results from Ref. [2] have been interpreted as evidence for a possible C_4 symmetry in the Hamiltonian [6–8]. This interpretation was originally based on the observation of an extended regular $\Delta I=2$ staggering in the yrast SD band of ^{149}Gd [1]. We have no clear evidence for such extended staggering in the SD bands of ^{194}Hg. However, it is not clear that C_4 symmetry must always generate extended regular staggering [20]. So, in fact, we are unable to use our new results to discuss the possible presence of a C_4 symmetry in ^{194}Hg. Many more systematic studies must be done in order to establish the presence or absence of C_4 symmetry.

Alternatively, we may compare the observed deviations with the patterns expected from a band crossing, as discussed in Refs. [9–11]. The insets in Fig. 2 show the deviations from a smooth reference that one would expect for the crossing of two bands. Two extreme cases were chosen where the crossing occurs either near levels in the bands (a) or at the midpoint between levels (b). The interaction between the bands is assumed to be so weak that the configuration mixing is extremely small and, as a result, no measurable cross talk between the bands occurs. Situation (a) can be approximated by the shift of only one level in the band and (b) by the shift of two levels. It is obvious that the staggering patterns in the insets are very similar to parts of the staggering plots for band 2 and 3. However, no single pattern can account for the experimentally observed deviations from the smooth reference. In addition, no evidence was found for additional SD bands that could be involved in a band crossing even though new SD bands could be identified [21] in ^{195}Hg and the strongest SD band in ^{193}Hg [22] was observed with an intensity of about 3% relative to the yrast SD band in ^{194}Hg. The full width at half maximum of all SD transitions was carefully checked and was found to be in agreement with the expected values taking intrinsic detector resolution and Doppler broadening into account. We conclude that there is no indication for a new SD band in ^{194}Hg with virtually identical transition energies to those of a known band. Therefore there is no experimental evidence for bands whose possible crossings could account for the observed deviations in bands 2 and 3.

In summary, we have performed a high statistics experiment to test for the previously reported [2] evidence for a $\Delta I=2$ staggering in the three SD bands in ^{194}Hg. The transition energies have been determined in this work with a precision of at least 60 eV for most transitions. With an improvement in the precision by a factor of two with respect to Ref. [2] we cannot confirm evidence for an extended regular $\Delta I=2$ staggering in any of the three SD bands of ^{194}Hg. However, we observe deviations from a smooth reference in the SD bands 2 and 3 that differ from those previously reported. We are unable to discuss the possible presence of a C_4 symmetry, since no extended staggering was observed, which was the basis of earlier discussions of this symmetry and no specific predictions are available for bands in ^{194}Hg. The oscillation patterns of the γ-ray energies that can be induced by a simple band crossing or level shift have been briefly discussed. While the similarities of these patterns with parts of the observed effects are significant, at least two such crossings or level shifts would be required in each band to explain the data. Even though such level shifts seem to provide a simple and straightforward explanation of the observed effects, it is apparent that other experimental signatures, such as a crossing band, are needed to fully understand the results of the present work. No such band has been found in the present data set. Thus the data does not settle the question of the origin of the observed effects.

This work is supported in part by the Department of Energy, Nuclear Physics Division, under Contract Nos. DE-AC03-76SF00098 and W-31-109-ENG-38.

[1] S. Flibotte *et al.*, Phys. Rev. Lett. **71**, 4299 (1993).
[2] B. Cederwall *et al.*, Phys. Rev. Lett. **72**, 3150 (1994).
[3] G. de Angelis *et al.*, Phys. Rev. C **53**, 679 (1996).
[4] S. M. Fischer *et al.*, Phys. Rev. C **53**, 2126 (1996).
[5] A. T. Semple *et al.*, Phys. Rev. Lett. **76**, 3671 (1996).
[6] I. Hamamoto and B. Mottelson, Phys. Lett. B **333**, 294 (1994).
[7] I. M. Pavlichenkov and S. Flibotte, Phys. Rev. C **51**, R460 (1995).
[8] A. O. Macchiavelli, B. Cederwall, R. M. Clark, M. A. Deleplanque, R. M. Diamond, P. Fallon, I. Y. Lee, F. S. Stephans, and S. Asztalos, Phys. Rev. C **51**, R1 (1995).
[9] Y. Sun, J. Zhang, and M. Guidry, Phys. Rev. Lett. **75**, 3398 (1995).
[10] M. A. Riley at the Workshop on Gammasphere Physics, Berkeley 1995 (to be published).
[11] W. Reviol, H.-Q. Jin, and L. L. Riedinger, Phys. Lett. B **371**, 19 (1996).
[12] Gammasphere Proposal, Report No. LBL-PUB-5202, 1988; I. Y. Lee, Nucl. Phys. **A520**, 641c (1990).
[13] F. A. Beck, Prog. Part. Nucl. Phys. **28**, 443 (1992); C. W. Beausang *et al.*, Nucl. Instrum. Methods Phys. Res. A **313**, 37 (1992).
[14] M. A. Riley *et al.*, Nucl. Phys. **A512**, 178 (1990).
[15] C. W. Beausang *et al.*, Z. Phys. A **335**, 325 (1990); E. A. Henry *et al.*, Z. Phys. A **335**, 361 (1990).
[16] C. W. Beausang, D. Prevost, M. H. Bergstrom, G. deFrance, B. Haas, J. C. Lisle, Ch. Theisen, J. Timár, P. J. Twin, and J. N. Wilson, Nucl. Instrum. Methods Phys. Res. A **364**, 560 (1995).
[17] B. Cederwall *et al.*, Nucl. Instrum. Methods Phys. Res. A **354**, 591 (1995).
[18] G. Hackman and J. C. Waddington, Nucl. Instrum. Methods Phys. Res. A **357**, 559 (1995).
[19] H. Hübel, A. P. Byrne, S. Ogaza, A. C. Stuchbery, and G. D. Dracoulis, Nucl. Phys. **A453**, 316 (1986).
[20] B. Mottelson, in Proceedings of the Conference on Physics from Large Gamma-Ray Detector Arrays, Berkeley CA, 1994, Report No. LBL-35687, p. 2.
[21] G. Hackman *et al.*, Bull. Am. Phys. Soc. **41**, 882 (1996), and to be published.
[22] D. M. Cullen *et al.*, Phys. Rev. Lett. **65**, 1547 (1990).