# The Quark Contributions to the Nucleon Spin From Semi-inclusive lp- DIS with Charged Current 

E.A.Degtyareva*, S.I.Timoshin<br>Gomel State Technical University


#### Abstract

Current paper considers the possibilities to determine quark contributions based on observable values - polarization asymmetries of semi-inclusive $\ell N$ - DIS. Numerical computations of asymmetries are provided.


## 1 Introduction

Investigation of nucleon spin structure is one of important problems of particle physics [1]. Spin nucleon problem is not yet solved completely and therefore further research of all the contributions into nucleon spin is necessary. Semi-inclusive lp- DIS with charged current can be a source of new data about the structure of nucleon. The mentioned process allows us to determine the contributions of valence quarks, antiquarks separately, which is a significant advantage. This process can be studied experimentally at electron-proton colliders. Current paper considers the possibilities to determine quark contributions based on observable values - polarization asymmetries. Numerical computations of asymmetries are provided.

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## 2 The semi-inclusive deep inelastic $l p$ - scattering

Let us consider the process of semi-inclusive deep inelastic lp-scattering with charged weak current

$$
\begin{equation*}
\ell+p \rightarrow \nu+h+X \tag{1}
\end{equation*}
$$

The differential cross sections of the process (1) in the case of a lepton:

$$
\begin{align*}
& \left(\frac{d^{3} \sigma_{\ell^{-}}}{d x d y d z}\right)^{h}= \\
& =2 \rho x\left\{\sum_{q_{i}, q_{j}} q_{i}\left(x, Q^{2}\right) D_{q_{j}}^{h}\left(z, Q^{2}\right)+y_{1}^{2} \sum_{\bar{q}_{j}, \bar{q}_{i}} \bar{q}_{j}\left(x, Q^{2}\right) D_{\bar{q}_{i}}^{h}\left(z, Q^{2}\right)+\right.  \tag{2}\\
& \left.+P_{N}\left(\sum_{q_{i}, q_{j}} \Delta q_{i}\left(x, Q^{2}\right) D_{q_{j}}^{h}\left(z, Q^{2}\right)-y_{1}^{2} \sum_{\bar{q}_{j}, \bar{q}_{i}} \Delta \bar{q}_{j}\left(x, Q^{2}\right) D_{\bar{q}_{i}}^{h}\left(z, Q^{2}\right)\right)\right\}
\end{align*}
$$

where $q=u, c, t, \bar{q}=\bar{d}, \bar{s}, \bar{b}$;

$$
\begin{align*}
& \left(\frac{d^{3} \sigma_{\ell^{+}}}{d x d y d z}\right)^{h}= \\
& =2 \rho x\left\{y_{1}^{2} \sum_{q_{i}, q_{j}} q_{i}\left(x, Q^{2}\right) D_{q_{j}}^{h}\left(z, Q^{2}\right)+\sum_{\bar{q}_{j}, \bar{q}_{i}} \bar{q}_{j}\left(x, Q^{2}\right) D_{\bar{q}_{i}}^{h}\left(z, Q^{2}\right)+\right.  \tag{3}\\
& \left.+P_{N}\left(\sum_{q_{i}, q_{j}} \Delta q_{i}\left(x, Q^{2}\right) D_{q_{j}}^{h}\left(z, Q^{2}\right)-\sum_{\bar{q}_{j}, \bar{q}_{i}} \Delta \bar{q}_{j}\left(x, Q^{2}\right) D_{\bar{q}_{i}}^{h}\left(z, Q^{2}\right)\right)\right\}
\end{align*}
$$

