Nuclear shapes of highly deformed bands in ^{171,172}Hf and neighboring Hf isotopes

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A Gammasphere experiment was carried out to search for triaxial strongly deformed (TSD) structures in ^{171,172}Hf and the wobbling mode, a unique signature of nuclei with stable triaxiality. Three strongly deformed bands in ^{172}Hf and one in ^{171}Hf were identified through ^{48}Ca(^{128}Te, xn) reactions. Linking transitions were established for the band in ^{171}Hf and, consequently, its excitation energies and spins (up to $111/2\hbar$) were firmly established. However, none of the ^{172}Hf sequences were linked to known structures. Experimental evidence of triaxiality was not observed in these bands. The new bands are compared with other known strongly deformed bands in neighboring Hf isotopes. Theoretical investigations within various models have been performed. Cranking calculations with the Ultimate Cranker code suggest that the band in ^{171}Hf and two previously proposed TSD candidates in ^{170}Hf and ^{175}Hf are built on proton ($i_{13/2}h_{9/2}$) configurations, associated with near-prolate shapes and deformations enhanced with respect to the normal deformed bands. Cranked relativistic mean-field calculations suggest that band 2 in ^{175}Hf has most likely a near-prolate superdeformed shape involving the $\pi i_{13/2} \otimes \nu j_{15/2}$ high-j intruder orbitals. It is quite likely that the bands in ^{172}Hf are similar in character to this band.

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I. INTRODUCTION

An important advancement in nuclear structure research is the observation of wobbling motion, a unique signature of nuclei with stable triaxiality. This collective excitation mode was predicted about 30 years ago [1], but only recently was established in experiments in the nuclei ^{163,165,167}Lu [2–5], and possibly in $^{161}_{71}$Lu [6], forming an island of triaxial strongly deformed (TSD) structures. Systematic cranking calculations [7] using the ULTIMATE CRANKER (UC) code [8] predicted high-spin TSD minima with $(\varepsilon_2, \gamma) \sim (0.40, \pm 20^\circ)$ in potential energy surfaces (PES) for nuclei with $Z \sim 72$ and $N \sim 94$, where they coexist with normal deformed (ND) structures $(\varepsilon_2, \gamma) \sim (0.25, 0^\circ)$. These TSD minima are caused by large single-particle shell gaps associated with proton numbers $Z = 71$ and 72 and neutron numbers $N = 94$ and 97 [9,10]. Further theoretical calculations based on the particle-rotor model [11,12] and cranked-shell model plus random-phase approximation [13] pointed out the essential role of the rotation aligned $i_{13/2}$ quasiproton that allows wobbling to compete in energy with quasiparticle excitations in these Lu nuclei. However, it is not understood why the "wobbling phonon bands" are not seen in odd-odd $^{162}_{71}$Lu$_{91}$ [14] and $^{164}_{71}$Lu$_{93}$ [15,16] where multiple TSD bands were observed. It is also unclear why the difference in excitation energy of the zero-phonon ($n = 0$) and one-phonon ($n = 1$) bands decreases with increasing spin, which implies a decreasing wobbling frequency.

More questions are raised from the proposed TSD bands in Hf nuclei. The wobbling mode was first predicted for even-even nuclei [1]. Early searches for this mode and the associated TSD structures concentrated on even-A Hf isotopes around $^{166}_{72}$Hf$_{94}$ [17], the center of the predicted TSD island [7]. After the first identification of TSD bands in ^{168}Hf [18], a number of strongly deformed bands were reported in ^{170}Hf [19], ^{173,174}Hf [20,21], and ^{175}Hf [22]. A majority of these bands was observed in the heavier Hf nuclei, e.g., eight in ^{174}Hf$_{102}$, which is far away from the predicted neutron shell gaps. In addition, the bands in $^{173-175}$Hf are more strongly populated than the bands in the lighter isotope ^{168}Hf$_{96}$. Furthermore, the measured transition quadrupole moments, $Q_t \approx 13$–14.5 eb, for the bands in $^{173-175}$Hf [21,22] are significantly larger than the values predicted by UC calculations for the TSD structures, e.g., $Q_t \approx 9.9$ eb in ^{174}Hf. The larger Q_t value implies a superdeformed (SD) nuclear shape or a smaller triaxiality than that of the predicted TSD structures, or a combination of both possibilities, because the Q_t moment is proportional to $\cos(30^\circ + \gamma)$ [8]. A convincing theoretical description, consistent with the experimentally measured transition quadrupole moments, is not available for any of proposed TSD/SD structures in the heavier Hf isotopes, the failure to observe wobbling excitations in Hf isotopes is considered

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a less severe discrepancy (see more detailed discussion in Sec. III D). These facts cast doubt on the interpretation that some of the observed strongly deformed bands in the Hf nuclei are built on triaxial shapes. Several high-j intruder orbitals play crucial roles in the UC calculations for Hf nuclei, including $\pi i_{13/2}$, $\nu j_{15/2}$, and $\nu i_{11/2}$ [10], but their energy locations are poorly known at large triaxiality. However, only a single aligned $i_{13/2}$ proton is involved in the UC calculations for the TSD structures in Lu nuclei around ^{163}Lu, where the $\nu j_{15/2}$ and $\nu i_{11/2}$ orbitals are located farther above the Fermi surface. The observed TSD structures in the Lu isotopes generally agree with the predicted TSD minima obtained in the UC calculations, although the measured Q_t values are generally smaller than the calculated ones.

It is also necessary to recognize that when seeking theoretical interpretations of the TSD/SD bands of quasiparticle nature in these Hf isotopes, it is insufficient to calculate only the PES and analyze the quasiparticle Routhian diagrams for the equilibrium deformation of the TSD/SD minimum in these surfaces. Such a process could be misleading considering the fact that the UC results do not properly reproduce the observed transition quadrupole moments [21,22]. The rotational properties of the observed bands (experimental alignments, moments of inertia, excitation energies as a function of spin, etc.) need to be investigated as well because they usually help to constrain theoretical interpretations. This has been done only in a few cases for neighboring nuclei, namely for the wobbling and TSD bands in ^{163}Lu [23,24] and ^{167}Lu [13] and for TSD bands in ^{163}Tm [25,26]. Only in Refs. [23–26] were the deformation and rotational properties of the TSD bands defined simultaneously in a way free from the parameters adjustable to the experimental data. More generally, the wobbling excitation and TSD structures in the Lu-Hf region require more extensive theoretical investigation. The situation is clear if the current theoretical studies in this region are compared with those in other regions of superdeformation, such as the $A \sim 60$, 130, 150, and 190 mass regions (see Refs. [27–33] and references therein). Considerable theoretical efforts were devoted to the understanding of the deformation and the rotational and single-particle properties of the observed SD bands in those regions, which include both macroscopic + microscopic [Cranked Nilsson-Strutinsky (CNS) and total Routhian surface (TRS) approaches] and microscopic (relativistic mean-field theory, Hartree-Fock approaches based on the Skyrme or Gogny forces) methods.

