2.5D SOLVER FOR 3D SCATTERING BY A LARGE 2D INHOMOGENEOUS OBJECT

S. Van den Bulcke, A. Franchois
INTEC Dept., Ghent Univ., St.-Pietersnieuwstraat 41, B-9000 Gent, Belgium.
sara.vandenbulcke@intec.UGent.be, ann.franchois@intec.UGent.be

Abstract: This paper presents a 2.5D exact forward solver to compute the 3D scattered field of a 2D inhomogeneous dielectric cylinder under a time-harmonic 3D illumination. This approach leads to a significant reduction in computational cost with respect to a full 3D solver, in case of quasi 2D objects with cross-sectional dimensions that are several to many wavelengths. The accuracy and efficiency of the solver are demonstrated by comparison to results obtained with a full 3D solver for plane wave oblique incidence. The solver will be used to study scattering in a free-space active millimeter-wave imaging system.

INTRODUCTION

An exact forward solver to compute the 3D scattered field of an infinitely long (lossy) dielectric inhomogeneous cylinder with an arbitrary cross-sectional shape under time-harmonic 3D illumination is presented. We intend to use this solver for modeling an active millimeter-wave imaging system, which is currently being developed by the Vrije Universiteit Brussel (VUB) for indoor security imaging applications, Volkov et al. [1]. Objects of interest, e.g. a concealed object carried on a body, are typically many wavelengths in size—the wavelength is 3mm at 100GHz—such that the use of a full-wave 3D solver is computationally expensive. We therefore have preferred in this paper to study quasi-2D objects under 3D illumination. This 2.5D problem is expanded in a number of 2D problems using a spatial Fourier transform with respect to the coordinate z along the cylinder's axis. The different 2D solutions are then recombined to one 2.5D solution using an inverse Fourier Transform. A similar approach is applied for modelling geophysical low frequency electromagnetic scattering in Abubakar et al. [2]. The 3D illumination capability yields more flexibility than a purely 2D approach, since it allows to accurately simulate multi-view imaging experiments. When the incident field is a plane wave under oblique incidence or a Gaussian beam, which have few spectral components, the total number of 2D problems to be solved is furthermore limited. The problem is formulated in section 2 and numerical results are presented in section 3.

FORMULATION

The problem is formulated in the frequency domain and the time dependence $\exp(-i\omega t)$ is omitted. Consider a 2D inhomogeneous dielectric object surrounded with free space and with complex permittivity $\epsilon(\mathbf{r}) = \epsilon_r(\mathbf{r})\epsilon_0 = \epsilon'(\mathbf{r}) + i\epsilon''(\mathbf{r})$, with $\epsilon'(\mathbf{r})$ and $\epsilon''(\mathbf{r})$ representing the real and imaginary part of $\epsilon(\mathbf{r})$ and $\mathbf{r} = (x, y)$ the 2D position vector. The object is illuminated with a 3D incident field $\mathbf{E}^i(\mathbf{r}, z) = [E_1^i(\mathbf{r}, z), E_2^i(\mathbf{r}, z), E_3^i(\mathbf{r}, z)]$ and the resulting scattered field is defined as

$$\mathbf{E}^s(\mathbf{r}, z) = \mathbf{E}(\mathbf{r}, z) - \mathbf{E}^i(\mathbf{r}, z), \tag{1}$$

with $\mathbf{E}(\mathbf{r}, z)$ the total field. The electric flux density $\mathbf{D}(\mathbf{r}, z) = [D_1(\mathbf{r}, z), D_2(\mathbf{r}, z), D_3(\mathbf{r}, z)]$ is chosen as the unknown of the scattering problem, Zwamborn et al. [3]. The spatial Fourier transform with respect to z is defined as $\hat{g}(\mathbf{r}, k_z) = \int_{-\infty}^{\infty} g(\mathbf{r}, z)e^{-ik_z z}\mathrm{d}z$. A contrast source integral equation (CSIE) in the space-spatial frequency domain $(x, y; k_z)$ is now derived. The transformed scattered fields satisfy the transformed Maxwell equations:

$$\begin{aligned}
\widehat{\nabla} \times \widehat{\mathbf{E}}^s(\mathbf{r}, k_z) &= i\omega\mu_0 \widehat{\mathbf{H}}^s(\mathbf{r}, k_z) \\
\widehat{\nabla} \times \widehat{\mathbf{H}}^s(\mathbf{r}, k_z) &= -i\omega\chi(\mathbf{r})\widehat{\mathbf{D}}(\mathbf{r}, k_z) - i\omega\varepsilon_0 \widehat{\mathbf{E}}^s(\mathbf{r}, k_z),
\end{aligned} \tag{2}$$

where $\chi(\mathbf{r}) = \frac{[\varepsilon(\mathbf{r}) - \varepsilon_0]}{\varepsilon(\mathbf{r})}$ is the normalized permittivity contrast and $\widehat{\nabla} = (\partial_x, \partial_y, ik_z)$. A vector potential $\widehat{\mathbf{A}}^s(\mathbf{r}, k_z)$ is introduced:

$$\widehat{\mathbf{A}}^s(\mathbf{r}, k_z) = \frac{1}{\varepsilon_0} \int_S \widehat{G}(\mathbf{r}, \mathbf{r}'; k_z)\chi(\mathbf{r}')\widehat{\mathbf{D}}(\mathbf{r}', k_z)\mathrm{d}\mathbf{r}', \tag{3}$$

with the Green's function given by $\widehat{G}(\mathbf{r}, \mathbf{r}'; k_z) = \frac{i}{4} H_0^{(1)}\left(\sqrt{k_0^2 - k_z^2}\,|\mathbf{r} - \mathbf{r}'|\right)$. The scattered field is expressed as:

$$\widehat{\mathbf{E}}^s(\mathbf{r}, k_z) = \left(k_0^2 \mathbf{I} + \widehat{\nabla}\widehat{\nabla}\right) \cdot \widehat{\mathbf{A}}^s(\mathbf{r}, k_z). \tag{4}$$

Introduction of (3)-(4) in (1) yields the CSIE for $\mathbf{r} \in S$:

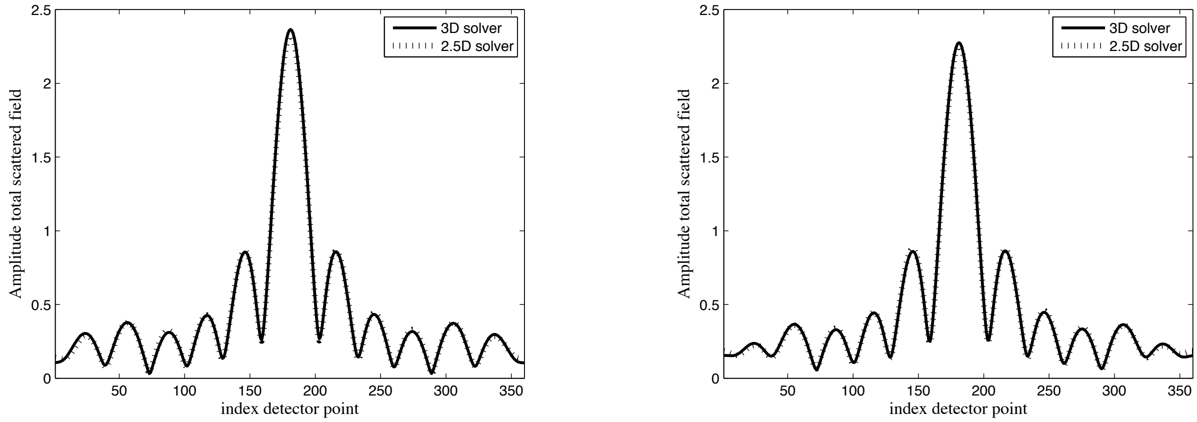
$$\widehat{\mathbf{E}}^i(\mathbf{r}, k_z) = \frac{\widehat{\mathbf{D}}(\mathbf{r}, k_z)}{\varepsilon(\mathbf{r})} - \left(k_0^2 \mathbf{I} + \widehat{\nabla}\widehat{\nabla}\right) \cdot \widehat{\mathbf{A}}^s(\mathbf{r}, k_z). \tag{5}$$

The object domain S is discretized in square cells with cell size Δ. The complex permittivity takes a constant value within each cell. A Method of Moments with Galerkin weighting is applied to discretize the CSIE, in which the vector potential $\widehat{\mathbf{A}}^s(\mathbf{r}, k_z)$ and the electric flux density $\widehat{\mathbf{D}}(\mathbf{r}, k_z)$ are expanded in basis functions [3]. The basis and testing functions are chosen as products of one-dimensional triangle functions and one-dimensional pulse functions. For the $x-$ $(y-)$ component of the fields we use a one-dimensional triangle function in the $x-$ $(y-)$ direction of support 2Δ and a one-dimensional pulse function in the $y-$ $(x-)$ direction of support Δ. For the z-component triangle functions of support 2Δ are used in both directions. The obtained linear set of equations is solved iteratively with a stabilized biconjugate gradient Fast Fourier Transform method (BiCGSTAB-FFT), Xu et al.[4], van der Vorst [5].

NUMERICAL EXAMPLE

The 2.5D solver is compared to a full 3D solver, Lewyllie et al. [6], for plane wave illumination under oblique incidence. The scatterer is a dielectric circular cylinder with relative permittivity $\epsilon_r = 2$ and a radius equal to one wavelength ($\lambda_0 = 1$mm, $f = 300$GHz). With the 2.5D solver, this cylinder is infinitely long, while with the 3D solver, only a finite 3D object can be modeled. The length l of this finite cylinder is chosen long enough, for example $l = 100\lambda_0 = 100$mm. In both simulations (2.5D and 3D), the scattered fields are calculated for 360 points on a circle with a radius of $2\lambda_0 = 2$mm. The propagation vector of the incident plane wave $\mathbf{k}$ makes an angle ψ with the xy-plane and the magnetic field is polarized parallel to this plane.

