Relativistic Generalization of the Threshold Resummation Factors in the Quasipotential Approach

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Abstract

The quasipotential approach in quantum field theory is used to derive relativistic threshold resummation factors in quantum chromodynamics.

1 Introduction

As is well known, for interection $V(r) = -\alpha/r$ (such as the electric-Coulomb and color-Coulomb interections) the nonrelativistic Schrödinger equation leads to the known S-wave Gamov–Sommerfeld–Sakharov factor [1–3]

$$S_{\rm nr} = \frac{X_{\rm nr}}{1-\exp(-X_{\rm nr})}, \qquad X_{\rm nr} = \frac{\pi\,\alpha}{v_{\rm nr}}, \tag{1}$$

which is related to the wave function of the continuous spectrum at the origin by $|\psi(0)|^2$. Here $2v_{\rm nr}$ is the relative velocity of two particles. For two particles of equal masses m the relative velocity is given in terms of their centre-of-mass energe $\sqrt{s}$ by [4–9]

$$2v_{\rm nr} = \frac{\sqrt{s^2-4sm^2}}{s-2m^2}. \tag{2}$$

This gives $2v_{\rm nr} \sim 2\sqrt{1-4m^2/s}$ when $\sqrt{s} \sim 2m$ and $2v_{\rm nr} \to 1$ when $s \to \infty$. An expansion of (1) in a power series in the coupling constant α reproduces the threshold singularities of the form $(\alpha/v)^n$, $v = \sqrt{1-4m^2/s}$. Thus, the real expansion parameter in the threshold region is α/v. Obviously, it becomes to be singular, when the velocity $v \to 0$. A description of quark-antiquark systems close to threshold thus does not permit us to cut off the perturbative series even if the expansion parameter α_S is small. The problem is well known from QED [10].

The resummation can be performed on the level of potential consideration. The corresponding nonrelativistic expression can also be obtained for higher ℓ states (see, e.g., [11]). In the relativistic theory the nonrelativistic approximation needs to be modified. The corresponding relativistic resummation of the S-factor has been found in [12]. Its applications for describing some hadronic processes can be found in [13–15]. The relativistic resummation of the P-factor ($\ell = 1$ state) has been found in [16]. In the same place suggest new model expression for $R(s)$ in which threshold singularities are summarized into a main potential contribution.

In this note we derive a relativistic L-factor for higher ℓ states.

2 Relativistic threshold resummation factors

The resummation factors appear in the parametrization of the imaginary part of corresponding quark current correlators, $R(s)$. In QED, the function $R(s)$ can be approximated by the Bethe-Salpeter (BS) amplitude of two charged particles, $\chi_{\rm BS}(x)$, at $x = 0$ [17]. The nonrelativistic replacement of this amplitude by the wave function, which obeys the Schrödinger equation with the Coulomb potential, leads to the formula (1) with the substitution $\alpha \to 4\alpha_S/3$, for QCD.

A starting point of our consideration is the quasipotential (QP) approach proposed by Logunov and Tavkhelidze [18], in the form suggested by Kadyshevsky [19]. The possibility of using the QP approach for our task is based on the fact that the BS amplitude, which parameterizes the physical quantity $R(s)$, is taken at $x = 0$, therefore, in particular, at the relative time $\tau = 0$. The QP wave function is defined as the BS amplitude at $\tau = 0$, and $R(s)$ can be expressed through the QP wave function $\psi_{\rm QP}(\mathbf{p})$ by using the relation

$$\chi_{\rm BS}(x = 0) = \int d\Omega_p \, \psi_{\rm QP}(\mathbf{p}) \,, \tag{3}$$

where $d\Omega_p = (d\mathbf{p})/[(2\pi)^3 \, E_p]$ is the relativistic three-dimensional volume element in the Lobachevsky space realized on the hyperboloid $E_p^2 - \mathbf{p}^2 = m^2$.

In the following we will consider the case of two particles with the same masses m and use the system of units $c = \hbar = m = 1$.

