Semipartial geometries from 1-systems of $W_5(q)$

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A *1-system* of $W_5(q)$ is a set $\mathcal{M}$ of q^3+1 lines $L_0, L_1, \ldots, L_{q^3}$ of $W_5(q)$ such that every generator of $W_5(q)$ which contains a line $L_i \in \mathcal{M}$, is disjoint from all lines $L_j \in \mathcal{M} \setminus \{L_i\}$.

Let $\mathcal{M}$ be a 1-system of $W_5(q)$ in $\mathsf{PG}(5,q)$ and embed $\mathsf{PG}(5,q) := H$ as a hyperplane in $\mathsf{PG}(6,q)$. Let $\mathcal{P}$ be the set of all points of $\mathsf{PG}(6,q) \setminus H$, $\mathcal{L}$ the set of planes of $\mathsf{PG}(6,q)$, not in H, which meet H in a line of $\mathcal{M}$, and I the natural incidence of $\mathsf{PG}(6,q)$. Then $\mathsf{SPG}(\mathcal{M}) := (\mathcal{P}, \mathcal{L}, \mathrm{I})$ is a semipartial geometry with parameters $s = q^2-1$, $t = q^3$, $\alpha = q$ and $\mu = q^2(q^2-1)$.

A *strong regulus* of $W_5(q)$ is a regulus of lines of $W_5(q)$, the opposite regulus of which entirely consists of lines of $W_5(q)$. If q is odd, then $W_5(q)$ does not have strong reguli, so let q be even. A 1-system $\mathcal{M}$ of $W_5(q)$, q even, is *locally hermitian* at a line $L \in \mathcal{M}$ if and only if for every line $M \in \mathcal{M} \setminus \{L\}$, the unique strong regulus containing L and M, is contained in $\mathcal{M}$.

A class of locally hermitian 1-systems of $W_5(q)$ is known to exist for all q even with $q > 2$, see [1]*. Concerning their semipartial geometries, the following theorem can be shown.

Theorem 1 *Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be two locally hermitian 1-systems of $W_5(q)$, q even and $q > 2$. Then $\mathsf{SPG}(\mathcal{M}_1)$ and $\mathsf{SPG}(\mathcal{M}_2)$ are isomorphic if and only if $\mathcal{M}_1$ and $\mathcal{M}_2$ are isomorphic for the stabilizer of $W_5(q)$ in $\mathsf{P\Gamma L}(6,q)$.*

From every locally hermitian 1-system of $W_5(q)$, q even, one can construct a new 1-system, which is not locally hermitian, by replacing a strong regulus by its opposite regulus. In this context, the following result holds.

Theorem 2 *Let $\mathcal{M}$ be a 1-system of $W_5(q)$, q even, which is locally hermitian at $L \in \mathcal{M}$. Let $\mathcal{M}'$ be obtained from $\mathcal{M}$ by replacing a strong regulus of $\mathcal{M}$ through L by its opposite regulus. Then $\mathsf{SPG}(\mathcal{M})$ and $\mathsf{SPG}(\mathcal{M}')$ are not isomorphic.*

From the above theorems, the existence of several new examples of semipartial geometries follows.

*[1] D. Luyckx and J. A. Thas, On 1-systems of $Q(6,q)$, q even. To appear in *Designs, Codes and Cryptography*, 2002.