

# Deep Inelastic Processes by a Charged Current and Proton Spin

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## Abstract

The possibilities of the extraction the new information about the different quark contributions in the proton spin with help a set of measurable quantities are duscussed for deep inelastic scattering by charged current at  $\vec{e} - \vec{p}$  HERA collider.

## 1 Introduction

In the near future the electron-proton collider HERA will to use in operating with both polarized beams [1] - [3].

Two inclusive reactions of deep inelastic scattering (DIS) can to be at HERA. Neutral current (NC):

$$e^{\pm} + P \rightarrow e^{\pm} + X. \quad (1)$$

Charged current (CC):

$$e^{-}(e^{+}) + P \rightarrow \nu(\bar{\nu}) + X. \quad (2)$$

The processes (1) are studied already over two gecade year in the experiments on fixed polarized targeds at SLAC [4] - [5] and DESY [6] - [7]. In the future experiments (1) by the  $\vec{e} - \vec{p}$  collisions at HERA will to measure the spin structure function (SF) of proton  $g_1^p$  in the range of very small  $x \sim 10^{-4}$ , the polarized valence quarks and gluon distributions. Moreover, there is at HERA the umque possibility for the investigation of spin effects in CC reactions (2). The most important property of the processes (2) is that they distinguish quark and antiquark flavours. This allow to extract the new data for the contributions of quark flavours and valence quarks in the proton spin because of other combinations of spin - dependent SF than electromagnetic processes (1).

Thus, the CC reactions (2) are a source of the new information about the spin properties of nucleons. Their study is one the main aims of the polarization program at HERA.

## 2 The asymmetries and the spin structure of proton

The differential cross section of the polarization ep DIS (2) can be written as

$$d^2\sigma_{\mp}/dxdy = \chi_w [xy^2 F_1^{\mp}(x) + y_1 F_2^{\mp}(x) \pm xy(1 - \frac{y}{2}) F_3^{\mp}(x) + P_N x (y_1^+ g_6^{\mp}(x) \pm y_1^- g_1^{\mp}(x))]. \quad (3)$$

Here  $x = Q^2/2pq$  and  $y = pq/pk$  are the scaling variables;  $Q^2 = -q^2 = -(k - \hat{k})^2$ ;  $k(\hat{k})$ ,  $P$  are the 4-moment of the incoming (outcoming) lepton and proton, respectively;

$$y_1 = 1 - y, y_1^{\mp} = 1 \mp y_1^2, \\ \chi_w = \frac{G^2 s}{2\pi} \left( \frac{1}{1 + Q^2/M_w^2} \right)^2, S = 2Pk,$$

$G$  is Fermi constant,  $M_w$  is mass of  $W$  boson,  $P_N$  is degree of longitudinal proton polarization; the  $F_{1,2,3}$  and  $g_{1,6}$  are spin - averaged and spin - dependent structure functions of nucleon. The measurable polarization asymmetries determine as following combinations of the cross sections (3)

$$A_{\mp} = \frac{(\sigma_{-}^{\downarrow\uparrow} \pm \sigma_{+}^{\uparrow\uparrow}) - (\sigma_{-}^{\downarrow\downarrow} \pm \sigma_{+}^{\uparrow\downarrow})}{(\sigma_{-}^{\downarrow\uparrow} \pm \sigma_{+}^{\uparrow\uparrow}) + (\sigma_{-}^{\downarrow\downarrow} \pm \sigma_{+}^{\uparrow\downarrow})}, \quad (4)$$

$$A_{e^-,e^+} = \frac{\sigma_{\mp}^{\downarrow\uparrow,\uparrow\uparrow} - \sigma_{\mp}^{\downarrow\downarrow,\uparrow\downarrow}}{\sigma_{\mp}^{\downarrow\uparrow,\uparrow\uparrow} + \sigma_{\mp}^{\downarrow\downarrow,\uparrow\downarrow}}, \quad (5)$$

where

$$\sigma \equiv d^2\sigma/dxdy.$$

The first arrow corresponds to the direction of the electron ( $\downarrow$ ) or positron ( $\uparrow$ ) spin, the second arrow corresponds to the direction of the proton spin:  $\uparrow$  ( $P_N = +1$ ) and  $\downarrow$  ( $P_N = -1$ ).

