## Orbifold projection in supersymmetric QCD at $N_f \leq N_c$

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## Abstract

Supersymmetric orbifold projection of  $\mathcal{N} = 1$  SQCD with relatively small number of flavors  $(N_f \leq N_c)$  is considered. The purpose is to check whether orbifolding commutes with the infrared limit. On the one hand, one considers the orbifold projection of SQCD and obtains the low-energy description of the resulting theory. On the other hand, one starts with the low-energy effective theory of the original SQCD, and only then perfoms orbifolding. It is shown that at finite  $N_c$  the two low-energy theories obtained in these ways are different. However, in the case of stabilized run-away vacuum these two theories are shown to coincide in the large  $N_c$  limit. In the case of quantum modified moduli space, topological solitons carrying baryonic charges are present in the orbifolded low-energy theory. These solitons may restore the correspondence between the two theories provided that the soliton mass tends to zero in the large  $N_c$ limit.

1. Recently, a new approach to the dynamics of strongly coupled gauge theories based on the correspondence between large  $N_c$  theories and supergravity in higher dimensions has been suggested [1]. One of impressive results obtained within this approach is a relation between Green's functions of two different gauge theories in the large  $N_c$  limit. The relation holds provided the gauge and matter contents are related by the so called "orbifold projection". Then all Green's functions of the projected (daughter) theory are equal to certain Green's functions of the original (parent) theory. Suggested in the framework of string theory [2, 3, 4, 5], this relation has been later supported by diagrammatic analysis at the level of field theory [6, 7].

By making use of supersymmetry breaking orbifold projection, several non-supersymmetric candidate dual pairs were suggested [7] (see [8] for string theoretical interpretation). In order to construct such a pair one starts with a supersymmetric model exhibiting the Seiberg duality [9]. Then one makes orbifold projections of the electric and magnetic theories and arrives at two apparently different gauge models. The two models, however, are claimed to be equivalent in the large  $N_c$  limit.

A potential caveat of this construction is the necessity to interchange large  $N_c$  and infrared limits. These limits may not commute if states with masses which scale as  $\Lambda/N_c^{\alpha}$ ,  $\alpha > 0$ , are present in the spectrum ( $\Lambda$  is the infrared scale of the theory). Such states do not show up in the effective low-energy theory at finite  $N_c$ ; however, they become massless in the large  $N_c$  limit. This situation is inherent, e.g., in conventional QCD where  $\eta'$ -meson becomes massless in the large  $N_c$  limit. Another example is provided by  $\mathcal{N} = 2$  supersymmetric theories [10].

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Yet another potential problem is that the relation between parent and daughter theories proven at the level of planar diagrams may be spoiled by non-perturbative effects.

Several arguments suggesting that these problems do not arise in  $\mathcal{N} = 1$  SQCD have been presented in Ref. [7]. For instance, it was shown that large  $N_c$  behavior of gluino condensate and of mesonic Green's functions agrees with the exact results.

Another possible check is to consider the situation where the low-energy descriptions of projected original and effective theories are known. Then one can explicitly check whether these two theories coincide in the large  $N_c$  limit. One example of this type has been already presented in Ref. [7]. Namely, supersymmetry preserving orbifold projection of SQCD in the region of Seiberg duality was considered there. This projection splits both the electric and magnetic theories into the sets of decoupled theories with smaller numbers of colors and flavors. These projected theories are again related to each other by the Seiberg duality, as expected. It is worth noting, that the equivalence between the projected theories holds even at finite  $N_c$  in this case.

The purpose of this letter is to study whether the equivalence of the above type holds in  $\mathcal{N} = 1$  SQCD with relatively small number of quark flavors,  $N_f \leq N_c$ . In order to make use of the advantages of supersymmetry and keep the dynamics under control we restrict our consideration to orbifold projection that preserves supersymmetry. It is shown that at finite  $N_c$  the equivalence between the projected theories does not hold. However, in the case of run-away vacuum stabilized by the quark mass term, the two theories are shown to coincide in the large  $N_c$  limit (with  $N_f/N_c$  kept constant). Consequently, this case serves as a non-trivial check of the commutativity of the large  $N_c$  and infrared limits in  $\mathcal{N} = 1$  SQCD. In the case  $N_f = N_c$ , when quantum deformation of the classical flat directions occurs, the orbifolded effective theory does not reproduce the vacuum manifold of the orbifolded elementary theory at finite  $N_c$ . However, it is shown that topological solitons exist in this case. If the solitons become massless in the large  $N_c$  limit, equivalence between two daughter theories may be restored. Finally, it is pointed out that the infrared limit and orbifold projection do not commute in the special case, when the parent theory belongs to the region of the Seiberg duality, while the daughter theory does not. This exception is due to the violation of the conditions of the theorem about orbifold projection in the parent magnetic theory.

