Form Factor of the Relativistic Two-Particle System in the Relativistic Quasipotential Approach: The Case of Arbitrary Masses and Scalar Current

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Abstract

The new relativistic form factor of the relativistic two-particle bound system in the case of scalar current is obtained. Consideration is conducted within the framework of relativistic quasipotential approach on the basis of covariant Hamiltonian formulation of quantum field theory by transition to the three-dimensional relativistic configurational representation in the case of two interacting relativistic particles with arbitrary masses.

1 Introduction

The study of hadrons electromagnetic form factors allows to obtain the information about spatial hadrons structure. The idea of the composite quark nature of hadrons and suggestion about scale invariant behavior in the region of large momentum transfers has allowed to reveal regularity of the elastic hadrons form factors behavior [1]. To describe the behavior of the form factors the different pole vector-dominance models (VDM) were used. These models successfully reproduce the behavior of the pion form factor as in space-like, so and at time-like regions [2], and behavior of the nucleon form factor in the space-like regions [3]. However the models VDM fail in description experimently of the observed for large importances of the momentum transfer of the system $-t = Q^2$ the quick decrease of electromagnetic form factor at time-like region according to the law of dipole $\sim t^{-2}$. The reason is that the model VDM assume that the virtual photon flying in the nucleon "sees" only the vector mesons which there are the quark-antiquark bound-states while the structure of nucleon study at small distances where the momentum transfer of the system there is enough large value and quarks move quasifree (the asymptotic freedom).

The other approach was suggested in [4–6] for description of the behavior of baryon and nucleon electromagnetic form factors at time-like region close to their threshold. In this approach the baryon (nucleon) electromagnetic form factor at time-like region close to $B\bar{B}$ ($N\bar{N}$) threshold introduces as a product of a factor corresponding to singularities of transition amplitude lying far from $B\bar{B}$ ($N\bar{N}$) threshold and a factor reflecting strong final state interaction. This last factor gives the energy dependency of the form factor.

However the problem of covariant description of form factor in the whole, rather then only in asymptotic region energy within the framework of relativistic quark model taking into account differences of their masses, continues remain interesting and at present. For this we must know the dynamics of the interacting quarks more in detail, in particular, we must know the covariant wave functions their of relative motion.

Within the quantum field theory the covariant wave functions of the relative motion can be obtained using the relativistic covariant two-particle quasipotential equations of Logunov–Tavkhelidze [7] and Kadyshevsky [8, 9]. The using of three-dimensional relativistic quasipotential (RQP) equation of Logunov-Tavkhelidze for description of the form factors of composite systems was executed in [10–14]. However, use of the equation Logunov-Tavkhelidze for wave function in the momentum representation has not allowed to research the behavior of the form factor in broad interval of importances of the momentum transfer of the relativistic two-particle bound system. The other model of the account of the contribution small the distances in form factor of the proton was considered in [15]. This model is based on invariant description of the structure of the particles in relativistic configurational space that was carried in [16] in the case of interaction between two relativistic spinless particles that have equal masses m in which the Compton wavelength of particle plays role of the natural scale. In this model is taken into account both the contribution to the proton form factor of vector mesons and the contribution from its the central part having radius of the Compton wavelength. The method of transition to the relativistic configurational representation in the case of interaction between two relativistic spinless particles with equal masses proposed in [16] was used in [17] to construct the three-dimensional covariant formalism for the description of relativistic two-particle systems. Within the framework of this formalism the expressions for the form factors of relativistic two-particle systems [18, 19] were obtained.

The aim of this work is to obtain the expression for the elastic form factor of relativistic two-particle system in the case of scalar current on the basis of covariant Hamiltonian formulation of quantum field theory [8, 9] by transition to the three-dimensional relativistic configurational representation for the interaction of two relativistic spinless particles having arbitrary masses m_1, m_2 [20, 21].

2 Equation for the vertex function

Applying the Kadyshevsky rules for the diagrams as given in [8, 9] allows to present the equation for the vertex function in the case of interaction between two relativistic spinless particles with arbitrary masses m_1 and m_2 (in full analogy with the equation for the scattering amplitude in [20, 21]) in the graphic form (Fig. 1). The graphic equation in accordance with the

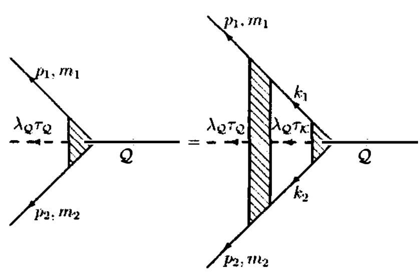

Figure 1: *The graphic equation for the vertex function in the case of interaction between two relativistic spinless particles with arbitrary masses that corresponds the Kadyshevsky rules for the diagrams.*

Kadyshevsky rules for the diagrams answers the integral equation [1)]

$$\Gamma_Q(p_1,p_2;\lambda_Q\tau_Q) = \frac{1}{(2\pi)^3}\int d\tau_K d^{(4)}k_1 d^{(4)}k_2 \theta(k_{10})\delta(k_1^2-m_1^2)\theta(k_{20})\times$$
$$\times\delta(k_2^2-m_2^2)\frac{1}{\tau_K - i\varepsilon}V(p_1,p_2;\lambda_Q\tau_Q|k_1,k_2;\lambda_Q\tau_K)\,\Gamma_Q(k_1,k_2;\lambda_Q\tau_K)\times$$
$$\times\delta^{(4)}(-Q+k_1+k_2+\lambda_Q\tau_K), \tag{1}$$

where all the momenta of the particles k_i (or p_i, q_i) belong to the mass shells

$$k_i^2 = k_{i0}^2 - \mathbf{k}_i^2 = m_i^2, i=1,2. \tag{2}$$

The equations (2) define the three-dimensional surfaces of a hyperboloids whose upper sheets serve as a models of the Lobachevsky space. Composite particle has the 4-momentum Q and moment $J=0$ moreover $Q^2=(q_1+q_2)^2 = Q_0^2 - \mathbf{Q}^2 = s_q = M_Q^2$. The solid lines correspond the components of composite particle, the dashed lines correspond quasi particles, and block of the diagrams that marked on fig. 1 as the trapezoid is considered as the quasipotential. As a vector λ_Q, it is convenient to choose the 4-velocity of the system $\lambda_Q = (\lambda_Q^0;\boldsymbol{\lambda}_Q) = Q/\sqrt{Q^2}$.

We notice that in given approach for the vertex function $\Gamma_Q(p_1,p_2;\lambda_Q\tau_Q)$ the two external of moment are parallel on the strength of choice of 4-velocity of the composite particle $\lambda_Q = Q/M_Q$. Consequently, for the bounded system of spinless particles which are found in the motion with moment $J=0$ the vertex function $\Gamma_Q(p_1,p_2;\lambda_Q\tau_Q)$ can depend only from the Lorentz scalars: $Q^2 = M_Q^2, p_i^2 = m_i^2, Qp_i = M_Q(\lambda_Q p_i), i = 1,2; (\lambda_Q\tau_Q)^2 = \tau_Q^2, p_1p_2, \lambda_Q Q$. Since all the momenta of the particles belong to the masses of hyperboloids (2), and $\lambda_Q Q = M_Q$, that only the four parameters are essential: $p_1p_2, \tau_Q, Qp_i, i=1,2$. Using the parallel condition $\lambda_Q \upuparrows Q$ and the conservation law $-Q+p_1+p_2+\lambda_Q\tau_Q = 0$, we find the three independent of correlations, connecting these four parameters:

$$M_Q - \tau_Q = \sqrt{m_1^2+m_2^2+2p_1p_2},$$
$$Qp_1 = \frac{M_Q(m_1^2+p_1p_2)}{M_Q-\tau_Q},\ Qp_2 = \frac{M_Q(m_2^2+p_1p_2)}{M_Q-\tau_Q}.$$

[1)] We use the system of units where $\hbar = c = 1$.