We carried out an experimental study for ^{171,172}Hf, two isotopes located between the lighter ^{168,170}Hf and heavier $^{173-175}$Hf nuclei. The motivation was to search for TSD structures and wobbling bands, to clarify the nature of these proposed TSD/SD bands in Hf nuclei, and to learn about their intrinsic configurations. Here, we report on the identification of three strongly deformed bands in ^{172}Hf and a single one in ^{171}Hf. The latter band was linked to known ND levels, yielding firm spin and parity assignments up to $111/2^+$. However, the decay pathways of the bands in ^{172}Hf could not be established. These ^{172}Hf bands have similar dynamic moments of inertia, $J^{(2)}$, but they do not resemble a family of wobbling bands. The new bands were compared with other highly deformed bands in $^{170-175}$Hf isotopes. UC and cranked relativistic mean-field (CRMF) calculations were performed with the goal to see whether the study of rotational properties can provide additional insight in the nature of these bands. Our study shows that the highly deformed bands in the Hf isotopes fall in two groups. The bands in the first group, including the new band in ^{171}Hf, start from spins as low as $I \sim 15$–$20\hbar$. A detailed comparison of the experimental properties of the bands in this group and the UC calculations suggests that these bands are associated with a prolate minimum with $Q_t \sim 8.5$ eb, which is an enhanced deformation (ED) with respect to the ND bands. The results are contrary to previous suggestions that band 1 in ^{170}Hf [19] and band 1 in ^{175}Hf [22] may be candidates for TSD structures. The calculations suggest that the proton $i_{13/2}h_{9/2}$ configuration is largely responsible for the enhanced deformation. The bands in the second group are located at higher spins than those in the first group. Our CRMF calculations suggest that band 2 in ^{175}Hf [22], and likely the similar bands in $^{172-174}$Hf, are associated with superdeformed prolate shapes ($Q_t \sim 11.6$ eb) with little triaxiality. The intrinsic configurations of these bands involve the $\pi i_{13/2}$, as well as the $\nu j_{15/2}$ orbitals originating above the $N = 126$ spherical shell closure. We cannot rule out the possibility that some of the bands in this group may be associated with triaxial shapes, and this will require further experimental and theoretical investigations.

The manuscript consists mainly of two parts: new experimental discoveries and theoretical discussions where the ED bands in ^{170,171,175}Hf are compared with UC calculations and the SD band in ^{175}Hf with CRMF calculations. Finally, the SD bands in ^{172}Hf are discussed separately.

II. EXPERIMENTAL PROCEDURE AND RESULTS

High-spin states in the ^{171,172}Hf nuclei were populated through the ^{128}Te(^{48}Ca,xn) reactions using the ATLAS facility at Argonne National Laboratory. With a beam energy of 209 MeV, the dominant evaporation residues ^{172}Hf and ^{171}Hf were produced in an approximate ratio of 1.8:1. The target consisted of $\sim$0.5 mg/cm^2 isotopically enriched ^{128}Te with 0.5 mg/cm^2 of Au in front of the Te and 50 μg/cm^2 Au on the back. A beam wobbling device and a target wheel were used to help with heat dissipation in the target, thus allowing a larger beam current ($\sim$2 pnA) to be deposited. Coincident γ rays were measured using the Gammasphere array [34], which consisted of 100 Compton-suppressed Ge detectors at the time of the experiment. A data set of approximately 2.1×10^9 three- and higher-fold coincidence events was collected. In the off-line analysis, the data were sorted into a database where the γ-ray energies and detector identification were stored for each event. The Radware software package [35] was used to construct three-dimensional (cube) and four-dimensional (hypercube) histograms and to analyze the γ-ray coincidence relationships. In addition, an analysis of directional correlation from oriented states (DCO ratios) [36] was performed on the data set to determine the multipolarity of the γ rays. Gated DCO matrices, with detectors at 31°, 37°, 143°, 148°, and 163° along the x axis, and detectors from 58° through 122° along the y axis, were constructed from the database for this purpose. The technique was calibrated with transitions of

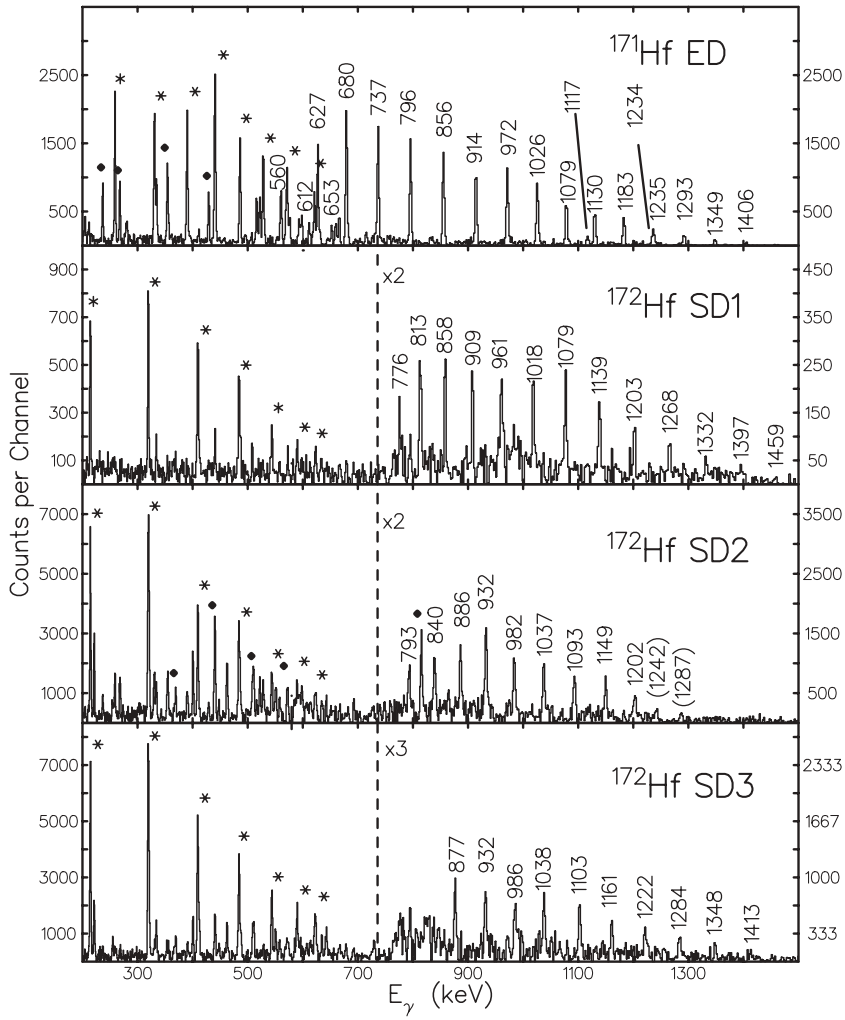

FIG. 1. Triple-gated (upper two) and double-gated (lower two) γ-ray coincidence spectra for the strongly deformed bands in ^{171,172}Hf observed in this work. Band members are labeled by energies, as are several decay-out transitions from the band in ^{171}Hf. For ^{171}Hf stars and filled diamonds denote the known transitions in the normal deformed 1/2[521] and 7/2[633] bands [38], and for ^{172}Hf these symbols relate to the ground (G) and AF bands [37], respectively.

known multipolarity. The extracted DCO ratios from E2-gated spectra fall into two distinct groups centered around 1.0 and 0.6 for stretched quadrupole and dipole transitions, respectively.