Figure 1 shows the amplitude of the scattered field $|\mathbf{E}_s|$ on the detector circle for incidences $\psi = 8°$ (a) and $\psi = 16°$ (b). An excellent agreement between the 2.5D solver (dotted line) and 3D solver (solid line) is observed. Differences due to the finite length of the 3D cylinder are not visible on this detector circle. For the simulations with the 2.5D solver, the BiCGS iterations were stopped when the relative error dropped below 10^{-8}, after 57 iterations. With the 3D solver the simulations were stopped when the relative error dropped below $10^{-2.5}$, after 360 iterations. The 3D simulation took 16 hours on a single AMD Opteron 270 processor of a 64 bit machine, the 2.5D



(a) (b)

Figure 1: Scattered field amplitude on the detector circle: (a) oblique incidence angle $\psi = 8°$ and (b) $\psi = 16°$.

simulation only 16 seconds. The 2.5D solver is not only much faster than the 3D solver, it also uses far less unknowns: the 3D problem consists of more than 5 million unknowns and occupies 2.4Gb of memory while the 2.5D problem has only 12 288 unknowns and uses no more than 42.652 Mb of memory.

CONCLUSION

A 2.5D exact forward solver to compute the 3D scattered field of a 2D inhomogeneous dielectric cylinder, with cross-sectional dimensions that are several to many wavelengths, under a time-harmonic 3D illumination was presented. For a dielectric cylinder under oblique plane wave illumination an excellent agreement with results from a 3D full-wave solver was demonstrated.

REFERENCES

[1] Volkov, L., and J. Stiens, US patent 6777684: Systems and methods for millimeter and sub-millimeter wave imaging, August 2004.
[2] Abubakar, A., P. M. van den Berg, and T. M. Habashy, "An integral equation approach for 2.5-dimensional forward and inverse electromagnetic scatterin", *Geophys. J. Int.*, **vol. 165**, pp.744-762, 2006.
[3]Zwamborn, P., and P. M. van den Berg, "A weak form of the conjugate gradient FFT method for two-dimensional TE scattering problems", *IEEE Transactions on Microwave Theory and Techniques*, **vol. 39**, no. 6, pp. 953-960, June 1991.
[4] Xu, X. M., Q. H. Liu and Z. Q. Zhang , "The stabilized biconjugate gradient fast Fourier transform method for electromagnetic scattering", *J. Appl. Computat Electromag. Soc.*, **vol. 17**, no. 1, pp. 97-103, March 2002.
[5] van der Vorst, H. A., "BI-CGSTAB: A fast and smoothly converging variant of BI-CG for the solution of non symmetric linear systems", *SIAM Journal on scientific and Statistical Computing*, **vol. 13**, no. 2, pp. 631-644, 1992.
[6] Lewyllie, P., A. Franchois, C. Eyraud, and J.-M. Geffrin, "Testing a 3D BCGS-FFT solver against experimental data", *Proceedings of 9th International Conference on Electromagnetics in Advanced Applications*, Torino, Italy, 2005.

R7-63: Inverse Scattering and Imaging

Saturday, July 28, 2007 • Québec Suite

Chair: E. Marengo, Northeastern University, USA

1:30 PM EMTS75: "2.5D SOLVER FOR 3D SCATTERING BY A LARGE 2D INHOMOGENEOUS OBJECT"
S. Van den Bulcke, A. Franchois, Ghent University, Belgium
Presenter: Sara Van den Bulcke, Ghent University, Belgium

1:50 PM EMTS122: "Multigrid Optimization Method for Inverse Scattering of a Lossy Dielectric Cylinder"
M. Tanaka, K. Yano, T. Takada, A. Kusunoki, Oita University, Japan
Presenter: Mitsuru Tanaka, Oita University, Japan

2:10 PM EMTS192: "Non-iterative imaging including multiple scattering"
E.A. Marengo, Northeastern University, USA
Presenter: Edwin Marengo, Northeastern University, USA

2:50 PM EMTS80: "IMAGING OF OBJECTS THROUGH SCATTERING RANDOM MEDIUM LAYERS USING A PULSED SCANNING FOCUSED BEAM"
A. Ishimaru, S. Jaruwatanadilok, Y. Yasuo, E.I. Thorsos, University of Washington, USA
Presenter: Akira Ishimaru, University of Washington, USA

3:10 PM EMTS187: "Some isuues with using the radiative transfer approach to scattering from layered random media with rough interfaces"
S. Mudaliar, Air Force Research Laboratory/SNHE, USA
Presenter: Saba Mudaliar, Air Force Research Laboratory/SNHE, USA

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