2. Consider  $\mathcal{N} = 1$  SQCD with gauge group  $SU(\Gamma N_c)$  and  $\Gamma N_f$  flavors of quarks<sup>2</sup>  $Q^a_{\alpha}$ and anti-quarks  $\tilde{Q}^{\beta}_b$ .  $\Gamma$  is some positive integer. By the large  $N_c$  limit we mean the limit  $N_f$ ,  $N_c \to \infty$ , with  $N_f/N_c$  kept constant. A general description of the orbifold projection can be found in Refs. [2–7]. Let us describe its specific version [7] adapted to the case of  $\mathcal{N} = 1$  SQCD which we consider throughout this paper. One makes use of the discrete group  $\mathbb{Z}_{\Gamma}$ . The action of the generator of this group on the quark superfields is defined as follows,

$$Q^a_{\alpha} \to \left(T_{N_f}\right)^a_b \left(T_{N_c}\right)^{\beta*}_{\alpha} Q^b_{\beta} , \qquad (1)$$

 $<sup>^{2}</sup>$ In what follows Latin and Greek letters stand for flavor and color indices, respectively.

where  $T_N$  is a  $\Gamma N \times \Gamma N$  diagonal matrix which consists of  $\Gamma$  blocks of the size  $N \times N$ ,

$$T_N = diag(1, e^{i/2\pi\Gamma}, \dots, e^{(\Gamma-1)i/2\pi\Gamma})$$

The extension of this action to the anti-quark and gauge superfields is straightforward. The Lagrangian of the orbifolded theory is obtained from the Lagrangian of the parent theory by removing all fields and interactions which are not invariant under the action<sup>3</sup> of  $\mathbb{Z}_{\Gamma}$ . The theorem proven in Refs. [6, 7] says that Green's functions of the  $\mathbb{Z}_{\Gamma}$ -invariant fields calculated in the parent and daughter theories are the same at the level of planar diagrams modulo rescaling of coupling constants. Generally, in order to obtain the relation between parameters of the parent and daughter theories, one should rescale the fields in both theories in such a way that the corresponding Lagrangians take the form

$$\mathcal{L}_p(g_{pi}, fields) = \Gamma N_c L_p(\bar{g}_{pi}, fields) \tag{2}$$

and

$$\mathcal{L}_o(g_{oi}, fields) = N_c L_o(\bar{g}_{oi}, fields) , \qquad (3)$$

where the subscripts p and o are assigned to parent and daughter (orbifolded) theories. Then the standard  $N_c$ -counting rules imply that couplings  $\bar{g}_{pi}$ ,  $\bar{g}_{oi}$  are constant in the large  $N_c$  limit. The equivalence between parent and daughter theories holds provided  $\bar{g}_{pi}$  and  $\bar{g}_{oi}$  are equal. In SQCD case under consideration this means that the canonical gauge coupling constants in the parent and orbifolded theories are related as follows,

$$\Gamma g_p^2 = g_o^2 \,. \tag{4}$$

**3.** Let us start from the case of run-away vacuum. The parent theory has  $\Gamma N_c$  colors and  $\Gamma N_f$  flavors with  $N_f/N_c < 1$ . At low energies the dynamics can be described in terms of mesons

$$M_b^a = Q_\alpha^a \tilde{Q}_b^\alpha \tag{5}$$

with the following effective superpotential [11, 12]

$$W_{eff} = \Gamma \cdot (N_c - N_f) \left(\frac{\Lambda_h^{\Gamma(3N_c - N_f)}}{\det M}\right)^{\frac{1}{\Gamma(N_c - N_f)}}, \qquad (6)$$

where  $\Lambda_h$  is the holomorphic infrared scale of the theory. The latter is related to the holomorphic coupling constant  $g_h$  in the following way,

$$\Lambda_h^{3N_c - N_f} = \mu \,\mathrm{e}^{-8\pi^2/g_h^2(\mu)} \,,$$

<sup>&</sup>lt;sup>3</sup>One can also consider the case when  $\mathbb{Z}_{\Gamma}$  is non-trivially embedded into the non-anomalous *R*-symmetry group. However, supersymmetry is broken in the daughter theory in the latter case, so we will restrict our consideration to the case of trivial embedding.