where $q=d, s, b, \bar{q}=\bar{u}, \bar{c}, \bar{t}$. Here $\rho=\frac{G^{2} s}{2 \pi}\left(\frac{m_{w}^{2}}{m_{w}^{2}+Q^{2}}\right)^{2}, y_{1}=1-y$, $G$ is Fermi constant, $m_{w}$ is the W-boson mass, $x=\frac{Q^{2}}{2 p \cdot q}, y=\frac{p \cdot q}{p \cdot k}$, $Q^{2}=-q^{2}=-\left(k-k^{\prime}\right)^{2}, s=2 p \cdot k, k\left(k^{\prime}\right)$ and $p$ are the initial (final) lepton and proton 4-momenta, respectively, $P_{N}$ is the degree of longitudinal polarization of proton, $q(x)(\Delta q(x)) / \bar{q}(x)(\Delta \bar{q}(x))$ are the unpolarized
(polarized) quark/antiquark distribution functions, $D_{q}^{h}\left(z, Q^{2}\right)\left(D_{\vec{q}}^{h}\left(z, Q^{2}\right)\right)$ are the fragmentation functions of quark (antiquark) with flavor $q$ to the hadron $h$.

We define the polarization asymmetries in the following form [2]

$$
\begin{align*}
& A_{\ell^{-}}^{h^{+} h^{-}}=\frac{\left(\frac{d^{3} \sigma_{\text {ell }}^{\downarrow \dagger}}{d x d y d z}\right)^{h^{+}-h^{-}}-\left(\frac{d^{3} \sigma_{\text {ell }}^{\downarrow \downarrow}}{d x d y d z}\right)^{h^{+}-h^{-}}}{\left(\frac{d^{3} \sigma_{\text {ell }}^{\downarrow \dagger}}{d x d y d z}\right)^{h^{+}-h^{-}}+\left(\frac{d^{3} \sigma_{\text {cll }}^{\dagger-}}{d x d y d z}\right)^{h^{+}-h^{-}}},  \tag{4}\\
& A_{\ell^{+}}^{h^{+}-h^{-}}=\frac{\left(\frac{d^{3} \sigma_{\text {el }}^{\dagger \dagger}}{d+}\right)^{h^{+}-h^{-}}-\left(\frac{d^{3} \sigma_{\text {ell }}^{\uparrow+}}{d x d y d z}\right)^{h^{+}-h^{-}}}{\left(\frac{d^{3} \sigma_{\text {ell }}^{\dagger}+}{d x d y d z}\right)^{h^{+}-h^{-}}+\left(\frac{d^{3} \sigma_{\text {ell }}++}{d x d y d z}\right)^{h^{+}-h^{-}}}, \tag{5}
\end{align*}
$$

$$
\begin{align*}
& A_{ \pm}^{h^{+}-h^{-}}= \\
& =\frac{\left[\left(\frac{d^{3} \sigma_{-}^{\downarrow \uparrow}}{d x d y d z}\right)^{h^{+}-h^{-}} \pm\left(\frac{d^{3} \sigma_{l^{\uparrow+}}^{d x}}{d x d y d z}\right)^{h^{+}-h^{-}}\right]-\left[\left(\frac{d^{3} \sigma_{l^{\downarrow \downarrow}}}{d x d y d z}\right)^{h^{+}-h^{-}} \pm\left(\frac{d^{3} \sigma_{l+}^{\uparrow \downarrow}}{d x d y d z}\right)^{h^{+}-h^{-}}\right]}{\left[\left(\frac{d^{3} \sigma_{l-}^{\dagger \uparrow}}{d x d y d z}\right)^{h^{+}-h^{-}} \pm\left(\frac{d^{3} \sigma_{l+}^{\uparrow \uparrow}}{d x d y d z}\right)^{h^{+}-h^{-}}\right]+\left[\left(\frac{d^{3} \sigma_{l^{-}}^{\downarrow \downarrow}}{d x d y d z}\right)^{h^{+}-h^{-}} \pm\left(\frac{d^{3} \sigma_{l+}^{\uparrow \downarrow}}{d x d y d z}\right)^{h^{+}-h^{-}}\right]} \tag{6}
\end{align*}
$$

The first arrow corresponds to the helicity of the initial lepton $(\downarrow)$ or antilepton $(\uparrow)$ and the second - to the polarization degree of the proton: $\uparrow\left(P_{N}=+1\right), \downarrow\left(P_{N}=-1\right)$.