Signifies, the vertex function $\Gamma_Q(p_1,p_2;\lambda_Q\tau_Q)$ under $\lambda_Q \upuparrows Q$ depends only from one the Lorentz invariant scalar parameter, as which we choose Qp_1 and introduce the notations

$$\Gamma_Q(p_1,p_2;\lambda_Q\tau_Q) = \Gamma_{M_Q}(Qp_1),$$
$$V(p_1,p_2;\lambda_Q\tau_Q|k_1,k_2;\lambda_Q\tau_K) = V(Qp_1,Qk_1;\sqrt{s_q}).$$

Then, taking into consideration that

$$d^{(4)}k_i\theta(k_{i0})\delta(k_i^2-m_i^2) = d\mathbf{k}_i dk_{i0}\frac{\delta(k_{i0}-\sqrt{m_i^2+\mathbf{k}_i^2})}{2k_{i0}},$$
$$k_{i0} = \sqrt{m_i^2+\mathbf{k}_i^2}, \quad i=1,2, \tag{3}$$

and accounting the choice of the 4-velocity vector λ_Q, we execute the integrations in (1) on $dk_{i0}, i=1,2$. As a result expression (1) takes the form

$$\Gamma_{M_Q}(Qp_1) = \frac{1}{(2\pi)^3}\int d\tau_K \frac{d\mathbf{k}_1}{2\sqrt{m_1^2+\mathbf{k}_1^2}}\frac{d\mathbf{k}_2}{2\sqrt{m_2^2+\mathbf{k}_2^2}}\frac{1}{\tau_K-i\varepsilon}\times$$
$$\times V(Qp_1,Qk_1;\sqrt{s_q})\Gamma_{M_Q}(Qk_1)\delta^{(4)}\left[\left(-1+\frac{\tau_K}{M_Q}\right)Q+k_1+k_2\right]. \tag{4}$$

Now in (4) we execute the integrations respecting of $d\mathbf{k}_2, d\tau_K$. For that in integral on $d\mathbf{k}_2$ we execute the pure Lorentz transformation $L = \Lambda_{\lambda_Q}^{-1}$ corresponding to the 4-velocity λ_Q of composite particle: $\Lambda_{\lambda_Q}^{-1}Q = (M_Q;\mathbf{0})$. The group of motions of the Lobachevsky space is the Lorentz group. Therefore, the measures of integrations $d\Omega_{\mathbf{k}_i}$ on the masses of hyperboloids (2) are invariants of the pure Lorentz transformation $\Lambda_{\lambda_Q}^{-1}$:

$$d\Omega_{\mathbf{k}_i} = \frac{m_i\, d\mathbf{k}_i}{\sqrt{m_i^2+\mathbf{k}_i^2}} = \Lambda_{\lambda_Q}^{-1}d\Omega_{\mathbf{k}_i} = d\Omega_{\mathbf{\Delta}_{k_i,m_i\lambda_Q}} = \frac{m_i\, d\mathbf{\Delta}_{k_i,m_i\lambda_Q}}{\sqrt{m_i^2+\mathbf{\Delta}_{k_i,m_i\lambda_Q}^2}}, i=1,2, \tag{5}$$

where $\mathbf{\Delta}_{k_i,m_i\lambda_Q}$ is a spatial component of the 4-vector $\Delta_{k_i,m_i\lambda_Q}$ from the Lobachevsky space:

$$\Lambda_{\lambda_Q}^{-1}\mathbf{k}_i = \mathbf{\Delta}_{k_i,m_i\lambda_Q} = \mathbf{k}_i(-)m_i\boldsymbol{\lambda}_Q = \mathbf{k}_i - \boldsymbol{\lambda}_Q\left(k_{i0}-\frac{\mathbf{k}_i\cdot\boldsymbol{\lambda}_Q}{1+\lambda_Q^0}\right),$$
$$(\Lambda_{\lambda_Q}^{-1}k_i)^0 = \Delta^0_{k_i,m_i\lambda_Q} = k_{i0}\lambda_Q^0 - \mathbf{k}_i\cdot\boldsymbol{\lambda}_Q = \sqrt{m_i^2+\mathbf{\Delta}^2_{k_i,m_i\lambda_Q}}, \; i=1,2. \tag{6}$$

Besides, will take into account that $\mathcal{Q}p_1$ is the Lorentz scalar: $\mathcal{Q}p_1 = \Lambda^{-1}_{\lambda_\mathcal{Q}}(\mathcal{Q}p_1) = M_\mathcal{Q}\Delta^0_{p_1,m_1\lambda_\mathcal{Q}}$, and the δ-function in (4) is invariant of the Lorentz transformation (6):

$$\delta^{(4)}\left[\left(-1+\frac{\tau_\mathcal{K}}{M_\mathcal{Q}}\right)\mathcal{Q}+k_1+k_2\right]=$$
$$=\delta^{(4)}\left[\left(-1+\frac{\tau_\mathcal{K}}{M_\mathcal{Q}}\right)\Lambda^{-1}_{\lambda_\mathcal{Q}}\mathcal{Q}+\Lambda^{-1}_{\lambda_\mathcal{Q}}k_1+\Lambda^{-1}_{\lambda_\mathcal{Q}}k_2\right]=$$
$$=\delta(-M_\mathcal{Q}+\tau_\mathcal{K}+\Delta^0_{k_1,m_1\lambda_\mathcal{Q}}+\Delta^0_{k_2,m_2\lambda_\mathcal{Q}})\delta^{(3)}(\mathbf{\Delta}_{k_1,m_1\lambda_\mathcal{Q}}+\mathbf{\Delta}_{k_2,m_2\lambda_\mathcal{Q}}). \quad (7)$$

Then, taking into consideration relations (5)–(7) the equation in (4) is converted to the form

$$\Gamma_{M_\mathcal{Q}}(\mathbf{\Delta}_{p_1,m_1\lambda_\mathcal{Q}})=\frac{1}{(2\pi)^3}\int\frac{d\mathbf{\Delta}_{k_1,m_1\lambda_\mathcal{Q}}}{4\sqrt{m_1^2+\mathbf{\Delta}^2_{k_1,m_1\lambda_\mathcal{Q}}}\sqrt{m_2^2+\mathbf{\Delta}^2_{k_1,m_1\lambda_\mathcal{Q}}}}\times$$
$$\times\frac{V(\mathbf{\Delta}_{p_1,m_1\lambda_\mathcal{Q}},\mathbf{\Delta}_{k_1,m_1\lambda_\mathcal{Q}};\sqrt{s_q})\Gamma_{M_\mathcal{Q}}(\mathbf{\Delta}_{k_1,m_1\lambda_\mathcal{Q}})}{M_\mathcal{Q}-\sqrt{s_{\Delta_{k_1,m_1\lambda_\mathcal{Q}}}}-i\varepsilon}, \quad (8)$$

where the total energy of two free relativistic particles of arbitrary masses, $\sqrt{s_k}=\sqrt{(k_1+k_2)^2}$, is invariant of the Lorentz transformation (6):

$$\sqrt{s_k}=\Lambda^{-1}_{\lambda_\mathcal{Q}}\sqrt{s_k}=\sqrt{m_1^2+\mathbf{\Delta}^2_{k_1,m_1\lambda_\mathcal{Q}}}+\sqrt{m_2^2+\mathbf{\Delta}^2_{k_1,m_1\lambda_\mathcal{Q}}}=\sqrt{s_{\Delta_{k_1,m_1\lambda_\mathcal{Q}}}}, \quad (9)$$

and are introduced the notations

$$\Gamma_{M_\mathcal{Q}}(\mathcal{Q}p_1)=\Gamma_{M_\mathcal{Q}}(\mathbf{\Delta}_{p_1,m_1\lambda_\mathcal{Q}}),$$
$$V(\mathcal{Q}p_1,\mathcal{Q}k_1;\sqrt{s_q})=V(\mathbf{\Delta}_{p_1,m_1\lambda_\mathcal{Q}},\mathbf{\Delta}_{k_1,m_1\lambda_\mathcal{Q}};\sqrt{s_q}).$$

