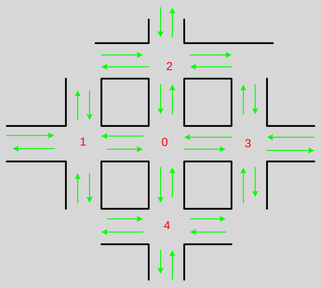
Perturbation Analysis and Sample-Path Optimization :

Stochastic Flow Models of Urban Traffic Networks Case

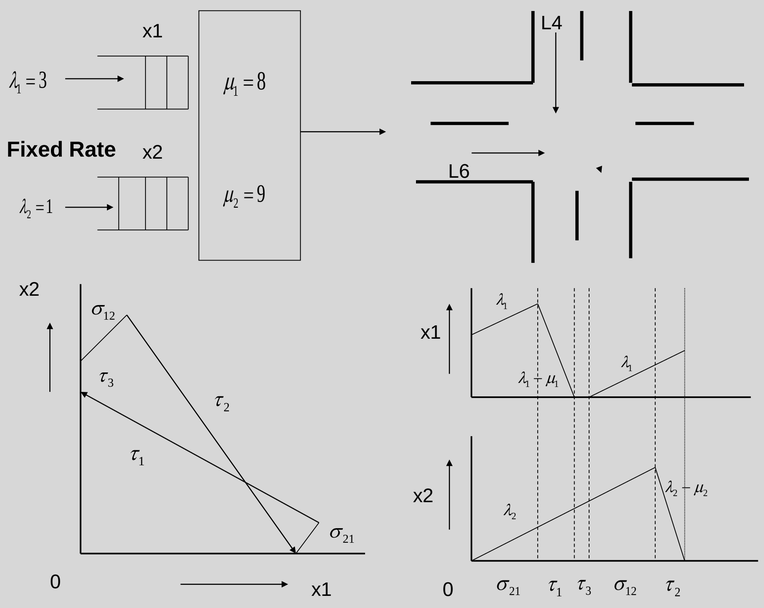
Herman Sutarto, René Boel
SYSTeMS Research Group, EESA Department
{herman.sutarto, Rene.Boel}@ ugent.be

Road Traffic as a Large Network

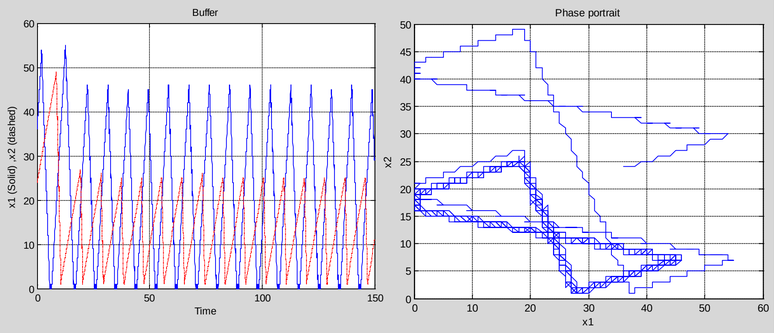
- Stochastic processes describing internal and external flows of vehicles
- Large network with huge traffic volume has many events that make it difficult to simulate and to control.
- We need an alternative modeling paradigm Stochastic Fluid Model (SFM) that **aggregates multiple events**
- The objective is to explore the use of SFM for the purpose of control and optimization rather than only for performance analysis.



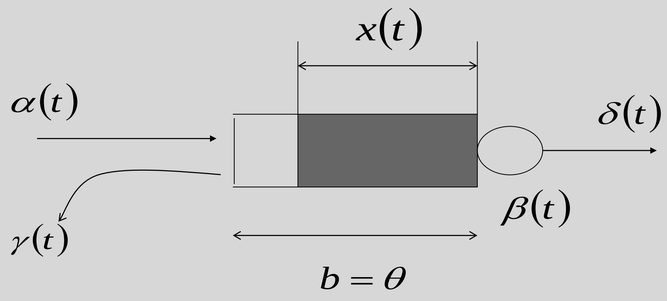
Flow Models : An Example (Lefeber)



$$J = \frac{1}{T}\int_0^T \left(c_1 x_1(s) + c_2 x_2(s)\right) ds$$



The Basic SFM : Single class of SFM



$$\frac{dx}{dt^+} = \begin{cases} 0 & if\ x = 0\ and\ A < 0 \\ 0 & if\ x = \theta\ and\ A > 0 \\ A(t,\theta)\ other \end{cases} \qquad A(t,\theta) = \alpha(t,\theta) - \beta(t)$$

$$\delta(t,\theta) = \begin{cases} \beta(t) & , \quad if\ x(t,\theta) > 0 \\ \alpha(t,\theta) & , \quad if\ x(t,\theta) = 0 \end{cases}$$

