Evidence for octupole vibration in the triaxial superdeformed well of ^{164}Lu

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High-spin states in ^{164}Lu were populated in the ^{121}Sb(^{48}Ca,5n) reaction at 215 MeV and γ-ray coincidences were measured with the Gammasphere spectrometer. Through this experiment the eight known triaxial superdeformed bands in ^{164}Lu could be confirmed. Some of these bands were extended to higher as well as to lower spins. Evidence is reported for the first time for weak $\Delta I = 1$, $E1$ transitions linking TSD3 and TSD1. This observation may imply coupling to octupole vibrational degrees of freedom. The decay mechanism is different from the one observed in the neighboring even-N isotopes, which exhibit wobbling excitations built on the $\pi i_{13/2}$ structure with $E2(M1)$, $\Delta I = 1$ interband decay. An additional sequence decaying at high spin into TSD1 was observed up to $I^\pi = (50^-)$. This band has a constant dynamic moment of inertia of $\sim 70\hbar^2\text{MeV}^{-1}$ and an alignment that is $\sim 2\hbar$ larger than that found for TSD1. A revision of the assumed spin-parity-assignment of TSD2 is based on the observed decay-out to normal-deformed structures. The parity and signature quantum numbers of TSD2 are now firmly assigned as $(\pi, \alpha) = (+, 0)$, in disagreement with the former assignment of $(\pi, \alpha) = (-, 1)$, which was based on the assumption that TSD2 is the signature partner of TSD1. TSD1 and TSD2 show an alignment gain at $\hbar\omega \sim 0.67$ and 0.60 MeV, respectively. In TSD1 the involvement of the $j_{15/2}$ neutron orbital is suggested to be responsible for the high-frequency crossing.

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I. INTRODUCTION

Recently, wobbling excitations have been observed in several even-N Lu isotopes [1–7]. Wobbling is a phenomenon uniquely related to the rotation of a triaxially deformed nucleus [8]. The observed wobbling bands are based on the strongly deformation-driving $i_{13/2}$ proton orbital. Transition quadrupole moments, derived from lifetime measurements [9,10], show that the deformation of the wobbling bands is indeed very large. However, it is surprising that wobbling was only observed in the odd-A Lu isotopes, and not in the odd-odd or even-even neighbors where similar triaxial superdeformed (TSD) bands are known.

In odd-odd nuclei, the various combinations of proton and neutron orbitals that are exploited are expected to sample the triaxial minima differently. In contrast to the even-N isotopes, signature-partner bands of the lowest neutron excitations are expected to lie close in energy and to be associated with identical shapes. Therefore, an odd-odd Lu isotope might offer the possibility of studying wobbling excitations coexisting with a neutron-signature partner to the band on which the wobbling is based. Whether wobbling can persist in such cases or whether the wobbling degree of freedom will be diluted into the neutron-signature partner is an interesting question. Calculations [11] indicate that in the even-N case, where only one proton-signature partner may exist, wobbling will be favored if the energy difference from signature splitting is large.

Bands that are presumably of TSD nature have been observed in the odd-odd nuclei ^{162}Lu [12] and ^{164}Lu [13], the latter nucleus showing the largest number of such bands. Out of the eight TSD bands in ^{164}Lu, two, namely TSD1 and TSD3, were connected to normal-deformed (ND) structures by both stretched and unstretched $E1$ and $E2$ transitions, most likely of "statistical" nature [14]. Spin and parity of bands TSD1

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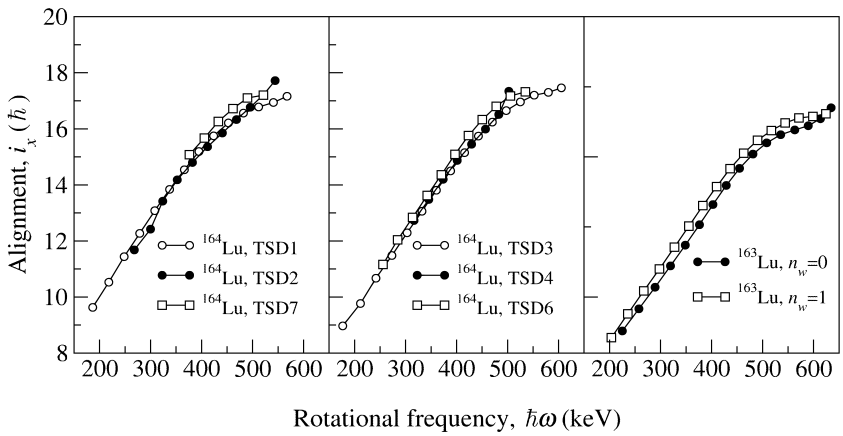

FIG. 1. Alignments of TSD bands grouped as suggested prior to this experiment. Left and center panels: Alignments of the candidates for signature-partner bands and the excited wobbling band built on one of the partners in ^{164}Lu. Right panel: The $n_w = 0$ and 1 phonon wobbling excitations in ^{163}Lu.

and TSD3 were in Ref. [13] based on measurements of γ-ray angular correlation (DCO) ratios of the strongest $\Delta I = 1$ decay-out transitions. Some of the bands were proposed to be signature partners, on the basis of their rotational properties, and candidates for wobbling bands were suggested, based on similarities with the even-N cases of wobbling excitations, as demonstrated in Fig. 1. The present experiment sheds new light on this situation. First, TSD2 has been connected to lower lying states and its character as a signature partner to TSD1 is now ruled out. Second, it has not been possible to find evidence for wobbling excitations. However, a new phenomenon was observed: namely, $\Delta I = 1, E1$ transitions between TSD1 and TSD3 of a strength that may imply coupling to octupole vibrational degrees of freedom. Two additional high-spin structures, labeled X1 and X2, were observed in connection with TSD1. The occurrence of these new states may be interpreted as a sign of the presence of different quasiparticle structures interacting with TSD1 at spin $I = 42$. A partial level scheme summarizing the new results established in this experiment is given in Fig. 2.

