Identification of the $I^\pi = 10^+$ Yrast Rotational State in ^{24}Mg

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(Received 5 February 2001; published 14 September 2001)

Spins and parities of high spin states above the particle-binding threshold in ^{24}Mg were determined with a basis expansion technique using triple and quadruple angular correlations between α particles and γ rays. The first unambiguous identification of a 10^+ state is reported. Located at 19.2(1) MeV, this state decays predominantly by α emission, although a candidate γ-decay branch with a 5.927 MeV transition connecting this 10^+ level to the rotational 8^+ state at 13.2 MeV was identified as well. The corresponding γ-α branching ratio is $7(3) \times 10^{-4}$.

DOI: 10.1103/PhysRevLett.87.142502 PACS numbers: 27.30.+t, 07.05.Kf, 21.10.Re, 23.20.En

The nucleus ^{24}Mg contains a sufficiently small number of nucleons that its structure can be calculated with the shell model using the full *sd*-configuration space. Yet, this nucleus also exhibits collective excitations associated with large, prolate deformations. In this context, the identification of states approaching $12\hbar$, the maximum spin that can be generated within the *sd* shell, is particularly relevant. Their properties test the microscopic basis of collective motion; i.e., either they become less collective than lower levels as the number of available configurations within the *sd* shell decreases, or they maintain their collectivity by gathering strength from sizable admixtures from higher lying orbitals, e.g., from the (fp) shell. Such issues are most readily addressed in light nuclei, where calculations remain tractable.

The study of levels beyond the established 8^+ states in ^{24}Mg is hampered by the fact that they are particle unbound by energies $\geq$5 MeV. In addition, their deexcitation often proceeds through complex paths and, as a result, angular correlation studies are difficult and quantum number assignments problematic. The interpretation of many measurements has been carried out by modeling the reaction mechanism using calculations of the Hauser-Feshbach type, see, e.g., [1–3]. With this technique, Lumpkin *et al.* [1] proposed four candidates for 10^+ levels between 20.1 and 28.2 MeV. Szanto de Toledo *et al.* [2,3] interpreted the state at 20.1 MeV as the lowest 10^+ level. These assignments, however, remain doubtful as the descriptions of the data contain adjustable parameters that could not be thoroughly checked with data on levels with established comparable spin values. Here it is shown that the rigorous angular correlation analysis of multistep decay paths results in unambiguous spin and parity assignments to high lying states in ^{24}Mg. In particular, the first firm identification of a 10^+ state at 19.2 MeV has been achieved.

The experiment was performed at the ATLAS accelerator at Argonne National Laboratory. The setup consisted of five 16×16 silicon strip detectors, located around the target for the detection of α particles, and of the Gammasphere array [4] for the coincident detection of γ rays. Excited states in ^{24}Mg were prepared with the ^{12}C(^{16}O, α)^{24}Mg reaction at $E_{\mathrm{c.m.}} = 26.6$ MeV. The α particles emitted from the ^{28}Si compound nucleus, denoted as α_0, populate the states of interest in ^{24}Mg and determine their excitation energy. These particles were measured upstream from the target in a segmented annular detector. The resulting excitation-energy spectra were calibrated with a magnetic spectrometer measurement [5] performed at the same beam energy. The α particles emitted from ^{24}Mg states (α_1) and their angular distributions were measured in four segmented silicon detectors positioned at 45° with respect to the beam axis. Subsequent γ transitions in ^{20}Ne were measured with Gammasphere. Details about the experiment can be found in Ref. [6].

In the analysis, a dynamic quantization axis for the ^{24}Mg states was introduced. This procedure is analogous to that described in Ref. [7]. It eliminates the dependence of the angular correlation on the angle of emission of α_0 by tilting the quantization axis slightly away from the beam axis in a manner depending on the α_0 emission angle and angular momentum. The observed α_0 correlation patterns consistently show a population from compound nuclear states with spin $16 \pm 1\hbar$.