Figure 1 presents spectra of the strongly deformed band in ^{171}Hf, labeled as band ED, and in ^{172}Hf, labeled as SD1-SD3. Band ED in ^{171}Hf has an intensity of 1.4(1)% relative to the total population in this reaction channel. The intensities of SD1-SD3 in ^{172}Hf are 0.7(2)%, 0.5(1)%, and 0.4(1)%, respectively, relative to the total intensity feeding the ground state. The coincidence relationships between each of the new bands in ^{172}Hf and the known transitions up to spin 18^+–20^+ in the ND yrast band [37] are clearly seen and firmly establish that these bands belong to ^{172}Hf. However, detailed decay pathways could not be established. DCO ratios were measured for all transitions in each band, except for those very weak transitions at the highest spins, and the results were consistent with expectations for an $E2$ cascade. It can be seen from the intensity profile of band SD1 that the decay out occurs in the lowest two levels, whereas in band SD2 the lowest four levels are involved in the process. Therefore, multiple decay pathways exist for each band and, consequently, the intensities of depopulating transitions are fragmented. For band SD3 the low-spin transitions below 877 keV could not be firmly identified.

A partial level scheme of ^{171}Hf is presented in Fig. 2 to show the decay pathways of band ED into the previously known ND band [38]. The DCO ratios of in-band transitions, measured up to spin $91/2\hbar$, are consistent with a stretched $E2$ character. Band ED decays to the $1/2^-[521]$ sequence through several, one-step, high-energy dipole transitions and to the $7/2^+[633]$ band (not shown in the figure) through more complex multistep pathways. The DCO ratios of the 1117- and 1235-keV linking transitions were extracted from DCO spectra gated on $E2$ transitions in the $1/2^-[521]$ band. The values are 0.51(4) and 0.53(6), respectively, consistent with stretched dipole transitions. There is a small probability that the two γ rays could correspond to $\Delta I = 0$, $M1$ transitions with strong $E2$ admixtures, because this would result in similar DCO values. In this case, however, stretched $E2$ transitions depopulating each level would be more competitive because of the energy factor. Such stretched $E2$ transitions are not observed. In addition, this scenario would make the excitation energy of band ED too high above other ND bands to account for its strong population. Therefore, such a possibility can be ruled out. Furthermore, we assign positive parity to band ED because the linking transitions probably have an $E1$ multipolarity. An $M1$ transition of such high energy would be expected to exhibit an $E2$ admixture, resulting in a larger

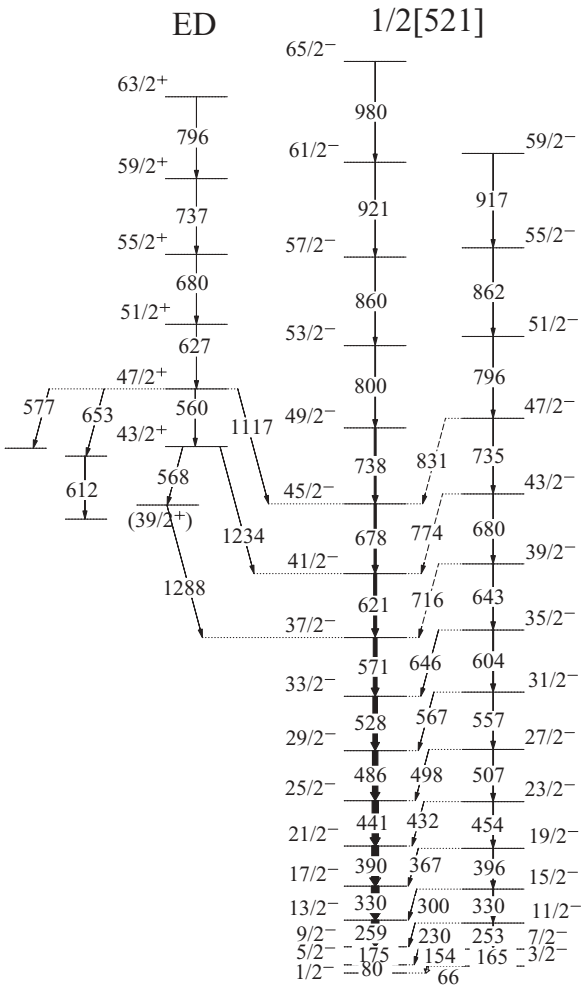

FIG. 2. Partial level scheme of ^{171}Hf showing the decay pathways out of the new ED band.

DCO ratio. As a result, band ED has a parity and signature $(\pi, \alpha) = (+, -1/2)$. The linking transitions from band ED to the $7/2^+[633]$ cascade include 653.4, 612.4, 577.0, and 568.4 keV γ rays. The positions of the first three of these transitions shown in the level scheme should be regarded as tentative because the full decay pathways could not be established. The 568.4-keV γ ray decays to a level tentatively suggested as $39/2^+$, feeding the $7/2^+[633]$ band as well as the $1/2^-[521]$ band through the 1288.2-keV transition. Unfortunately, no reliable DCO ratios could be extracted for these two weak γ rays.

III. DISCUSSION

A. Superdeformed (SD) bands and bands with enhanced deformation (ED)

The large deformation of the strongly deformed bands in ^{171}Hf and ^{172}Hf has been inferred from the fact that their dynamic moments of inertia, $J^{(2)}$, are larger than those of the ND bands ($\sim 60\hbar^2$/MeV) in each nucleus, and are similar to those of known strongly deformed bands in neighboring Hf isotopes where quadrupole moments have been measured.

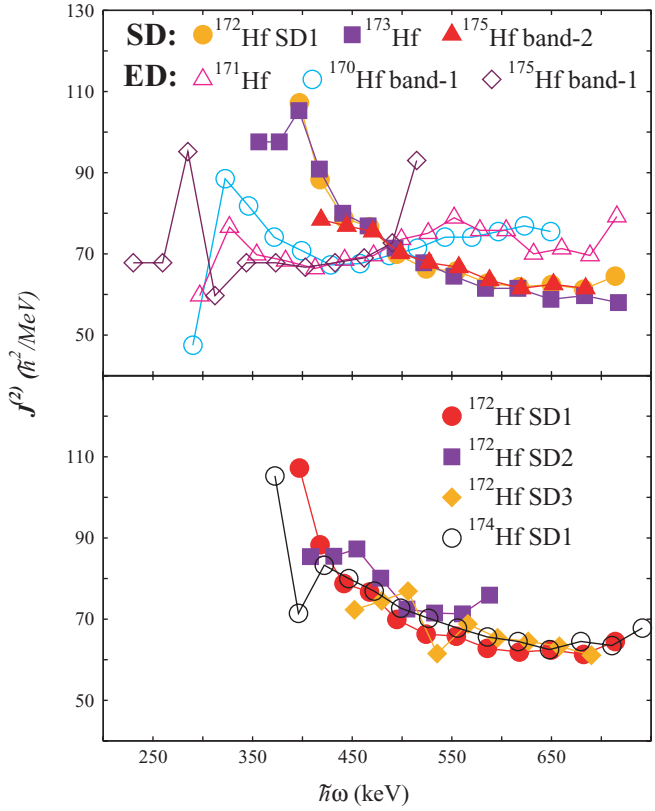

FIG. 3. (Color online) Dynamic moments of inertia $J^{(2)}$ as a function of rotational frequency for the new bands in ^{171,172}Hf compared to strongly deformed bands in other Hf isotopes.