3 Equation for the wave function

The total energy of two free relativistic particles of arbitrary masses (9) can be expressed through the energy of one effective relativistic particle, $\Delta^0_{k',m'\lambda_\mathcal{Q}}$, having mass $m'=\sqrt{m_1m_2}$ and the relative 3-momentum $\mathbf{\Delta}_{k',m'\lambda_\mathcal{Q}}$

in the form (see Ref. [20, 21])

$$\sqrt{s_k} = \sqrt{s_{\Delta_{k_1,m_1\lambda_Q}}} = \frac{m'}{\mu}\Delta^0_{k',m'\lambda_Q}, \ \Delta^0_{k',m'\lambda_Q} = (\Lambda^{-1}_{\lambda_Q}k')^0 =$$
$$= k'_0\lambda^0_Q - \mathbf{k}'\cdot\boldsymbol{\lambda}_Q = \sqrt{m'^2 + \boldsymbol{\Delta}^2_{k'm'\lambda_Q}},$$
$$\boldsymbol{\Delta}_{k',m'\lambda_Q} = \Lambda^{-1}_{\lambda_Q}\mathbf{k}' = \mathbf{k}'(-)m'\boldsymbol{\lambda}_Q = \mathbf{k}' - \boldsymbol{\lambda}_Q\left(k'_0 - \frac{\mathbf{k}'\cdot\boldsymbol{\lambda}_Q}{1+\lambda^0_Q}\right), \tag{10}$$

where $\mu = m_1m_2/(m_1+m_2)$ is the usual reduced mass. The 4-vector k' is chosen as

$$k' = (k'_0;\mathbf{k}') = \sqrt{\frac{\mathcal{K}^2}{\mathcal{K}^2_\perp}}\mathcal{K}_\perp, \tag{11}$$

where $\mathcal{K} = (m_2k_1 - m_1\dot{k}_2)/(m_1+m_2)$, the vector $\mathcal{K}_\perp = \mathcal{K} - \lambda_\mathcal{K}(\lambda_\mathcal{K}\mathcal{K})$ is the Wightman–Gording vector, and $\lambda_\mathcal{K} = (k_1+k_2)/\sqrt{s_k} = \lambda_Q$. Signifies, $(\lambda_\mathcal{K}\mathcal{K}_\perp) = 0$, but from (11) we find:

$$k'^2 = {k_0}'^2 - \mathbf{k}'^2 = \mathcal{K}^2 = \frac{m_1m_2}{(m_1+m_2)^2}\left[(m_1+m_2)^2 - s_k\right]. \tag{12}$$

Under the Lorentz transformation (6) from the correlation (7) follows that

$$\boldsymbol{\Delta}_{k_2,m_2\lambda_Q} = -\boldsymbol{\Delta}_{k_1,m_1\lambda_Q}, \ \Lambda^{-1}_{\lambda_Q}\lambda_Q = (1;\mathbf{0}), \ \Lambda^{-1}_{\lambda_Q}\mathcal{K}_\perp = (0;\boldsymbol{\Delta}_{k_1,m_1\lambda_Q}),$$
$$\Lambda^{-1}_{\lambda_Q}\mathcal{K} = \left(\frac{m_2\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_1,m_1\lambda_Q}} - m_1\sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_1,m_1\lambda_Q}}}{m_1+m_2};\boldsymbol{\Delta}_{k_1,m_1\lambda_Q}\right). \tag{13}$$

Then from (9), (12) and (13) we get expression $(\Lambda^{-1}_{\lambda_Q}k'_0 = 0)$

$$\boldsymbol{\Delta}^2_{k',m'\lambda_Q} = -(\Lambda^{-1}_{\lambda_Q}k')^2 = -(\Lambda^{-1}_{\lambda_Q}\mathcal{K})^2 =$$
$$= \frac{m_1m_2}{(m_1+m_2)^2}\left[s_{\Delta_{k',m'\lambda_Q}} - (m_1+m_2)^2\right],$$

whence and follows the formula (10).

As direction of the vector $\boldsymbol{\Delta}_{k',m'\lambda_Q}$ in correspondence to (11) and (13), we choose the direction of the vector $\boldsymbol{\Delta}_{k_1,m_1\lambda_Q}$:

$$\boldsymbol{\Delta}_{k',m'\lambda_Q} = \sqrt{\frac{(\Lambda_{\lambda_Q}^{-1}\mathcal{K})^2}{(\Lambda_{\lambda_Q}^{-1}\mathcal{K}_\perp)^2}}(\Lambda_{\lambda_Q}^{-1}\mathcal{K}_\perp) = \frac{\boldsymbol{\Delta}_{k_1,m_1\lambda_Q}}{|\boldsymbol{\Delta}_{k_1,m_1\lambda_Q}|}\times$$

$$\times\left[\boldsymbol{\Delta}^2_{k_1,m_1\lambda_Q} - \left(\frac{m_2\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_1,m_1\lambda_Q}} - m_1\sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_1,m_1\lambda_Q}}}{m_1+m_2}\right)^2\right]^{1/2}. \tag{14}$$

The inverse transformation have the form

$$\boldsymbol{\Delta}_{k_1,m_1\lambda_Q} = \boldsymbol{\Delta}_{k',m'\lambda_Q}\frac{m'}{2\mu}\sqrt{\frac{4\mu^2+\boldsymbol{\Delta}^2_{k',m'\lambda_Q}}{m'^2+\boldsymbol{\Delta}^2_{k',m'\lambda_Q}}}. \tag{15}$$

From the correlations (9) and (10) follows

$$\begin{aligned}\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_1,m_1\lambda_Q}} &= \frac{m'}{2\mu}f_\mp^{-1}(\boldsymbol{\Delta}_{k',m'\lambda_Q}),\\ \sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_1,m_1\lambda_Q}} &= \frac{m'}{2\mu}f_\pm^{-1}(\boldsymbol{\Delta}_{k',m'\lambda_Q}),\end{aligned} \tag{16}$$

where

$$f_\pm(\boldsymbol{\Delta}_{k',m'\lambda_Q}) = \frac{\sqrt{m'^2+\boldsymbol{\Delta}^2_{k',m'\lambda_Q}}}{m'^2+\boldsymbol{\Delta}^2_{k',m'\lambda_Q} \pm m'\sqrt{m'^2-4\mu^2}}.$$

The relativistic three-dimensional volume element in the Lobachevsky space (5) is converted as

$$\frac{d\boldsymbol{\Delta}_{k_1,m_1\lambda_Q}}{\sqrt{m_1^2+\boldsymbol{\Delta}^2_{k_1,m_1\lambda_Q}}\sqrt{m_2^2+\boldsymbol{\Delta}^2_{k_1,m_1\lambda_Q}}} = \frac{m'}{2\mu}\frac{d\boldsymbol{\Delta}_{k',m'\lambda_Q}}{\sqrt{m'^2+\boldsymbol{\Delta}^2_{k',m'\lambda_Q}}}f(\boldsymbol{\Delta}_{k',m'\lambda_Q}), \tag{17}$$

where

$$f(\boldsymbol{\Delta}_{k',m'\lambda_Q}) = \frac{\sqrt{4\mu^2+\boldsymbol{\Delta}^2_{k',m'\lambda_Q}}}{m'^2+\boldsymbol{\Delta}^2_{k',m'\lambda_Q}}.$$

Now in (8) we perform the change of variables as in (14), (15) and take into account expressions (10) and (17). Then equation (8) is converted to the form

$$\Gamma_{M_Q}(\mathbf{\Delta}_{p',m'\lambda_Q}) = \frac{2\mu}{m'}\frac{1}{(2\pi)^3}\times$$
$$\times\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_Q}} \frac{V(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{\Delta}_{k',m'\lambda_Q}; \Delta^0_{q',m'\lambda_Q})}{8\mu}\frac{f(\mathbf{\Delta}_{k',m'\lambda_Q})\Gamma_{M_Q}(\mathbf{\Delta}_{k',m'\lambda_Q})}{2\Delta^0_{q',m'\lambda_Q} - 2\Delta^0_{k',m'\lambda_Q} - i\varepsilon}, \tag{18}$$

where are introduced the notations

$$\Gamma_{M_Q}(\mathbf{\Delta}_{p_1,m_1\lambda_Q}) = \Gamma_{M_Q}(\mathbf{\Delta}_{p',m'\lambda_Q}),\ \sqrt{s_q} = M_Q = \frac{m'}{\mu}\Delta^0_{q',m'\lambda_Q},$$
$$V(\mathbf{\Delta}_{p_1,m_1\lambda_Q}, \mathbf{\Delta}_{k_1,m_1\lambda_Q}; \sqrt{s_q}) = V(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{\Delta}_{k',m'\lambda_Q}; \Delta^0_{q',m'\lambda_Q}),$$

and $d\Omega_{\mathbf{\Delta}_{k',m'\lambda_Q}} = m' d\mathbf{\Delta}_{k',m'\lambda_Q}/\Delta^0_{k',m'\lambda_Q}$ is the relativistic three-dimensional volume element in the Lobachevsky space. The now all the momenta of the particles belong to the mass shell

$$\Delta^{0^2}_{k',m'\lambda_Q} - \mathbf{\Delta}^2_{k',m'\lambda_Q} = m'^2, \tag{19}$$

which defines the three-dimensional surface of a hyperboloid whose upper sheet serves as a model of the Lobachevsky space momentum.