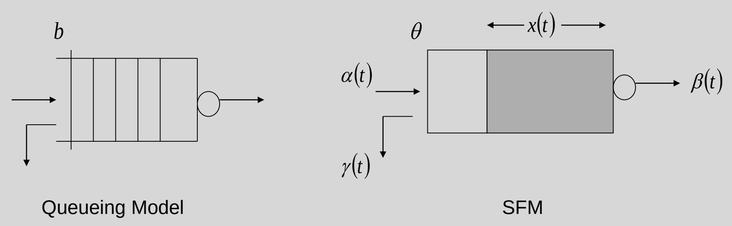
$$\gamma(\theta;t) = \begin{cases} \max\{\alpha(t) - \beta(t), 0\}, & if\ x(\theta;t) = \theta \\ 0 & if\ x(\theta;t) < \theta \end{cases}$$

$\{\alpha(t)\}$ *and* $\{\beta(t)\}$ Piecewise Constant and Right-Continous

$x(\theta;t)$ Piecewise linear and Continous in t

$\gamma(\theta;t)$ Piecewise Constant

Buffer Control and SFM Counterpart



When a vehicle arrives and the queue length is below a given level b,it is accepted;otherwise it is rejected.

In Discrete Event System (DES),both x(t) and b are integers but in SFM both are treated as real numbers

Optimization Problems in SFM

$$J(\theta; x(0), T) = E\left[\ell(\theta; x(0), T)\right]$$

Where $\ell(\theta; x(0), T)$ is a sample function evaluated in the interval $[0, T]$ with initial condition $x(0)$

It is difficult to obtain closed-form expression for $J(\theta; x(0), T)$

Therefore,we will resort to **iterative methods such as stochastic approximation algorithms** which are driven by estimates of the cost function gradient w.r.t the parameter vector of interest.

We seek to obtain θ^* minimizing through an iterative scheme of the form :

$$\theta_{n+1} = \theta_n - \eta_n H_n(\theta_n; x(0), T, \omega_n), \quad n = 0,1.....$$

Where $H_n(\theta_n; x(0), T, \omega)$ is an estimate of $dJ/d\theta$ evaluated at $\theta = \theta_n$ and based on information obtained from a sample path denoted by ω_n . It is assumed that stationary condition apply to this system.However, we shall consider T as a fixed time horizon and evaluate performance over $[0, T]$.

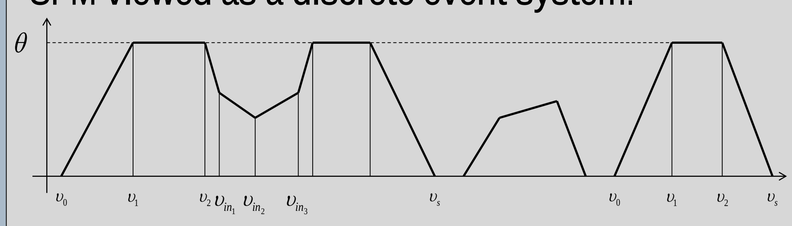
We need to estimate $H_n(\theta_n)$ or $dJ/d\theta$ and the IPA (Infinitesimal Perturbation Analysis) approach is based on using the sample derivative $d\ell/d\theta$

as an estimate of $dJ/d\theta$.

The strength of this approach is that $d\ell/d\theta$ can be obtained from observable sample path data alone and can be implemented on line.

Sample Path of SFM

Since solving problem relies on gradient information of given cost function wrt threshold parameters become an essential task,PA methods are therefore suitable if appropreately adapted SFM viewed as a discrete event system.



An event in a sample-path of the SFM may be either exogenous or endegenous and induced event.