Considering the case where α_1 feeds the ^{20}Ne ground state, the angular distributions of α_1 relative to the quantization axis follow the familiar squared Legendre polynomials and provide an effective method for spin determination. Unfortunately, this favorable situation is limited to states below $\approx$14 MeV, since decays towards ^{20}Ne excited states dominate at higher ^{24}Mg excitation energies. The population of the 2^+ or 4^+ levels in ^{20}Ne is characterized by an incoherent superposition of several m-substates washing out the characteristic angular distributions, and making spin assignments impossible. This difficulty can be overcome by detecting the direction of the subsequent ^{20}Ne, $2^+ \rightarrow 0^+$ γ ray [8,9]. This restores coherence to the 2^+ m-substate population and reestablishes the characteristic oscillatory patterns of the α_1 angular distributions. Here, this approach is generalized to states in ^{24}Mg which decay to the 4^+ level of ^{20}Ne. The coherence between the 4^+ m-substates is achieved by detecting the direction of emission of the entire $4^+ \rightarrow 2^+ \rightarrow 0^+$ γ cascade.

Under the present coincidence conditions, the α_1 particles are emitted from a state of spin $I_A, m = 0$ in ^{24}Mg and feed a level of spin $I_B = 0^+, 2^+, 4^+$ in ^{20}Ne. The observed correlations were found to be consistent with "stretched" α_1 decays, i.e., all deexcitations were assumed to correspond to $L_{\alpha_1} = I_A - I_B$. Hence, the observed states in ^{24}Mg must be of natural parity $\pi = (-1)^I$. The triple and quadruple angular correlations can be represented in the form of Eqs. (1) and (2) below, where all terms depending on the α_1 angles and quantum numbers are collected in the $A^{\lambda_0 q_0}(I_A, I_B)$ statistical tensors resulting from the angular momentum coupling between I_A, L_{α_1}, and I_B. All terms describing the γ-ray angular distributions are collected in the $B^{\lambda_0 q_0}$ tensors. The exact definition of these A, B_2, and B_4 terms is given in [6].

$$W(\theta_{\alpha_1}\theta_\gamma\phi_\gamma) = \sum_{\lambda_0 q_0} A^{\lambda_0 q_0}(I_A, 2^+)\,(\theta_{\alpha_1}) \times B_2^{\lambda_0 q_0}(2^+, 0^+)\,(\theta_\gamma\phi_\gamma), \tag{1}$$

$$W(\theta_{\alpha_1}\theta_{\gamma_1}\phi_{\gamma_1}\theta_{\gamma_2}\phi_{\gamma_2}) = \sum_{\lambda_0 q_0} A^{\lambda_0 q_0}(I_A, 4^+)\,(\theta_{\alpha_1}) \times B_4^{\lambda_0 q_0}(4^+, 2^+, 0^+)\,(\theta_{\gamma 1}\phi_{\gamma 1}\theta_{\gamma 1}\phi_{\gamma 1}). \tag{2}$$

Equations (1) and (2) suggest an expansion of the correlation function into the basis $B_i^{\lambda_0 q_0}$ with the coefficients $A^{\lambda_0 q_0}$. Indeed, the $B_2^{\lambda_0 q_0}$ and $B_4^{\lambda_0 q_0}$ tensors form a basis of their respective spaces, which is orthogonal in the indices λ_0 and q_0. λ_0 is the rank of the orientation tensor of the 2^+ or, respectively, 4^+ intermediate ^{20}Ne state. The orthogonality is then used to generate a basis expansion of the experimental data, which is obtained by folding the geometrical correlations over all available θ_γ angles with the respective basis tensors. The resulting transformed representations are functions of θ_{α_1} only and are expected to fit the calculated $A^{\lambda_0 q_0}(I_A, 2^+ \text{ or } 4^+)\,(\theta_{\alpha_1})$ for all relevant $\lambda_0 q_0$ indices. Note that Eq. (1) is analogous to the triple correlation formula of [8,9].