II. EXPERIMENTAL DETAILS

The experiment has been performed using the Argonne Tandem-Linear-Accelerator System (ATLAS) facility at Argonne National Laboratory. High-spin states of ^{164}Lu were populated by using the reaction ^{121}Sb(^{48}Ca,5n)^{164}Lu at a beam energy of 215 MeV. The target consisted of three self-supporting ^{121}Sb foils each with a thickness of $\sim$500 μg/cm^2. The emitted γ rays were detected by the Gammasphere spectrometer [15]. To select high-fold events, the trigger condition for the Ge detectors was set on five or more signals after Compton-suppression coupled to 10 or more signals from BGO Compton-shield modules.

The spectrometer consisted of 101 Ge detectors at the time of the experiment. After the data were presorted, a total of 2.36×10^9 events remained for the analysis, from which 1.36×10^9 were five- or higher fold coincidence events.

These events were sorted into three- and four-dimensional coincidence arrays (cubes and hypercubes, respectively) and analyzed with programs from the RADWARE package [16].

III. RESULTS

A. The TSD1 band

The band TSD1 was established with firm parity and signature quantum numbers, $(\pi, \alpha) = (-, 0)$, up to spin $I^\pi = 42^-$ in Ref. [13]. In the present experiment, several additional transitions with energies greater than that of the top $42^- \rightarrow 40^-$ transition of 1135.2 keV were found in coincidence with this band. A coincidence spectrum is presented in Fig. 3 documenting the new transitions, which could be placed at the top of the band. Since the regularity of the energy spacing of $\sim$59 keV within the band is broken, it becomes more difficult to place them in the level scheme. However, coincidence spectra show that the 1219-keV transition is not in coincidence with the 1162- and 1193-keV transitions. Therefore, a new structure, labeled X1, composed of only one level that is connected to TSD1 via the two transitions of 1193 and 1162 keV, is proposed, as shown in Fig. 2. In addition, another structure, X2, also decays to TSD1 at $I^\pi = 42^-$. This band exhibits a regular spacing of $\sim$56 keV. Angular correlation measurements could not be performed for these new transitions, but by the observed interband transitions above and below $I^\pi = 42^-$, spin and parity values are firmly locked to TSD1 for both structures at this level. The intensity balances seen in Fig. 3 support this decay scheme. An additional transition of 1552 keV was observed in coincidence with TSD1; however, it was not possible to establish its position in the level scheme.

From the eight decay-out transitions to ND structures proposed in Ref. [13], five were confirmed by the present experiment and analysis. In addition, two new decays have been found, a $\Delta I = 1$ transition of 1541 keV established from the 14^- to the 13^- level, and a $\Delta I = 2$ transition of 1678 keV linking the 16^- and 14^- states, which provides important support for the parity and signature quantum numbers determined for TSD1 in Ref. [13]. Decay transitions to the ND structures are also present in Fig. 3.

B. The TSD2 band

The TSD2 band was firmly established in Ref. [13] with 11, and possibly 13, presumably $\Delta I = 2$ transitions. However, no firm connection to ND structures was proposed in the previous work. The present experiment confirms the two previously