Let us consider the case $h=\pi$.Equations (4),(5), (6) for the case of a proton targets for asymmetries give us

$$
\begin{align*}
A_{\ell^{-}}^{\pi^{+}-\pi^{-}}= & \frac{\Delta u\left(x, Q^{2}\right) D_{d}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)-y_{1}^{2} \Delta \bar{d}\left(x, Q^{2}\right) D_{\bar{u}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)}{u\left(x, Q^{2}\right) D_{d}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)+y_{1}^{2} \bar{d}\left(x, Q^{2}\right) D_{\bar{u}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)},  \tag{7}\\
A_{\ell^{+}}^{\pi^{+-} \pi^{-}}= & \frac{y_{1}^{2} \Delta d\left(x, Q^{2}\right) D_{u}^{\pi^{+-} \pi^{-}}\left(z, Q^{2}\right)-\Delta \bar{u}\left(x, Q^{2}\right) D_{\bar{d}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)}{y_{1}^{2} d\left(x, Q^{2}\right) D_{u}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)+\bar{u}\left(x, Q^{2}\right) D_{\bar{d}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)},  \tag{8}\\
A_{+, p}^{\pi^{+}-\pi^{-}}= & \left\{\Delta u\left(x, Q^{2}\right) D_{d}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)-y_{1}^{2}\left(\Delta \bar{d}\left(x, Q^{2}\right) D_{\bar{u}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)+\right.\right. \\
& \left.\left.+\Delta d\left(x, Q^{2}\right) D_{u}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)\right)+\Delta \bar{u} D_{\bar{d}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)\right\} / \\
& /\left\{u\left(x, Q^{2}\right) D_{d}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)+y_{1}^{2}\left(\bar{d}\left(x, Q^{2}\right) D_{\bar{u}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)-\right.\right. \\
& \left.\left.-d\left(x, Q^{2}\right) D_{u}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)\right)-\bar{u}\left(x, Q^{2}\right) D_{\bar{d}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)\right\}, \tag{9}
\end{align*}
$$

$$
\begin{align*}
A_{-, p}^{\pi^{+}-\pi^{-}}= & \left\{\Delta u\left(x, Q^{2}\right) D_{d}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)-y_{1}^{2}\left(\Delta \bar{d}\left(x, Q^{2}\right) D_{\bar{u}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)-\right.\right. \\
& \left.\left.-\Delta d\left(x, Q^{2}\right) D_{u}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)\right)-\Delta \bar{u} D_{\bar{d}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)\right\} / \\
& /\left\{u\left(x, Q^{2}\right) D_{d}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)++y_{1}^{2}\left(\bar{d}\left(x, Q^{2}\right) D_{\bar{u}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)+\right.\right. \\
& \left.\left.+d\left(x, Q^{2}\right) D_{u}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)\right)+\bar{u}\left(x, Q^{2}\right) D_{\bar{d}}^{\pi^{+}-\pi^{-}}\left(z, Q^{2}\right)\right\} . \tag{10}
\end{align*}
$$

With the correlations for $\pi$ - meson fragmentation functions [3], [4]

$$
D_{d}^{\pi^{+}-\pi^{-}}=-D_{u}^{\pi^{+}-\pi^{-}}, D_{u}^{\pi^{+}-\pi^{-}}=-D_{\bar{u}}^{\pi^{+}-\pi^{-}}
$$