The proper Lorentz transformation means a translation in the Lobachevsky space. The role of the plane waves corresponding to these translations are played by the following functions

$$\xi(\mathbf{p}, \mathbf{r}) = (E_p - \mathbf{p} \cdot \mathbf{n})^{-1-ir} , \tag{4}$$

where $\mathbf{r} = \mathbf{n}r$ and $\mathbf{n}^2 = 1$. These functions correspond to the principal series of unitary representations of the Lorentz group and in the nonrelativistic limit ($p \ll 1$, $r \gg 1$) $\xi(\mathbf{p}, \mathbf{r}) \to \exp(i\mathbf{p} \cdot \mathbf{r})$. The functions (4) obey the following conditions of completeness and orthogonality

$$\begin{gathered} \int d\Omega_p \, \xi(\mathbf{p}, \mathbf{r}) \, \xi^*(\mathbf{p}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') , \\ \int d\mathbf{r} \, \xi(\mathbf{p}, \mathbf{r}) \, \xi^*(\mathbf{k}, \mathbf{r}) = (2\pi)^3 \, \delta(\mathbf{p}(-)\mathbf{k}) , \end{gathered} \tag{5}$$

where the relativistic momentum-spase δ-function is $\delta(\mathbf{p}(-)\mathbf{k}) = \sqrt{1 + \mathbf{p}^2} \, \delta(\mathbf{p} - \mathbf{k})$. The QP wave functions in the momentum and relativistic configuration representations are related as follows:

$$\begin{gathered} \psi(\mathbf{r}) = \int d\Omega_p \, \xi(\mathbf{p}, \mathbf{r}) \, \psi_{\mathrm{QP}}(\mathbf{p}) , \\ \psi_{\mathrm{QP}}(\mathbf{p}) = \int d\mathbf{r} \, \xi^*(\mathbf{p}, \mathbf{r}) \, \psi(\mathbf{r}). \end{gathered} \tag{6}$$

The QP equation in the momentum spase has the form

$$(2E - 2E_p) \, \psi_{\mathrm{QP}}(\mathbf{p}) = \int d\Omega_k \, V(\mathbf{p}(-)\mathbf{k}) \, \psi_{\mathrm{QP}}(\mathbf{k}) . \tag{7}$$

For a spherically symmetric potential the ξ-transform of (7) leads to the equation

$$\int d\Omega_p \, d\mathbf{r}' \; (2E - 2E_p) \; \xi(\mathbf{p}, \mathbf{r}) \, \xi^*(\mathbf{p}, \mathbf{r}') \, \psi(\mathbf{r}') = V(r) \, \psi(\mathbf{r}) , \tag{8}$$

where the right hand side is local. Here the transform of the potential is given in terms of the same relativistic plane wave by

$$V(\mathbf{p}(-)\mathbf{k}) = \int d\mathbf{r}\, \xi^*(\mathbf{p}(-)\mathbf{k},\mathbf{r})\, V(\mathbf{r}). \tag{9}$$

We note that the left hand side of equation (8) can be rewritten in a non-integral form by using the relatios (6) and the operator of the free Hamiltonian [20, 21]

$$\hat{H}_0 = \cosh\left(i\frac{\partial}{\partial r}\right) + \frac{i}{r}\sinh\left(i\frac{\partial}{\partial r}\right) - \frac{\Delta_{\theta,\varphi}}{2r^2}\exp\left(i\frac{\partial}{\partial r}\right), \tag{10}$$

where $\Delta_{\theta,\varphi}$ is the angular part of the Laplacian operator. The relation

$$\hat{H}_0\, \xi(\mathbf{p},\mathbf{r}) = E_p\, \xi(\mathbf{p},\mathbf{r})$$

allows us to express the equation (8) in terms finite differences

$$\left(2E - 2\hat{H}_0\right) \psi(\mathbf{r}) = V(r)\, \psi(\mathbf{r}). \tag{11}$$

Solutions of this equation, in principle, can contain arbitrary functions of r with period i, the so-called the i-periodic constants, which appear in the solutions due to the finite difference nature of the Hamiltonian (10). For some problems, such as defining the bound state spectrum, this i-periodic constant is not important. However, for the purpose of extracting resummation factors, one must develop a method which avoids this ambiguity. For this instead of the equation (11) we will to use the equation (8).