The wave function of system in momentum space we define as

$$\Psi_{M_Q}(\mathbf{\Delta}_{p',m'\lambda_Q}) = \frac{f(\mathbf{\Delta}_{p',m'\lambda_Q})\Gamma_{M_Q}(\mathbf{\Delta}_{p',m'\lambda_Q})}{2^{3/2}\sqrt{m'}(2\Delta^0_{q',m'\lambda_Q} - 2\Delta^0_{p',m'\lambda_Q})} \tag{20}$$

and we introduce the notation

$$\widetilde{V}(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{\Delta}_{k',m'\lambda_Q}; \Delta^0_{q',m'\lambda_Q}) =$$
$$= \frac{1}{8\mu} f(\mathbf{\Delta}_{p',m'\lambda_Q}) V(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{\Delta}_{k',m'\lambda_Q}; \Delta^0_{q',m'\lambda_Q}).$$

Then instead of equation (18) we obtain the completely covariant RQP-equation [2)]:

$$(2\Delta^0_{q',m'\lambda_Q} - 2\Delta^0_{p',m'\lambda_Q})\Psi_{M_Q}(\mathbf{\Delta}_{p',m'\lambda_Q}) =$$
$$= \frac{2\mu}{m'}\frac{1}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_Q}} \widetilde{V}(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{\Delta}_{k',m'\lambda_Q}; \Delta^0_{q',m'\lambda_Q})\Psi_{M_Q}(\mathbf{\Delta}_{k',m'\lambda_Q}). \tag{21}$$

[2)] This equation was received by other way in [20, 21].

Eq. (21) can be considered as a relativistic generalization of the Schrödinger equation in the spirit of the Lobachevsky geometry realized on upper sheet of the mass-shell hyperboloid (19). This equation describes the scattering on quasipotential $\widetilde{V}(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{\Delta}_{k',m'\lambda_Q}; \Delta^0_{q',m'\lambda_Q})$ of effective relativistic particle that plays the role of two-particle system, has a mass m' and a relative 3-momentum $\mathbf{\Delta}_{q',m'\lambda_Q}$, and carries the total energy $\sqrt{s_q} = M_Q$ of interacting particles, proportional to the energy $\Delta^0_{q',m'\lambda_Q}$ of one effective relativistic particle of the mass m' (10).

In the equation (21) it is convenient to expand over the complete system of functions [20, 21]

$$\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}) = \left(\frac{\Delta^0_{p',m'\lambda_Q} - \mathbf{\Delta}_{p',m'\lambda_Q} \cdot \mathbf{n}}{m'} \right)^{-1-ir/\lambda'}, \tag{22}$$

which realize the principal series of unitary irreducible representations of the Lorentz group, i.e. the group of motions of the Lobachevsky space momentum, realized on upper sheet of the mass hyperboloid (19). In the nonrelativistic limit ($|\mathbf{\Delta}_{p',m'\lambda_Q}| \ll 1/\lambda'$, $r \gg \lambda'$) $\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}) \to \exp(i\mathbf{\Delta}_{p',m'\lambda_Q} \cdot \mathbf{r})$. The group parameter r in (22) plays the role of the modulus of the relativistic relative coordinate $\mathbf{r}$ ($\mathbf{r} = r\mathbf{n}$, $|\mathbf{n}| = \mathbf{1}$), and $\lambda' = 1/m'$ is the Compton wavelength associated with the effective relativistic particle of mass m' [16, 21]. This parameter enumerates the eigenvalues of the invariant Casimir operator of the Lorentz group $\hat{C}_L = (1/4)M_{\mu\nu}M^{\mu\nu}$ ($M_{\mu\nu} = p_\mu\partial/\partial p^\nu - p_\nu\partial/\partial p^\mu$ are the group generators):

$$\widehat{C}_L\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}) = \left(\frac{1}{m'^2} + r^2\right) \xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}), 0 \le r < \infty, \tag{23}$$

and, therefore, it is a relativistic invariant.

The functions in (22) obey the following conditions of completeness and orthogonality [21]:

$$\frac{1}{(2\pi)^3} \int d\Omega_{\mathbf{\Delta}_{p',m'\lambda_Q}} \xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r})\xi^*(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}') = \delta(\mathbf{r}' - \mathbf{r}),$$

$$\frac{1}{(2\pi)^3} \int d\mathbf{r}\xi(\mathbf{\Delta}_{q',m'\lambda_Q}, \mathbf{r})\xi^*(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}) = \frac{\Delta^0_{q',m'\lambda_Q}}{m'}\delta(\mathbf{\Delta}_{p',m'\lambda_Q} - \mathbf{\Delta}_{q',m'\lambda_Q}), \tag{24}$$

and these the functions satisfy the equation in terms of finite differences [21]

$$(2\Delta^0_{p',m'\lambda_Q} - \widehat{H}_0)\xi(\mathbf{\Delta}_{p',m'\lambda_Q}, \mathbf{r}) = 0. \tag{25}$$

Here

$$\widehat{H}_0 = 2m'\left[\cosh\left(i\lambda'\frac{\partial}{\partial r}\right) + \frac{i\lambda'}{r}\sinh\left(i\lambda'\frac{\partial}{\partial r}\right) - \frac{\lambda'^2}{2r^2}\Delta_{\theta,\varphi}\exp\left(i\lambda'\frac{\partial}{\partial r}\right)\right] \tag{26}$$

is the operator of the free Hamiltonian, while $\Delta_{\theta,\varphi}$ is its angular part.

The wave RQP-functions in the momentum space and the $\mathbf{r}$-representation, called the relativistic configuration representation [20, 21], are related by

$$\psi_{M_Q}(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{p',m'\lambda_Q}}\xi(\mathbf{\Delta}_{p',m'\lambda_Q},\mathbf{r})\Psi_{M_Q}(\mathbf{\Delta}_{p',m'\lambda_Q}),$$
$$\Psi_{M_Q}(\mathbf{\Delta}_{p',m'\lambda_Q}) = \int d\mathbf{r}\xi^*(\mathbf{\Delta}_{p',m'\lambda_Q},\mathbf{r})\psi_{M_Q}(\mathbf{r}). \tag{27}$$

For the local quasipotential

$$\widetilde{V}(\mathbf{\Delta}_{p',m'\lambda_Q},\mathbf{\Delta}_{k',m'\lambda_Q};\Delta^0_{q',m'\lambda_Q}) \equiv \widetilde{V}\left((\mathbf{\Delta}_{p',m'\lambda_Q}(-)\mathbf{\Delta}_{k',m'\lambda_Q})^2;\Delta^0_{q',m'\lambda_Q}\right) \tag{28}$$

square of the vector of momentum transfer in the Lobachevsky space $\mathbf{\Delta}_{p',k'} = \mathbf{p}'(-)\mathbf{k}'$ is the Loretz invariant that allows to present it in the form

$$\mathbf{\Delta}^2_{p',k'} = \left(\Delta^0_{p',k'}\right)^2 - m'^2 = \left(\frac{\Delta_{p',m'\lambda_Q}\Delta_{k',m'\lambda_Q}}{m'}\right)^2 - m'^2 =$$
$$= \left(\left(\mathbf{\Delta}_{p',m'\lambda_Q}(-)\mathbf{\Delta}_{k',m'\lambda_Q}\right)^0\right)^2 - m'^2 = \left(\mathbf{\Delta}_{p',m'\lambda_Q}(-)\mathbf{\Delta}_{k',m'\lambda_Q}\right)^2 =$$
$$= \mathbf{\Delta}^2_{\mathbf{\Delta}_{p',m'\lambda_Q},\mathbf{\Delta}_{k',m'\lambda_Q}}.$$

Thus, the quasipotential (28) depends on the invariant quantity the square of vector of difference in the Lobachevsky space of two momentum vectors $\mathbf{\Delta}_{\mathbf{\Delta}_{p',m'\lambda_Q},\mathbf{\Delta}_{k',m'\lambda_Q}} = \mathbf{\Delta}_{p',m'\lambda_Q}(-)\mathbf{\Delta}_{k',m'\lambda_Q}$. With this quasipotential, the right-hand side of equation (21) represents a convolution in the Lobachevsky space that allows to use the expansion over the matrix elements of group of motions of this space, i.e. transformations (27). By using transformations (27) and eq. (25), equation (21) with the quasipotential (28) local in the Lobachevsky space takes the form

$$(2\Delta^0_{q',m'\lambda_Q} - \widehat{H}_0)\psi_{M_Q}(\mathbf{r}) = \frac{2\mu}{m'}V(\mathbf{r};\Delta^0_{q',m'\lambda_Q})\psi_{M_Q}(\mathbf{r}), \tag{29}$$

where the quasipotential $V(\mathbf{r};\Delta^0_{q',m'\lambda_Q})$ is given in terms of the same relativistic plane waves as

$$V(\mathbf{r};\Delta^0_{q',m'\lambda_Q}) = \frac{1}{(2\pi)^3}\int d\Omega_{\mathbf{\Delta}_{p',k'}}\xi(\mathbf{\Delta}_{p',k'},\mathbf{r})\widetilde{V}\left((\mathbf{\Delta}_{p',k'})^2;\Delta^0_{q',m'\lambda_Q}\right).$$