where  $\mu$  is the normalization scale. The value of the holomorphic coupling constant  $g_h$  is determined by the Shifman-Vainstein relation [13] between the holomorphic and canonical coupling constants,

$$\operatorname{Re}\left(\frac{8\pi^2}{g_h^2}\right) = \frac{8\pi^2}{g^2} + N_c \ln g^2 \,. \tag{7}$$

Besides mesons, the low-energy effective theory contains pure Yang-Mills sector corresponding to the unbroken  $SU(\Gamma(N_c - N_f))$  gauge group. In order to protect mesons from acquiring infinite vacuum expectation values, we consider a deformation of the theory by the mass term  $mQ^a\bar{Q}_a = m\text{Tr}M$  added to the superpotential. We take this mass term in the flavor-symmetric form to preserve the vectorial  $SU(\Gamma N_f)$  symmetry relevant to  $1/N_c$ expansion.

Now we apply the orbifold projection to the high-energy theory and study whether the obtained daughter theory corresponds at low-energies to the orbifolded effective theory. Upon orbifolding the high-energy theory one obtains  $\Gamma$  decoupled theories. Each of them has the gauge group  $SU(N_c)$  and  $N_f$  quark flavors. The rescaling rule (4) implies that holomorphic infrared scale of the orbifolded theory is equal to

$$\Lambda_o = \Gamma^{-\frac{N_c}{3N_c - N_f}} \Lambda_h \ . \tag{8}$$

The quark mass is left unchanged under the orbifold projection.

Consequently, at low energies the daughter theory is described by the following effective superpotential,

$$W_1 = \sum_{i=1}^{\Gamma} (N_c - N_f) \left( \frac{\Lambda_h^{3N_c - N_f}}{\Gamma^{N_c} \det M^{(i)}} \right)^{\frac{1}{N_c - N_f}} + \sum_{i=1}^{\Gamma} m \operatorname{Tr} M^{(i)} , \qquad (9)$$

where mesons referring to different gauge sectors are distinguished by the superscript (i). In addition there is a pure gluonic sector described by  $[SU(N_c - N_f)]^{\Gamma}$  gauge group.

Let us now consider the orbifold projection of the effective theory, the latter consisting of the mesonic sector described by the superpotential (6) and of the gluonic sector described by the gauge group  $SU(\Gamma(N_c - N_f))$ . The action of the discrete group  $\mathbb{Z}_{\Gamma}$  on the meson superfields is determined by Eqs. (1) and (5). In the effective meson theory,  $\Lambda_h$  serves as the coupling constant. Consequently, one rescales it according to the general rule described above. The weak coupling form of the mesonic Kahler potential,  $K(M^{\dagger}, M) = 2 \operatorname{Tr}(M^{\dagger}M)^{1/2}$ [14] (or, equivalently, the explicit relation between the meson and quark superfields (5)), implies that rescaling  $M \to \Gamma N_c M$  is needed to cast the Lagrangian of the effective theory in the form (2). Then, the general rescaling rule described above implies that  $\Lambda_h$  again rescales according to Eq. (8).

After the orbifold projection of the low-energy theory one obtains the gluonic sector described by the same  $[SU(N_c - N_f)]^{\Gamma}$  gauge group as above and the following meson su-

perpotential,

$$W_2 = \Gamma \cdot (N_c - N_f) \prod_{i=1}^{\Gamma} \left( \frac{\Lambda_h^{3N_c - N_f}}{\Gamma^{N_c} \det M^{(i)}} \right)^{\frac{1}{\Gamma(N_c - N_f)}} + \sum_{i=1}^{\Gamma} m \operatorname{Tr} M^{(i)} .$$
(10)

Obviously, the two superpotentials (9) and (10) are different. However, the vacuum expectation values of mesons are the same in both cases,

$$\langle M^{(i)} \rangle = \Gamma^{-1} \Lambda_h^{\frac{3N_c - N_f}{N_c}} m^{\frac{N_f - N_c}{N_c}} \equiv \langle M \rangle .$$
 (11)