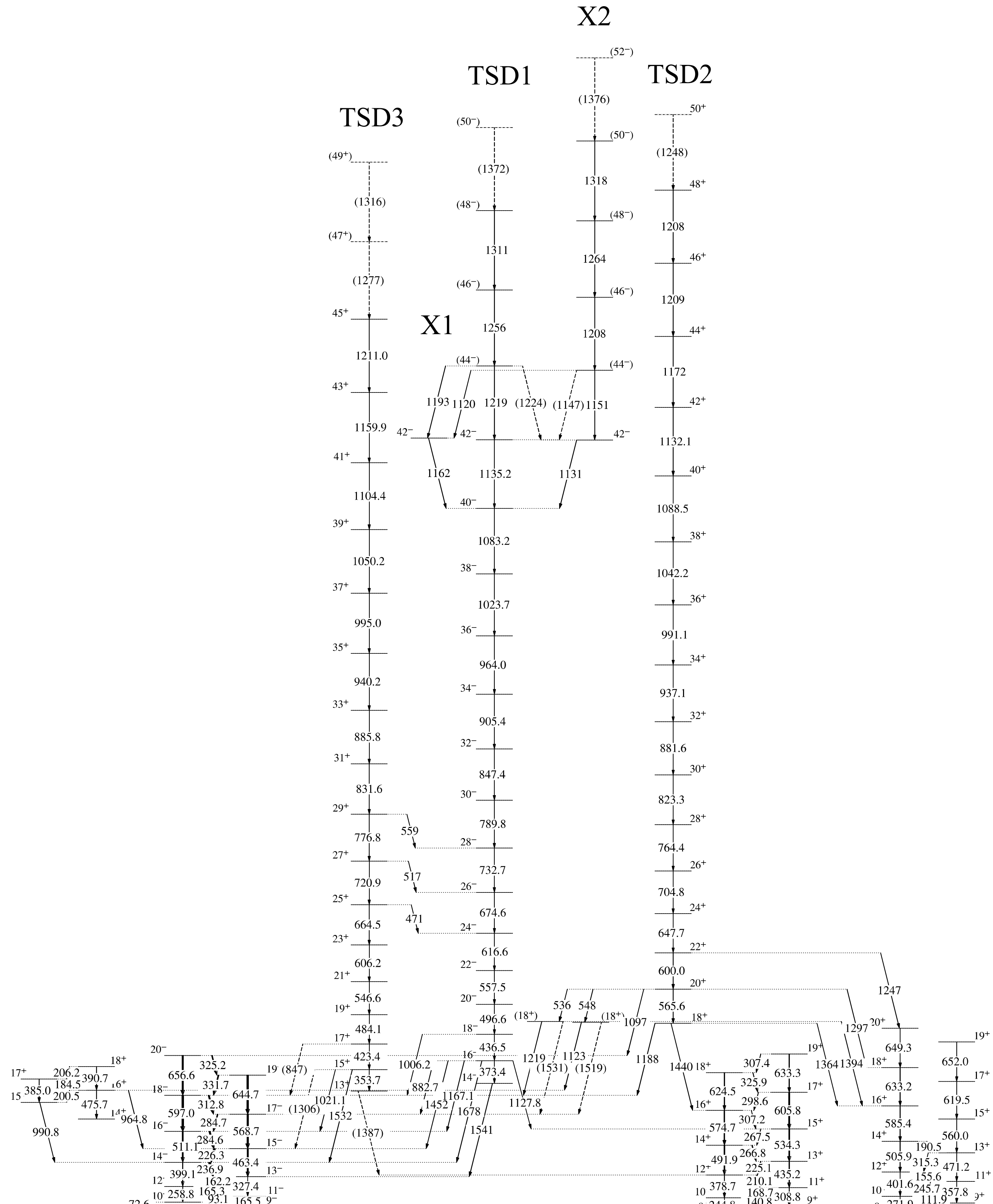

FIG. 2. Partial level scheme of ^{164}Lu. The TSD bands 1, 2, and 3 are shown together with the newly established excited state X1 and the sequence X2 as well as with the ND structures observed in their decay.

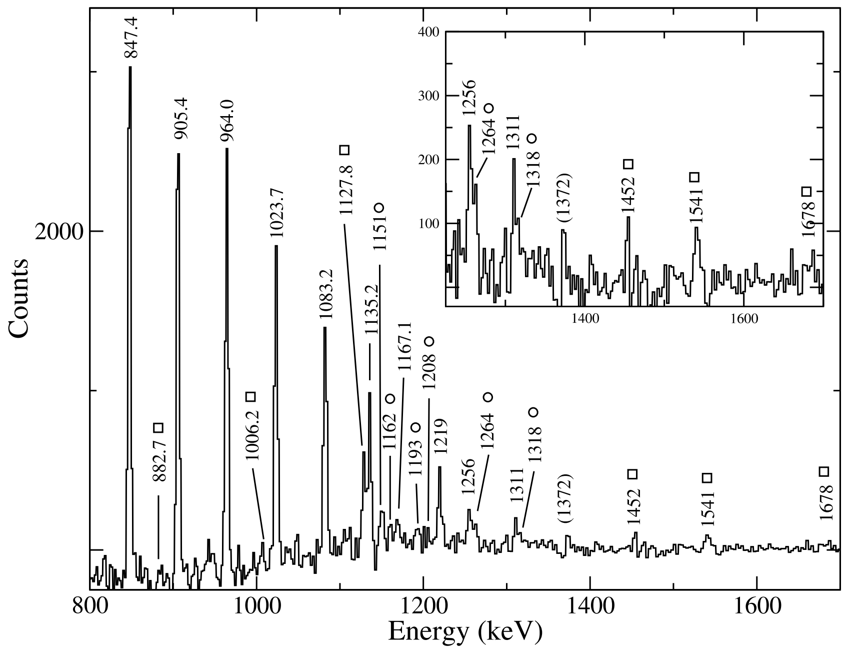

FIG. 3. High-energy part of a γ-ray coincidence spectrum of TSD1 in ^{164}Lu. The spectrum was obtained by using triple coincidence gates on all transitions of the band up to and including the 1135-keV transition. The inset shows the energy spectrum in the range between 1230 and 1685 keV. The γ rays marked with open squares correspond to decay-out transitions populating ND bands. The γ rays marked with open circles correspond to the new established transitions at high spin of the structures labeled X1 and X2.

uncertain transitions of 1172 and 1209 keV and extends the band by adding one, and possibly two, new transitions at the top. The 1208-keV transition is clearly in coincidence with the 1209-keV line in a triple coincidence gate of 1209 keV with a list of all transitions in TSD2 up to and including the 1172-keV transition, used twice. A γ-ray coincidence spectrum documenting this band is displayed in Fig. 4. Links to the ND structures are suggested for TSD2 based on a multitude of decay-out transitions to several of the ND bands. They are all rather weak, but together they form a consistent scheme, in which the parity and spin values for TSD2 comply with $(\pi, \alpha) = (+, 0)$, with $I^\pi = 18^+$ being assigned to the lowest state. The decay-out passes through two additional 18^+ states of unknown origin. The strongest transition of 1188 keV, shown in Fig. 4, is clearly observed in coincidence with the band and feeds into the yrast negative-parity ND band at $I = 18$. By placing this transition as a $\Delta I = 0$, $18^+ \rightarrow 18^-$ decay, several weak $\Delta I = 1$ and $\Delta I = 2$ transitions seen in Fig. 4 have the correct energy as decays and are observed in coincidence with the band as well as with the relevant ND structures. The various decay paths are shown in Fig. 2. The observed transitions of stretched and unstretched $E2$ and $E1$ character are most likely of similar nature as the decays observed from TSD1 and TSD3.

