Breakdown of K Selection in ^{178}Hf

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Coulomb activation of the four quasiparticle $K^{\pi} = 16^+$ ^{178}Hf isomer ($t_{1/2} = 31$ y) has led to the measurement of a set of $E\lambda$ matrix elements coupling the isomer band to the ground band. The present data combined with earlier ^{178}Hf Coulomb excitation data have probed the K components in the wave functions and revealed the onset and saturation of K mixing in low-K bands, whereas the mixing is negligible in the high-K bands. The implications can be applied to other quadrupole-deformed nuclei.

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Studies of electromagnetic (EM) excitation and deexcitation of high-K isomeric states [1–5] have demonstrated significant violations of the K-selection rule in axially symmetric, quadrupole-deformed nuclei. The purpose of the present Letter is to probe K selection experimentally as a function of spin by investigating the EM matrix elements coupling rotational bands in ^{178}Hf. The results indicate that considerable K mixing occurs at higher spin in the low-K bands.

The K-selection rule [6] does not allow EM transitions between two states $|I_iM_iK_i\rangle$ and $|I_fM_fK_f\rangle$ of an axially symmetric nucleus for which the forbiddenness $\nu \equiv |\Delta K| - \lambda$ is greater than zero, where λ is the multipole order and $\Delta K \equiv K_f - K_i$. The degree of hindrance of a K-forbidden transition can be expressed in terms of the "reduced hindrance" $f_\nu \equiv [B(\mathcal{M}\lambda)_{\mathrm{W.u.}}/B(\mathcal{M}\lambda)]^{1/\nu}$, where $B(\mathcal{M}\lambda)_{\mathrm{W.u.}}$ is the Weisskopf single-particle estimate of the EM reduced transition probability. The EM population of high-K states from the ground state is unlikely, either through highly hindered K-forbidden transitions or through multiple-step transitions of low or zero forbiddenness. For K-forbidden transitions, f_ν is expected to be $\gg 1$.

Predictions of the hindrance values of measured K-isomer decays [7] have been made based on mechanisms including γ-barrier tunneling [1,8] and softness to γ deformation [9,10], and it has been suggested that more than one mechanism may need to be considered [2,11,12]. Allusions to high-K components in the yrast band have been under consideration for decades [2]. K mixing calculations for high-K isomer states, based on density of states considerations, have reproduced some of the observed systematics, without discounting the existence of *some* mixing in the low-K bands [11] and demonstrated the need to consider other effects to account for some f_ν values which are smaller than expected. For instance, γ-barrier tunneling and density of states calculations have both overpredicted and underpredicted the measured reduced hindrance values [12].

An earlier ^{178}Hf(^{136}Xe, ^{136}Xe)^{178}Hf Coulomb excitation experiment [3] populated the $K^{\pi} = 6^+$ and 8^- isomer bands in ^{178}Hf from the ground state band (GSB) and measured a remarkably high $19^+ \rightarrow 18^+$ yield in the known $K^{\pi} = 16^+$ isomer band [Fig. 1 and [3,13]]. A new activation experiment was devised to measure the Coulomb excitation of the 16^+ isomer as a function of collision energy, in order to confirm the 16^+ band population and to extract model-independent $\langle I_{K=16} \| E2 \| I_{\mathrm{GSB}}\rangle$ matrix elements. A stack of five 1 mg/cm^2 natural Ta targets was irradiated at normal incidence by a ≈ 10 pnA ^{178}Hf^{24+} 858 MeV beam from ATLAS, providing an excitation function over a centroid bombarding energy range of 73% (target 5) to 86% (target 1) of the Coulomb barrier E_{Coul}. [Nuclear effects are small at 86% E_{Coul} [14–16] and insignificant for $E_{\mathrm{beam}} \leq 80\% E_{\mathrm{Coul}}$.] Hollow cylindrical 42 mg/cm^2 tantalum "catchers" collected scattered Hf ions over $40° < \theta^{\mathrm{lab}}_{\mathrm{scat}} < 90°$, so that $\ll 1\%$ of the nuclei in the 16^+ state were lost or embedded in downstream targets. Faraday cup data and scattering data from a silicon detector determined the absolute activities.