Moreover, one can straightforwardly check that the Lagrangians describing dynamics in these vacua are the same for both superpotentials in the large  $N_c$  limit. To see this, let us compare *F*-terms originating from Eqs. (9) and (10), which determine the scalar potentials in the two cases. Making use of the first superpotential (9) we obtain

$$F_1^{(i)} \equiv \frac{\partial W_1}{\partial M^{(i)}} = -\left(\frac{\Lambda_h^{3N_c - N_f}}{\Gamma^{N_c} \det M^{(i)}}\right)^{\frac{1}{N_c - N_f}} \left(M^{(i)T}\right)^{-1} + m \; .$$

In the case of the second superpotential (10) we have

$$F_2^{(i)} \equiv \frac{\partial W_2}{\partial M^{(i)}} = -\left(\prod_{j=1}^{\Gamma} \frac{\Lambda_h^{3N_c - N_f}}{\Gamma^{N_c} \det M^{(j)}}\right)^{\frac{1}{\Gamma(N_c - N_f)}} \left(M^{(i)T}\right)^{-1} + m \; .$$

We write the meson fields in the form  $M^{(i)} = \langle M \rangle + \delta M^{(i)}$ , then  $F_1^{(i)}$  and  $F_2^{(i)}$  differ by terms coming from the expansion of determinants in the denominators. However, these terms are suppressed by extra powers of  $N_c - N_f$ . Hence, both *F*-terms in the large  $N_c$  limit are reduced to

$$F^{(i)} = -\Gamma^{-1} m^{\frac{N_f}{N_c}} \Lambda^{\frac{3N_c - N_f}{N_c}} \left( \langle M \rangle + \delta M^{(i)T} \right)^{-1} + m .$$

We conclude that  $\mathcal{N} = 1$  SQCD with  $N_f < N_c$  provides a non-trivial check of the technique suggested in Ref. [7].

4. Let us now turn to the case of SQCD with the gauge group  $SU(\Gamma N_c)$  and  $\Gamma N_c$  quark flavors. This theory exhibits quantum deformation of the moduli space [15]. Namely, the space of vacua of the microscopic theory is described by the set of holomorphic gauge invariants constructed of quark and anti-quark fields. These invariants are mesons (5), and (anti)-baryons

$$B = \epsilon^{\alpha_1 \dots \alpha_{\Gamma N_c}} Q^1_{\alpha_1} \dots Q^{\Gamma N_c}_{\alpha_{\Gamma N_c}} ,$$
  

$$\tilde{B} = \epsilon_{\beta_1 \dots \beta_{\Gamma N_c}} \tilde{Q}^{\beta_1}_1 \dots \tilde{Q}^{\beta_{\Gamma N_c}}_{\Gamma N_c} ,$$
  
det  $M = P \tilde{P} = \Lambda^{2\Gamma N_c}$ 

subject to the constraint

$$\det M - B\tilde{B} = \Lambda_h^{2\Gamma N_c} . \tag{12}$$

The r.h.s of Eq. (12) is of purely quantum origin and indicates the difference between the topologies of the quantum and classical spaces of vacua. At low energies this theory is described by the non-linear sigma model with the field space parameterized by mesonic  $M_{\tilde{b}}^a$  and (anti-)baryonic  $(\tilde{B})B$  coordinates that satisfy the constraint (12).

Let us take the orbifold projection of this theory with respect to the discrete group  $\mathbb{Z}_{\Gamma}$ as described above. Then, in analogy to the case of smaller number of flavors, the orbifolded theory splits into  $\Gamma$  copies of SQCD with gauge groups  $SU(N_c)$  and  $N_c$  quark flavors. At low energy the latter theory is described in terms of sigma-model fields  $M_b^{a(i)}$ ,  $B^{(i)}$ ,  $\tilde{B}^{(i)}$ , where  $a, b = 1, \ldots, N_c$  and  $i = 1, \ldots, \Gamma$ . These fields satisfy the constraints

$$\det M^{(i)} - B^{(i)}\tilde{B}^{(i)} = \Lambda_a^{2N_c}$$
(13)

for every i.