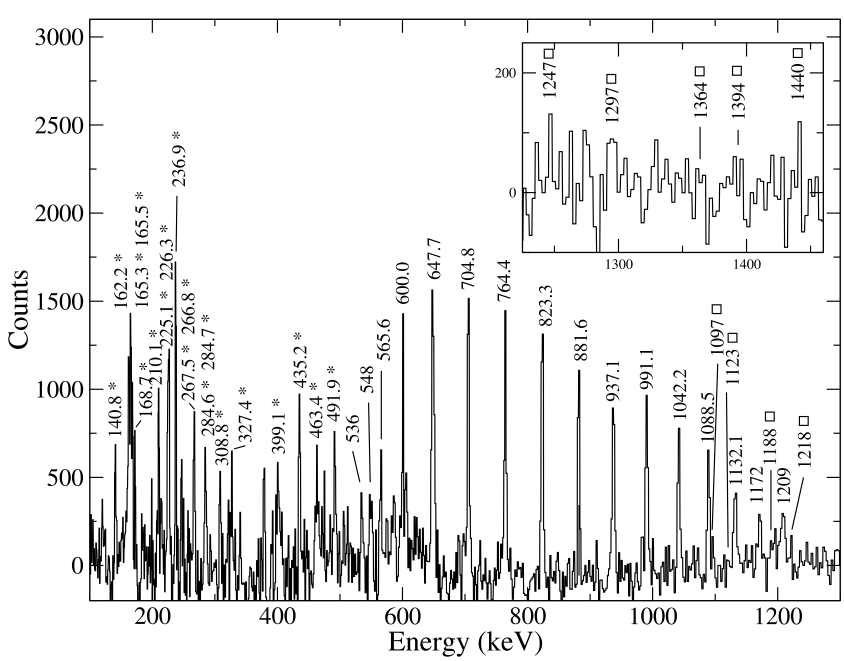

FIG. 4. Gamma-ray coincidence spectrum of TSD2 in ^{164}Lu. The spectrum was obtained by using triple gates of a list containing all transitions from 565.6 up to 1208 keV. The ND transitions populated in its decay are marked by asterisks. The transitions marked with an open square correspond to the decays to ND structures.

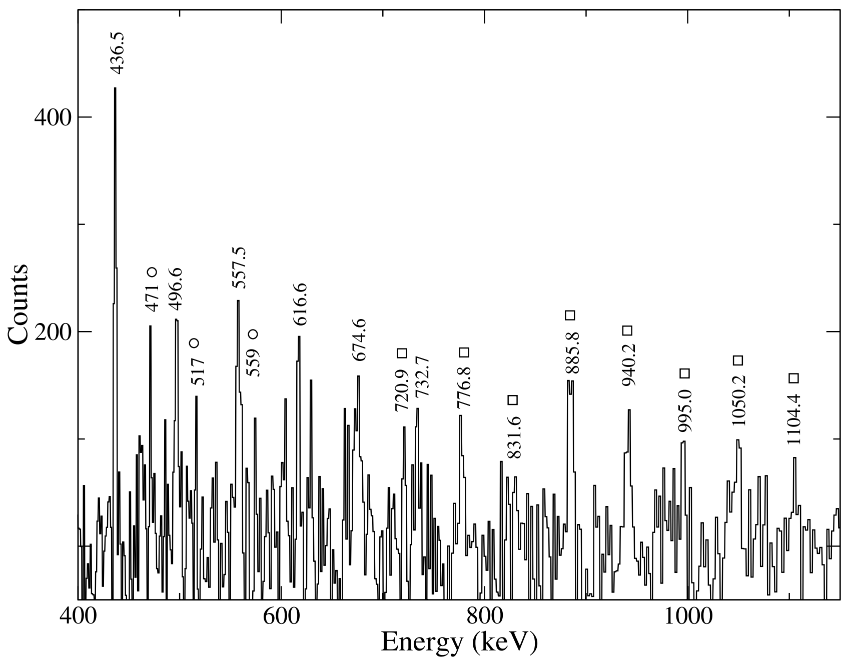

FIG. 5. Gamma-ray coincidence spectrum demonstrating the connection between TSD3 and TSD1 in ^{164}Lu obtained as a sum of two triple-gated spectra. Two different lists containing all transitions from TSD3 above spin-parity $I^\pi = 23^+$ and all transitions from TSD1 below $I^\pi = 24^-$, respectively, were applied. The two triple-gated spectra were built by using one list once and the other list twice and vice versa. Transitions from TSD3 are marked with open squares; transitions from TSD1 are unmarked. The suggested interband transitions are marked with open circles.

C. The TSD3 band

In Ref. [13] the band TSD3 was established up to $I^\pi = 45^+$ with firm parity and signature quantum numbers, $(\pi, \alpha) = (+, 1)$. The band was extended in the present work as seen in Fig. 2. The addition of two new transitions at the top of the band extends it tentatively up to spin-parity $I^\pi = 49^+$. Additional decay to ND structures has also been found. A new transition from the 15^+ to the 16^- state with an energy of 1021 keV has been established together with a possible 17^+ to 17^- transition at 1060 keV. The observation of these transitions confirms the decay path proposed by Törmänen *et al.* [13]. Several additional weak transitions of 1080, 1204, 1226, and 1334 keV are also observed in coincidence with TSD3. They are probably involved in the decay-out of this band. However, no clear placement could be established for these transitions owing to their low intensity.

