

## A Multiplicative Calderón-Based Preconditioner for the Coupled Surface and Volume Electric Field Integral Equations

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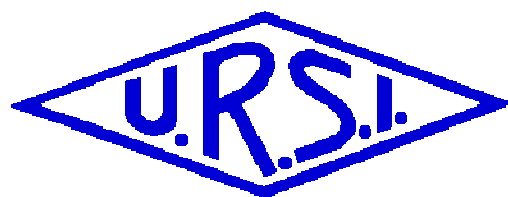
The need for dense spatial discretizations commonly arises when analyzing electromagnetic wave interactions on composite structures with sub-wavelength geometric features, e.g. millimeter and microwave integrated circuits, antenna feeds and beam-forming networks, aircraft fuselage details, etc. In this paper, a well conditioned coupled set of surface and volume electric field integral equations (SEFIE and VEFIE) for analyzing wave interactions with such structures is presented. SEFIE and VEFIE, which enforce fundamental boundary and field consistency conditions on perfect electrically conducting (PEC) surfaces and dielectric volumes, are discretized using the well-known Rao-Wilton-Glisson (RWG) (S. M. Rao et al., IEEE Trans. Antennas Propagat., 30(3), 1982, 409-418) surface and Schaubert-Wilton-Glisson (SWG) (D. H. Schaubert et al., IEEE Trans. Antennas Propagat., 32(1), 1984, 77-85) volume basis functions; and the resulting coupled method of moments (MOM) linear system is solved iteratively. It is known that the VEFIE operator, even when applied to the analysis of wave interactions with densely discretized volumes, is well-posed. On the other hand, the SEFIE operator becomes ill-posed when applied to the analysis of wave interactions with densely discretized PEC surfaces. This increases the condition number of the coupled MOM system and renders its iterative solution inefficient or even impossible.

The ill-posedness of the SEFIE operator stems from the fact that its eigenvalues accumulate at zero and infinity. For this reason the MOM matrix obtained upon discretizing the SEFIE is highly ill-conditioned when the discretization is dense. In recent years, many techniques that leverage Calderón identities to alleviate the ill-posedness of SEFIE operators have been proposed (H. Contopanagos et al., IEEE Trans. Antennas Propagat., 50(12), 2002, 1824-1830, R. J. Adams and N. J. Champagne, IEEE Trans. Antennas Propagat., 52(9), 2004, 2262-2266), S. Borel, D. Levaux, and F. Alouges, IEEE Trans. Antennas Propagat., 83(9), 2005, 2995-3004). These techniques exploit the self-regularizing property of the SEFIE, i.e. the fact that its square has a bounded spectrum, thus giving rise to MOM matrices that are well-conditioned independent of the surface mesh used. Unfortunately, none of the Calderón preconditioners introduced to date can easily be integrated into existing codes. Invariably, implementation bottlenecks can be traced to the need to properly discretize the spaces in between the two SEFIE operators, and the difficulty in constructing well-conditioned Gram matrices linking them.

This paper presents a multiplicative Calderón-based preconditioner for the coupled set of SEFIE and VEFIE. The proposed preconditioner regularizes the SEFIE and VEFIE using a multiplicative Calderón and a simple diagonal preconditioner respectively. The Calderón Multiplicative Preconditioner (CMP) described in (F. P. Andriulli et al., accepted for publication in IEEE Trans. Antennas Propagat.) is adopted here. The preconditioned SEFIE is obtained by multiplying two standard SEFIE matrices that are constructed using RWG basis functions and weighted by sparse Gram and projection matrices. The Gram and projection matrices contain only  $O(N)$  elements evaluated analytically using a simple relationship between ordinary and barycentric RWG basis functions. Here, barycentric RWG basis functions (A. Buffa and S. Christiansen, Comptes rendus. Mathématique, 340(6), pp. 461-464) are used to properly discretize the range of the SEFIE operator. This results in a well-conditioned Gram matrix.

The effectiveness of the new preconditioner in alleviating the ill-posedness of the coupled SEFIE and VEFIE operator and the number of iterations required to solve the coupled MOM system will be demonstrated via its application to the analysis of electromagnetic interactions with realistic structures.

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