The orbifold projection of the low-energy theory described by Eq. (12) is the non-linear sigma-model with the fields  $M_b^{a(i)}$ , B,  $\tilde{B}$  subject to the constraint

$$\prod_{i=1}^{\Gamma} \det M^{(i)} - B\tilde{B} = \Lambda_o^{2\Gamma N_c} .$$
(14)

Manifolds  $Q_1$  and  $Q_2$  determined by Eqs. (13) and (14), respectively, are different. For instance, the former has complex dimension  $\Gamma(N_c^2 + 1)$  whereas the complex dimension of  $Q_2$  is ( $\Gamma N_c^2 + 1$ ). Consequently, the orbifolded original theory, described at low energies by Eq. (13), has extra ( $\Gamma - 1$ ) massless superfields as compared to the orbifolded sigma model. This is due to the fact that baryons of the parent sigma model do not carry flavor or color indices and, as a result, they are not affected by the orbifold projection. Hence, the orbifolded effective theory has one conserved baryonic charge and one pair of massless baryons instead of  $\Gamma$ .

Furthermore, there is a profound difference between  $Q_1$  and  $Q_2$ . To see this difference, let us study their topology in more detail. The manifold  $Q_1$  is merely a direct product of  $\Gamma$ copies of the manifold Q defined by one of the equations listed in Eqs. (13). The topology of Q has been studied in Ref. [16]. It was found there that this space is homotopically equivalent to the double suspension of  $SL(N_c, \mathbb{C})$  group,  $\Sigma(\Sigma(SL(N_c, \mathbb{C})))$ .

The suspension of the manifold  $\mathcal{X}$  is the cylinder  $\mathcal{X} \times [0,1]$  with all points on the lower base  $\mathcal{X} \times 0$  identified and all points on the upper base  $\mathcal{X} \times 1$  identified as well. For example, the suspension of *d*-dimensional sphere  $S^d$  is (d+1)-dimensional sphere  $S^{(d+1)}$ . The latter observation is the basis of the theorem about suspension (see., e.g., Ref. [17], p.79),  $\pi_{q+1}(\Sigma(\mathcal{X})) = \pi_q(\mathcal{X})$  for all  $q \leq 2n-2$ , provided that  $\pi_i(\mathcal{X}) = 0$  for i < n. In particular, this theorem implies that the lowest non-trivial homotopy group of  $\mathcal{Q}$  is  $\pi_5(\mathcal{Q}) = \mathbb{Z}$ , so that  $\pi_5(\mathcal{Q}_1) = \mathbb{Z}^{\Gamma}$ .

Let us now consider the manifold  $Q_2$ . A straightforward generalization of the arguments of Ref. [16] shows that this space is homotopically equivalent to the double suspension of the manifold  $\mathcal{Y}$  determined by the following equation,

$$\prod_{i=1}^{\Gamma} \det M^{(i)} = \Lambda_o^{2\Gamma N_c} .$$
(15)

At  $\Gamma = 1$ , the manifold  $\mathcal{Y}$  is  $SL(N_c, \mathbb{C})$  in accordance with the above discussion.

Let us calculate the lowest homotopy group of  $\mathcal{Q}_2$ . Every non-degenerate matrix  $M^{(i)}$ can be decomposed into a product of a matrix with unit determinant and of the diagonal matrix of the form  $diag(t_i, 1, ..., 1)$ . Then Eq. (15) implies that the manifold  $\mathcal{Y}$  is a direct product  $SL(N_c, \mathbb{C})^{\Gamma} \times T_{\Gamma-1}$ . Here  $T_{\Gamma-1}$  is  $(\Gamma-1)$ -dimensional complex torus  $\mathbb{C}_*^{\Gamma-1}$  determined by the equation

$$\prod_{i=1}^{\Gamma} t_i = \Lambda_o^{2\Gamma N_c}$$

Therefore, the manifold  $Q_2$  is homotopically equivalent to

$$\mathcal{Q}_2 \sim \Sigma(\Sigma(\mathcal{Y})) \sim \mathcal{Q}_1 \times (S^3)^{\Gamma-1}$$

The lowest non-trivial homotopy group of  $\mathcal{Q}_2$  is

$$\pi_3(\mathcal{Q}_2) = \mathbb{Z}^{\Gamma-1} . \tag{16}$$

As a result, contrary to the case of  $Q_1$  sigma-model, there may exist topological solitons in the  $Q_2$  sigma-model, provided that the stabilizing higher-derivative terms are present in the Kahler potential of the effective theory. These solitons are distinguished by  $(\Gamma - 1)$ conserved topological charges.