For the first time, evidence for mutual connections between TSD bands in ^{164}Lu is obtained. From Fig. 5 it is clear that transitions between high-spin states of TSD3 are seen in coincidence with the lower energy transitions of TSD1. Since spins, parities, and excitation energies for both bands are firmly established, the decay must take place through $\Delta I = 1$, $E1$ transitions from the positive-parity band TSD3 to the negative-parity band TSD1. Accordingly, by looking at the transition energies for the expected decays, three weak interband transitions could be observed, as illustrated with open circles in Fig. 5. These correspond to $24^+ \rightarrow 23^-$ at 471 keV, $25^+ \rightarrow 24^-$ at 517 keV, and $26^+ \rightarrow 25^-$ at 559 keV.

Because of the low intensity of these transitions, no branching ratios could be directly measured. However, by gating on the top transitions of TSD3, it is possible to obtain an estimate by measuring the relative loss in intensity between the lower lying successive in-band transitions. This naturally relies on the assumption that no other decay paths occur. Such an estimate gives an average branching-out ratio of 7(5)% over the three states where decay-out is observed. Since this result lies close to the experimental limit of measurable intensities, the presence of a new, presumably TSD, band with transition energies equal to or very close to both bands may be considered as an alternative explanation for the observed coincidences. This hypothetical band would have to consist of the five transitions at low spin close to TSD1 from 373 to 616 keV and transitions with energies close to TSD3 above and including 721 keV. Such an identity between bands is rather rare. Usually, when it occurs, distinct energy differences for some transitions exist, which disturb the identity to both sides. The fact that this hypothetical new band would have no transitions different in energy from those of TSD1 and TSD3 is, indeed, very unlikely. Therefore, this alternative explanation has been discarded.

IV. DISCUSSION

The three connected TSD bands as well as the new short X2 band are displayed together with the lowest negative- and positive-parity ND bands in a plot of excitation energy as a function of spin in Fig. 6. With its new parity and signature, TSD2 can possibly be viewed as a signature partner to TSD3. However, the rotational properties of the two sequences are somewhat different and no connections linking them have been found. Note that the band X2 is yrast in the spin range of $I = 42$–50.

A. Search for wobbling

An extensive search for the characteristic "wobbling" decay between TSD bands in the two potential families, TSD7 with TSD1 and TSD6 with TSD3, was not successful. The depopulation from the $n_w = 1$ to the $n_w = 0$ phonon band is, according to the even-N Lu isotopes, expected to consist

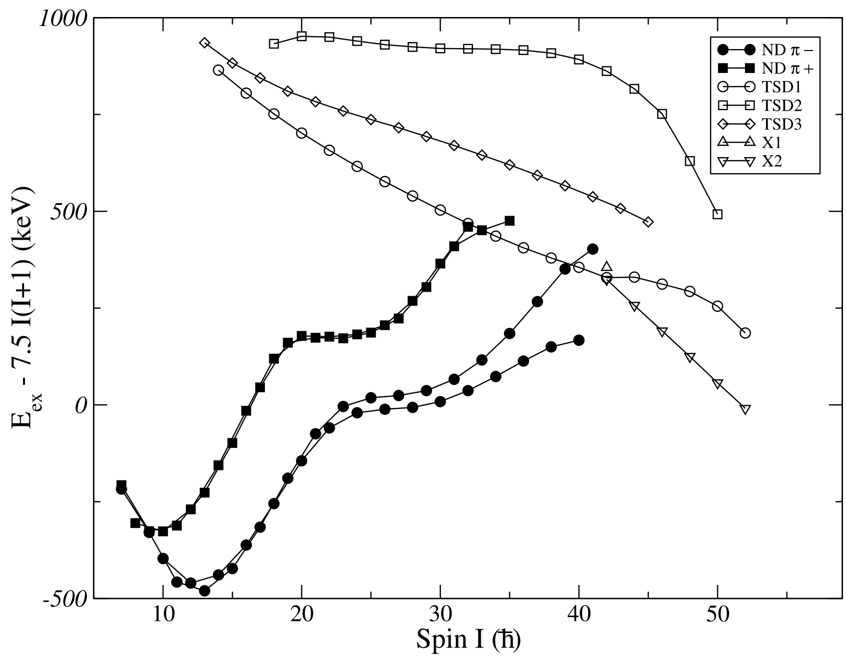

FIG. 6. Excitation energies relative to a rigid-rotor reference as a function of spin for TSD1, TSD2, TSD3, X1, and X2 in ^{164}Lu compared to the lowest ND bands of positive and negative parity.

of several $\Delta I = 1$ transitions. The interband transitions occur in a region where the observed ratio $B(E2)_{\rm out}/B(E2)_{\rm in} \sim 0.2$ and the $(E_{\gamma,\rm out}/E_{\gamma,\rm in})^5$ factor make it possible for the decay-out to compete with the in-band decay. The energy of the $\Delta I = 1$ interband transitions, $E_{\gamma,\rm out}$, is directly related to the wobbling frequency. If it is similar to that observed in its neighbors, ^{163}Lu and ^{165}Lu, one would expect to see the decays in the region with $E_\gamma \sim 500$–800 keV. Accordingly, a one-phonon wobbling band would be expected with γ-ray energies extending down to ∼500 keV and a gradually decreasing intensity in the decay-out region.

