1-systems of $Q^+(7,q)$ and trialities

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A *1-system* $\mathcal{M}$ of the hyperbolic quadric $Q^+(7,q)$ in $\mathsf{PG}(7,q)$ is a set $\{L_0, L_1, \ldots, L_{q^3}\}$ consisting of q^3+1 lines on $Q^+(7,q)$ with the property that the tangent space of $Q^+(7,q)$ at L_i has no point in common with $(L_0 \cup L_1 \cup \ldots \cup L_{q^3}) \setminus L_i$, for $i = 0, 1, \ldots, q^3$. If $\mathcal{M}$ consists of q^2 reguli through a common line $L_i \in \mathcal{M}$, then the 1-system is called *locally hermitian* at L_i.
We will consider the action of a triality τ of $Q^+(7,q)$ on a 1-system $\mathcal{M}$ of an induced $Q(6,q) \subseteq Q^+(7,q)$. Special attention will be paid to the case of a locally hermitian, semiclassical 1-system of $Q(6,q)$. In particular, it will be explained that its image under a triality is again locally hermitian and semiclassical and that it is contained in a hyperplane of $\mathsf{PG}(7,q)$ if and only if $\mathcal{M}$ is a spread of a classical generalized hexagon $\mathsf{H}(q)$ on $Q(6,q)$. Since we are able to show that whenever $\mathcal{M}$ is not a spread of some $Q^-(5,q) \subseteq Q(6,q)$, nor a spread of a classical generalized hexagon $\mathsf{H}(q)$ on $Q(6,q)$, the 1-system $\mathcal{M}^\tau$ was not previously known, this construction using a triality yields new examples of locally hermitian, semiclassical 1-systems of $Q^+(7,q)$.

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