By using of the expandings

$$\begin{aligned} \xi(\mathbf{p},\mathbf{r}) &= \sum_{\ell=0}^{\infty}(2\ell+1)\, i^{\ell}\, p_{\ell}(\cosh\chi_p,\, r)\, P_{\ell}\left(\frac{\mathbf{p}\cdot\mathbf{r}}{pr}\right), \\ \psi(\mathbf{r}) &= \sum_{\ell=0}^{\infty}(2\ell+1)\, i^{\ell}\, \frac{\varphi_{\ell}(r,\chi)}{r}\, P_{\ell}\left(\frac{\mathbf{q}\cdot\mathbf{r}}{qr}\right), \end{aligned} \tag{12}$$

and also formula [21]

$$p_{\ell}(\cosh\chi, r) = \frac{(-1)^{\ell}\,(\sinh\chi)^{\ell}}{r^{(\ell+1)}}\left(\frac{d}{d\cosh\chi}\right)^{\ell}\left(\frac{\sin r\chi}{\sinh\chi}\right),$$

the equation (8) transformed to the form

$$\frac{2}{\pi}\int_0^\infty d\chi' \frac{(\sinh\chi')^{2\ell+2}\,(-1)^{\ell+1}}{r^{(\ell+1)}}\,(2\cosh\chi - 2\cosh\chi')\left(\frac{d}{d\cosh\chi'}\right)^{\ell}\left(\frac{\sin r\chi'}{\sinh\chi'}\right)\times$$

$$\times\left(\frac{d}{d\cosh\chi'}\right)^{\ell}\frac{1}{\sinh\chi'}\int_0^\infty dr' \frac{r'\sin r'\chi'}{(-r')^{(\ell+1)}}\,\varphi_\ell(r',\chi) = \frac{V(r)\,\varphi_\ell(r,\chi)}{r}, \tag{13}$$

where $P_\ell(z)$ is a Legendre function of the first kind, χ is the rapidity which related to E by $E = \cosh\chi$, and

$$p_\ell(\cosh\chi, r) = \frac{(-1)^{\ell+1}}{r}\sqrt{\frac{\pi}{2\sinh\chi}}\,(-r)^{(\ell+1)}\,P^{-1/2-\ell}_{-1/2+ir}(\cosh\chi) \tag{14}$$

is the solution of equation (11) in the case when the interaction is switched off ($V(r) \equiv 0$). Here $(-r)^{(\ell+1)}$ is the generalized power [21, 22]

$$(-r)^{(\ell+1)} = i^{\ell+1}\frac{\Gamma(ir+\ell+1)}{\Gamma(ir)},$$

where $\Gamma(z)$ is gamma-function.

Consider the Coulomb potential defined in relativistic configuration space as

$$V(r) = -\frac{\alpha}{r}. \tag{15}$$

The ξ-transformation of (15) gives the potential in momentum space

$$V(\Delta) \sim \frac{1}{\chi_\Delta \sinh\chi_\Delta}, \tag{16}$$

where the relative rapidity χ_Δ corresponds to $\mathbf{\Delta} = \mathbf{p}(-)\mathbf{k}$ and is defined in terms of the square of the momentum transfer by $Q^2 = -(p-k)^2 = 2(\cosh\chi_\Delta - 1)$. For large Q^2 the potential $V(\Delta)$ behaves as $(Q^2\ln Q^2)^{-1}$, which reproduces the principal behavior of the QCD potential proportional to $\bar\alpha_S(Q^2)/Q^2$ with $\bar\alpha_S(Q^2)$ being the QCD running coupling. This property of the quasipotential (15), its QCD-like behavior, has been noted by Savrin and Skachkov in [23].

Note that solutions of Eq. (11) for the Coulomb potential have been investigated in [24]. Other forms of the QP equation with the Coulomb potential have been considered in [25].