The validity of this new formalism has been thoroughly tested using known ^{24}Mg states [8–10] feeding the ^{20}Ne 2^+ level. This is illustrated in Fig. 1 where the transformed experimental data for the 8^+ state at 16.5(1) MeV are compared with the relevant $A^{\lambda_0 q_0}(I_A, 2^+)\,(\theta_{\alpha_1})$ curves: the distributions calculated for a spin 8 initial state reproduce the data in all $(\lambda_0 q_0)$ values, while those calculated for spins 10 and 6 (not shown) can be ruled out. A single normalization parameter $[N(I_A)]$ was adjusted in this analysis.

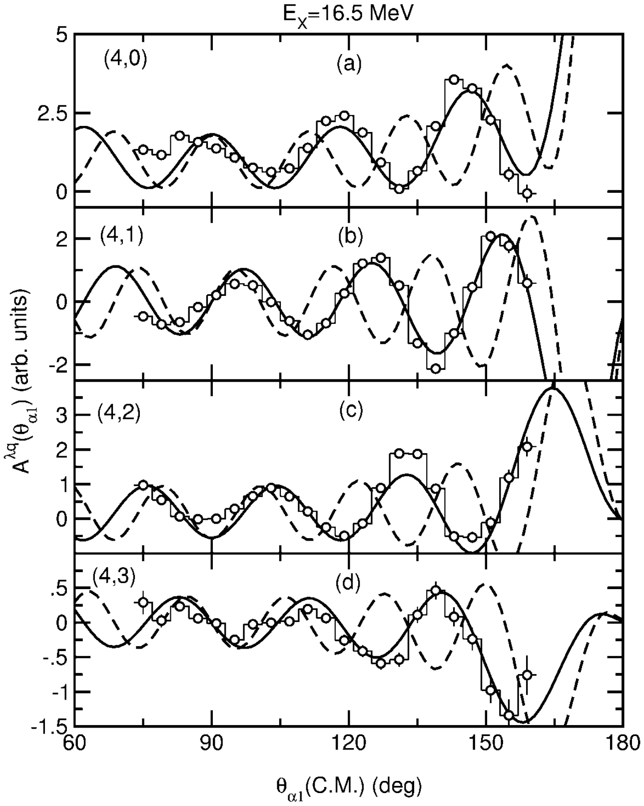

FIG. 1. Transformed triple angular correlation data observed for the α_1-γ decay of the 16.5(1) MeV state. Plotted are the data for the most relevant correlations, e.g., $(\lambda_0 q_0) = (4,0)\,(4,1)\,(4,2)\,(4,3)$. The thick full (dashed) line refers to a $I_A = 8$ (10) hypothesis.

The method was then extended to the analysis of quadruple correlations. The correlation data were transformed by folding them with the basis tensors $B_4^{\lambda_0 q_0}$ over all available angles $(\theta_{\gamma_1}\phi_{\gamma_1}\theta_{\gamma_2}\phi_{\gamma_2})$. The most relevant $(\lambda_0 q_0)$ indices are $(8,0)$, $(8,1)$, and $(8,2)$, and the characteristics of the $A^{\lambda_0 q_0}(I_A, 4^+)\,(\theta_{\alpha_1})$ curves are very similar to those seen in the triple-correlation case. Figure 2 presents the data for the 19.2(1) MeV state. In this case, the correlation pattern fits the distributions calculated for a 10^+ state very well. Note that this analysis is only possible because of the near 4π coverage provided by Gammasphere.