The activities were counted five months later at Yale University's Wright Nuclear Structure Laboratory. The targets were positioned between two four-crystal "clover" Ge detectors. A target foil and its catcher were positioned between the clovers and counted for times ranging from 16.5 h to 237 h. (Target 2, excited at 83%E_{Coul} was not measured.) Relative γ-ray efficiency data were taken using a ^{152}Eu source. The absolute efficiencies ($\approx$3% at $\approx$400 keV) and the detection probabilities of relevant combinations of γ rays were calculated, including angular

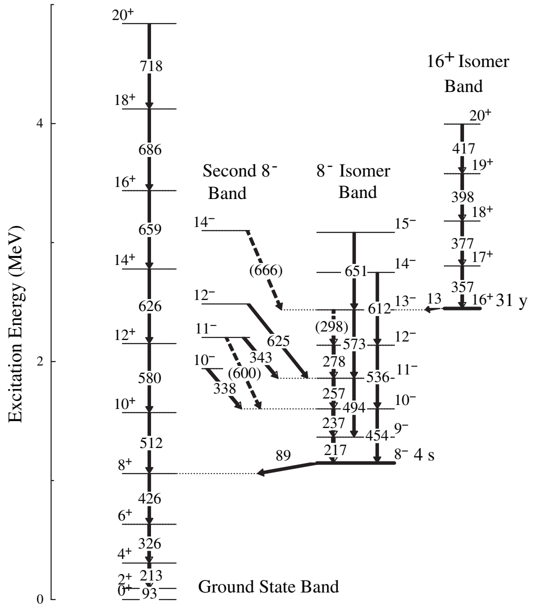

FIG. 1. A partial level diagram for ^{178}Hf from the ^{178}Hf(^{136}Xe, ^{136}Xe)^{178}Hf Coulomb excitation experiment. The converted 13 keV transition from the $K^{\pi} = 16^{+}$ isomer was not observed.

correlation and summing effects. Count rates were obtained from a >1-fold matrix by gating on the 326 keV $6^{+} \rightarrow 4^{+}$ GSB transition and counting the coincident 426 keV $8^{+} \rightarrow 6^{+}$ γ rays in the GSB (Fig. 2).

Four measured 16^{+} isomer activities and three prompt $19^{+}_{K=16}$ yields of the first experiment [3] were combined in an effort to find a single consistent set of GSB → 16^{+} matrix elements. Because only 7 data points were available, attempts were made to reduce the number of fit parameters. The spin-dependent mixing (SDM) model [Eq. (4–95) of [6]] for K-forbidden EM transitions failed to reproduce the measured population of the $K > 6$ bands, because the perturbation breaks down for strong mixing, predicting unrealistic $B(E\lambda)$ values of hundreds of W.u. coupling observed higher spin states. Ultimately, the GSB → $K^{\pi} = 16^{+}$ matrix elements were adjusted individually to reproduce the yields, observing the physical constraints such as the measured upper limits on both the GSB → 16^{+} feeding intensity [$\approx 10^{-4}$ normalized to the $8^{+}_{\mathrm{GSB}} \rightarrow 6^{+}_{\mathrm{GSB}}$ yield in the Hf(Xe,Xe)Hf experiment] and the $B(E2; \mathrm{GSB} \rightarrow K^{\pi} = 16^{+})$ values, etc.

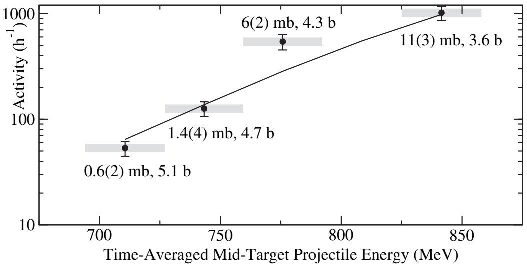

FIG. 2. Measured 16^{+} isomer activity vs bombarding energy (points with $\sigma_{16^{+}}/\sigma_{\mathrm{Rutherford}}$) and reproduced activity (line) from the direct fit of the GSB → $K^{\pi} = 16^{+}$ matrix elements with $\chi^{2} = 3.5$.

There was insufficient sensitivity to determine the matrix elements individually with correlated errors, but a coherent set of matrix elements with upper limits, several lower limits, and diagonal (uncorrelated) errors was found that meets the physical constraints described above. $B(E2; K = 0 \rightarrow K = 16)$ values ranging from 0.04–1.4 W.u. (Fig. 3) simultaneously reproduced the measured activity (Fig. 2) and the $19^{+}_{K=16}$ prompt γ-ray yields from the ^{178}Hf(^{136}Xe, ^{136}Xe)^{178}Hf experiment.