Substituting in (4) and (5) the expressions (3), we obtain for the asymmetries

$$A_{\mp} = \frac{x\chi_w [y_1^+ (g_{6(x)}^- \pm g_{6(x)}^+) + y_1^- (g_{1(x)}^- \mp g_{1(x)}^+)]}{\sigma_{-}^{\pm} \pm \sigma_{+}^{\pm}}, \quad (6)$$

$$A_{e^-,e^+} = \frac{x\chi_w[y_1^+ g_{6(x)}^{\mp} \pm y_1^- g_1^{\mp}(x)]}{\sigma_{\mp}^a}, \quad (7)$$

where  $\sigma_{\mp}^a = \chi_w[x y^2 F_1^{\mp}(x) + y_1 F_2^{\mp}(x) \pm x y \left(1 - \frac{y}{2}\right) F_3^{\mp}(x)]$

is the unpolarized cross sections of the processes (2). The spin - dependent  $SF_s g_{1,6}^{\mp}(x)$  obtained in the quark-parton model (QPM) in the form

$$\begin{aligned} g_1^{\mp}(x) &= \sum_q \Delta q(x) + \sum_{\bar{q}} \Delta \bar{q}(x), \\ g_6^{\mp}(x) &= \sum_q \Delta q(x) - \sum_{\bar{q}} \Delta \bar{q}(x). \end{aligned} \quad (8)$$

Here  $q = u, c, t(d, s, b)$  and  $\bar{q} = \bar{d}, \bar{s}, \bar{b}(\bar{u}, \bar{c}, \bar{t})$  for the electron (positron) and  $\Delta q(x)$  ( $\Delta \bar{q}(x)$ ) are the polarized (anti) quark densities.

Using (8) and neglecting the contributions of the heavy quarks ( $c, b, t$ ) the asymmetries (6) can be written as

$$A_+ = \frac{2x\chi_w[\Delta u_v(x) + y_1^2 \Delta d_v(x)]}{\sigma_-^a + \sigma_+^a}, \quad (9)$$

$$A_- = \frac{2x\chi_w[\Delta u(x) + \Delta \bar{u}(x) - y_1^2(\Delta d(x) + \Delta \bar{d}(x) + \Delta s(x) + \Delta \bar{s}(x))]}{\sigma_-^a - \sigma_+^a}, \quad (10)$$

where  $\Delta q_v(x) = \Delta q(x) - \Delta \bar{q}(x)$  are the polarized valence quark densities.

Integrating (10) over  $x$  one has

$$\Delta u - y_1^2(\Delta d + \Delta s) = \int_0^1 \frac{dx}{2x\chi_w} A_- (\sigma_-^a - \sigma_+^a) \quad (11)$$

where  $\Delta q = \int_0^1 [\Delta q(x) + \Delta \bar{q}(x)] dx$  is the contribution quark flavour  $q = u, d, s$  in the proton spin. If two of the supplementary measurable quantities, for example, the axial charges  $g_A$  and  $a_8$

$$g_A = F + D = 1.2670 \pm 0.0035$$

$$a_8 = 3F - D = 0.585 \pm 0.025$$

which are in QPM

$$g_A = \Delta u - \Delta d, \quad a_8 = \Delta u + \Delta d - 2\Delta s \quad (12)$$

are used, equation (11) and (12) allows one to determine the quark flavour contributions  $\Delta u, \Delta d, \Delta s$  in the proton spin.

If to assume that the flavour symmetry of the light quark sea is, i.e.  $\Delta \bar{u} = \Delta \bar{d}$ , then

$$g_A = \Delta u_v - \Delta d_v. \quad (13)$$

From (9) it follows that

$$\Delta u_v + y_1^2 \Delta d_v = \int_0^1 \frac{dx}{2x\chi_w} A_+(\sigma_-^a + \sigma_+^a). \quad (14)$$

The expression (13) and (14) allow one to find the contributions of the valence quarks  $\Delta u_v$  and  $\Delta d_v$  to the proton spin.

Thus, the scheme of extraction of the quark flavours and valence quark contributions to the proton spin is suggested by a set of measurable quantities: the asymmetries  $A_+$  and  $A_-$ , the unpolarized cross sections  $\sigma_-^a$  and  $\sigma_+^a$  (or  $SF_s$ ,  $F_2$  and  $xF_3$ ), the axial charges  $g_A$  and  $a_8$ .

### 3 The virtual polarization asymmetries

Here we consider the other approach for the investigation the spin of the proton in CC reactions (2). We propose to use the polarization asymmetries, which constructed from the total cross sections for the absorption of a virtual  $W$  boson by a polarized proton.