we received

$$
\begin{align*}
A_{\ell^{-}}^{\pi^{+}-\pi^{-}} & =\frac{\Delta u(x)-y_{1}^{2} \Delta \bar{d}(x)}{u(x)+y_{1}^{2} \bar{d}(x)}  \tag{11}\\
A_{\ell^{+}}^{\pi^{+}-\pi^{-}} & =\frac{y_{1}^{2} \Delta d(x)-\Delta \bar{u}(x)}{y_{1}^{2} d(x)+\bar{u}(x)}  \tag{12}\\
A_{+, p}^{\pi^{+}-\pi^{-}} & =\frac{\Delta u(x)+\Delta \bar{u}(x)-y_{1}^{2}(\Delta d(x)+\Delta \bar{d}(x))}{u_{V}(x)-y_{1}^{2} d_{V}(x)}  \tag{13}\\
A_{-, p}^{\pi^{+}-\pi^{-}} & =\frac{\Delta u_{V}(x)+y_{1}^{2} \Delta d_{V}(x)}{u(x)+\bar{u}(x)+y_{1}^{2}(d(x)+\bar{d}(x)} \tag{14}
\end{align*}
$$

where $u_{V}(x)=u(x)-\bar{u}(x), d_{V}(x)=d(x)-\bar{d}(x)$.
In the same way we receive asymmetries for the case $h=K^{+}$. For $K-$ meson fragmentation function look like [3], [4] $D_{d}^{K^{+}-K^{-}}=0$, hence the asymmetries (3)-(5) have the following form

$$
\begin{align*}
A_{\ell^{-}}^{K^{+} K^{-}} & =-\frac{\Delta \bar{d}(x)}{\bar{d}(x)}  \tag{15}\\
A_{\ell^{+}}^{K^{+}-K^{-}} & =\frac{\Delta d(x)}{d(x)}  \tag{16}\\
A_{+, p}^{K^{+}-K^{-}} & =-\frac{\Delta d_{V}(x)}{d_{V}(x)}  \tag{17}\\
A_{-, p}^{K^{+}-K^{-}} & =\frac{\Delta d_{V}(x)}{d(x)+\bar{d}(x)} \tag{18}
\end{align*}
$$

where $\Delta d_{V}(x)=\Delta d(x)-\Delta \bar{d}(x)$.
The considered asymmetries have no dependence on fragmentation function which is very useful for the analysis of proton spin structure.

For nucleon spin structure analysis we introduce the first moments of parton distributions as follows

$$
\begin{equation*}
\Delta q(\Delta \bar{q})=\int_{0}^{1} \Delta q(x)(\Delta \bar{q}(x)) d x \tag{19}
\end{equation*}
$$

which correspond to the quark $q($ antiquark $\bar{q}))$ contributions to the spin of nucleon.

Then the antiquark contributions to the nucleon spin can be defined from the asymmetries (19), (20), (12) as

$$
\begin{gather*}
\Delta \bar{d}=-\int_{0}^{1} \bar{d}(x) A_{\ell^{-}}^{K^{+}-K^{-}} d x  \tag{20}\\
\Delta \bar{u}=\int_{0}^{1}\left(-A_{\ell^{+}}^{\pi^{+}-\pi^{-}}\left(y_{1}^{2} d(x)+\bar{u}(x)\right)+y_{1}^{2} A_{\ell^{+}}^{K^{+}-K^{-}} d(x)\right) d x \tag{21}
\end{gather*}
$$

The quark contributions $(\Delta u+\Delta \bar{u}),(\Delta d+\Delta \bar{d})$ can be determined using the asymmetry (11),(12),(15),(16)

$$
\begin{align*}
& \Delta u+\Delta \bar{u}= \int_{0}^{1}\left(A_{\ell^{-}}^{\pi^{+}-\pi^{-}}\left(u(x)+y_{1}^{2} \bar{d}(x)\right)-A_{\ell^{+}}^{\pi^{+}-\pi^{-}}\left(y_{1}^{2} d(x)+\bar{u}(x)\right)+\right.  \tag{22}\\
&\left.+y_{1}^{2}\left(A_{\ell^{+}}^{K^{+}-K^{-}}-A_{\ell^{-}}^{K^{+}-K^{-}}\right)\right) d x \\
& \Delta d+\Delta \bar{d}=\int_{0}^{1}\left(d(x) A_{\ell^{+}}^{K^{+}-K^{-}}-\bar{d}(x) A_{\ell^{-}}^{K^{+} K^{-}}\right) d x \tag{23}
\end{align*}
$$