To solve quasipotential equation (13) with the potential (15) we use the method developed in [12] and [26]. In this case a solution of quasipotential equation (13) with the Coulomb potential (15) one can seek in the form

$$\frac{r\,\varphi_\ell(r,\chi)}{(-r)^{(\ell+1)}} = \int\limits_\alpha^\beta d\zeta \exp(ir\zeta)\, R_\ell(\zeta,\chi)\,, \tag{17}$$

where the ζ-integration is performed in the complex plane over a contour with end points α and β as in [12, 26]: $\alpha = -R - i\varepsilon$, $\beta = -R + i\varepsilon$, $R \to \infty\,, \varepsilon \to +0$. Substituting (17) into (13) and taking into account that

$$\frac{1}{i\pi}\int\limits_0^\infty dr' \sin(r'\chi')\, \exp(ir'\zeta) = \frac{1}{i\pi}\frac{\chi'}{\chi'^2 - \zeta^2}\,,$$

we arrive at the equation

$$(-1)^\ell \int\limits_\alpha^\beta d\zeta\, R_\ell(\zeta,\chi)\left(\frac{d}{d\cosh\zeta}\right)^\ell \Bigg[(\sinh\zeta)^{2\ell+1}\;(2\cosh\chi - 2\cosh\zeta)\,\times$$

$$\times\left(\frac{d}{d\cosh\zeta}\right)^\ell \left(\frac{\exp(ir\zeta)}{\sinh\zeta}\right)\Bigg] = -\frac{\alpha}{r}\prod_{n=1}^{\ell}(r^2+n^2)\int\limits_\alpha^\beta d\zeta\, \exp(ir\zeta)\, R_\ell(\zeta,\chi)\,. \tag{18}$$

For $\ell = 0$ state this approach leads to the following relativistic S-factor [12]:

$$S(\chi) = \frac{X(\chi)}{1-\exp\left[-X(\chi)\right]}\,, \qquad X(\chi) = \frac{\pi\,\alpha}{\sinh\chi}\,, \tag{19}$$

where χ is the rapidity which related to s by $2\cosh\chi = \sqrt{s}$. The function $X(\chi)$ in Eq. (19) can be expressed in terms of the velocity $v = \sqrt{1-4/s}$, where $\sqrt{s}$ is the center of mass energy, as $X(\chi) = \pi\alpha\sqrt{1-v^2}/v$. The S-factor is involved into the expression for the function $R(s)$ that corresponds to the vector quark current. However, to perform a threshold resummation in the axial-vector case one has to use the relativistic P-factor [16], which corresponds to $\ell = 1$ state.

To derive P-factor for $\ell = 1$ state [16] we note that in the nonrelativistic case the S-factor is defined by the wave function at $r = 0$. In the relativistic case one has to use the value of QP wave function at $r = i$ [12]. Indeed, according to Eqs. (3), (4) and (6), one finds a relation between the required BS amplitude and the QP wave function in the form

$$\chi_{\mathrm{BS}}(x = 0) = \psi(r = i)\,.$$

The P-factor in the nonrelativistic case [11] is defined by derivative of the partial wave function for $\ell = 1$ state at $r = 0$. In the relativistic case, instead of the derivative, one has to use its finite difference analog [21, 22]

$$\Delta^* = \frac{1}{i}\left[\exp\left(i\frac{\partial}{\partial r}\right) - 1\right]. \tag{20}$$

Thus, the relativistic P-factor is connected, as one can expect, with QP partial wave function $\varphi_1(r,\chi)$ ($\ell = 1$) and it is defined by

$$P(\chi) = \lim_{r\to i}\left|\frac{3}{\sinh\chi}\,\Delta^*\left[\frac{\varphi_1(r,\chi)}{r}\right]\right|^2. \tag{21}$$