Figure 3(a) shows the ^{24}Mg excitation energy spectrum obtained when gating on the α_1 decays feeding the 4^+ state in ^{20}Ne, in coincidence with the $4^+ \rightarrow 2^+ \rightarrow 0^+$ γ-ray cascade. The lower three histograms [Figs. 3(b)–3(d)] present the normalization parameter $N(I_A)$ as a function of the excitation energy E_X in ^{24}Mg for $I_A = 8$, 9, and 10. A large positive value of $N(I_A)$ at the same E_X value as a peak in Fig. 3(a) identifies the presence of a state with spin I_A. Negative values of $N(I_A)$ arise from levels with spin $I \neq I_A$ [$N(I_A)$ would be zero if complete 4π coverage were achieved]. Figure 3(b) confirms that the 17.2 MeV level is characterized by 8^+ quantum numbers, while the two peaks at 20.1 and 20.3 MeV [Fig. 3(c)] are dominated

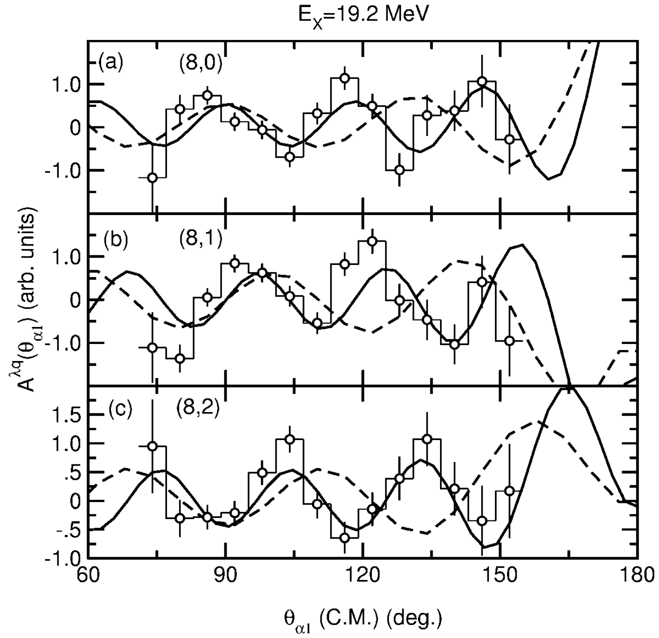

FIG. 2. Transformed quadruple angular correlation data observed for the α_1-γ-γ decay of the 19.2(1) MeV state. Plotted are the correlation data for $(\lambda_0 q_0) = (8,0)\,(8,1)\,(8,2)$ (see text). The thick full (dashed) line refers to a $I_A = 10$ (8) hypothesis.

by 9^- excitations. A clear signal for 10^+ strength is noted for the state at 19.2 MeV [Fig. 3(d)] for which the relevant correlations were presented in Fig. 2.

It is worth noting that the 9^- state at 20.2(1) MeV was mentioned as a candidate for the lowest 10^+ level in [1–3], while the present analysis yields an unambiguous assignment of $I_A^\pi = 9^-$. In total, new spin and parity assignments were made for nine states between 15.7 and 20.3 MeV; these results will be published elsewhere [11].

In order to gain more insight into the properties of the 19.2(1) MeV 10^+ state, a search was undertaken for any possible γ-decay branch, as any such transition can be expected to link levels associated with closely related configurations. The data were scanned with the following condition on the $(\alpha_0\gamma\gamma)$ events: (i) a 19.2 ± 0.15 MeV excitation energy in coincidence with (ii) any of the ^{24}Mg transitions below the 6_1^+ state. The coincident γ-ray spectrum is presented in Fig. 4. Also shown is the corresponding background spectrum. A peak of 11 ± 4 counts is visible at 5927(5) keV. This energy matches the value expected for the γ transition linking the new 19.2 MeV, 10^+ level to the rotational 8_2^+ state at 13.212 MeV. The 5099 keV γ line deexciting this 8_2^+ state is also present in the spectrum. The 5927 keV transition establishes a state at 19.139(5) MeV matching the energy from the α_0 spectrum. Since the uncertainty in the particle channel is approximately ± 100 keV, the presence of a doublet of levels cannot, however, be entirely ruled out. Under the assumption that the same state is seen in both decay channels, the γ-α branching ratio is $7(3) \times 10^{-4}$.