As demonstrated in Fig. 3, the $J^{(2)}$ values of these bands fall into two distinct groups: they decrease with rotational frequency in one group, while they increase slightly in the other. When defining these two groups of bands, we excluded from consideration the low-spin region of bands where the $J^{(2)}$ values are affected by either paired band crossings and/or by interactions with ND bands. For bands in the first group, the measured Q_t values are $\sim$14.5 eb in ^{173}Hf and 12.6–13.8 eb in ^{174}Hf [21]. A preliminary value of $\sim$13 eb has also been reported for band 2 in ^{175}Hf [22], which is clearly larger than the estimated $\sim$7 eb for the ND states [39]. For the convenience of the discussion below, bands in this group are labeled as SD bands. Among them, only band 2 in ^{175}Hf is linked to known levels and, as a result, the spin and parity quantum numbers in all other bands are unknown. All of these bands start from $\hbar\omega \gtrsim 0.35$ MeV. Inspection of the $J^{(2)}$ moments of bands SD1–SD3 in ^{172}Hf indicates that these sequences fall in this SD group.

The second group includes the new ED band in ^{171}Hf, band 1 in ^{170}Hf [19] and band 1 in ^{175}Hf [22]. Notably, all of them are linked to known ND levels. They start at lower rotational frequencies, and correspondingly at lower spins, as compared to band 2 in ^{175}Hf (and most likely also to the majority of bands in the SD group). A tentative value of the quadrupole moment $Q_t \sim 9$ eb is measured for band 1 in ^{175}Hf [22], indicating an enhanced deformation; i.e., larger than that of ND bands in this nucleus, and similar to those of TSD bands in Lu nuclei [40],

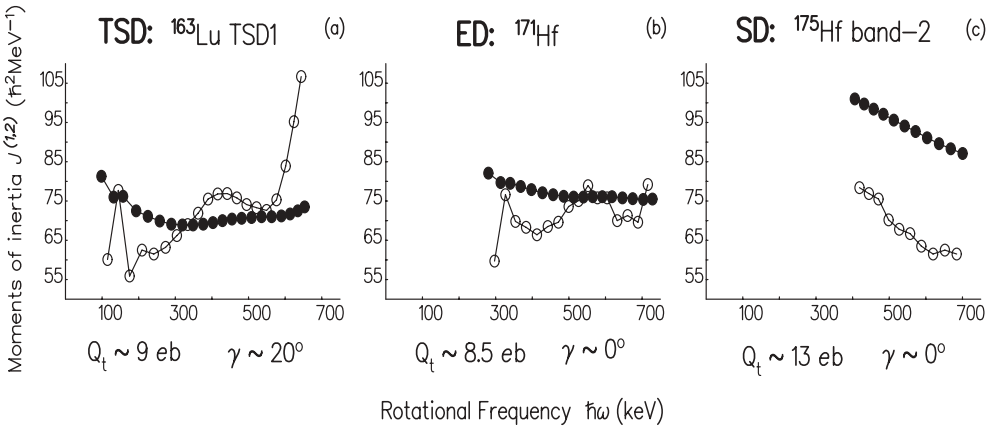

FIG. 4. Measured kinematic ($J^{(1)}$, filled circles) and dynamic ($J^{(2)}$, open circles) moments of inertia of (a) the TSD band in ^{163}Lu [2], (b) the band ED in ^{171}Hf, and (c) the band 2 in ^{175}Hf [22]. The quoted values of quadrupole moment Q_t and triaxiality parameter γ for ^{171}Hf are calculated with the UC code, and the γ value for ^{175}Hf is from our CRMF calculations. See text for a detailed discussion.

but less than that of the SD bands seen in $^{173-175}$Hf. Based on the similar properties of the bands in this group, including the aligned angular momentum to be discussed later, it is plausible that these bands may be associated with similar deformations. The bands in this group are labeled as ED bands.

Figure 4 compares the kinematic, $J^{(1)}$, and the dynamic, $J^{(2)}$, moments of inertia of linked ED and SD bands observed in ^{171,175}Hf. The fact that the $J^{(1)}$ values of band 2 in ^{175}Hf are considerably larger than the $J^{(2)}$ values and that both of them decrease smoothly with increasing rotational frequency clearly indicates that the pairing interaction is negligible in this band. This is a typical feature of rotational bands in a regime of weak pairing [41,42]. Based on the observed features of the dynamic moments of inertia, the same is also expected in the other SD bands. The bands showing this type of relation between the $J^{(1)}$ and $J^{(2)}$ moments are usually very well described in the cranked formalism without pairing, as was illustrated by the study of smooth terminating and superdeformed bands in different mass regions (see, e.g., Refs. [41,42] and references therein). The relative properties of the $J^{(1)}$ and $J^{(2)}$ moments of band ED in ^{171}Hf (Fig. 4) suggest that pairing may be stronger in this band as compared to the SD bands. The same conclusion can be applied to the other ED bands. The increases in $J^{(2)}$ moments seen at $\hbar\omega \sim 0.5$ MeV and at $\hbar\omega \sim 0.7$ MeV may be caused by a paired band crossing. An alternative explanation for these jumps in $J^{(2)}$ values may involve an unpaired interaction with unobserved bands. The available theoretical calculations do not allow to distinguish between these two possibilities. The observed spin ranges, the properties of the moments of inertia, the difference in Q_t moments, and other features suggest that the SD bands in Hf nuclei are built on particle-hole excitations on the top of the ED bands. Such particle-hole excitations lead to additional quenching of pairing and, thus, to the observed features of the moments of inertia of SD bands. Wobbling bands in the Lu isotopes display features similar to the group of ED bands. For ^{163}Lu, the $J^{(2)}$ moment of the TSD bands exhibits a bump in the frequency range $\hbar\omega \approx 0.35$–0.5 MeV caused by a gradual alignment of the first pair of $i_{13/2}$ neutrons [23]. Such a bump is not seen in any Hf band. The UC predicted $i_{13/2}$ neutron crossing frequency associated with ED band in ^{171}Hf is greater than 0.55 MeV.