Solving the equation (18) for $\ell = 1$ state we arrive at the following expression for the function $\varphi_1(r,\chi)$:

$$\varphi_1(r,\chi) = C_1(\chi)\frac{r}{r^{(2)}}\int\limits_{\alpha}^{\beta} d\zeta\frac{\exp[(ir+2)\zeta]}{(\exp\zeta - \exp\chi)^4}\left[\frac{\exp\zeta - \exp(-\chi)}{\exp\zeta - \exp\chi}\right]^{-2+iA}, \tag{22}$$

where

$$A = \frac{\alpha}{2\sinh\chi}\,. \tag{23}$$

Performing in (22) ζ-integration in the complex plane along a contour with end points α and β one obtain the resulting solution which does not contain the i-periodic constant in the form

$$\varphi_1(r,\chi) = C_1(\chi)\frac{2r\,\sinh(\pi r)}{r^{(2)}}\int\limits_{-\infty}^{\infty} dx\frac{\exp[(ir+2)x]}{(\exp x + \exp\chi)^4}\left[\frac{\exp x + \exp(-\chi)}{\exp x + \exp\chi}\right]^{-2+iA}. \tag{24}$$

The function (24) can also be represented in terms of hypergeometrical function obtained in [21, 24] by

$$\varphi_1(r,\chi) = N_1(\chi)(-r)^{(2)} \exp(ir\chi + iA\chi)\, F\,(2-iA, 2-ir; 4; 1-\exp(-2\chi))\ . \tag{25}$$

The normalization constant $N_1(\chi)$ in (25) can be obtained also as in [12]. By using Eqs. (20), (21) and (25) we finally find [16]

$$P(\chi) = \frac{X(\chi)}{1-\exp\left[-X(\chi)\right]}\left(1 + \frac{\alpha^2}{4\sinh^2\chi}\right)\ , \tag{26}$$

where $X(\chi)$ is defined in (19).

The relativistic threshold resummation factors (19) and (26) have the following important properties. In the nonrelativistic limit, $v \ll 1$, they reproduce the known nonrelativistic result. In the ultrarelativistic limit, as it has been argued in [27, 28], the bound state spectrum vanishes as a mass $m \to 0$ because the particle mass is the only dimensional parameter. This feature reflects an essential difference between potential models and quantum field theory, where an additional dimensional parameter appears. One can conclude that within a potential model, the S- and P-factors which correspond to the continuous spectrum should go to unity in the limit $m \to 0$. Thus, in contrast to the nonrelativistic case, the relativistic resummation factors, the S-factor (19) and P-factor (26), reproduce both the known nonrelativistic and the expected ultrarelativistic limits.

The relativistic L-factor for higher ℓ states is connected with QP partial wave function $\varphi_\ell(r,\chi)$ by the relation

$$L(\chi) = \lim_{r\to i}\left|\frac{\Gamma(2\ell+2)}{(2\sinh\chi)^\ell\,\Gamma^2(\ell+1)}\,(\Delta^*)^\ell\left[\frac{\varphi_\ell(r,\chi)}{r}\right]\right|^2\ , \tag{27}$$

where the function $\varphi_\ell(r,\chi)$ that one can obtain solving the equation (18), gives by means of the expression

$$\begin{aligned}\varphi_\ell(r,\chi) &= N_\ell(\chi)(-r)^{(\ell+1)}\exp[ir\chi + iA\chi + i\pi(\ell+1)]\,\times \\ &\times F\,(\ell+1-iA\,, \ell+1-ir\,; 2\ell+2\,; 1-\exp(-2\chi))\ .\end{aligned} \tag{28}$$

The normalization constant $N_\ell(\chi)$ in (28) can be obtained (also as in [12]) from the condition

$$\lim_{\alpha\to 0}\varphi_\ell(r,\chi) = r\,p_\ell(\cosh\chi, r) \longrightarrow \frac{\sin(r\chi - \pi\ell/2)}{\sinh\chi} \quad \text{at} \quad r\to\infty.$$

By using Eqs. (20), (27) and (28) we finally find

$$L(\chi) = \frac{X(\chi)}{1 - \exp\left[-X(\chi)\right]} \prod_{n=1}^{\ell} \left[1 + \left(\frac{\alpha}{2n \sinh \chi}\right)^2\right], \ \ell = 0, 1, \ldots . \tag{29}$$

3 Summing up of the threshold singularities

In comparing theoretical results with experimental data it is important to use the "simplest" objects which allow one to check direct consequences of the theory without using model assumptions.0 Some single-argument functions which have a straightforward connection with experimentally measured quantities can play the role of these objects. The corresponding functions can be defined with the Euclidean and the Minkowskian arguments [29].