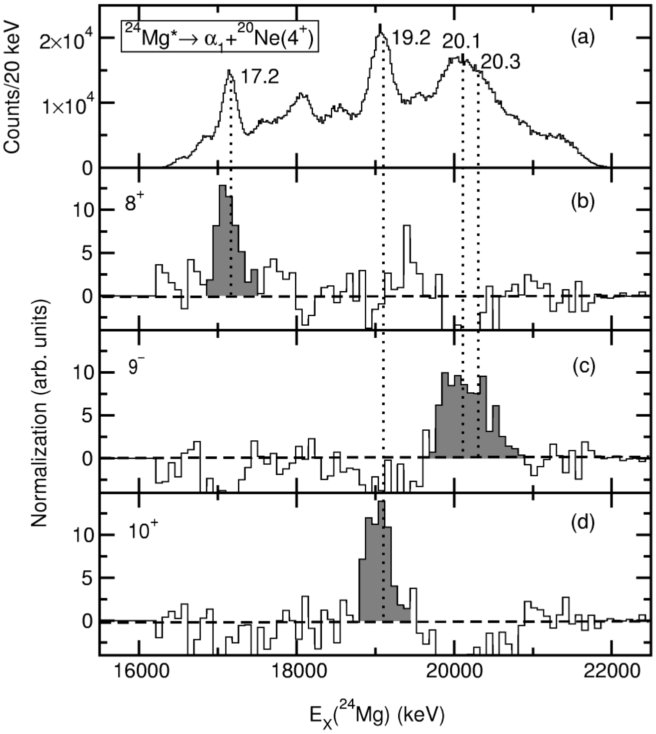

FIG. 3. (a) Observed excitation energy spectrum of ^{24}Mg, gated on α_1 decays towards the 4^+ of ^{20}Ne. (b)–(d): Normalization parameter $N(I_A)$ as a function of the excitation energy E_X for the values of $I_A^\pi = 8^+$, 9^-, and 10^+, respectively.

Theoretical descriptions of the high spin structure of ^{24}Mg have been discussed in detail in Ref. [12]. In particular, this reference addresses the issue that the 8_2^+ level rather than 8_1^+ yrast state is a member of the ground state rotational band, a feature which came initially as a surprise to experimentalists and theorists alike. The 8_1^+ state

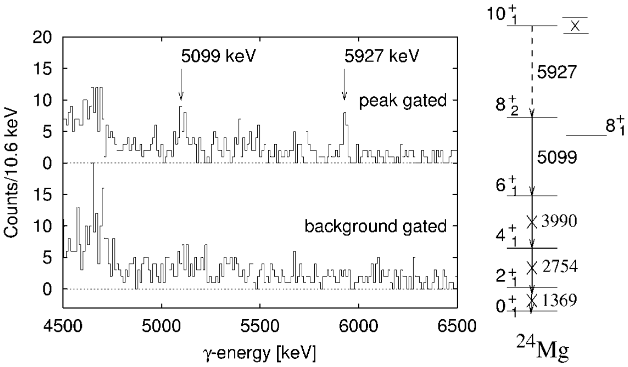

FIG. 4. Upper part: γ spectrum gated on a 19.2 ± 0.15 MeV excitation energy in coincidence with either the 1369 ± 20, 2754 ± 30, or 3990 ± 40 keV ground state band transitions. The decay scheme illustrates the gating conditions with a cross. Lower part: γ spectrum gated on background regions; i.e., sum of spectra gated either on (1) a 19.2 ± 0.15 MeV excitation energy measured in coincidence with either of the γ-background regions 1711 ± 20, 3175 ± 30, or 4200 ± 40 keV or (2) a 18.6(15) MeV excitation energy in coincidence with the above mentioned γ-peak regions.