It is tempting to identify these charges with the missing  $(\Gamma - 1)$  baryonic charges. An argument in favor of this identification is the possibility to construct corresponding topological currents by formal extension of the algebra of the Noether currents; in analogy to the case of the conventional QCD [18]. This is possible due to the existence of the Wess–Zumino term in SQCD with  $N_f = N_c$ . In the parent theory this term reads as follows [16],

$$\Gamma_p = \frac{-1}{12\pi^2 \Lambda^{4\Gamma N_c}} \operatorname{Im} \int d\Omega \, \det M \cdot \epsilon^{\mu\nu\lambda\rho\sigma} \partial_{\mu} B \, \partial_{\nu} \tilde{B} \times \operatorname{Tr} \left( M^{-1} \partial_{\lambda} M M^{-1} \partial_{\rho} M M^{-1} \partial_{\sigma} M \right) \, .$$

After the orbifold projection it takes the following form,

$$\Gamma_{o} = \frac{-1}{12\pi^{2}\Lambda_{o}^{4\Gamma N_{c}}} \operatorname{Im} \int d\Omega \left(\prod_{i=1}^{\Gamma} \det M^{(i)}\right) \cdot \epsilon^{\mu\nu\lambda\rho\sigma} \partial_{\mu}B \,\partial_{\nu}\tilde{B}$$
$$\times \sum_{i=1}^{\Gamma} \operatorname{Tr} \left(M^{(i)-1} \partial_{\lambda}M^{(i)}M^{(i)-1} \partial_{\rho}M^{(i)}M^{(i)-1} \partial_{\sigma}M^{(i)}\right) .$$

The contribution of this term to the flavor current in the daughter theory reads as follows,

$$j^{\mu} = \frac{1}{4\pi^2 \Lambda_o^{4N_c}} \prod_{i=1}^{\Gamma} \det M^{(i)} \cdot \epsilon^{\mu\nu\lambda\rho} \partial_{\nu} B \,\partial_{\lambda} \tilde{B} \operatorname{Tr} \left( T^f \sum_{i=1}^{\Gamma} \partial_{\rho} M^{(i)} M^{(i)-1} \right) + h.c.$$

where  $T^f$  is a generator of the flavor group. In complete analogy to the case of QCD, one formally substitutes here  $T^f$  in the form  $diag(0, \ldots, i, \ldots, 0)$  where 0, i stand for the blocks of length  $\Gamma$ . In this way one obtains  $\Gamma$  conserved topological currents,

$$j^{(i)\mu} = \frac{1}{2\pi^2 \Lambda_o^{4N_c}} \operatorname{Im} \prod_{i=1}^{\Gamma} \det M^{(i)} \cdot \epsilon^{\mu\nu\lambda\rho} \partial_{\nu} B \, \partial_{\lambda} \tilde{B} \operatorname{Tr} \left( \partial_{\rho} M^{(i)} M^{(i)-1} \right) ,$$

which are subject to the constraint  $\sum j^{(i)\mu} = 0$ . These currents correspond precisely to the topological property (16), and the above argument indeed shows that they are naturally identified with the missing baryonic currents.

To prove the equivalence of the  $Q_1$  and  $Q_2$  sigma-models in the large  $N_c$  limit one would have to study the behavior of the soliton mass in this limit. The above consideration indicates that these two models may be equivalent at large  $N_c$  provided that the soliton mass tends to zero as  $N_c \to \infty$ . We leave the analysis of this point for future.

5. Finally, let us present an example in which the orbifold projection does not commute with the infrared limit. Let us consider  $\mathcal{N} = 1$  SQCD with  $\Gamma N_c$  colors and  $\Gamma(N_c+1)$  flavors. This theory belongs to the region of the Seiberg duality and its low-energy behavior can be described in terms of a magnetic theory with  $\Gamma$  colors and  $\Gamma(N_c+1)$  flavors. In addition, the magnetic theory contains gauge-singlet meson fields  $M_h^a$  with the following superpotential

$$W_m = q M \tilde{q}$$

where q and  $\tilde{q}$  are dual quark and anti-quark superfields; a dimensionful coefficient is omitted.