The $R(s)$-function discussed above, which is determined by the imaginary part of the correlator of the vector or axial-vector quark current, $R_{V/A}(s)$, plays the role of such kind object. Corresponding perturbative expressions can be written as (see [30, 31])

$$R^{PT}_{V/A}(s) = T_{V/A}(v) \left[1 + \frac{\alpha_S}{\pi} g_{V/A}(v)\right], \tag{30}$$

where

$$T_V(v) = v\,\frac{3 - v^2}{2}, \qquad g_V(v) = \frac{4\pi}{3}\left[\frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right], \tag{31}$$

$$T_A(v) = v^3, \qquad g_A(v) = \frac{4\pi}{3}\left[\frac{\pi}{2v} - \left(\frac{19}{10} - \frac{22}{5}v + \frac{7}{2}v^2\right)\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right]. \tag{32}$$

The functions $g_V(v)$ [10] and $g_A(v)$ [32] approximate the corresponding exact two loop expressions.

The perturbative representation (30) is not applicable in the threshold area because it does not contain the resummation of the threshold singularities. The expressions including the resummation factors have the form

$$\mathcal{R}_{V/A}(s) = \mathcal{R}^{(0)}_{V/A}(s) + \mathcal{R}^{(1)}_{V/A}(s) = \mathcal{R}^{(0)}_{V/A}(s)\left[1 + \delta_{V/A}(s)\right], \tag{33}$$

where

$$\mathcal{R}_V^{(0)}(s) = T_V(v)\, S(\chi), \qquad \mathcal{R}_A^{(0)}(s) = T_A(v)\, P(\chi), \tag{34}$$

$$\mathcal{R}_{V/A}^{(1)}(s) = T_{V/A}(v) \left[\frac{\alpha_S}{\pi}\, g_{V/A}(v) - \frac{1}{2} X(\chi) \right]. \tag{35}$$

The function $\mathcal{R}^{(0)}(s)$ in (33) is the product of the factor $T(v)$ and the threshold resummation factor for corresponding state. It describes the principle "potential" contribution. The next term $\mathcal{R}^{(1)}(s)$ in (33) relates to a QCD correction. In the limit $m \to 0$ the resummation factors $S, P \to 1$, the vector and axial-vector corrections become asymptotically equal, $\delta_{V/A}(s) \to \alpha_S/\pi$, and the expression (34) reproduces the known massless formula.

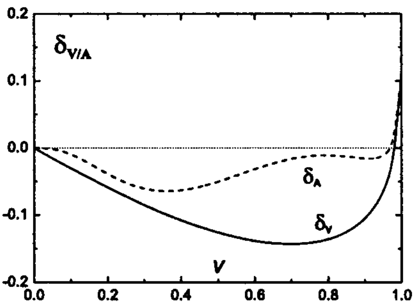

Figure 1: *The relative corrections $\delta_{V/A}$ vs. v.*

The relative correction in (33)is described by $\delta_{V/A}(s)$. Its behavior we show in Fig. 1. The curves were obtained for $\alpha_S = 0.35$ that corresponds to the value of the strong coupling extracted from the τ-decay data (see [33]). Fig. 1 demonstrates that the correction to the principle potential contribution is small for a wide energy-interval: $|\delta(s)| \lesssim 15\%$.

4 Conclusions

To summarize the threshold singularities and find corresponding resummation factors we have used the quasipotential approach in quantum field theory. These relativistic resummation factors appear in the function $R(s)$, which is proportional to the imaginary part of the quark current correlator, and could have a significant impact in interpreting strong-interaction physics. Indeed, in many physically interesting cases, the function $R(s)$ appears as a factor in an integrand, as, for example, for the case of inclusive τ decay, for smearing quantities, and for the Adler D-function.

The relativistic threshold resummation factors obtained here reproduce both the known nonrelativistic and expected ultrarelativistic limits and correspond to the QCD-like Coulomb potential.

We have suggested new expressions for $R(s)$ in which threshold singularities are summarized by a potential contribution. We have demonstrated that the QCD correction to the principle potential contribution is rather small for a wide interval of energies.

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