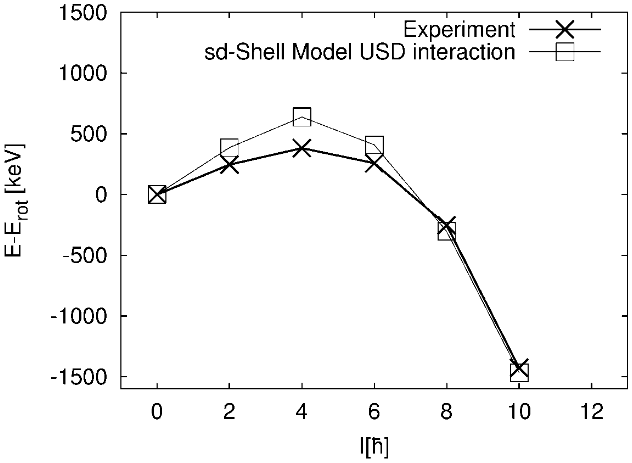

FIG. 5. Excitation energies of the ground state band in ^{24}Mg, with a rotational reference $E_{\mathrm{rot}}[\mathrm{keV}] = 187 \times I(I + 1)$ subtracted, compared with the results of a sd-shell model calculation with the USD Hamiltonian [13].

was described in shell model calculations using empirical interactions within the full sd-shell basis. These same interactions, labeled PW and CWC in Ref. [12], predict the 10_1^+ level around 19.5 MeV with decay properties depending sensitively on the choice of the interaction. While the PW interaction predicts the 10^+ state to be most strongly connected to the $K = 0$ ground state band, the CWC interaction favors a strong link with the $K = 2$ band. In addition, the same Ref. [12] also discussed the results of a Nilsson-Strutinsky calculation where the lowest 10^+ level is understood as a member of the ground state band. In this context, the present result for the 19.2(1) MeV state favors its interpretation as a member of the ground state rotational band if the 5927 keV γ ray is identified with the $10_1^+ \rightarrow 8_2^+$ transition.

To explore the nature of the new 10^+ state further, new shell model calculations were performed using the USD interaction [13] in the full sd-shell basis. The results are compared in Fig. 5 with the data after subtraction of a rotational energy reference defined as $E_{\mathrm{rot}}[\mathrm{keV}] = 187 \times I(I + 1)$. These calculations reproduce the overall evolution of the ground state band with spin rather well. The predicted energy for the 10_1^+ level of 19.104 MeV is in excellent agreement with the measurement. The curvature seen in the figure can be viewed as a signature for the onset of band termination, i.e., the level energy decreases with respect to the rotational reference as one approaches the 12^+ state obtained by aligning the spins of the eight valence nucleons. In fact, the wave function of the USD 10^+ state is dominated (50%) by the $[(d_{5/2})^7 \times d_{3/2}]$ configuration, which can be interpreted as a particle-hole excitation built from the 8_1^+ level which has a 46% $(d_{5/2})^8$ component in its configuration. Thus, the 10_1^+ excitation energy is largely determined by the effective $d_{5/2} - d_{3/2}$ spin-orbit splitting. Nevertheless, the calculation also shows a strong parentage with the rotational character of the ground state band via its large $B(E2)$ transition probability of $22.4e^2$ fm^4 to the 8_2^+ state, a value significantly larger than those for decays towards the 8_1^+ and the 8_3^+ levels (5.2 and $11.3e^2$ fm^4, respectively).

To summarize, the power of quadruple angular correlations combined with a new analysis method has been utilized to identify the first 10^+ state at an energy of 19.2(1) MeV in ^{24}Mg. Evidence for the γ decay of this state has also been reported. These results indicate that the rotational band is yrast in ^{24}Mg at spin $10\hbar$ and is associated with a rotational frequency of $\hbar\omega \approx 2.9$ MeV.

This work was supported by the U.S. Department of Energy, Nuclear Physics Division, under Contracts No. W31-109-ENG-38 and No. DE-FG05-88ER40406, and by the National Science Foundation.

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