This cross section are given by

$$\sigma_{tot} = k\epsilon_\mu^*(q)\epsilon_\nu(q)W_{\mu\nu}, \quad (15)$$

Where

$$K = \frac{\pi^2\alpha}{2\sin^2\Theta_w\sqrt{\nu^2 - q^2}}$$

Here  $\epsilon_\mu(q)$  is the polarization 4-vector of virtual  $W$ -boson,  $\nu = pq/M$ ;  $M$  is the proton mass and  $\Theta_w$  is the Weinberg angle,  $W_{\mu\nu}$  is the hadron tensor [9]. Calculating in c.m.frame the helicity amplitudes of the compton scattering of a virtual  $W$ -boson on a polarized proton we obtain for  $\sigma_{tot}$  in scaling limit the following expressions:

$$\begin{aligned} \sigma_{\frac{1}{2}}^{\frac{3}{2}} &= \frac{K}{M} \left[ F_1(x) - \frac{F_3(x)}{2} \pm g_1(x) \mp g_6(x) \right], \\ \sigma_{\frac{-1}{2}}^{\frac{-3}{2}} &= \frac{K}{M} \left[ F_1(x) + \frac{F_3(x)}{2} \pm g_1(x) \pm g_6(x) \right]. \end{aligned} \quad (16)$$

From (16) can to construct a virtual  $W$  boson-proton polarization asymmetries

$$A_1(x) = \frac{(\sigma_{1/2} + \sigma_{-1/2}) - (\sigma_{3/2} + \sigma_{-3/2})}{(\sigma_{1/2} + \sigma_{-1/2}) + (\sigma_{3/2} + \sigma_{-3/2})} = \frac{2xg_1(x)}{F_2(x)}, \quad (17)$$

$$A_6(x) = \frac{(\sigma_{1/2} - \sigma_{-1/2}) - (\sigma_{3/2} - \sigma_{-3/2})}{(\sigma_{1/2} - \sigma_{-1/2}) + (\sigma_{3/2} - \sigma_{-3/2})} = \frac{2xg_6(x)}{xF_3(x)}. \quad (18)$$

Taking into account (17) and (18), we can express the observable asymmetries  $A_{e^-,e^+}$  in terms of the  $A_1$  and  $A_6$  as

$$A_{e^-,e^+} = \frac{y_1^+ A_x^{e^-,e^+} F_3^{\mp}(x) \pm y_1^- A_1^{e^-,e^+} F_2^{\mp}(x)}{y_1^+ F_2^{\mp}(x) \pm y_1^- x F_3^{\mp}(x)}. \quad (19)$$

Let us now consider a polarized deuteron target. The deuteron SF is defined in the conventional way:

$$g_j^d(x) = \frac{1}{2} \left[ g_j^p(x) + g_j^n(x) \right] (1 - 1.5w_D). \quad (20)$$

Here  $j = 1$  for NC reactions (1) and  $j = 1, 6$  for CC reactions (2);  $w_D \simeq 0.05$  is the probability of  $D$ -wave state in the deuteron wave function.

Then with help (8) and (20) we obtain CC:

$$g_1^{e^-d}(x) = g_1^{e^+d}(x) \equiv g_1^{ed}(x) = \frac{1}{2} [\Delta q(x) + \Delta \bar{q}(x)] (1 - 1.5w_D) \quad (21)$$

where  $\Delta q(x) = \Delta u(x) + \Delta d(x) + \Delta s(x)$ .

On other hand it is known that for NC reactions (1)

$$\frac{18}{5} g_1^d(x) = \left[ \frac{1}{2} (\Delta q(x) + \Delta \bar{q}(x)) - \frac{3}{5} \Delta s(x) \right] (1 - 1.5w_D). \quad (22)$$

Comparing (21) and (22), we arrive at the relation

$$g_1^d(x) \simeq \frac{5}{18} g_1^{ed}(x) \quad (23)$$

as for the spin-averaged SF  $F_2(x)$  in case of deuteron target.

From (21) we obtain the total quark contribution in the proton spin

$$\Delta \Sigma \equiv \Delta q + \Delta \bar{q} = \frac{2\Gamma_1^{ed}}{1 - 1.5w_D} \quad (24)$$

where  $\Gamma_i = \int_0^1 \Delta q(x)(\Delta \bar{q}(x))dx$ .

Let us now consider another the spin SF  $g_6(x)$ , which does not have an analog in the electromagnetic processes (1).