Both TSD6 and TSD7 are rather weakly populated, and out-of-band transitions from these sequences would be hard to observe. Nonetheless, although the individual $\Delta I = 1$ decay branches may not be found, the coincidence between high-spin transitions in the proposed $n_w = 1$ band and lower spin transitions in the $n_w = 0$ band should be present.

In the case of TSD6, for which the energy of the lowest transition is as high as 752 keV, we estimate that, given the value $B(E2)_{\rm out}/B(E2)_{\rm in} \sim 0.2$, the energy of the decay-out transitions would have to be $\geqslant 1$ MeV; that is, the wobbling frequency would be more than twice that for ^{163}Lu and ^{165}Lu. Alternatively, a much larger value of $B(E2)_{\rm out}/B(E2)_{\rm in}$ would be needed. Such a scenario would require a much larger value of the triaxiality parameter γ [17] and, therefore, seems unlikely.

In any event, our limit of coincidence intensity implies that the suggestion of TSD6 and TSD7 being wobbling excitations built on TSD3 and TSD1, like those in ^{163}Lu and ^{165}Lu, can be ruled out. If wobbling excitations really exist, the corresponding bands must be of even weaker population than TSD6 and TSD7 and, therefore, should lie at higher excitation energy. This in turn implies a considerably larger wobbling frequency, and a question here is if the wobbling degree of freedom would persist in the presence of a higher density of quasiparticle excitations.

B. Connection between TSD3 and TSD1

The bands TSD1 and TSD3 have parity and signature $(\pi, \alpha) = (-, 0)$ and $(+, 1)$, respectively, based on the stretched dipole character measured [13] for the $16^- \rightarrow 15^+$, 1128-keV and the $15^+ \rightarrow 14^-$, 1532-keV transitions. This assignment is supported by the additional decay-out transitions found in the present experiment, as illustrated in Fig. 2. The configurations $\pi i_{13/2} \otimes \nu h_{9/2}$ and $\pi i_{13/2} \otimes \nu i_{13/2}$ have been assigned to TSD1 and TSD3, respectively. With the opposite parity of TSD1 and TSD3, the $\Delta I = 1$ connecting transitions between the bands have to be of $E1$ multipolarity. Therefore, they cannot be interpreted in terms of wobbling.

The $\Delta I = 1$ connecting $E1$ transitions are indeed very weak, and no direct measurement of out-of-band to in-band intensity ratios could be performed. Nevertheless, the presence of these $E1$ transitions competing with highly collective $E2$ transitions is already a sign of large $B(E1)$ values. The relative reduced transition probabilities between the $\Delta I = 1$, $E1$ and $\Delta I = 2$, $E2$ transitions can be expressed as

$$\frac{B(E1)}{B(E2)} = \frac{1}{1.3 \times 10^6} \frac{E_\gamma^5(E2)}{E_\gamma^3(E1)} \frac{I_\gamma(E1)}{I_\gamma(E2)} \quad [\mathrm{fm}^{-2}], \quad (1)$$

where E_γ is given in MeV.

Since the two bands, TSD1 and TSD3, belong to the same triaxial well, the assumption can be made that both bands have the same deformation corresponding to the transition quadrupole moment of $Q_t = 7.1^{+0.5}_{-0.6}$ b measured for TSD1 [9]. With the estimated branching ratio of 0.07(5), a reduced transition probability $B(E1)$ in the order of $\sim 1.3 \times 10^{-3} e^2$ fm^2 ($= 6.5 \times 10^{-4}$ Wu) can then be derived. The expected $E1$ strength for low-energy transitions between single-quasiparticle states depends on the orbitals involved, but, in general, there is a considerable reduction of the $E1$ effective charge (see [18] and references therein). Together with hindrance effects from nuclear shell structure and relevant

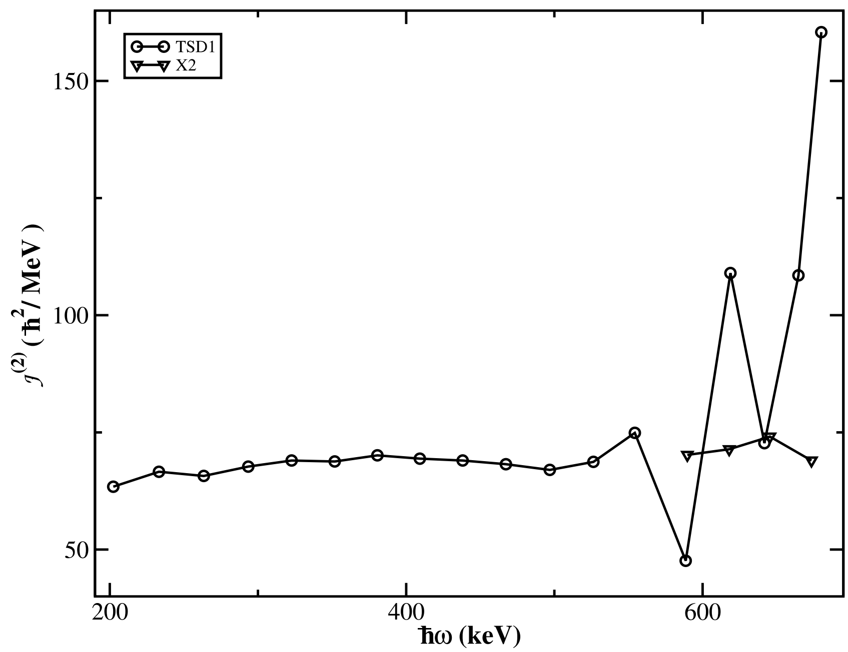

FIG. 7. Dynamic moments of inertia $\mathcal{J}^{(2)}$ as a function of rotational frequency $\hbar\omega$ for TSD1 and X2 in ^{164}Lu.