For spherically symmetric potentials, expanding the quasipotential wave RQP-function $\psi_{M_Q}(\mathbf{r})$ in the Legendre functions $P^\nu_\mu(z)$ of the first kind as

$$\psi_{M_Q}(\mathbf{r}) = \sum_{\ell=0}^{\infty}(2\ell+1)i^\ell\frac{\varphi_\ell(r,\chi)}{r}P_\ell\left(\frac{\mathbf{\Delta}_{q',m'\lambda_Q}\cdot\mathbf{r}}{|\mathbf{\Delta}_{q',m'\lambda_Q}|r}\right), \tag{30}$$

we obtain equation for the partial wave function in the form

$$\left[\cosh\left(i\lambda'\frac{d}{dr}\right) + \frac{\lambda'^2\ell(\ell+1)}{2r(r+i\lambda')}\exp\left(i\lambda'\frac{d}{dr}\right) - X(r)\right]\varphi_\ell(r,\chi) = 0, \tag{31}$$

where

$$X(r) = \frac{\mu}{m'^2}\left(M_Q - V(r;\chi)\right),$$

and χ is the rapidity related with the relative 3-momentum and energy of effective relativistic particle by the formulas

$$\mathbf{\Delta}_{q',m'\lambda_Q} = m'\sinh\chi\mathbf{n}_{\Delta_{q',m'\lambda_Q}}, \quad |\mathbf{n}_{\Delta_{q',m'\lambda_Q}}| = 1,$$
$$M_Q = \frac{m'}{\mu}\Delta^0_{q',m'\lambda_Q}, \quad \Delta^0_{q',m'\lambda_Q} = m'\cosh\chi.$$

4 Form factor of the relativistic two-particle system

For simplicity we consider here only the case of spinless field when the Hamiltonian density is given by the expression

$$H(x) = -z_1\varphi_1^+(x)\varphi_1(x)A(x) - z_2\varphi_2^+(x)\varphi_2(x)A(x). \tag{32}$$

In ref. [18] founded on refs. [10–14], the form factor of two-particle system was defined as the matrix element of the local current operator between bound states with the 4-momentum $\mathcal{P}$, $\mathcal{Q}$ through the covariant wave RQP-functions satisfying eq. (21). Then, as follows from refs. [18, 19], the invariant expression in the momentum representation for the matrix element of the local current operator near poles of bound states for the interaction of two relativistic spinless particles with arbitrary masses m_1, m_2 has the form

$$< \mathcal{P}|J(0)|\mathcal{Q} >= z_1 \frac{1}{(2\pi)^3} \int d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d^{(4)}k_2 d^{(4)}k_1 d^{(4)}k_1' \theta(k_{20})\delta(k_2^2 - m_2^2)\times$$
$$\times \Gamma^+_{\mathcal{P}}(k_2, k_1'; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) \frac{1}{(\tau_{\mathcal{P}} + i\varepsilon)(\tau_{\mathcal{Q}} - i\varepsilon)} \Gamma_{\mathcal{Q}}(k_2, k_1; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}})\, \theta(k_{10})\delta({k_1}^2 - m_1^2)\times$$
$$\times \theta(k_{10}')\delta(k_1'^2 - m_1^2)\delta^{(4)}(-\mathcal{Q} + k_1 + k_2 + \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}) \times$$
$$\times \delta^{(4)}(\mathcal{P} - k_2 - k_1' - \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) + (1 \leftrightarrow 2), \qquad (33)$$

where all the momenta of the particles belong to the mass shells (2). As a vectors $\lambda_{\mathcal{P}}$ and $\lambda_{\mathcal{Q}}$, it is convenient to choose the 4-velocities of the system: $\lambda_{\mathcal{P}} = \mathcal{P}/\sqrt{\mathcal{P}^2}, \mathcal{P}^2 = (p_1 + p_2)^2 = s_p = M^2_{\mathcal{P}}$ and $\lambda_{\mathcal{Q}} = \mathcal{Q}/\sqrt{\mathcal{Q}^2}, \mathcal{Q}^2 = (q_1 + q_2)^2 = s_q = M^2_{\mathcal{Q}}$.

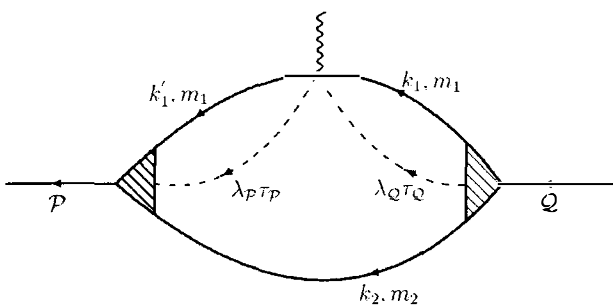

Figure 2: *The diagram for the matrix element of the local current operator between bound states with the 4-momentum $\mathcal{P}$, $\mathcal{Q}$ for the interaction of two relativistic spinless particles with arbitrary masses.*

This equation answers the diagram on fig. 2. Here follows to emphasize that because of transition to different own timeses of the system before ($\tau_{\mathcal{Q}} = \lambda_{\mathcal{Q}}X, X = x_1 + x_2$) and after interaction ($\tau_{\mathcal{P}} = \lambda_{\mathcal{P}}X$) the diagram on fig. 2 differ from diagrams, which appear in approach of the Kadyshevsky for S-matrix. The 4-velocities of the composite particle before, $\lambda_{\mathcal{Q}}$, and after interaction, $\lambda_{\mathcal{P}}$, will differ also.

As it was installed in section 2, for the bounded system of spinless particles which are found in the motion with moment $J = 0$ the vertex functions $\Gamma_{\mathcal{Q}}(k_2, k_1; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}})$ and $\Gamma_{\mathcal{P}}(k_2, k_1'; \lambda_{\mathcal{P}}\tau_{\mathcal{P}})$ when $\lambda_{\mathcal{Q}} \upuparrows \mathcal{Q}$ and $\lambda_{\mathcal{P}} \upuparrows \mathcal{P}$ will depend each only on one the Lorentz invariant scalar parameter, as which we choose accordingly $\mathcal{Q}k_2$ and $\mathcal{P}k_2$ and introduce the notations

$$\Gamma_{\mathcal{Q}}(k_2, k_1; \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}) = \Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2),\ \Gamma_{\mathcal{P}}(k_2, k_1'; \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) = \Gamma_{M_{\mathcal{P}}}(\mathcal{P}k_2).$$

Then, taking into consideration (3) and accounting choice of the 4-velocities vectors $\lambda_{\mathcal{Q}} = \mathcal{Q}/M_{\mathcal{Q}}$ and $\lambda_{\mathcal{P}} = \mathcal{P}/M_{\mathcal{P}}$, we execute the integrations in (33) respecting of variables $dk_{i0}, i = 1, 2, dk_{10}'$. As a result expression for current (33) takes the form

$$\begin{aligned} < \mathcal{P}|J(0)|\mathcal{Q} >= \frac{z_1}{(4\pi)^3} \int \frac{d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d\mathbf{k}_2 d\mathbf{k}_1 d\mathbf{k}_1'}{\sqrt{m_2^2 + \mathbf{k}_2^2}\sqrt{m_1^2 + \mathbf{k}_1^2}\sqrt{m_1^2 + \mathbf{k}_1'^2}} \Gamma^+_{M_{\mathcal{P}}}(\mathcal{P}k_2) \times \\ \times \frac{1}{(\tau_{\mathcal{P}} + i\varepsilon)(\tau_{\mathcal{Q}} - i\varepsilon)} \Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2) \delta^{(4)} \left[\left(-1 + \frac{\tau_{\mathcal{Q}}}{M_{\mathcal{Q}}} \right) \mathcal{Q} + k_1 + k_2 \right] \times \\ \times \delta^{(4)} \left[\left(1 - \frac{\tau_{\mathcal{P}}}{M_{\mathcal{P}}} \right) \mathcal{P} - k_2 - k_1' \right] + (1 \leftrightarrow 2). \end{aligned} \quad (34)$$