Using (8) and (20) we obtain

$$g_6^\mp(x) = \frac{1}{2}[\Delta q(x) - \Delta \bar{q}(x) \mp (\Delta s(x) + \Delta \bar{s}(x))]\left(1 - \frac{3}{2}w_D\right) \quad (25)$$

and for the deuteron target

$$\begin{aligned} g_6^{e^-d}(x) + g_6^{e^+d}(x) &= \Delta q_v(1 - 1.5w_D), \\ g_6^{e^+d}(x) - g_6^{e^-d}(x) &= [\Delta s(x) + \Delta \bar{s}(x)](1 - 1.5w_D). \end{aligned} \quad (26)$$

Therefore, the relations (26) determine the contributions of the valence quarks

$$\Delta q_{val} = (\Gamma_6^{e^-d} + \Gamma_6^{e^+d})(1 - 1.5w_D) \quad (27)$$

and the strange quarks

$$\Delta s + \Delta \bar{s} = \frac{1}{2}(\Gamma_6^{e^+d} - \Gamma_6^{e^-d})(1 - 1.5w_D). \quad (28)$$

Using (24), (28) and  $g_A$  can to obtain also the contributions of the quark flavours  $u$  and  $d$

$$\begin{aligned} \Delta u + \Delta \bar{u} &= \frac{1}{2}\left[\left(\frac{36}{5}\Gamma_1^d - \Gamma_6^{e^+d} + \Gamma_6^{e^-d}\right)(1 - 1.5w_D) + g_A\right], \\ \Delta d + \Delta \bar{d} &= \frac{1}{2}\left[\left(\frac{36}{5}\Gamma_1^d - \Gamma_6^{e^+d} + \Gamma_6^{e^-d}\right)(1 - 1.5w_D) - g_A\right]. \end{aligned} \quad (29)$$

Here we take into condaration that from (23) follow the relation

$$\Gamma_1^{ed} \simeq \frac{18}{5}\Gamma_1^d.$$

Note that the use of equation (27), (28) and (29) assumes the know of the SF  $g_6(x)$ . We propose the following scheme for the definition this SF. So, from (23) and  $A_1^d = 2xg_1^d/F_2^d$  follow the relation

$$A_1^d = A_1^{e^-d} = A_1^{e^+d}. \quad (30)$$

This allow to replace  $A_1^{e^-,e^+}$  on  $A_1^d$  in (19) and to find the asymmetries  $A_6$  as

$$A_6^{e^-d, e^+d} = \left( \frac{F_2^{e^-d, e^+d}}{xF_3^{e^-d, e^+d}} \pm \frac{y_1^-}{y_1^+} \right) A_{e^-d, e^+d} \mp \frac{y_1^- F_2^{e^-d, e^+d}}{y_1^+ x F_3^{e^-d, e^+d}} A_1^d. \quad (31)$$

Then from (18) can to determine SF  $g_6(x)$

$$g_6^{e^-d, e^+d} = \frac{1}{2x} A_6^{e^-d, e^+d} \cdot x F_3^{e^-d, e^+d}. \quad (32)$$

## 4 Conclusions

Thus, two approach are proposed for a determination the quark contributions in the proton spin using the following a sets of the measurable quantities:

- |      |   |  |
|------|---|--|
| (I)  | <ul style="list-style-type: none"> <li>• the assymetries <math>A_+</math> and <math>A_-</math></li> <li>• the unpolarized cross sections <math>d^2\sigma_{\mp}^a/dxdy</math> (or <math>SF_s</math>, <math>F_2</math>, <math>xF_3</math>)</li> </ul>   | $\left. \vphantom{\begin{array}{l} \bullet \\ \bullet \end{array}} \right\} \begin{array}{l} \text{CC} \\ \text{reactions} \end{array}$                            |
|      |   | (2)  |
|      | <ul style="list-style-type: none"> <li>• the axial charges <math>g_A</math>, <math>a_8</math></li> </ul>  |  |
| (II) | <ul style="list-style-type: none"> <li>• the moments of SF <math>g_6</math> (<math>\Gamma_6^{e^-}, \Gamma_6^{e^+}</math>)</li> <li>• the <math>SF_s</math>, <math>F_2</math> and <math>xF_3</math></li> <li>• <math>\Gamma_1^d</math> and <math>A_1^d</math> from NC reactions</li> <li>• <math>g_A</math></li> </ul> | $\left. \vphantom{\begin{array}{l} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array}} \right\} \begin{array}{l} e^\pm d \text{ DIS} \\ \text{(CC)} \end{array}$ |

An analysis of the numerical calculations show that  $g_6$  and  $A_6$  for  $e^-$  and  $e^+$  differ significantly at small  $x$ . This circumstance is associated with the contribution of polarized strange quarks to the quantities (see (25)).

Therefore, measurements  $g_6$  and  $A_6$  could probe the scale of the polarization of strange quarks and, hence, the role of then contribution to the spin structure of proton.

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