A new analysis of the $K^{\pi} = 8^{-}$ band Coulomb excitation data from the Hf(Xe,Xe)Hf experiment produced a set of matrix elements populating the 8^{-} bands which were <5 W.u., whereas the previous analysis using the SDM model gave some (less effective) matrix elements which were as large as hundreds of W.u., which is unrealistic. The experimental data were reproduced most accurately, and with the lowest $B(E3)$ values, by two-step excitations to both 8^{-} bands through the γ band in conjunction with single-step excitations from the GSB, both using Alaga rule coupling for $K = 5$ admixtures in the low-K bands. Other single K admixtures gave similar results. The $B(E3)$ values coupling the GSB to both 8^{-} bands are shown in Fig. 3. Strong mixing between the two 8^{-} bands and the small set of yields necessitated coupling the intrinsic matrix elements connecting the GSB to both 8^{-} bands as a single intrinsic matrix element, giving

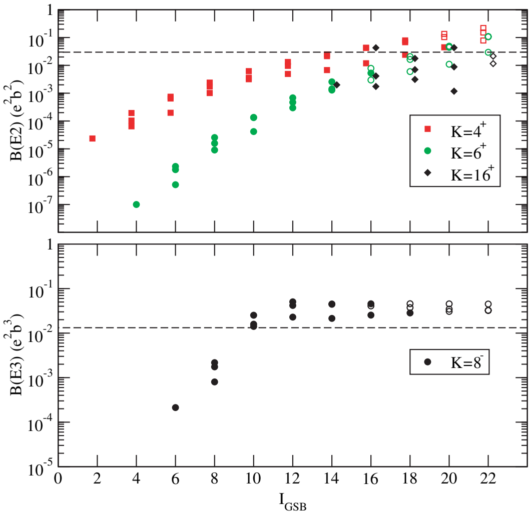

FIG. 3 (color). The three strongest reduced transition probabilities from each GSB level for GSB → K^{π} transitions. GSB → $4^{+}, 6^{+}$ matrix elements follow the SDM model. GSB → 8^{-} matrix elements follow the Alaga rule, attenuated at low spin. Transitions to unobserved high-spin levels (hollow) are extrapolated to clarify the spin dependence of the intrinsic matrix elements in the models used. Weisskopf estimates (dashed lines): $B(E2\uparrow)_{\mathrm{W.u.}} = 0.0297e^{2}$ b^{2}. $B(E3\uparrow)_{\mathrm{W.u.}} = 0.0132e^{2}$ b^{3}.

$\langle 8^- | E3 | \mathrm{GSB} \rangle = 0.37^{+0.07}_{-0.01} e$ b$^{3/2}$. (The errors include correlations.) The fit also yielded the γ-band matrix element $\langle 8^- | E3 | \gamma \rangle = 0.36^{+0.00}_{-0.06} e$ b$^{3/2}$ for both 8^- bands.

It was necessary to attenuate the $\langle 8^- \| E3 \| \mathrm{GSB} \rangle$ and $\langle 8^- \| E3 \| K^\pi = 2^+ \rangle$ matrix elements smoothly with decreasing spin by approximately an order of magnitude per $2\hbar$ for $I_{\mathrm{GSB}} < 10\hbar$ (Fig. 3) in order to keep the isomer cross section from growing unreasonably large and to preserve the 4.0(2) s half life. This resulted in matrix elements <4 W.u. The strength of the $3^-_{K=2} \rightarrow 0^+_{\mathrm{GSB}}$ transition in ^{178}Hf has been measured at 4 W.u. [17], indicating that the maximum values here are reasonable. Since the calculated yields are not extremely sensitive to reduction of the few matrix elements with $B(E3) \approx 4$ W.u., additional measurements might show that the largest matrix elements are actually smaller.

In the Xe beam experiment, it can be argued that the isomer bands could be populated through transfer reactions involving the ^{177,179}Hf contaminants in the target (4% and 3%, respectively). An upper limit on ^{178}Hf(^{136}Xe, ^{135}Xe)^{179}Hf transfer reactions was set using the only observed possible ^{135}Xe transition (288 keV) in coincidence with a double gate on ^{178}Hf GSB transitions. In the safe Coulomb excitation region, $25° < \theta_{\mathrm{scat}} < 52°$, where significant populations of the $K^\pi = 6^+_{\mathrm{isom}}, 8^-_{\mathrm{isom}}$ bands are already seen, an upper limit on ^{177}Hf(^{136}Xe, ^{135}Xe)^{178}Hf transfer was set at 10^{-5} of the ^{178}Hf GSB excitation. Assuming that the cross sections for ^{177}Hf(^{136}Xe, ^{135}Xe)^{178}Hf ($Q = -0.4$ MeV) and ^{178}Hf(^{136}Xe, ^{135}Xe)^{179}Hf ($Q = -1.9$ MeV) are similar, the upper limit on ^{177}Hf(^{136}Xe, ^{135}Xe)^{178}Hf reactions in the 4% ^{177}Hf impurity is $\sim \frac{1}{10}$ of the observed 16^+ isomer band yield in the unsafe region $52° < \theta_{\mathrm{scat}} < 78°$. Moreover, transfer must be divided among several bands, and transfer to a 4 quasiparticle state (e.g., the 16^+_{isom} band) is very unlikely, since breaking a pair of nucleons is a higher-order effect. In the ^{178}Hf beam experiment 16^+ isomer activation was observable at 73% E_{Coul}, consistent with the Coulomb excitation function (Fig. 2).