pairing factors, the reduced $E1$ strength between quasiparticle states in deformed rare earth nuclei may be expected [18] to be of the order of $10^{-8} - 10^{-6}$ Wu. With experimental values much larger than this, the inclusion of coupling to octupole vibrational degrees of freedom has been proposed, but a clear picture of the nature and strength of such octupole couplings has not yet emerged [18].

The estimated average value of $B(E1) \sim 6.5 \times 10^{-4}$ Wu for the transitions from TSD3 to TSD1 is indeed very large. In comparison, the reduced strengths of the $E1$ decay of TSD1 and TSD3 to ND bands are $\sim 0.4 \times 10^{-4}$ Wu [13] (i.e., one-tenth as strong, but still quite large). Octupole enhancement was proposed to explain the large $B(E1)$ values observed in the decay-out to ND structures of TSD1 and TSD3 [13].

Coupling to octupole vibrational degrees of freedom has been taken into account in the explanation of observed large $E1$ transition strengths between superdeformed (SD) signature-partner bands in the mass $A \sim 190$ region [19]. From the suggested neutron configurations involved in TSD3 ($\nu i_{13/2}$) and TSD1 ($\nu h_{9/2}$), a spin-flip is present, which implies very small octupole matrix elements. However, ultimate cranker (UC) [20] calculations predict for the TSD well components of approximately 20% of $\nu f_{7/2}$ in the wave function of TSD1, which may explain a possible octupole enhancement in the $E1$ transitions between the two TSD bands of opposite parity.

C. High-spin properties of TSD1, TSD2, TSD3, and X2

The present experiment has provided new information about the properties of the known bands at the highest spins and added a new band, X2, with γ-ray energies increasing regularly with spin (see Figs. 2 and 6). The band TSD1 exhibits an irregularity at $I^\pi = 42^-$ that is caused mainly by an interaction with the level X1 located 26 keV higher in excitation energy that must also have $I^\pi = 42^-$. The third $I^\pi = 42^-$ level, associated with X2, is as close as $\sim$4.5 keV to TSD1. The $I^\pi = (44^-)$ level of X2 decays to all three 42^- levels. A weak branch from the $I^\pi = (44^-)$ level of TSD1 to the $I^\pi = 42^-$ level of X2 is also suggested, illustrating the mutual mixing of all three $I^\pi = 42^-$ levels. This interaction causes an irregularity in the dynamic moment of inertia $\mathcal{J}^{(2)}$ of TSD1, which is considerably reduced by a shift of $\sim$11 keV of its $I^\pi = 42^-$ state. The band X2 is much closer to TSD1 at $I^\pi = 42^-$ and, accordingly, the interaction can be expected to be much smaller. A possible shift of $\leqslant 2$ keV cannot be traced in the dynamic moment of inertia $\mathcal{J}^{(2)}$ for X2 in Fig. 7.

TSD1 and TSD2 experience up-bends at the rather high rotational frequencies of 0.67 and 0.60 MeV, respectively. TSD3 is not extended to sufficiently high frequency to investigate whether a possible alignment occurs there as well. The short band, X2, covering a frequency range of 0.58–0.69 MeV, shows no alignment gain. The possible cause of a high-frequency alignment, based on calculations with the UC code [20] may be a so-called mixed crossing where the $j_{15/2}$ neutron with $\alpha = -1/2$ aligns together with the $\alpha = +1/2$ signature partner of the $h_{9/2}$ orbital.

To the lowest TSD bands, TSD1 and TSD3, the configurations $\pi[i_{13/2}, \alpha = +1/2] \otimes \nu[h_{9/2}, \alpha = -1/2]$ and $\pi[i_{13/2}, \alpha = +1/2] \otimes \nu[i_{13/2}, \alpha = +1/2]$, respectively, are assigned; that is, they involve the strongly shape-driving $i_{13/2}$ proton coupled to different neutron excitations. We can understand the alignment gain in the frequency range of 0.3–0.6 MeV as being due to the alignment of the first pair of $i_{13/2}$ neutrons in TSD1 and the second pair in TSD3, which comply with the observed small shift in the very gradual alignments of the two bands. The up-bend in TSD1 at $\hbar\omega \sim 0.67$ MeV cannot be due to the $\nu j_{15/2} h_{9/2}$ crossing just mentioned, which is blocked, but instead, the $\nu h_{9/2}$ may change to the $\nu j_{15/2}$ orbital with the same signature.