An exogenous event is jumping in either $\{\alpha(t)\}$ or $\{\beta(t)\}$

An endogenous event is defined when the buffer becomes full or empty

An induced event is indicated by υ_{in_1} .... υ_{in_3}

An Induced-event $\omega(t)$ denote the amount of time required to process the workload

$$\int_t^{t+\omega(t)} C(\tau)\, d\tau = x(t) \qquad C(t) = \beta_1(t) + \beta_2(t)$$

$$x(t) = \sum_{i=1}^{2} x_i(t)$$

Performance Matrices of SFM

Buffer Overflow $L_T(\theta) = \int_0^T \gamma(\theta;t)\, dt$

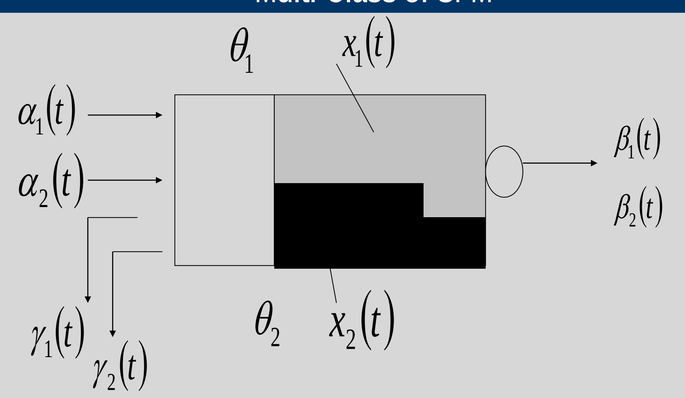
Workload $Q_T(\theta) = \int_0^T x(\theta;t)\, dt$

$$J(\theta) = \frac{1}{T}E[Q_T(\theta)] + \frac{R}{T}E[L_T(\theta)] \equiv \frac{1}{T}J_Q(\theta) + \frac{R}{T}J_L(\theta)$$

Expected Buffer Overflow $(1/T)E[L_T(\theta)]$

Expected Buffer Content $(1/T)E[Q_T(\theta)]$

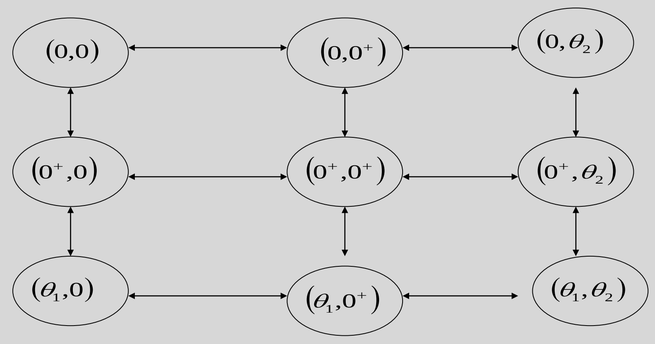
Multi-Class of SFM



$\beta_i(t)$ in two-class SFM depends on the queue contents and inflow process.

Define $s_i(t)$ discrete aggregates state and the corresponding three values by $0, 0^+, \theta_i$ and for two-class we have $\Phi_i = \{0, 0^+, \theta_i\}$

$$\Phi = \{(0,0),(0,0^+),(0,\theta_2),(0^+,0),(0^+,0^+),(0^+,\theta_2),(\theta_1,0),(\theta_1,0^+),(\theta_1,\theta_2)\}$$

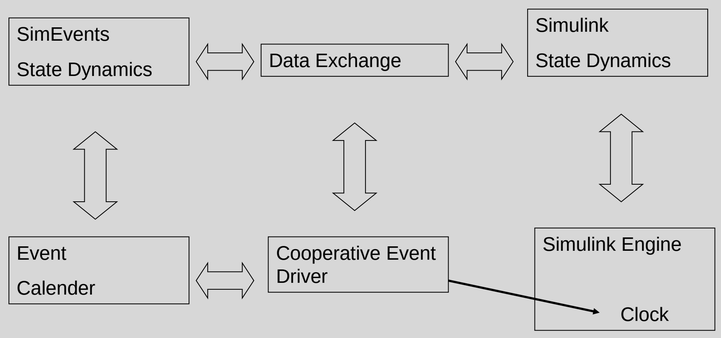


State Transition Diagram of Stochastic Hybrid Automata

Simulation Tools

If we consider a model DES/Hybrid for simulating server and queues ,there is suitable tools from MATLAB called SimEvents.

The users request resources in order to perform various tasks,occupy these resources for a certain amount of time and a relinquish them so that the other users may acces them.



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