The systematic decrease with increasing spin of the hindrance of K-forbidden transitions is apparent from Fig. 3 and Table I. For each of the high-K isomer bands observed, reproduction of the measured yields requires that the interband $B(E\lambda)$ values increase with increasing spin and saturate at ≈ 1 W.u. for $I \gtrsim 12$ in the GSB and the γ band. This saturation point represents the maximum mixing of K. For $I \geq 12$, reduced hindrance values of K-forbidden transitions from low-K to high-K bands are as low as $f_\nu \sim 1$, showing that the K-selection rule has little predictive power at high spin, i.e., highly K-forbidden transitions have similar strength to allowed interband transitions. A notable exception is the unobserved $12^+_{\mathrm{GSB}} \xrightarrow{E4} 16^+_{K=16}$ excitation whose hindrance (Table I) suggests that the $K = 12$ admixture in the GSB is insignificant for $I < 14$. The $8^+_{\mathrm{GSB}} \xrightarrow{E3} 8^-_{K=8}$ ($f_\nu > 9$) and $6^+_{\mathrm{GSB}} \xrightarrow{E3} 8^-_{K=8}$ ($f_\nu > 70$) transitions [20] suggest that the Alaga rule may not describe well all of the K-forbidden couplings.

TABLE I. Values of f_ν given in the direction $I_i \rightarrow I_f$ for selected K-forbidden transitions in ^{178}Hf. Weisskopf estimates $B(\mathcal{M}\lambda \downarrow)_{\mathrm{W.u.}}$ are $0.020e^2$ b (E1), $6.0 \times 10^{-3} e^2$ b^2 (E2), $2.0 \times 10^{-3} e^2$ b^3 (E3), $2.2 \times 10^{-4} e^2$ b^5 (E5), $1.8\mu_N^2$ (M1), $0.52\mu_N^2$ b (M2), $0.055\mu_N^2$ b^3 (M4). Defined $B(\mathcal{M}\lambda \uparrow)_{\mathrm{W.u.}} \equiv (2\lambda + 1) B(\mathcal{M}\lambda \downarrow)_{\mathrm{W.u.}}$.

Bands	I_i	I_f	$\mathcal{M}\lambda$	ν	f_ν
GSB → $K^\pi = 4^+$	2	4	E2	2	35
	6	8	E2	2	12
	12	14	E2	2	18
GSB → $K^\pi = 6^+$	4	6	E2	4	24
	8	10	E2	4	8
	12	14	E2	4	5
GSB → $K^\pi = 8^-$	8	8	E1	7	[a]67(1)
	6	8	M2	6	[a]>130
	8	11	E3	5	1.5
	10	13	E3	5	1.0
	12	15	E3	5	0.9
GSB → $K^\pi = 16^+$	12	16	E4	12	[a]>9
	14	16	E2	14	1.2
	16	18	E2	14	1.0
	18	20	E2	14	1.0
	20	21	E2	14	1.0
$K^\pi = 2^+ \rightarrow K^\pi = 6^+$	4	6	E2	2	14
	6	8	E2	2	6
	8	10	E2	2	3.9
	10	12	E2	2	2.7
$K^\pi = 2^+ \rightarrow K^\pi = 8^-$	5	8	E3	3	6
	7	10	E3	3	1.0
	9	12	E3	3	0.7
	11	14	E3	3	0.8
$K^\pi = 16^+ \rightarrow K^\pi = 8^-$	16	11	E5	3	[b]165(5)
(Isomer decays)	16	12	M4	4	[b]72(2)
	16	13	E3	5	[b]66(1)
$K^\pi = 14^- \rightarrow K^\pi = 8^-$	14	13	M1	5	[c]90
(Isomer decays)	14	12	E2	4	[c]33

[a] Calculated from Reference [18].
[b] Reference [18].
[c] Reference [19].