B. Band ED in ^{171}Hf

At the lowest level, band ED is ~900 keV higher than the favored signatures of the bands built on the $1/2^-[521]$ and $7/2^+[633]$ configurations. Band ED becomes yrast at $I = 67/2\hbar$ and crosses the unfavored signature partner of the $7/2^+[633]$ band around $I = 55/2\hbar$. The two $55/2^+$ states are separated by 30 keV. The strength of a possible mixing between the bands therefore is less than 15 keV and most likely much smaller because no irregularity is observed in the dynamic moments of inertia of band ED at the appropriate rotational frequency. Moreover, no cross-talk between the two bands is observed. This could indicate that band ED has a very different intrinsic structure than the ND bands and that band ED is most likely built on a well-separated potential energy minimum, with a deformation that differs from that associated with the ND minimum.

The aligned angular momentum, i_x, of band ED in ^{171}Hf is compared in Fig. 5 with the other two ED bands in ^{170,175}Hf, several ND bands in these nuclei, and the SD band 2 in ^{175}Hf. Other SD bands in the Hf nuclei are not shown because their level spins are not established. All ND bands are affected by the first $i_{13/2}$ neutron band crossing, at a frequency of ~0.3 MeV, where they gain about $7\hbar$ in aligned angular momentum. Above this crossing, the $1/2^-[521]$ bands change to three-quasiparticle configurations. The ED band in ^{171}Hf exhibits a large initial alignment, $13.8\hbar$, at low frequencies. Such a feature is typical for structures with aligned high-j quasiparticles. The alignment is higher by $5.2\hbar$ than the one of the three-quasiparticle $1/2^-[521]$ band (with a pair of aligned $i_{13/2}$ neutrons). It is also worth noting that the TSD bands in ^{163,165,167}Lu have initial alignments of about $6\hbar$, involving only a single aligned $i_{13/2}$ proton [11]. The other two ED bands, band 1 in ^{170}Hf and band 1 in ^{175}Hf, behave similarly; each has an aligned angular momentum ~$5\hbar$ higher than the

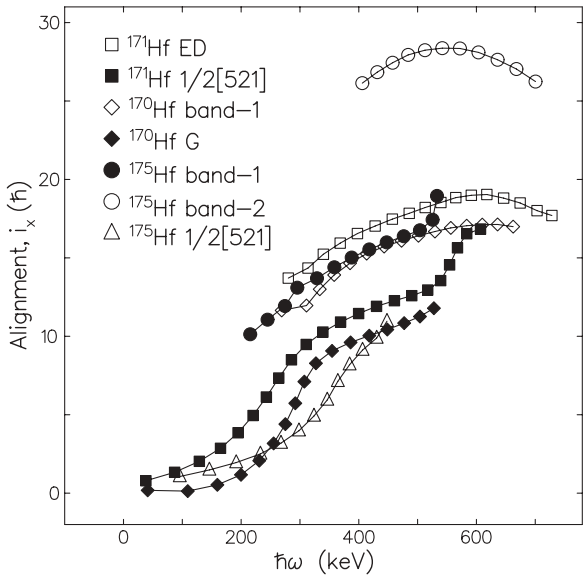

FIG. 5. Aligned angular momenta as a function of rotational frequency for the ED bands in ^{170,171,175}Hf and the SD band in ^{175}Hf. Identical Harris parameters, $I_0 = 30\hbar^2$/MeV and $I_1 = 40\hbar^4$/MeV^3, were used for all bands.

ground-state band and the $1/2^-[521]$ band, respectively. The first proton alignment observed around $\hbar\omega \sim 0.55$ MeV in the ND bands of neighboring nuclei, e.g., ^{169}Hf [43], is clearly missing in the ED bands.

To understand the intrinsic configuration of band ED in ^{171}Hf, we performed cranking calculations using the UC code. Pairing is taken into account in the code, and the standard parameters [44] were used for the Nilsson potential. The total energy at each spin is minimized in the $(\varepsilon_2, \gamma, \varepsilon_4)$ deformation plane. Figure 6 shows a representative potential energy surface for $(\pi, \alpha) = (+, -1/2)$ at $I = 59/2\hbar$ in ^{171}Hf. The minimum at $(\varepsilon_2, \gamma) \sim (0.3, 4^\circ)$ persists from $I \sim 20\hbar$ to the highest spins. As seen in the lower panel of the figure, the UC calculated excitation energies minus a rigid-rotor reference for such a band fit quite well those of band ED. The $\pi(i_{13/2}h_{9/2}) \otimes \nu(h_{9/2})$ configuration is suggested for band ED by these calculations. The calculated initial aligned angular momenta are 6.4, 3.2, and $1\hbar$ for the $\pi i_{13/2}$, $\pi h_{9/2}$, and $\nu h_{9/2}$ orbitals, respectively, with a total alignment of $10.6\hbar$. This amount is comparable to the $13.8\hbar$ initial alignment of band ED measured in Fig. 5. An accurate value of the alignment for band ED is difficult to extract from the plots because a set of common Harris parameters was used for the ND and the ED bands while they are likely associated with different deformations.

Previously, two other ED bands, band 1 in ^{170}Hf [19] and band 1 in ^{175}Hf [22], were proposed as candidates for TSD bands. A configuration of $\pi(i_{13/2})^2 \otimes \nu(i_{13/2})^2(h_{9/2})$ was suggested for the band in ^{175}Hf. Because all three ED bands have very similar properties in terms of the quantities i_x, $J^{(1)}$, and $J^{(2)}$ it is reasonable to expect that they be associated with the same high-j intruder orbitals. Indeed, our UC calculations for ^{170}Hf and ^{175}Hf yielded similar PES minima, $(\varepsilon_2, \gamma) \sim (0.3, 0^\circ)$, as seen in Fig. 6 for ^{171}Hf, and the calculated

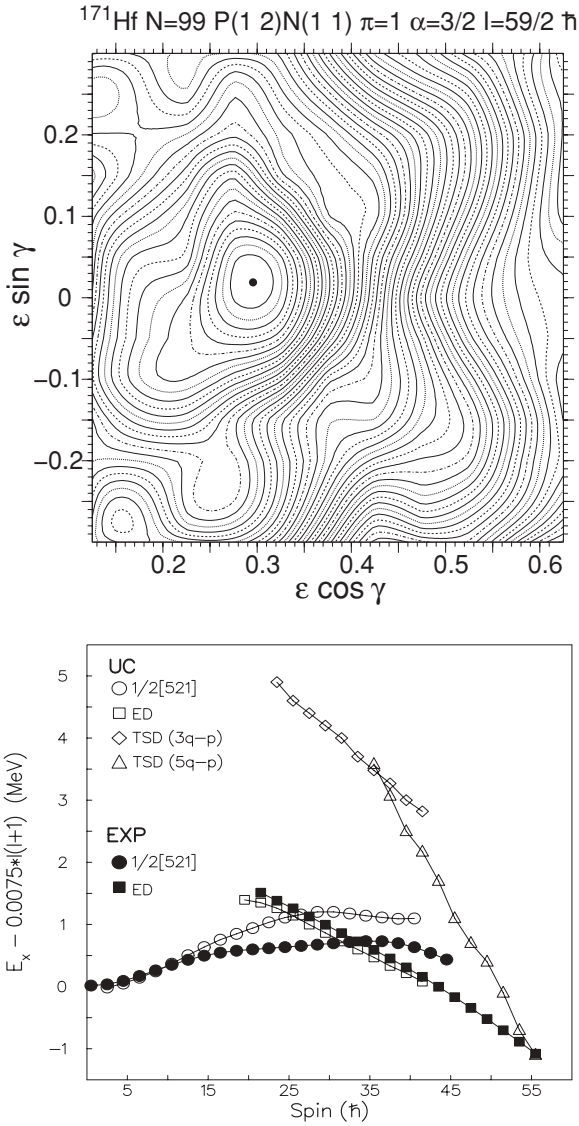

FIG. 6. (Upper panel) Potential energy surfaces for the configuration $(\pi, \alpha) = (+, -1/2)$ at $I = 59/2$ in ^{171}Hf calculated with the UC code. The contour interval is 0.2 MeV. (Lower panel) Experimental and UC calculated excitation energies minus a rigid-rotor reference for bands in ^{171}Hf.