Upon orbifolding the electric theory one obtains  $\Gamma$  decoupled theories with the gauge group  $SU(N_c)$  and  $(N_c+1)$  quark flavors. At low energies this theory confines and describes mesons and baryons interacting through the following superpotential [15]

$$W_1 = \sum_{i=1}^{\Gamma} \left( B^{(i)} M^{(i)} \tilde{B}^{(i)} - \det M^{(i)} \right) .$$
(17)

The orbifold projection of the magnetic theory, on the other hand, leaves  $\Gamma$  pairs of the gauge-singlet superfields  $q^{(i)}$  and  $\tilde{q}^{(i)}$  with the same quantum numbers as  $B^{(i)}$  and  $\tilde{B}^{(i)}$  and splits the meson multiplet in the same manner as above. The projected superpotential reads as follows,

$$W_2 = \sum_{i=1}^{\Gamma} q^{(i)} M^{(i)} \tilde{q}^{(i)} , \qquad (18)$$

which is different from Eq. (17).

The reason of this discrepancy is that  $1/N_c$  expansion does not work in the magnetic theory, as the number of magnetic colors does not depend on  $N_c$ . Consequently, the relation between the planar diagrams in the parent and daughter theories does not lead to the relation between corresponding Green's functions in this case.

6. To conclude, we have considered orbifold projection of SQCD with relatively small number of quark flavors. It was found that in the case of stabilized run-away vacuum, the orbifold projection serves as a non-trivial check of the commutativity of the large  $N_c$  and infrared limits.

We discussed in sect. 5 also a specific way of taking  $N_c \to \infty$  such that two daughter theories obtained in different limits are not equivalent. The reason is that the conditions of the theorem about the orbifold projection are violated in the effective theory.

The most intriguing situation occurs in SQCD with quantum modified moduli space,  $N_f = N_c$ . In this case it was found that upon orbifolding the high-energy and low-energy theories, one obtains a pair of sigma models which are not equivalent to each other at finite  $N_c$ . Namely, the vacuum manifold of the orbifolded elementary theory has larger dimension than the space of vacua of the orbifolded effective theory. As a result, the latter has smaller number of massless fields in every particular vacuum. Moreover, the number of the Noether currents in the orbifolded effective theory is too small to reproduce the correct current algebra of the sigma model corresponding to orbifolded SQCD. However, the two sigma models may become equivalent at large  $N_c$  due to the presence of topological solitons in the orbifolded effective theory. Namely, the topological currents restore the structure of the current algebra, and the solitons may provide the correspondence between the spectra of light fields in the two theories provided that the soliton mass tends to zero at large  $N_c$ . The correct structure of the space of vacua may be restored due to the presence of topologically non-trivial field configurations corresponding to non-zero "soliton condensate". Further analysis of this model from both field theoretical and brane points of view may provide new insights into orbifold projection.

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## References

- J.M. Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231, Int.J.Theor.Phys. 38 (1999) 1113
- [2] S. Kachru, E. Silverstein, Phys. Rev. Lett. 80 (1998) 4855
- [3] A. Lawrence, N. Nekrasov, C. Vafa, Nucl. Phys. **B533** (1998) 199

- [4] M. Bershadsky, Z. Kakushadze, C. Vafa, Nucl. Phys. B523 (1998) 59
- [5] Z. Kakushadze, Nucl. Phys. **B529** (1998) 157
- [6] M. Bershadsky, A. Johansen, Nucl. Phys. B536 (1998) 141
- [7] M. Schmaltz, Phys. Rev. **D59** (1999) 105018
- [8] A. Armoni, B. Kol, JHEP 9907 (1999) 011
- [9] N. Seiberg, Nucl. Phys. **B435** (1995) 129
- [10] M.R. Douglas, S.H. Shenker, Nucl. Phys. B447 (1995) 271
- [11] I. Affleck, M. Dine, N. Seiberg, Nucl. Phys. **B241** (1984) 493
- [12] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B260 (1985) 157
- [13] M.A. Shifman and A.I. Vainshtein, Nucl. Phys. **B277** (1986) 456
- [14] I. Affleck, M. Dine, N. Seiberg, Nucl. Phys. **B256** (1985) 557
- [15] N. Seiberg, Phys. Rev. **D49** (1994) 6857
- [16] S.L. Dubovsky, D.S. Gorbunov, Induced charge matching and Wess-Zumino term on the quantum modified moduli space, to appear in Phys. Rev. D, hep-th/9909155
- [17] A.T. Fomenko, D.B Fuchs, A course in homotopy theory, Moscow, Nauka, 1989
- [18] E. Witten, Nucl. Phys. **B223** (1983) 422