Now in (34) we execute the integrations respecting of $d\mathbf{k}_1, d\mathbf{k}_1', d\tau_{\mathcal{P}}, d\tau_{\mathcal{Q}}$. For that in integrales on $d\mathbf{k}_1$ and $d\mathbf{k}_1'$ we execute the pure Lorentz transformations $\Lambda_{\lambda_{\mathcal{Q}}}^{-1}$ and $\Lambda_{\lambda_{\mathcal{P}}}^{-1}$ accordingly: $\Lambda_{\lambda_{\mathcal{Q}}}^{-1}\mathcal{Q} = (M_{\mathcal{Q}}; \mathbf{0})$, $\Lambda_{\lambda_{\mathcal{P}}}^{-1}\mathcal{P} = (M_{\mathcal{P}}; \mathbf{0})$. We take into account that $\mathcal{Q}k_2$ and $\mathcal{P}k_2$ are the Lorentz scalars: $\mathcal{Q}k_2 = M_{\mathcal{Q}}\Delta^0_{k_2, m_2\lambda_{\mathcal{Q}}}$, $\mathcal{P}k_2 = M_{\mathcal{P}}\Delta^0_{k_2, m_2\lambda_{\mathcal{P}}}$, and the measures of integrations $d\Omega_{\mathbf{k}_i}$ and δ-functions in (34) on the masses of hyperboloids (2) are invariants of the Lorentz transformations $\Lambda^{-1}_{\lambda_{\mathcal{Q},\mathcal{P}}}$ (5), (7). Then expression for current in (34) is converted to the form

$$\begin{aligned} &< \mathcal{P}|J(0)|\mathcal{Q} >= \\ &= \frac{z_1}{(4\pi)^3} \int \frac{d\mathbf{\Delta}_{k_2, m_2\lambda_{\mathcal{Q}}}}{\sqrt{m_2^2 + \mathbf{\Delta}^2_{k_2, m_2\lambda_{\mathcal{Q}}}}} \frac{\Gamma^+_{M_{\mathcal{P}}}(\mathbf{\Delta}_{k_2, m_2\lambda_{\mathcal{P}}})}{\sqrt{m_1^2 + \mathbf{\Delta}^2_{k_2, m_2\lambda_{\mathcal{P}}}} \left(M_{\mathcal{P}} - \sqrt{s_{\Delta_{k_2, m_2\lambda_{\mathcal{P}}}}} \right)} \times \\ &\times \frac{\Gamma_{M_{\mathcal{Q}}}(\mathbf{\Delta}_{k_2, m_2\lambda_{\mathcal{Q}}})}{\sqrt{m_1^2 + \mathbf{\Delta}^2_{k_2, m_2\lambda_{\mathcal{Q}}}} \left(M_{\mathcal{Q}} - \sqrt{s_{\Delta_{k_2, m_2\lambda_{\mathcal{Q}}}}} \right)} + (1 \leftrightarrow 2), \end{aligned} \quad (35)$$

where the total energy of two free relativistic particles of arbitrary masses is invariant of the Lorentz transformation (6),

$$\sqrt{s_{k'}} = \sqrt{(k_2 + k_1')^2} = \sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{P}}}}} = \sqrt{m_1^2 + \mathbf{\Delta}^2_{k_2,m_2\lambda_{\mathcal{P}}}} + \sqrt{m_2^2 + \mathbf{\Delta}^2_{k_2,m_2\lambda_{\mathcal{P}}}},$$
$$\sqrt{s_k} = \sqrt{(k_2 + k_1)^2} = \sqrt{s_{\Delta_{k_2,m_2\lambda_{\mathcal{Q}}}}} = \sqrt{m_1^2 + \mathbf{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}}}} + \sqrt{m_2^2 + \mathbf{\Delta}^2_{k_2,m_2\lambda_{\mathcal{Q}}}},$$

and were introduced the notations

$$\Gamma_{M_{\mathcal{P}}}(\mathcal{P}k_2) = \Gamma_{M_{\mathcal{P}}}(\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{P}}}),\ \Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2) = \Gamma_{M_{\mathcal{Q}}}(\mathbf{\Delta}_{k_2,m_2\lambda_{\mathcal{Q}}}).$$

Farther, similarly previous section, in expression (35) we shall perform the change of variables as in (14), (15). We take into account correlations (10), (16), (17) and determination (20) too. Then expression (35) takes the form ($M_{\mathcal{P}} = M_{\mathcal{Q}} = M$)

$$< \mathcal{P}|J(0)|\mathcal{Q} >= \frac{z_1 + z_2}{(2\pi)^3} \left(\frac{2\mu}{m'}\right)^2 \int d\Omega_{\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}} \Psi^*_M(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}}) \times$$
$$\times \frac{f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}}) + f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})}{2f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})} \Psi_M(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}), \tag{36}$$

where factor $(f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}}) + f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}}))/2f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})$ possible to simplify to the form

$$\frac{f_+(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}}) + f_-(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})}{2f(\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}})} \approx 1 + \frac{m'^2 - 4\mu^2}{2(m'^2 + \mathbf{\Delta}^2_{k',m'\lambda_{\mathcal{P}}})},$$
$$\frac{m'\sqrt{m'^2 - 4\mu^2}}{m'^2 + \mathbf{\Delta}^2_{k',m'\lambda_{\mathcal{P}}}} < 1, \quad \frac{m'^2 - 4\mu^2}{4\mu^2 + \mathbf{\Delta}^2_{k',m'\lambda_{\mathcal{P}}}} < 1. \tag{37}$$

We shall notice that the 3-vector $\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}}$ is a spatial component of the 4-vector $\Delta_{k',m'\lambda_{\mathcal{P}}} = \Lambda^{-1}_{\lambda_{\mathcal{P}}} k'$ from the momentum space with the Lobachevsky geometry, belonging to the same mass hyperboloid (19). This a 3-vector can be presented in the manner of

$$\mathbf{\Delta}_{k',m'\lambda_{\mathcal{P}}} = \mathbf{k}'(-)m'\lambda_{\mathcal{P}} = \Lambda^{-1}_{\lambda_{\mathcal{P}}}\mathbf{k}' = \Lambda^{-1}_{\lambda_{\mathcal{P}}}\Lambda_{\lambda_{\mathcal{Q}}}\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}} =$$
$$= \left(\Lambda^{-1}_{\lambda_{\mathcal{P}}}\Lambda_{\lambda_{\mathcal{Q}}}\Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}\right)\left(\Lambda^{-1}_{\Delta_{\mathcal{P},\mathcal{Q}}}\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}\right) = V(\Lambda_{\lambda_{\mathcal{Q}}}, \mathcal{P})\mathbf{\Delta}_{k',m'\lambda_{\mathcal{Q}}}(-)\frac{m'}{M}\mathbf{\Delta}_{\mathcal{P},\mathcal{Q}}. \tag{38}$$

Here $\Delta_{\mathcal{P},\mathcal{Q}} = \Lambda^{-1}_{\lambda_{\mathcal{Q}}}\mathcal{P}$ is the 4-vector of momentum transfer of the system in the relativistic Lobachevsky space momentum:

$$\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}} = \Lambda^{-1}_{\mathcal{Q}}\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{P}}(-)\boldsymbol{\mathcal{Q}} = \boldsymbol{\mathcal{P}} - \frac{\boldsymbol{\mathcal{Q}}}{M}\left(\mathcal{P}_0 - \frac{\boldsymbol{\mathcal{P}}\cdot\boldsymbol{\mathcal{Q}}}{\mathcal{Q}_0 + M}\right) = M\sinh\chi_\Delta \mathbf{n}_\Delta,$$

$$\Delta^0_{\mathcal{P},\mathcal{Q}} = \left(\Lambda^{-1}_{\mathcal{Q}}\mathcal{P}\right)^0 = \frac{\mathcal{P}_0\mathcal{Q}_0 - \boldsymbol{\mathcal{P}}\cdot\boldsymbol{\mathcal{Q}}}{M} = \frac{\mathcal{P}\mathcal{Q}}{M} = M\cosh\chi_\Delta,$$

$$\boldsymbol{\mathcal{P}} = M\sinh\chi_{\mathcal{P}}\mathbf{n}_{\mathcal{P}},\ \boldsymbol{\mathcal{Q}} = M\sinh\chi_{\mathcal{Q}}\mathbf{n}_{\mathcal{Q}}, \mathcal{P}_0 = M\cosh\chi_{\mathcal{P}},\ \mathcal{Q}_0 = M\cosh\chi_{\mathcal{Q}},$$