excitation energies of the corresponding bands fit those of the observed ED bands in ^{170}Hf and ^{175}Hf quite well. The level of agreement between experiments and UC calculations for these bands is similar to that obtained for band ED in ^{171}Hf (see the lower panel in Fig. 6), and, thus, those bands are not shown in a separate figure. The same proton configuration of $\pi(i_{13/2}h_{9/2})$ is largely responsible for the ED bands in all three isotopes. In the case of ^{175}Hf, it couples to a $\nu(i_{13/2})$ orbital, whereas in ^{170}Hf there is no excited neutron. The calculated quadrupole moments for the ED bands, $Q_t \sim 8.5$ eb, are in line with a tentative experimental value of $\sim$9 eb for the ED band in ^{175}Hf [22]. A more precise measurement of the transition quadrupole moment in band ED of ^{171}Hf would provide further support for this interpretation. In general, the UC calculations provide consistent descriptions for the ED bands in all three isotopes.

The UC calculations also predict a TSD minimum around $(0.43, 20^\circ)$ for ^{171}Hf in a broad spin range. At lower spins, this minimum is associated with a three-quasiparticle configuration $\pi(i_{13/2}h_{9/2}) \otimes \nu(j_{15/2})$, or $\pi 6^1 \nu 7^1$ in a shorthand notation for the high-j intruder orbitals involved, with $Q_t \sim 8$ eb. For $I \gtrsim 35.5\hbar$, a five-quasiparticle structure $\pi(i_{13/2})^2 \otimes \nu(i_{13/2}h_{9/2}j_{15/2})$, or $\pi 6^2 \nu 7^1$, with $Q_t \sim 9.7$ eb becomes favored. In the UC calculations, the $N = 94$ neutron TSD shell gap is penetrated by the $\nu j_{15/2}$ orbital and two orbitals originating from the $g_{9/2}$ subshell with mixed wave functions dominated by $i_{11/2}$ components [10]. Therefore, all PES minima at large deformation have the $j_{15/2}$ neutron involved in their respective configuration for the spin range of interest. As seen in Fig. 6, the $\pi 6^1 \nu 7^1$ band is located $\sim$3 MeV above the yrast line, and the $\pi 6^2 \nu 7^1$ band crosses the yrast line at $I \approx 55.5\hbar$. Such predicted TSD bands were not observed in the experiment.

C. Band 2 in ^{175}Hf

To a large degree, the interpretation of SD band 2 in ^{175}Hf is central to the understanding of the structures of the observed strongly deformed bands in heavier Hf nuclei. The spins and parity of this band are known experimentally, and the relative properties of the $J^{(1)}$ and $J^{(2)}$ moments of inertia (see Fig. 4) strongly indicate that the impact of pairing is negligible in this band. CRMF calculations without pairing were performed for this band employing the formalism of Ref. [27] and the NL1 parametrization of the RMF Lagrangian. The results of these calculations for band 2 are presented in Fig. 7. The data for $J^{(1)}$ are best described by the $\pi 6^1 \nu 7^1$ configuration

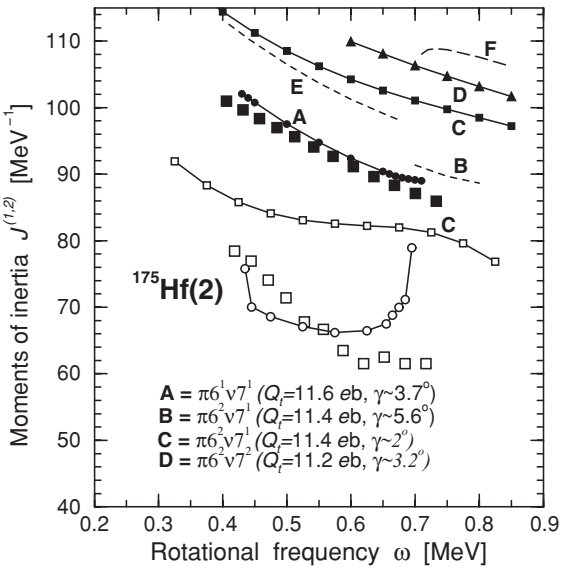

FIG. 7. Experimental and calculated kinematic $[J^{(1)}]$ and dynamic $[J^{(2)}]$ moments of inertia in ^{175}Hf. Experimental data for band 2 are presented by unlinked symbols. The results of the CRMF calculations are shown by linked symbols. Solid (open) symbols are used for the kinematic (dynamic) moments of inertia. Configurations E and F have the $\pi 6^2 \nu 7^1$ and $\pi 6^2 \nu 7^2$ structures, respectively.

(conf. A). The calculated $J^{(2)}$ moments of conf. A are also close to the experimental values. The calculated crossings at $\hbar\omega \sim 0.43$ MeV and $\hbar\omega \sim 0.68$ MeV, however, do not have experimental counterparts, which may suggest that the single-particle spectra obtained in these CRMF calculations are not optimal. The latter crossing leads to the $\pi 6^2 \nu 7^1$ configuration (conf. B). The CRMF calculations usually describe the $J^{(1)}$ moments with an accuracy of 5–10% [27,28,41]. Thus, the assignment of the $\pi 6^2 \nu 7^1$ configuration (conf. C) to SD band 2 cannot be completely excluded, but it is a less likely assignment considering the fact that conf. C is higher in energy than conf. A until spins higher than $58\hbar$. In addition, the calculated $J^{(2)}$ moments for this configuration differ considerably from the observed values. The assignment of the $\pi 6^2 \nu 7^2$ structure (conf. D) to band 2 in ^{175}Hf is very unlikely, because the moments of inertia of such a configuration are too large compared to experimental values. Furthermore, in the spin range of interest, the configurations with two neutrons in the $j_{15/2}$ orbital are located at higher energy than those with only one neutron in this $j_{15/2}$ state. They become energetically favored over conf. A only at spins around $70\hbar$. We may conclude that the CRMF results do not support configuration assignments of $\pi 6^2 \nu 7^2$, or $\pi(i_{13/2})^2 \otimes \nu(i_{13/2})^2(j_{15/2})^2(h_{9/2})$, that were proposed earlier for this band [22].