With identical parity and signature (π, α) one may compare TSD1 and X2. Their dynamic moments of inertia are similar,

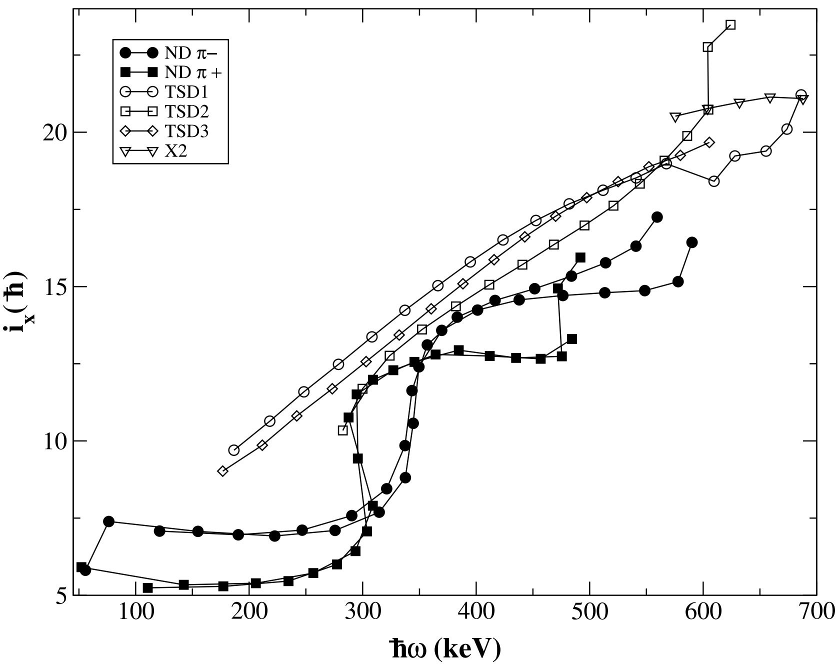

FIG. 8. Aligned angular momenta i_x relative to a reference as a function of rotational frequency $\hbar\omega$ for TSD1, TSD2, TSD3, and X2, in comparison with the ND bands in ^{164}Lu. The chosen reference was $i_{\text{ref}} = \mathcal{J}_0\omega + \mathcal{J}_1\omega^3$ with $\mathcal{J}_0 = 30\hbar^2$ MeV^{-1} and $\mathcal{J}_1 = 40\hbar^4$ MeV^{-3}.

as seen in Fig. 7, except for the irregularity at $\hbar\omega \sim 0.6$ MeV and alignment at $\hbar\omega \sim 0.67$ MeV in TSD1. With its 0.56–0.68 MeV frequency range, the larger alignment and a very small interaction with TSD1, X2 probably has a rather different structure than TSD1. At such a high frequency one may expect that a negative-parity proton, $h_{9/2}$, and a pair of aligned $i_{13/2}$ protons coupled to an $i_{13/2}$ neutron could compete with the configuration of TSD1 in which the alignment of a pair of $i_{13/2}$ protons is blocked.

TSD2 turned out to have $(\pi, \alpha) = (+, 0)$ and could, therefore, in principle be interpreted as a signature partner to TSD3. However, with the difference in rotational properties exhibited in Figs. 6 and 8, this possibility can be ruled out. The energy difference with respect to TSD3 is quite large, whereas one would have expected a partner with little energy difference since, according to the UC calculations, the $i_{13/2}$ neutrons have a rather small signature splitting at the triaxial shape. We suggest instead that the structure of TSD2 is different. At low frequency the $i_{13/2}$ neutron of TSD3 could be replaced by a $g_{7/2}$ neutron. The strong alignment increase at $\hbar\omega \sim 0.6$ MeV could possibly be caused by a change to a configuration involving two negative-parity neutrons or a negative-parity particle in both systems, like $\pi h_{9/2} \otimes \nu h_{9/2}$ with additional excited pairs of $i_{13/2}$ protons and neutrons. In this way the difference at the highest frequencies between TSD2 and X2 is found in the exchange of an $i_{13/2}$ neutron in X2 with an $h_{9/2}$ neutron. Unfortunately, we have no information about possible differences in shape between these so-called TSD bands, and the proposed configurations for TSD2 and X2 remain somewhat speculative.

The rather strong population of TSD2 relative to TSD1 and TSD3 may have its explanation in the large alignment gain of TSD2 at high spin, which results in TSD2 approaching the other bands close to $I = 50$.

V. SUMMARY

From the present experiment it was possible to add new information to the level scheme of TSD bands in ^{164}Lu. Three of the existing bands were extended and a new high-frequency TSD band was discovered. A total of four TSD bands are connected to ND structures, all of them with firm signature and parity assignments. We suggest that a high-frequency alignment in TSD1 could be caused by an exchange of an $h_{9/2}$ neutron with a $j_{15/2}$ one.

The search for wobbling-excitation relationships between the lowest TSD bands and potential candidates among the weaker populated unconnected bands gave no results. At this stage, we can rule out wobbling with frequencies ω_w similar to those found in the even-N neighbors exists. In the case of much higher values of ω_w and, therefore, higher relative energy of the wobbling excitations, it remains an open question whether the wobbling degree of freedom can compete in the presence of a higher density of quasiparticle excitations.

Most importantly, we have discovered a new type of connection between TSD bands in a Lu nucleus, namely by strongly enhanced $E1$ transitions. The reduced $E1$ strength implies that coupling to octupole vibrational degrees of freedom plays a role. These transitions most likely can be understood as connecting the $i_{13/2}$ neutron of TSD3 to the $f_{7/2}$ component of the negative-parity neutron in the wave function of TSD1. TSD1 and TSD3 in turn decay by several $E1$ transitions to the ND structures with a reduced $E1$ strength (by nearly an order of magnitude), but also with a suggested octupole enhancement. The reduction of a factor of 10 may reflect the fact that the two types of $E1$ transitions take place between states in minima of identical (TSD) shape and between states in minima of different (TSD and ND) shapes, respectively.

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