$$|\mathbf{n}_{\mathcal{P}}| = |\mathbf{n}_{\mathcal{Q}}| = |\mathbf{n}_\Delta| = 1, {\Delta^0}^2_{\mathcal{P},\mathcal{Q}} - \boldsymbol{\Delta}^2_{\mathcal{P},\mathcal{Q}} = M^2, \tag{39}$$

where $\chi_\Delta, \chi_{\mathcal{P}}, \chi_{\mathcal{Q}}$ are the corresponding rapidities, and

$$V(\Lambda_{\lambda_{\mathcal{Q}}}, \mathcal{P}) = \Lambda^{-1}_{\lambda_{\mathcal{P}}}\Lambda_{\lambda_{\mathcal{Q}}}\Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}$$

is the Wigner rotation matrix. Herewith from (38) and (39) follows that

$$\Delta^0_{k',m'\lambda_{\mathcal{P}}} \approx \frac{m'^2\Delta^0_{\mathcal{P},\mathcal{Q}}}{2M\Delta^0_{k',m'\lambda_{\mathcal{Q}}}}, \tag{40}$$

and the square of 4-momentum transfer of the system $t = (\mathcal{P} - \mathcal{Q})^2 = -Q^2$ is connected with the vector of momentum transfer $\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}}$ by expression

$$Q^2 = -t = -2M^2 + 2M\sqrt{M^2 + \boldsymbol{\Delta}^2_{\mathcal{P},\mathcal{Q}}} = 2M^2\left(\cosh\chi_\Delta - 1\right). \tag{41}$$

The elastic form factor $F(t)$ for the system of two relativistic spinless particles with arbitrary masses in the case of $J = 0$ and the scalar current we define as

$$F(t) = < \mathcal{P}|J(0)|\mathcal{Q} > . \tag{42}$$

Consequently, the form factor $F(t)$ possible consider as an invariant function which depends only on the invariant quantity the square of modulus of vector $\boldsymbol{\Delta}^2_{\mathcal{P},\mathcal{Q}}$ in the Lobachevsky space. Then, taking into consideration correlations (36)–(42), expression for the elastic form factor represents a convolution of covariant wave RQP-functions in this space:

$$F(\boldsymbol{\Delta}^2_{\mathcal{P},\mathcal{Q}}) = \frac{z_1+z_2}{(2\pi)^3}\left(\frac{2\mu}{m'}\right)^2 \times$$
$$\times \left[\int d\Omega_{\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}} \Psi^*_M\left(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}(-)\frac{m'}{M}\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}}\right)\Psi_M(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}) + \right.$$
$$+\frac{8M^4(m'^2-4\mu^2)}{m'^4(2M^2-t)^2}\times$$
$$\left.\times \int d\Omega_{\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}} \Psi^*_M\left(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}(-)\frac{m'}{M}\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}}\right)\Delta^{0\,2}_{k',m'\lambda_{\mathcal{Q}}}\Psi_M(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}})\right]. \tag{43}$$

By using the transformations (27), the addition theorem of relativistic plane waves (22) [21],

$$\int d\omega_n \xi\left(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}}(-)\frac{m'}{M}\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right) = \int d\omega_n \xi(\boldsymbol{\Delta}_{k',m'\lambda_{\mathcal{Q}}},\mathbf{r})\xi^*\left(\frac{m'}{M}\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right), \tag{44}$$

the condition of completeness in (24), equation (25), and the Hermitian of operator of the free Hamiltonian (26), the form factor in (43) can be represented in the form of relativistic Fourier image of covariant waves RQP-functions in the configuration representation [3)]:

$$F(\boldsymbol{\Delta}^2_{\mathcal{P},\mathcal{Q}}) = (z_1+z_2)\left(\frac{2\mu}{m'}\right)^2\left[\int d\mathbf{r}\xi^*\left(\frac{m'}{M}\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right)|\psi_M(\mathbf{r})|^2 + \right.$$
$$\left. +\frac{2M^4(m'^2-4\mu^2)}{m'^4(2M^2-t)^2}\int d\mathbf{r}\xi^*\left(\frac{m'}{M}\boldsymbol{\Delta}_{\mathcal{P},\mathcal{Q}},\mathbf{r}\right)|\widehat{H}_0\psi_M(\mathbf{r})|^2\right], \tag{45}$$

where possibility to applicability of the addition theorem (44) follows from independence of the wave RQP-function $\psi_M(\mathbf{r})$ in the case of $J=0$ from direction of the vector $\mathbf{r}$.

For s-state of the composite system the integrations in (45) respecting

3) An analogous expression in the case of two particles of equal masses was obtained by other way in [19].

of angles gives

$$F_{\ell=0}(Q^2) = 4\pi(z_1+z_2)\left(\frac{2\mu}{m'}\right)^2 \frac{\chi_\Delta}{\sinh\chi_\Delta}\left[\int\limits_0^\infty dr\frac{\sin(rm'\chi_\Delta)}{rm'\chi_\Delta}|\varphi_0(r,\chi_n)|^2 + \right.$$

$$\left. + \frac{2M_n^4(m'^2-4\mu^2)}{m'^4(2M_n^2+Q^2)^2}\int\limits_0^\infty drr^2\frac{\sin(rm'\chi_\Delta)}{rm'\chi_\Delta}\left|\widehat{H}_{0,\ell=0}\frac{\varphi_0(r,\chi_n)}{r}\right|^2\right], \qquad (46)$$

where are used decompositions (30) for wave function $\psi_M(\mathbf{r})$ and relativistic plane wave (22):

$$\xi(\mathbf{p}',\mathbf{r}) = \sum_{\ell=0}^{\infty}(2\ell+1)i^\ell p_\ell(r,\cosh\chi_{p'})P_\ell\left(\frac{\mathbf{p}'\cdot\mathbf{r}}{p'r}\right).$$

Here the rapidity χ_n corresponds to the level n bound state with energy $M = M_n = (m'^2/\mu)\cosh\chi_n$; the function

$$p_\ell(\rho,\cosh\chi_{p'}) = \sqrt{\frac{\pi}{2\sinh\chi_{p'}}}\frac{(-1)^{\ell+1}}{\rho}(-\rho)^{(\ell+1)}P^{-1/2-\ell}_{-1/2+i\rho}(\cosh\chi_{p'}),\ \rho = rm',$$

is a solution of the equation (25), where the function $(-\rho)^{(l+1)} = i^{l+1}\Gamma(l+1+i\rho)/\Gamma(i\rho)$ is called the generalized power [21], and $\Gamma(z)$ is a gamma function.

5 Results for the Coulomb and linear interaction

Now let us consider the expression for the invariant r.m.s. $< r_0^2 >$ of the composite system, which has the group-theoretical meaning of an eigenvalue of the Casimir operator of the Loretz group and in terms of the modulus square of wave function s-state according to (23), (45) and (46) is given by correlation [15]

$$< r_0^2 >= \frac{\widehat{C}_L F_{\ell=0}(t)|_{t=0}}{F_{\ell=0}(0)} = \frac{6\partial F_{\ell=0}(t)/\partial t|_{t=0}}{F_{\ell=0}(0)} = \frac{1}{M^2}+$$

$$+\left(\frac{m'}{M}\right)^2 \frac{\int\limits_0^\infty drr^2 \left[1+\frac{2\mu^2}{m'^4}\left(1-\frac{4\mu^2}{m'^2}\right)|M-V(r)|^2\right]|\varphi_0(r,\chi)|^2}{\int\limits_0^\infty dr \left[1+\frac{2\mu^2}{m'^4}\left(1-\frac{4\mu^2}{m'^2}\right)|M-V(r)|^2\right]|\varphi_0(r,\chi)|^2}+$$

$$+\frac{12\mu^2}{M^2 m'^4}\left(1-\frac{4\mu^2}{m'^2}\right)\frac{\int\limits_0^\infty dr|M-V(r)|^2|\varphi_0(r,\chi)|^2}{\int\limits_0^\infty dr \left[1+\frac{2\mu^2}{m'^4}\left(1-\frac{4\mu^2}{m'^2}\right)|M-V(r)|^2\right]|\varphi_0(r,\chi)|^2}. \tag{47}$$

Thus, wave function of s-state describes not all structure of the composite particle, but only the region which be upon distances that larger its of the Compton wavelength $1/M$. For the central sphere of radius $r_0 = 1/M$ and with the non-singular potential $V(r)$ ($V(0) < \infty$) it be correspond function of spatial distribution in the form $|\varphi_0(r,\chi)|^2 = \delta(r)/4\pi$. This distribution brings about the value of contribution to form factor from this sphere that equal

$$F_{\ell=0}(Q^2)|_{r_0=1/M} =$$
$$= (z_1+z_2)\left(\frac{2\mu}{m'}\right)^2 \frac{\chi_\Delta}{\sinh\chi_\Delta}\left[1+\frac{8M^4\mu^2(m'^2-4\mu^2)}{m'^6(2M^2+Q^2)^2}|M-V(0)|^2\right]. \tag{48}$$

We shall note that the second summands in (46) and (48), either as the third summand in (47), represent the dipole contribution accordingly in the form factor and r.m.s. of the composite system, because of difference of the particles masses: these the summands under $m_1 = m_2$ be equal to zero.