All of the calculated configurations are near prolate with γ-deformation values of only $\sim 4^\circ$. It is necessary to mention that the calculated transition quadrupole moment of the $\pi 6^1 \nu 7^1$ configuration (conf. A) is $Q_t \sim 11.7$ eb in the spin range of interest, which is somewhat lower than the preliminary measured value of $Q_t \sim 13$ eb [22] but well within the uncertainties associated with the measurements. At present, it is difficult to estimate how significant this discrepancy is.

It is well known that the deformation of the nuclear system in a specific configuration is predominantly defined by the shell structure. However, the deformation driving properties of active orbitals have some impact on the position of the minima in the potential energy surfaces. Figure 8 provides the Nilsson diagram obtained in the RMF calculations for axially symmetric shapes. At the values of the quadrupole moment of interest ($Q \sim 12$ eb), a large proton gap is seen at $Z = 72$ and somewhat smaller neutron gaps are observed at $N = 104$, 102 and $N = 100$. The presence of these gaps defines the SD nature of band 2 in ^{175}Hf. The rotation will affect the position and the size of these shell gaps because the lowest $j_{15/2}$ neutron and $i_{13/2}$ proton orbitals have large alignment and are, thus, strongly downsloping as a function of rotational frequency. In the $\pi 6^1 \nu 7^1$ configuration, as a result, the $N = 105$ gap is present and the $Z = 72$ gap is smaller than in the Nilsson diagram of Fig. 8. In addition to the deformation-driving $j_{15/2}$ neutron and $i_{13/2}$ proton orbitals, this configuration contains two neutron holes in the 11/2[505] orbital. Like the holes in the proton 9/2[404] orbital, which are important in the stabilization of superdeformation in the $A \sim 130$ mass region [45,46], these neutron holes in the extruder orbital drive the deformation of the nuclear system toward a larger value.

The neutron Nilsson diagram allows us to understand the development of the ED and SD shapes in the Hf isotopes using the $N = 104$ shell gap at $Q \sim 11$ eb as a starting point, see Fig. 8. The paths A and B lead to SD shapes. Path A, associated

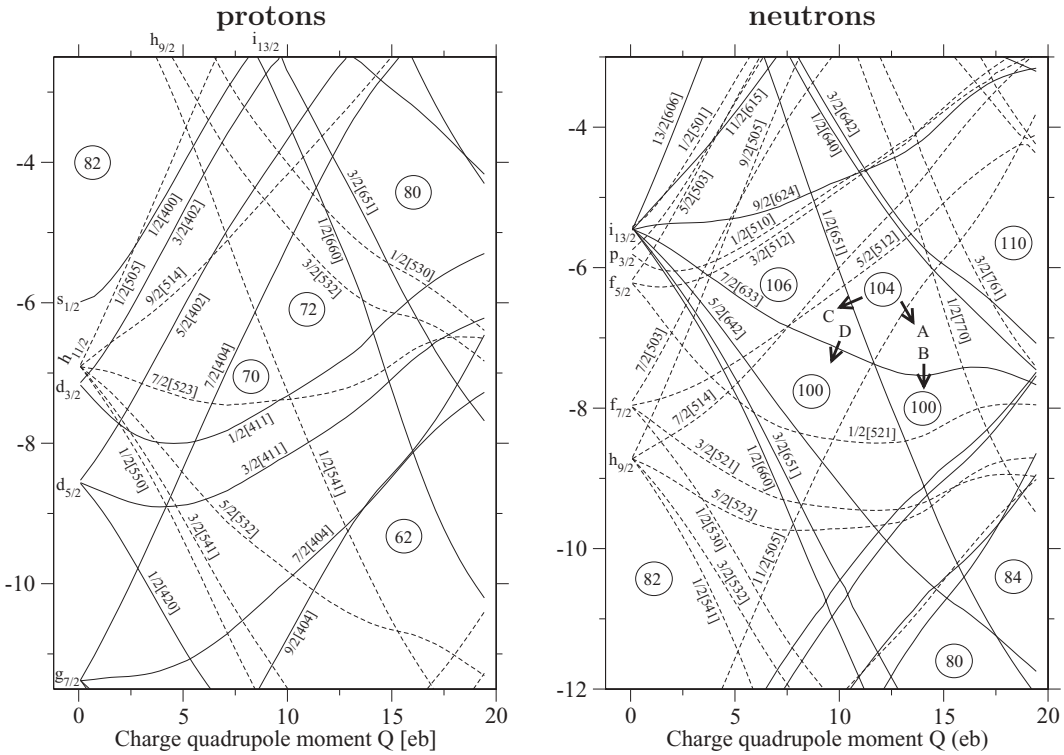

FIG. 8. Proton and neutron single-particle energies (in MeV) in ^{174}Hf, as a function of the quadrupole moment Q, obtained from the axially symmetric RMF calculations. Solid and dashed lines denote positive- and negative-parity orbitals, respectively.

with the creation of two holes in the 11/2[505] orbital, leads to a $N = 102$ shell gap and a larger deformation of the system. In contrast, removing two neutrons from the 7/2[633] orbital (path B) does not affect considerably the deformation of the system. It is quite likely that these paths will allow us to explain the fact that the quadrupole moments in the Hf isotope show a maximum at ^{173}Hf with $Q_t \approx 14.5$ eb [21]. Paths C and D lead to the ED shapes. In the configurations lying on these paths, the neutron 11/2[505] orbital is expected to be occupied with the 1/2[651] orbital being empty. Thus, the existence of the neutron gaps at $Q \sim 9$ eb and at $Q \sim 13$ eb allow us to understand the coexistence of ED and SD shapes in ^{175}Hf. A similar coexistence is expected also in lighter Hf isotopes. Although the UC calculations with the standard parameters for the Nilsson potential are able to reproduce the ED bands in ^{170,171,175}Hf, they fail to reproduce band 2 in ^{175}Hf [22], as well as the SD bands in ^{173,174}Hf [21]. Contrary to the CRMF calculations, the UC calculations do not show near-prolate minima at large deformation [22], which can be associated with the observed SD bands.

D. Bands SD1–SD3 in ^{172}Hf

The $J^{(2)}$ moments of bands SD1–SD3 in ^{172}Hf fall in the SD group, although the $J^{(2)}$ values of band SD2 are ∼10% larger than the ones of SD1 and SD3 and there are also some irregularities. The difference in $J^{(2)}$ moments, together with the fact that no linking transitions between the bands were observed, suggests that the three SD bands in ^{172}Hf are not members of the same wobbling family as seen in Lu isotopes, where very similar $J^{(2)}$ moments and interband $\triangle I = 1$ enhanced $E2$ transitions were found [3], in agreement with expectations. To understand the intrinsic configurations of these bands SD1–SD3 in ^{172}Hf, it is helpful to compare them with other SD bands in $^{173-175}$Hf, noting the following similarities between bands: (i) their $J^{(2)}$ moments resemble each other; (ii) band SD1 in ^{174}Hf and band 2 in ^{175}Hf are isospectral; (iii) similarly, band SD1 in ^{172}Hf and the band in ^{173}Hf are isospectral as well; and (iv) the band in ^{173}Hf and band SD1 in ^{174}Hf have similar quadrupole moments [21]. Isospectral bands, especially those with large deformation, are often associated with closely related configurations. In particular, they often contain the same high-j orbitals [47]. It is likely that the SD bands in ^{172}Hf have configurations similar to those of their heavier neighbors. Among all bands in the SD group, only band 2 in ^{175}Hf was linked to known levels, and a configuration of $\pi(i_{13/2})^2 \otimes \nu(i_{13/2})^2(j_{15/2})^2(h_{9/2})$ was proposed for this band in Ref. [22]. Similar configurations were suggested for the SD bands in ^{173,174}Hf [21]. However, such a configuration assignment for band 2 in ^{175}Hf was questioned based on the CRMF calculations in the previous section.