Strange quarks contributions $(\Delta s+\Delta \bar{s})$ can be determined using the additional observable quantities - axial charged $a_{8}$ [5]:

$$
\begin{align*}
& a_{8}=(\Delta u+\Delta \bar{u})+(\Delta d+\Delta \bar{d})-2(\Delta s+\Delta \bar{s})  \tag{24}\\
& \Delta s+\Delta \bar{s}=\frac{1}{2}\left[(\Delta u+\Delta \bar{u})+(\Delta d+\Delta \bar{d})-a_{8}\right] \tag{25}
\end{align*}
$$

Valence $d-, u-$ quarks contribution can be determined from (17) and (14) as

$$
\begin{align*}
\Delta d_{V}= & -\int_{0}^{1} d_{V}(x) A_{+, p}^{K^{+}-K^{-}} d x  \tag{26}\\
\Delta u_{V}= & \int_{0}^{1}\left(\left(u(x)+\bar{u}(x)+y_{1}^{2}(d(x)+\bar{d}(x))\right) A_{-, p}^{\pi^{+}-\pi^{-}}+\right. \\
& \left.+y_{1}^{2} d_{V}(x) A_{+, p}^{K^{+}-K^{-}}\right) d x \tag{27}
\end{align*}
$$



Figure 1: Obtained asymmetries (solid line for $y=0.2$, dotted line for $y=0.5$, dashed-doted line for $y=0.9$ ): a) $A_{\ell^{-}}^{\pi^{+} \pi^{-}}(x, y)$, b) $A_{\ell^{+}}^{\pi^{+}-\pi^{-}}(x, y)$, c) $A_{-, p}^{\pi^{+}-\pi^{-}}(x, y)$ d) $A_{+, p}^{\pi^{+}-\pi^{-}}(x, y)$.

## 3 Numerical results and conclusions

The numerical results of obtained asymmetries are presented on Fig.1.The asymmetries $A_{\ell^{-}}^{\pi^{+}-\pi^{-}}, A_{\ell^{+}}^{\pi^{+}-\pi^{-}}$(Fig.1(a,b)) almost not depend from the variable $y$. However, $A_{\ell^{-}}^{\pi^{+}-\pi^{-}}$increase and $A_{\ell^{+}}^{\pi^{+}-\pi^{-}}$decrease in medium and large $x$ region. The asymmetries $A_{-, p}^{\pi^{+}-\pi^{-}}$and $A_{+, p}^{\pi^{+}-\pi^{-}}$(Fig.1(c,d)) significantly depend from $y$ in all the $x$ region and achieve $60 \%-70 \%$ in measurable kinematic region.

Thus, spin asymmetries are determined for the case of $\pi-, K-$ meson production in semi-inclusive $\ell N$ - DIS. The obtained asymmetries do not depend on fragmentation functions. Here the quarks and antiquarks contribution $\left(\Delta u+\Delta \bar{u}, \Delta d+\Delta \bar{d}, \Delta s+\Delta \bar{s}, \Delta u_{V}, \Delta d_{V}, \Delta \bar{u}, \Delta \bar{d}\right)$ in the nucleon spin are obtained through the measurable asymmetries of semiinclusive $\ell p-$ DIS with charged current for $\pi-, K-$ mesons production. Numerical results of this asymmetries are provided and show, that this asymmetries significant in all $(x, y)$ measurable kinematic region.

## References

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[^0]:    *E-mail:dekaterinaa@mail.ru