Band interactions are reflected in the measured moments of inertia by an increase in slope of the moment of inertia $I(\omega)$, seen at $I \approx 6$ and $I \approx 10$ in the γ and GS bands, respectively (Fig. 4), while the $B(E\lambda)$ values saturate at $\sim$1 W.u. as low as $I \approx 8$ and $I \approx 10$ for transitions from the γ band and the GSB, respectively (Fig. 3), in order to reproduce the measured γ ray yields in the $K^\pi = 4^+, 6^+, 8^-$, and 16^+ bands. Moreover, Coriolis alignment is expected to happen at much lower spin in low-K bands than in high-K bands [21], which are strongly deformation

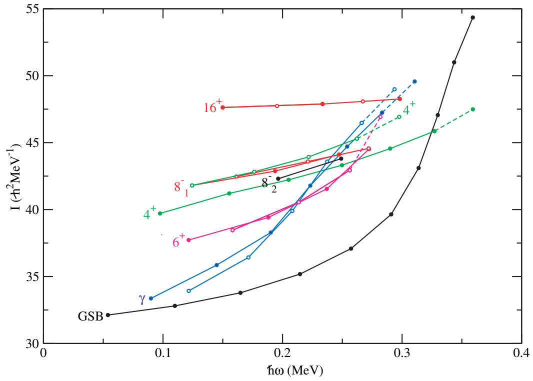

FIG. 4 (color). Measured moments of inertia of ^{178}Hf rotational bands. Dashed lines connect to tentatively assigned levels. The first transition in each trace is the $I_i = K + 2 \rightarrow I_f = K$ transition. The 16^+ band data are from Mullins *et al.* [13].

coupled. The moments of inertia of the high-K bands are relatively constant in slope, with the exception of the 6^+ band at $I \approx 12$, suggesting that the high-K bands are not K mixed to the same degree as the low-K bands. The 16^+ band has a remarkably constant moment of inertia [13] up to $I = 22$. In contrast with the transitions from the $K = 0, 2$ bands to the high-K isomer bands, the $16^+_{\text{isom}} \rightarrow K^\pi = 8^-$ and $14^-_{\text{isom}} \rightarrow K^\pi = 8^-$ γ decays are strongly hindered with $33 \leq f_\nu \leq 165(5)$ in all of the five known branches, showing that the onset of significant high-K admixtures in the 8^- band must occur at $I > 13$, if at all, whereas less hindered $f_\nu \sim 1$ transitions from the γ and GS bands are required to reproduce the present measured yields. That is, the strongly hindered decays of the 16^+ and 14^- isomers to the $11^- \leq I^\pi_{K=8} \leq 13^-$ states are consistent with K being a good quantum number for the high-K bands, suggesting that mixing in the low-K bands is primarily responsible for the K-selection violations and that the EM matrix elements coupling to the high-K bands are sensitive probes of the K distributions in the low-K bands. Coulomb excitation of a band with projection K, assuming that it is reasonably pure, would require admixtures K' in the low-K (nominally K_i) bands of $K - \lambda \leq K' \leq K + \lambda$. Hence, the mixing fractions of the $2 \leq K' \leq 6$ components are depicted in Fig. 3 as a function of spin by the $B(E2; K_i \rightarrow K = 4)$ values, the $4 \leq K' \leq 8$ components by the $B(E2; K_i \rightarrow K = 6)$ values, etc.

This first qualitative measurement of the K distribution with respect to nuclear spin in low-K bands has revealed the rapid breakdown of the goodness of the K quantum number as the low-K bands are excited to higher rotational frequencies. The rapid increase in the interband $E\lambda$ matrix elements coincides with the rotational alignment of low-K bands which has a noticeable effect on the moment of inertia above the $I \approx 10$ levels of the γ band and the GSB. Higher-K components are admixed in the nominally low-K bands with increasing spin, until the reduced transition probabilities saturate for $I \gtrsim 12\hbar$ near $\sim$1 W.u. signifying the total breakdown of the K quantum number. The present work, initially focused on explaining the K-forbidden Coulomb excitation of the ^{178}Hf isomers, has reached a conclusion of broader significance: for the first time, the loss of purity of K with increasing angular momentum in nuclear states has been revealed. The K mixing is largest in low-K bands, while the high-K bands remain very pure, even at the same spin (I) levels where the low-K bands are completely mixed.

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