A definitive interpretation for bands SD1–SD3 in ^{172}Hf is difficult because they are not linked to the low-spin level scheme. Without the knowledge of spins, parity, and excitation energies of the bands, a detailed comparison between calculations and experimental values cannot be performed. Representative PES obtained in the UC calculations for

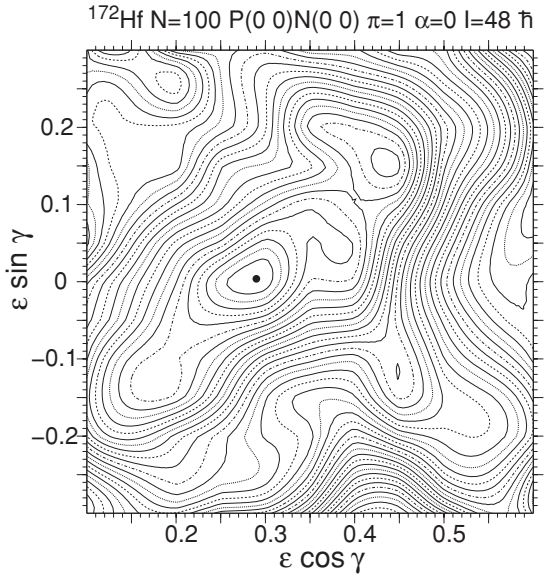

FIG. 9. Potential energy surfaces for $(\pi, \alpha) = (+, 0)$ at $I = 48$ in ^{172}Hf calculated with the UC code. The contour interval is 0.2 MeV.

^{172}Hf are given in Fig. 9. Similar to the calculated PES in ^{174}Hf [21], no prolate minimum with large deformation ($\varepsilon_2 \gtrsim 0.4$) is present. A high-spin TSD minimum around $(\epsilon_2, \gamma) \approx (0.43, 16^\circ)$ is predicted for all combinations of parity and signature (π, α) above spin $I \sim 34\hbar$. The other minimum at $(\epsilon_2, \gamma) \approx (0.45, -12^\circ)$ is located at higher excitation energy and it is not well developed. Therefore, any TSD band (if observed) should be related to the minimum with a positive γ value. The calculations indicate that many closely lying bands located in this minimum should be observed. The predicted intrinsic configuration for the minimum with $(\pi, \alpha) = (+, 0)$ is $\pi(i_{13/2})^2 \otimes \nu(j_{15/2})(h_{9/2})$ or $\pi 6^2 \nu 7^1$. The configurations with other combinations of parity and signature are associated with the same high-j intruder orbitals, $\pi 6^2 \nu 7^1$. The predicted TSD bands approach the yrast line above spin $I \sim 50\hbar$, similarly to the $\pi 6^2 \nu 7^1$ band depicted in Fig. 6 for ^{171}Hf. The UC calculated Q_t values, $10.5 - 11.5$ eb, for the TSD structures in ^{172}Hf are close to the values predicted for the TSD structures in ^{174}Hf. However, as mentioned above, the UC calculations do not predict the deformations for SD bands in ^{173,174}Hf correctly. The same may occur in ^{172}Hf. A measurement of quadrupole moments for the SD bands in ^{172}Hf is likely to provide further information about the validity of the UC calculations and the nature of the observed bands.

It is not clear whether the three SD bands in ^{172}Hf should be associated with a TSD minimum (as suggested by UC calculations) or with a near-prolate SD minimum (as suggested by the CRMF calculations in ^{175}Hf). More detailed investigation of these bands calls for a systematic study of the SD bands in the $^{172-174}$Hf nuclei within the framework of the CRMF theory using an effective alignment approach; this is beyond the scope of current manuscript. At present, the possibility of these bands being TSD structures cannot be ruled out. Indeed, the intensity of each band is low, and the observation of collective wobbling excitations based on these bands might be difficult, even if they are triaxial. Such a result may stem from two mechanisms. In the first one, the interaction between wobbling band and surrounding quasiparticle bands leads to a considerable fragmentation of the wobbling amplitude in its wave function, so it looses to a large degree its unique properties. This mechanism is expected at high excitation energies for wobbling bands. In the second mechanism, the population of the wobbling bands drops below the observational limit because the total feeding intensity has to be distributed among many excited bands. This mechanism is the same as the one proposed in Refs. [25,26]. The situation in odd-Z, even-N Lu isotopes, where wobbling excitations were observed, is very different in comparison to the Hf isotopes. The excitation energy of the lowest quasiparticle TSD bands ($n_w = 0$ bands) in ^{163,165,167}Lu nuclei is relatively low in comparison to that of other configurations; in particular, it is lower than that of TSD bands based on quasiparticle excitations [23,25]. Therefore, it is easier to populate with significant strength wobbling bands built on these $n_w = 0$ bands, because other configurations are likely located at higher excitation energy and, consequently, the interaction between the wobbling bands and quasiparticle bands is weak, and does not lead to a significant fragmentation of the wobbling amplitude.

IV. CONCLUSION

Three strongly deformed bands in ^{172}Hf and one in ^{171}Hf were identified and the latter was linked to known ND structures. The wobbling mode, an experimental fingerprint of triaxiality, was not observed. The strongly deformed bands in $^{170-175}$Hf fall into two groups: the ED bands and the SD bands. The results of cranking calculations using the ULTIMATE CRANKER code are consistent with the observed properties of the ED bands, including the new band in ^{171}Hf and two previously proposed TSD candidates in ^{170}Hf and ^{175}Hf. The calculations suggest that the ED bands are associated with near-prolate shapes and an enhanced deformation with respect to the ND bands. Their configurations involve a proton $i_{13/2}h_{9/2}$ structure. However, the UC calculations with standard parameters for the Nilsson potential failed to reproduce the deformation of previously observed SD bands in ^{173,174}Hf. The cranked relativistic mean-field calculations indicate that the SD band in ^{175}Hf, and very likely the similar bands in $^{172-174}$Hf, are near-prolate superdeformed bands with configurations involving $\pi i_{13/2} \otimes \nu j_{15/2}$ high-j orbitals. The measurement of quadrupole moments for the new bands in ^{171,172}Hf would be of help in confirming the above assessments. Further experimental and theoretical efforts are necessary to investigate the possibility that some weak bands in the SD group may still be associated with TSD shapes.

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