As example, we consider the form factor of meson in the case of the Coulomb field of attraction between quarks:

$$V(r) = -\frac{\alpha}{r}, \alpha > 0, \tag{49}$$

i.e. we consider that inside of meson the interaction between two quarks is realized by the exchange of the massless scalar boson (gluon).

The radial wave function of exact solution of the RQP-equation (31) with interaction (49) for the s-state and ground level $n = 0$ with the energy M_0 has the form [23–25]

$$\varphi_0(r, i\kappa_0) = N_{0,0}(\kappa_0) r m' \exp\left[-rm'\kappa_0 + \frac{i\tilde{\alpha}\kappa_0}{2\sin\kappa_0}\right],$$

where $\tilde{\alpha} = 2\mu\alpha/m'$, $M_0 = (m'^2/\mu)\cos\kappa_0$, κ_0 defines by the following quantization condition $\tilde{\alpha}/(2\sin\kappa_0) = 1, 0 \leq \kappa_0 < \pi/2$, and the normalization factor $N_{0,0}(\kappa_0)$ was found from condition

$$4\pi \int_0^\infty dr |\varphi_0(r, i\kappa_0)|^2 = 1. \tag{50}$$

Then the form factor (46) for the ground level of bound s-state with the energy M_0 is given as

$$F_{\ell=0,n=0}(Q^2) =$$

$$= 16(z_1 + z_2)\left(\frac{2\mu}{m'}\right)^2 \frac{\kappa_0^4 \chi_\Delta}{\sinh\chi_\Delta(\chi_\Delta^2 + 4\kappa_0^2)^2}\left\{1 + \frac{8M_0^4(m'^2 - 4\mu^2)\cos^2\kappa_0}{m'^2(2M_0^2 + Q^2)^2}\times\right.$$
$$\left.\times\left[1 + \frac{\tan\kappa_0(\chi_\Delta^2 + 4\kappa_0^2)}{2\kappa_0} + \frac{\tan^2\kappa_0(\chi_\Delta^2 + 4\kappa_0^2)^2}{4\kappa_0\chi_\Delta}\left(\frac{\pi}{2} - \arctan\frac{2\kappa_0}{\chi_\Delta}\right)\right]\right\}. \tag{51}$$

For large Q^2 the rapidity behaves as $\chi_\Delta \approx \ln(Q^2/M^2)$ and, consequently, the leading behavior of form factor (51) gives by expression

$$F_{\ell=0,n=0}(Q^2) \approx 32(z_1 + z_2)\left(\frac{2\mu}{m'}\right)^2 \frac{\kappa_0^4}{(Q/M_0)^2\left[\ln(Q/M_0)^2\right]^3}, \tag{52}$$

i.e. either as in [18]. Such behavior of the form factor under large $t = -Q^2$ differs from prediction of the nonrelativistic model based on the Coulomb potential, which gives the dipole decrease of the pion form factor: $F_\pi \sim t^{-2}$. However, the nonrelativistic result contradicts the prediction of the dimensional quark counting rules [1], which gives the decrease of the pion form factor under the law $F_\pi \sim t^{-1}$.

As the second example, we consider the form factor in the case of linear quasipotential

$$V(\rho) = \frac{\beta\rho}{m'}, \rho = m'r, \beta > 0. \qquad (53)$$

For the calculation of form factor in (46) we use the relativistic quasiclassical approach (WKB-approach) for the regular solution with the linear quasipotential (53) that for given level n of bound s-state with the energy M_n has the form [25, 26]

$$\varphi_0(\rho, \chi_n) = \frac{C_0(\chi_n)}{\sqrt[4]{\left(\cosh\chi_n - \rho/\tilde{\beta}\right)^2 - 1}} \sin\left[\int\limits_0^{\rho} d\rho' \operatorname{arcosh}(\cosh\chi_n - \rho'/\tilde{\beta})\right],$$
$$\tilde{\beta} \gg 1, \qquad (54)$$

where $M_n = (m'^2/\mu)\cosh\chi_n, n = 1, 2, \ldots; \tilde{\beta} = m'^3/(\mu\beta)$, and the rapidity χ_n defines from the quasiclassical quantization condition for s-state

$$\int\limits_0^{\tilde{\beta}(\cosh\chi_n - 1)} d\rho \operatorname{arcosh}(\cosh\chi_n - \rho/\tilde{\beta}) = \pi\left(n - \frac{1}{4}\right), n = 1, 2, \ldots, \qquad (55)$$

from which directly find:

$$\chi_n \cosh\chi_n - \sinh\chi_n = \frac{\pi}{\tilde{\beta}}\left(n - \frac{1}{4}\right), n = 1, 2, \ldots. \qquad (56)$$

From expressions (54)– (56), we find:

$$C_0^2(\chi_n) = \frac{m' \sinh\chi_n}{2\pi^2}\frac{d\chi_n}{dn} = \frac{m'}{2\pi\tilde{\beta}\chi_n}. \qquad (57)$$

Finally, taking into consideration correlations (54) and (57), the expression (46) for the form factor in the case of linear quasipotential (53) for given level n of bound s-state with the energy M_n in the WKB-approach takes the form

$$F_{\ell=0,n}(Q^2) = (z_1 + z_2)\left(\frac{2\mu}{m'}\right)^2 \frac{\pi}{2\tilde{\beta}\chi_n \sinh\chi_n \sinh\chi_\Delta}\left[1 + O\left(\frac{1}{\tilde{\beta}\chi_\Delta}\right)\right],$$
$$\tilde{\beta}\chi_\Delta \gg 1. \qquad (58)$$

Obviously that for large Q^2 the leading behavior of form factor (58) gives by expression

$$F_{\ell=0,n}(Q^2) \approx (z_1 + z_2)\left(\frac{2\mu}{m'}\right)^2 \frac{\pi}{\tilde{\beta}\chi_n \sinh\chi_n (Q/M)^2},$$

i.e. it decreases under the law $F_\pi \sim t^{-1}$, that predicts the dimensional quark counting rules [1].

6 Conclusions

In this paper, the new covariant elastic form factor of relativistic two-particle bound system in the case of scalar current is obtained. The consideration is conducted within the framework of relativistic quasipotential approach on the basis of covariant Hamiltonian formulation of quantum field theory [8, 9] by transition to the three-dimensional relativistic configurational representation in the case of two interacting relativistic spinless particles of arbitrary masses m_1, m_2 [20, 21].

The elastic form factor $F(\mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}})$ was represented in the form of relativistic Fourier image of covariant waves RQP-functions in the configuration representation and its possibly consider as an invariant function which depends only on the invariant quantity the square of modulus of vector $\mathbf{\Delta}^2_{\mathcal{P},\mathcal{Q}}$ in the Lobachevsky space (the expressions (45) and (46)). The invariant relativistic relative coordinate r is conjugated to the rapidity $m'\chi_\Delta$, and it is the distance in the Lobachevsky space.

Using of the three-dimensional relativistic configurational representation for the system of two relativistic spinless particles of arbitrary masses has allowed to install that the wave function of s-state describes not whole structure of the composite particle, but only the region which be upon distances that larger its of the Compton wavelength $1/M$. The executed analysis has shown, that the leading contribution to structure of the composite particle from the central sphere of radius $r_0 = 1/M$ be proportional $\chi_\Delta/\sinh\chi_\Delta$. In the nonrelativistic limit this the relativistic geometric factor go to 1. The corrective member in (46) corresponds the dipole contribution bound to the difference of particles masses. This the member under $m_1 = m_2$ be equal to zero.

As examples, the expressions for the form factors of relativistic two-particles bound systems in the case of Coulomb and linear quasipotentials were obtained (expressions (51) and (58)). It is installed that the covariant wave RQP-functions of Coulomb and linear quasipotentials for larges Q^2 give the decrease for these form factors under the law $F_\pi \sim t^{-1}$, which predicts the dimensional quark counting rules [1].

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